CE231: Strength of Materials
Spring 2003
Thornton Hall D221, Tuesday/Thursday, 9.30-10.45

Instructor: Prof. Matthew R. Begley
begley@virginia.edu
B229E Thornton Hall (across from CE office)
(434) 243 8728 or D112

Office Hours:
T: ~~10.45 – 11.45 a.m.~~ 2-3 pm
W: 9.00 – 11.00 a.m.
F: 11-12 am

Teaching Assistant: Ms. Kyle Maner
kcm6r@virginia.edu
B123 ~~D112~~ Thornton Hall (CE Structures Lab)

Office Hours:
M: 3.00 – 5.00 p.m.
R: 12.00 – 2.00 p.m.*
*Thursdays hours held at E~~x~~spresso Royale Caffe[1] on the Corner.

Course Description (catalogue): Stress and strain definitions: Normal stress and strain, thermal strain, shear stress, shear strain; transformations of stress and strain; Mohr's circle for plane stress and strain; stresses due to combined loading; axially loaded members; torsion of circular and thin-walled closed sections; deformation, strains and stresses in beams; deflections of beams; stability of columns; energy concepts in mechanics.

Course Description (Begley): a study of the fundamental concepts used to describe the behavior of materials and predict the behavior of (usually) solid structures: course topics provide the foundation to: (*i*) predict the (mechanical) performance and lifetime of applications ranging from bridges and stadiums to microelectronics and biomedical devices, (*ii*) serve as the cornerstone for advanced research into the behavior materials and structures.

Course Objectives:

(1) to develop an understanding of fundamental concepts used to describe the behavior of solid materials and structures, such as stress, strain, energy, etc.
(2) to develop a proficiency in mechanics and structural analysis in order to predict the performance of engineering structures and designs
(3) to further develop the ability to critically analyze engineering problems, with a particular emphasis on the physical behavior of materials.

These objectives also support broader or more 'global' objectives that relate to civil engineering practice and core ABET outcome areas, such as:

(a) an ability to apply knowledge in of mathematics, ~~since~~ science and engineering
(b) an ability to design a system, component or process to meet desired needs
(c) an ability to identify, formulate and solve engineering problems
(d) a knowledge of contemporary issues
(e) proficiency in the analysis of structures (one of the competency areas of ABET)

[1] Kyle insists that this is the spelling used by this café.

Course Policies:

Text (required): *Mechanics of Materials*, R.C. Hibbeler, 5th Edition, 2003, Prentice Hall.

Grading:

	Midterm #1:	15 %
	Midterm #2:	20 %
	Final Exam:	35 %
	Homeworks (8 to 10):	20 %
	Mini-projects:	10 %

Homework Assignments:

- The plan is to distribute homework assignments on Thursdays and have them turned in by the Friday of the following week. Specific details will be included with each assignment. Current plans are for ten (10) homework assignments of 5-8 problems (an unspecified fraction will be graded), and two mini-projects consisting of the analysis of one problem and experiment for comparison.

- Homework is not pledged but should never be copied. Students may consult each other and share ideas in the ***early*** stages of homework solutions, but each student must complete his/her own homework independently. Homework that is identical in all ***significant*** respects is unacceptable.

- A due date will be given when homework is assigned, and will typically be the Friday of the week following the homework's distribution . Homework turned in after the due date will be accepted only if arrangements are made prior to the original due date; this will only be possible under extremely extenuating circumstances.

- Students should regard homework problems as professional presentations. Homework should be neatly organized, with the answers to problems summarized at the appropriate locations. Sufficient detail must be included to permit satisfactory evaluation of student performance, but excessive detail should be avoided.

Tests:

1. All tests and examinations will be administered under the University of Virginia honor system. Students will be assumed to be familiar with the honor system, and will be bound by it. The honor system is a very important attribute of the University of Virginia, but only works if the concept of honor is taken seriously by all involved.

2. The detailed formats of tests will be announced prior to their administration, and specific limits within which the student is permitted to work will be announced. Time permitting, review sessions will be held in the evening, prior to each exam.

Miscellaneous:

- Please turn off cell phones.
- Use e-mail to make an appointment with me if you can not attend office hours.

Course Content:

Previous topics that will be useful:

- Force resultants in two and three dimensions and equilibrium
- Moments, couples and equivalent force systems
- Analysis of trusses: method of joints, sections
- Internal forces: axial force, shear torsion, shear bending and moment diagrams
- Moment of inertia
- Distributed forces
- Principal of virtual work
- Stability of equilibrium
- Calculus - *all of it.*

HW #1: Stress and strain
HW #2: Material behavior
HW #3: Axial loads
HW #4: Torsion
HW #5: Bending
HW #6: Transverse /Combined loads
HW #7: Mohr's circle
HW #8: Mohr's circle, maximum shear, principle stress
HW #9: Deflection of beams
HW #10: Energy methods, buckling

Mini-Project #1 - Mechanical Measurements at the Nanoscale:

Students will be asked to analyze the behavior of a beam with a load acting at a single point, then verify this analysis with experiments on a micro-beam (with dimensions only a fraction of a human hair. The ultimate goal is to illustrate the application of strength of materials in a cutting edge application, such as micro-mirror arrays used for optical switching of fiber optic signals.

Mini-Project #2 - Failure Prognostics: "Squash You Like An Egg":

Students will be asked to analyze a beam structure that will support your instructor's weight, then specify the clearance between the beam and the underlying surface. The goal will be to set the clearance such that the deflected beam just contacts an egg – naturally without breaking it. The project is designed to illustrate the role of strength of materials in practice, i.e. in situations where 'lethal' damage can occur.

Course schedule:

WEEK	DATE	TOPIC	Reading	Notes
1	R: 1/16	Introduction/review	1.1, 1.2	*In-class survey*
2	T: 1/21	Stress	1.3, 1.4	
	R: 1/23	Stress	1.5 – 1.7	HW #1 Out, Due 1/31
3	T: 1/28	Strain	2.1 – 2.2	
	R: 1/30	Material behavior	3.1 – 3.4	HW #2 Out, Due 2/7
4	T: 2/4	Axial loads	3.5 – 3.8	
	R: 2/6	Axial loads	4.1 – 4.3	HW #3 Out, Due 2/14
5	T: 2/11	Torsion	4.4 – 4.7	
	R: 2/13	Torsion	5.1, 5.2	HW #4 Out, Due at Midterm
	T: 2/18	Torsion	5.4, 5.5	
6	**MIDTERM #1: week of February 17-21**			
	R: 2/20	TBA	6.1, 6.2	HW #5 Out: Due, 2/28
7	T: 2/25	Bending	6.3, 6.4	*In-class survey*
	R: 2/27	Bending	6.5, 6.6	
SPRING BREAK: March 3-7				
8	T: 3/11	Bending	6.7, 6.10	
	R: 3/13	Transverse shear	7.1, 7.2	Mini-Project #1 Out, Due: 3/20
9	T: 3/18	Transverse shear	7.3	
	R: 3/20	Combined loads	8.1, 8.2	HW #6 Out, Due: 3/28
10	T: 3/25	Stress transformations	9.1-9.2	
	R: 3/27	Principal stresses	9.3-9.4	HW #7 Out, Due at Midterm
11	T: 4/1	Mohr's circle	9.4, 9.5	
	MIDTERM #2: week of March 31 – April 4th			
	R: 4/3	TBA	9.7	HW #8 Out, Due: 4/11
12	T: 4/8	Plane strain	10.1, 10.2	*In-class survey*
	R: 4/10	Mohr's circle for strain	10.3–10.5	Mini-Project #2 Out, Due: 4/18
13	T: 4/15	Deflections of beams	12.1-12.2	
	R: 4/17	Deflection of beams	12.5-12.7	HW #9 Out: Due: 4/25
14	T: 4/22	Energy methods	14.1–14.3	
	R: 4/24	Energy methods	14.6 – 9	HW #10 Out, Due: 5/2
15	T: 4/29	Buckling of columns	13.1 – 3	
	R: 5/1	Buckling of columns		*In-class survey*

CE 231 Final: ~~Wednesday, May 7th~~, 2.00 – 5.00 p.m.
Thursday, May 8th

Average Mechanical Properties of Typical Engineering Materials[a]

(U.S. Customary Units)

Materials		Specific Weight γ (lb/in^3)	Modulus of Elasticity E (10^3) ksi	Modulus of Rigidity G (10^3) ksi	Yield Strength (ksi) σ_Y Tens.	Comp.[b]	Shear	Ultimate Strength (ksi) σ_u Tens.	Comp.[b]	Shear	% Elongation in 2 in. specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°F$
Metallic													
Aluminum Wrought Alloys	2014-T6	0.101	10.6	3.9	60	60	25	68	68	42	10	0.35	12.8
	6061-T6	0.098	10.0	3.7	37	37	19	42	42	27	12	0.35	13.1
Cast Iron Alloys	Gray ASTM 20	0.260	10.0	3.9	–	–	–	26	97	–	0.6	0.28	6.70
	Malleable ASTM A-197	0.263	25.0	9.8	–	–	–	40	83	–	5	0.28	6.60
Copper Alloys	Red Brass C83400	0.316	14.6	5.4	11.4	11.4	–	35	35	–	35	0.35	9.80
	Bronze C86100	0.319	15.0	5.6	50	50	–	95	95	–	20	0.34	9.60
Magnesium Alloy	[Am 1004-T61]	0.066	6.48	2.5	22	22	–	40	40	22	1	0.30	14.3
Steel Alloys	Structural A36	0.284	29.0	11.0	36	36	–	58	58	–	30	0.32	6.60
	Stainless 304	0.284	28.0	11.0	30	30	–	75	75	–	40	0.27	9.60
	Tool L2	0.295	29.0	11.0	102	102	–	116	116	–	22	0.32	6.50
Titanium Alloy	[Ti-6A1-4V]	0.160	17.4	6.4	134	134	–	145	145	–	16	0.36	5.20
Nonmetallic													
Concrete	Low Strength	0.086	3.20	–	–	–	1.8	–	–	–	–	0.15	6.0
	High Strength	0.086	4.20	–	–	–	5.5	–	–	–	–	0.15	6.0
Plastic Reinforced	Kevlar 49	0.0524	19.0	–	–	–	–	104	70	10.2	2.8	0.34	–
	30% Glass	0.0524	10.5	–	–	–	–	13	19	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.017	1.90	–	–	–	–	0.30[c]	3.78[d]	0.90[d]	–	0.29[e]	–
	White Spruce	0.130	1.40	–	–	–	–	0.36[c]	5.18[d]	0.97[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both ension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

Average Mechanical Properties of Typical Engineering Materials[a]

Average Mechanical Properties of Typic[illegible] [illegible]gineering Materials[a]

(SI Units)

Materials		Density ρ (Mg/m^3)	Modulus of Elasticity E (GPa)	Modulus of Rigidity G (GPa)	Yield Strength (MPa) σ_Y Tens.	Yield Strength (MPa) σ_Y Comp.[b]	Yield Strength (MPa) σ_Y Shear	Ultimate Strength (MPa) σ_u Tens.	Ultimate Strength (MPa) σ_u Comp.[b]	Ultimate Strength (MPa) σ_u Shear	% Elongation in 50mm specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°C$
Metallic													
Aluminum Wrought Alloys	2014-T6	2.79	73.1	27	414	414	172	469	469	290	10	0.35	23
	6061-T6	2.71	68.9	26	255	255	131	290	290	186	12	0.35	24
Cast Iron Alloys	Gray ASTM 20	7.19	67.0	27	–	–	–	179	669	–	0.6	0.28	12
	Malleable ASTM A-197	7.28	172	68	–	–	–	276	572	–	5	0.28	12
Copper Alloys	Red Brass C83400	8.74	101	37	70.0	70.0	–	241	241	–	35	0.35	18
	Bronze C86100	8.83	103	38	345	345	–	655	655	–	20	0.34	17
Magnesium Alloy	[Am 1004-T61]	1.83	44.7	18	152	152	–	276	276	152	1	0.30	26
Steel Alloys	Structural A36	7.85	200	75	250	250	–	400	400	–	30	0.32	12
	Stainless 304	7.86	193	75	207	207	–	517	517	–	40	0.27	17
	Tool L2	8.16	200	75	703	703	–	800	800	–	22	0.32	12
Titanium Alloy	[Ti-6A1-4V]	4.43	120	44	924	924	–	1,000	1,000	–	16	0.36	9.4
Nonmetallic													
Concrete	Low Strength	2.38	22.1	–	–	–	12	–	–	–	–	0.15	11
	High Strength	2.38	29.0	–	–	–	38	–	–	–	–	0.15	11
Plastic Reinforced	Kevlar 49	1.45	131	–	–	–	–	717	483	20.3	2.8	0.34	–
	30% Glass	1.45	72.4	–	–	–	–	90	131	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.47	13.1	–	–	–	–	2.1[c]	26[d]	6.2[d]	–	0.29[e]	–
	White Spruce	3.60	9.65	–	–	–	–	2.5[c]	36[d]	6.7[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both ension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

Axial Load

Normal Stress

$$\sigma = \frac{P}{A}$$

Displacement

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\delta = \Sigma \frac{PL}{AE}$$

$$\delta_T = \alpha \Delta T L$$

Torsion

Shear stress in circular shaft

$$\tau = \frac{T\rho}{J}$$

where

$$J = \frac{\pi}{2} c^4 \text{ solid cross section}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) \text{ tubular cross section}$$

Power

$$P = T\omega = 2\pi f T$$

Angle of twist

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma \frac{TL}{JG}$$

Average shear stress in a thin-walled tube

$$\tau_{\text{avg}} = \frac{T}{2tA_m}$$

Shear Flow

$$q = \tau_{\text{avg}} t = \frac{T}{2A_m}$$

Bending

Normal stress

$$\sigma = \frac{My}{I}$$

Unsymmetric bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}, \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$

Shear

Average direct shear stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

Transverse shear stress

$$\tau = \frac{VQ}{It}$$

Shear flow

$$q = \tau t = \frac{VQ}{I}$$

Stress in Thin-Walled Pressure Vessel

Cylinder

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

Sphere

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

Material Property Relations

Poisson's ratio

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1 + \nu)}$$

Relations Between w, V, M

$$\frac{dV}{dx} = -w(x), \qquad \frac{dM}{dx} = V$$

Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

Buckling

Critical axial load

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

Critical stress

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{(KL/r)^2}, \; r = \sqrt{I/A}$$

Secant formula

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

Energy Methods

Conservation of energy

$$U_e = U_i$$

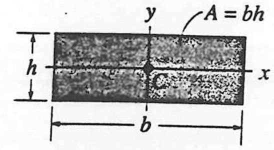
Strain energy

$$U_i = \frac{N^2 L}{2AE} \quad \textit{constant axial load}$$

$$U_i = \int_0^L \frac{M^2 dx}{EI} \quad \textit{bending moment}$$

$$U_i = \int_0^L \frac{f_s V^2 dx}{2GA} \quad \textit{transverse shear}$$

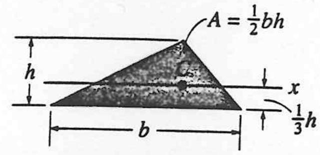
$$U_i = \int_0^L \frac{T^2 dx}{2GJ} \quad \textit{torsional moment}$$



$I_x = \frac{1}{12}bh^3$

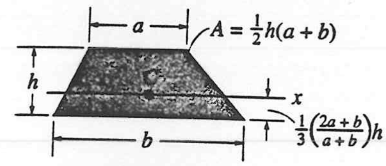
$I_y = \frac{1}{12}hb^3$

Rectangular area

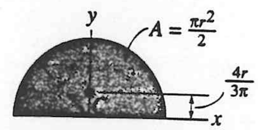


$I_x = \frac{1}{36}bh^3$

Triangular area



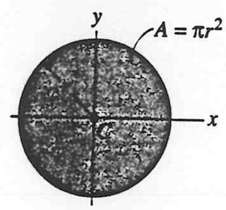
Trapezoidal area



$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$

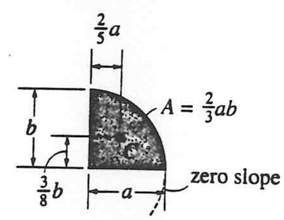
Semicircular area



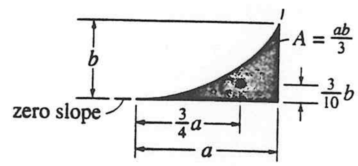
$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$

Circular area



Semiparabolic area



Exparabolic area

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C. Hovell

CE231 Strength of Materials **Spring 2003**

MIDTERM #1

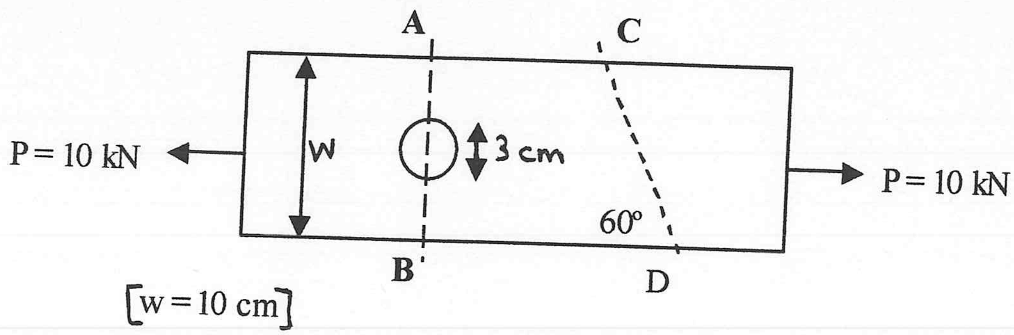
Wednesday, February 19th, 2003
7.00 p.m. – 8.15 p.m, D221 Thornton Hall

- **WRITE ON ONLY ONE SIDE OF A PIECE OF PAPER, AND WRITE YOUR NAME ON EACH PAGE THAT YOU USE.**
- **CLEARLY CROSS OUT THOSE PARTS OF YOUR RESPONSE YOU DO NOT WANT GRADED; IF IT IS NOT CLEAR, EVERYTHING WILL FACTOR INTO THE GRADE.**
- **IF YOU ARE RUNNING SHORT OF TIME, SKETCH AN OUTLINE OF THE STEPS YOU WOULD TAKE TO SOLVE THE PROBLEM (as opposed to attempting to grind out the solution).**
- **PLEASE WRITE AND SIGN THE HONOR PLEDGE ON THE FIRST SHEET OF YOUR EXAM.**

8
10
10
9
___
37

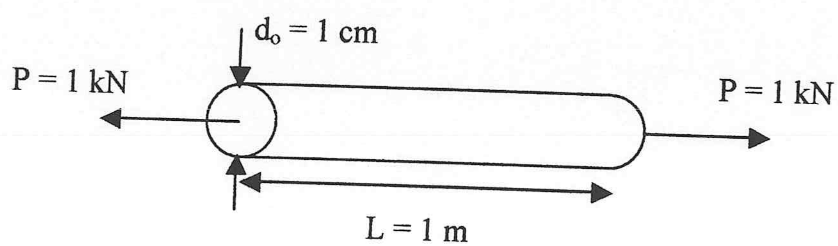
Problem One:

Calculate the ***average*** normal and shear stresses on planes A-B and C-D in the plate below: the thickness (into the page) is 5 mm.



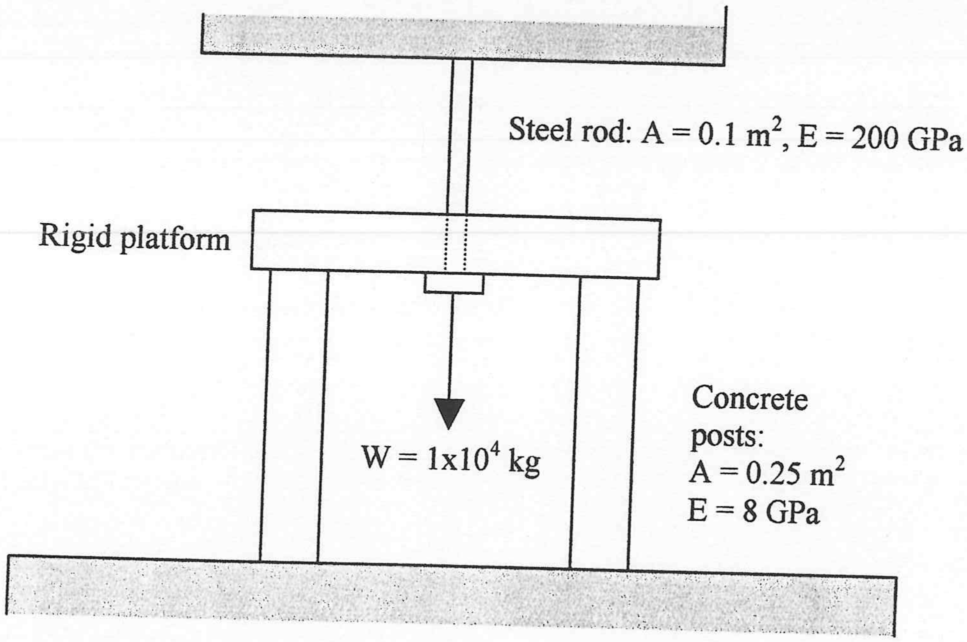
Problem Two:

Solve for the elongation of the bar and the new diameter of the bar after the load is applied; assume the elastic modulus is E = 200 GPa, and the shear (or bulk) modulus is G = 150 GPa.



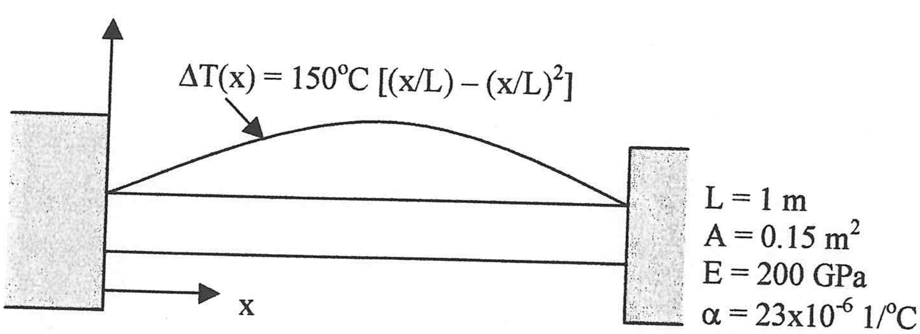
Problem Three:

A rigid platform is supported from below by two concrete posts, and from above by a steel rod. A load hangs from middle of the structure, as shown. Solve for the fractions of the weight that are carried by the posts and rod. Assume the posts and rod are one meter long, and the concrete posts are one meter apart from center-to-center.



Problem Four:

A metal bar is fixed between rigid supports. The temperature change of the bar is a function of position along the bar, as shown. Solve for the reaction forces supplied by the rigid supports. [**Bonus**: solve for the displacement as a function of position in the bar, i.e. u(x)].

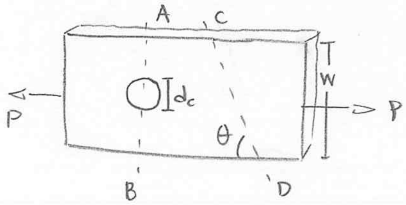


Strength of Materials - Exam #1

Catherine Hovell
February 19, 2003

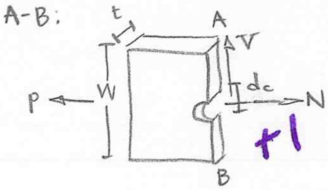
on my honor as a student, I have neither given nor received aid on this exam.

1.



$P = 10\ kN$
$\theta = 60°$
$d_c = 3\,cm$
$w = 10\,cm$
$t = 0.5\,cm$

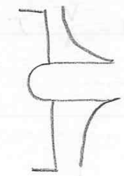
A-B:



$\Sigma F_x = 0 \quad N = P = 10\,kN$

$\Sigma F_y = 0 \quad V = 0$

+1



$\tau = \frac{V}{A} = 0$

+1

$\sigma = \frac{N}{A}$ $\quad A = t \cdot w$

$A = t(w - d_c)$

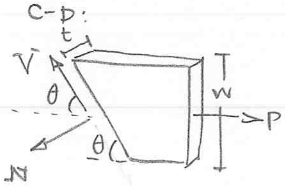
$\sigma = \frac{10\ kN}{(10\,cm)(0.5\,cm)}$

$\sigma = \frac{10\ kN}{(0.5\,cm)(10\,cm - 3\,cm)}$

$\boxed{\begin{array}{l}\text{Plane A-B:} \\ \tau = 0 \\ \sigma = 20\ MPa\end{array}}$

$\to \sigma = 28.6\ MPa$

C-D:



$\Sigma F_x: \; P = V\cos\theta + N\sin\theta$

$\Sigma F_y: \; V\sin\theta = N\cos\theta$

$P = V\cos\theta + V\sin\theta\tan\theta$

$V = \frac{P}{\cos\theta + \sin\theta\tan\theta}$

$V = 5\ kN$

$N = 8.66\ kN$

$\sigma = N/A' \;,\; \tau = V/A'$

$A = w \cdot t \qquad \sin\theta = \frac{A}{A'}$

$\sigma = \frac{8.66\ kN}{\frac{(10\,cm)(0.5\,cm)}{\sin\theta}} \qquad \sigma = 15\ MPa$

$\tau = \frac{5\ kN \cdot \sin\theta}{(10\,cm)(0.5\,cm)} \qquad \tau = 8.66\ MPa$

$\boxed{\begin{array}{l}\text{Plane C-D:} \\ \sigma = 15\ MPa \\ \tau = 8.66\ MPa\end{array}}$

+6

8

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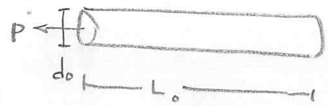
2.



$P = 1\,kN$
$d_o = 1\,cm$
$L_o = 1\,m$

$E = 200\ GPa$
$G = 150\ GPa$

$$G = \frac{E}{2(1+\nu)} \qquad 2G(1+\nu) = E$$

$\Sigma F_x:\ P = P$

$$\nu = \frac{E}{2G} - 1 \qquad \nu = -0.33 = -\frac{\varepsilon_{tra}}{\varepsilon_{lon}}$$

$$\sigma = \frac{F}{A} = E\varepsilon = E\frac{\delta}{L_o}$$

$$\nu = -0.33 = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\delta y / L_o}{\delta x / d_o}$$

$$\delta = \frac{F L_o}{EA}$$

$$\frac{0.33\,\delta x}{d_o} = \frac{\delta y}{L_o} \qquad \delta x = \frac{\delta y\, d_o}{0.33 L_o}$$

$$\delta = \frac{1\,kN \cdot (1\,m)}{200\,GPa \cdot \frac{\pi}{4}(0.1\,m)^2}$$

$$= \frac{6.366\times10^{-7}\,m \cdot (0.1\,m)}{0.33\,(1\,m)}$$

$\delta = 6.366\times10^{-7}\,m$

$\delta x = 1.929\times10^{-7}\,m$

$d' = d_o + \delta x = 0.1 + 1.93\times10^{-7}\,m$

elongation:
$6.37\times10^{-7}\,m$ longer

new diameter:
$0.1\,m + 1.93\times10^{-7}\,m$

or $0.100000193\,m$

+10

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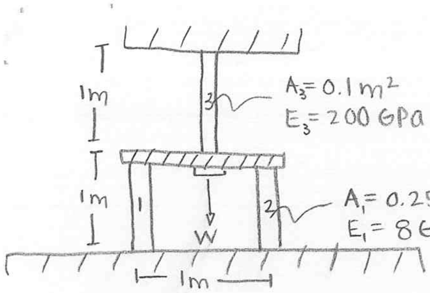
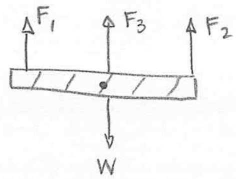
3.



$W = 1\times10^4\,kg$ $\quad L = 1m$



$\Sigma M_o = 0 \quad F_1 = F_2$

$\Sigma F_y: \; 2F_1 + F_3 = W$

$\delta_1 = \delta_2 = \delta_3$

$2(0.1F_3) + F_3 = W$

$1.2F_3 = W$

$F_3 = 8333\,kg$

$F_1 = 833.3\,kg$

$$\delta_1 = \varepsilon_1 \cdot L = \frac{\sigma}{E} L = \frac{F_1 L}{E_1 A_1} = \delta_2$$

$$\delta_3 = \varepsilon_3 L = \frac{\sigma}{E} \cdot L = \frac{F_3 \cdot L}{E_3 \cdot A_3}$$

$$\delta_1 = \delta_3$$

$$\frac{F_1 \cancel{L}}{E_1 A_1} = \frac{F_3 \cdot \cancel{L}}{E_3 \cdot A_3} \qquad F_1 = \frac{E_1 A_1}{E_3 A_3} F_3$$

$$F_1 = \frac{8\,GPa \cdot 0.25\,m^2}{200\,GPa \cdot 0.1\,m^2} F_3$$

$$F_1 = 0.1F_3$$

$F_1 = F_2 = 0.1F_3$

$$\boxed{\begin{array}{l} F_1 = F_2 = \tfrac{1}{12}W = 833.3\,kg \\ F_3 = \tfrac{5}{6}W = 8333\,kg \end{array}}$$

10

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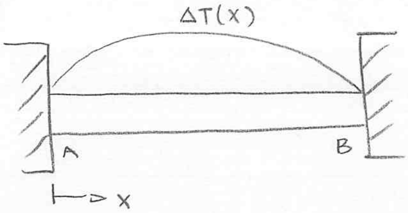
4.



$$\Delta T(x) = 150°C\left[(x/L) - (x/L)^2\right]$$

$L = 1\ m$
$A = 0.15\ m^2$
$E = 200\ GPa$
$\alpha = 23 \times 10^{-6}\ 1/°C$

$$\varepsilon = \frac{d}{dx}u(x) = \frac{\delta}{L_o}$$

$$\delta = L_o \frac{d}{dx}u(x)$$

$$\delta\, dx = L_o\, d(u(x))$$

$$u(x) = \int \frac{\delta}{L_o} dx$$

$F_A = F_B = F$

$\varepsilon_{th} = \alpha\, \Delta T(x)$ $\qquad \varepsilon_{tot} = \varepsilon_{th} + \frac{\sigma}{E} = 0 \rightarrow$ zero

so $\varepsilon_{th} = \frac{-\sigma}{E}$

$$d\delta = \alpha\, \Delta T(x)\, dx$$

$$\delta = 150°\alpha \int_0^L x/L - (x/L)^2\, dx$$

$$\delta = 150°\alpha\left[\frac{x^2}{2L} - \frac{x^3}{3L^2}\Big|_0^L\right]$$

$$= 150°\alpha\left[\frac{L}{2} - \frac{L}{3}\right] = 0.575\ mm$$

$$\delta = \frac{FL}{EA} \qquad F = \frac{(0.575\ mm)(200\ GPa)(0.15\ m^2)}{1\ m}$$

$F = 17.25\ MN$

$\boxed{F = 17.3\ MN \text{ tension}}$

So close. Compression.

$\delta_{mech} = -\delta_{thermal}$!!

or $\delta_{TOTAL} = 0 = \delta_{mech} + \delta_{th}$

LOSER! δ (+), so walls PUSH back!

9

57/80

Catherine Hovell

CE231 Strength of Materials

Spring 2003

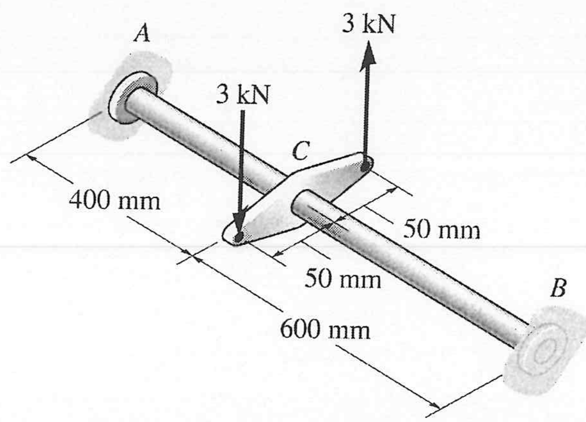
MIDTERM #2

Thursday, April 4th, 2003
9.30-10.45 a.m., D221 Thornton Hall

- WRITE ON ONLY ONE SIDE OF A PIECE OF PAPER, AND WRITE YOUR NAME ON EACH PAGE THAT YOU USE.
- CLEARLY CROSS OUT THOSE PARTS OF YOUR RESPONSE YOU DO NOT WANT GRADED; IF IT IS NOT CLEAR, EVERYTHING WILL FACTOR INTO THE GRADE.
- IF YOU ARE RUNNING SHORT OF TIME, SKETCH AN OUTLINE OF THE STEPS YOU WOULD TAKE TO SOLVE THE PROBLEM (as opposed to attempting to grind out the solution).
- PLEASE WRITE AND SIGN THE HONOR PLEDGE ON THE FIRST SHEET OF YOUR EXAM.

Problem One:

(1) Solve for the angle of twist at location C for the rod below, which is built-in on both ends labeled A and B. The bar has a torsional modulus of 120 GPa and a radius of 10 mm. (2) Sketch how the shear stress is distributed along the rod and along a radial line of the cross section.



Problem Two:

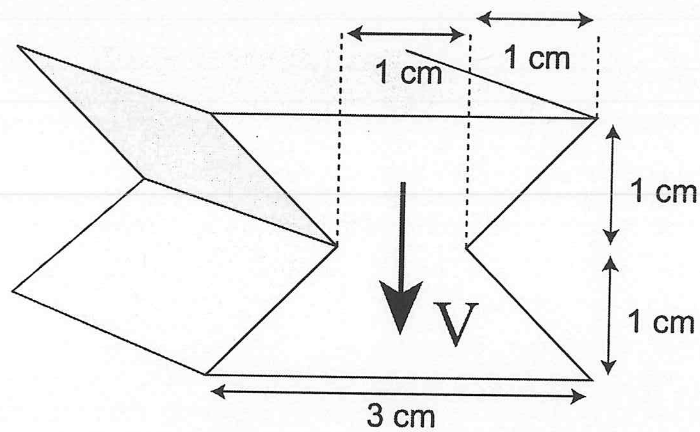
Explain the assumptions and procedure by which one obtains the result that the bending stress in a beam has a linear distribution from top to bottom. In other words, explain how one obtains the following relation:

$$\sigma(y) = -\frac{M_z y}{I_z}$$

Be sure to clearly indicate the meaning of your variables and how they are calculated (or defined). A couple of well-labeled schematics or sketches are highly recommended. Your response will be evaluated first in terms of the big picture - e.g. how the derivation invokes (or does not invoke) the three keys of any solids analysis – and then in terms of detail.

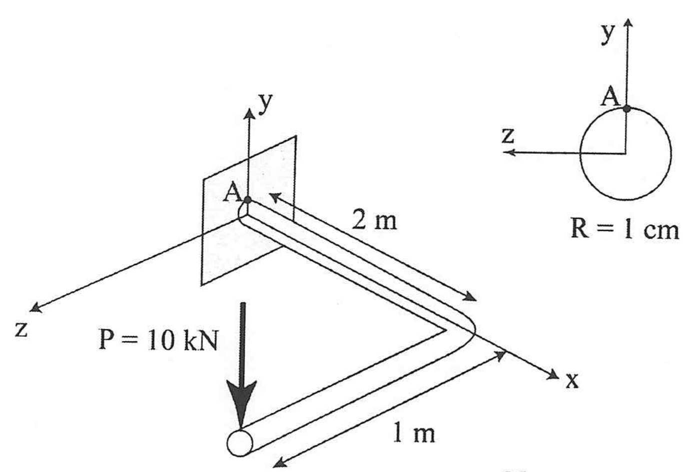
Problem Three:

Solve for the variation in transverse shear stress from top to bottom in the bow-tie shaped beam illustrated below, which is subjected to a vertical resultant shear force denoted as V. What is the maximum shear stress in terms of V? Note: you do not need to sketch or simplify the distribution!.



Problem Four:

A right-angled circular bar is built into a wall at one end, and loaded in the y-direction with a point load on the other end. Find the stresses acting at point A located at the wall. (The bar is in the x-z plane: it is fixed perpendicular to the y-z plane).



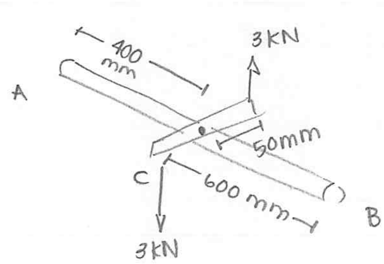
Note: not to scale!

Materials - Test #2

Catherine Hovell
April 3, 2003

On my honor as a student, I have neither given nor received aid on this exam. Catherine S. Hovell

1.



$G = 120\ GPa$

$r = 10\ mm$

$J = \frac{\pi}{2} r^4 = \frac{\pi}{2}(0.010\ m)^4 = 1.571 \times 10^{-8}\ m^4$ +3

$\phi_{AC} = \frac{(\cancel{300\ N\cdot m})(0.40 m)}{(1.571\times10^{-8} m^4)(120 GPa)} = 0.0637$ ($T_A = 180$)

= limits twist

$\phi_{BC} = \frac{(\cancel{300\ N\cdot m})(0.60 m)}{(1.571\times10^{-8} m^4)(120 GPa)} = 0.0955$ ($T_B = 120$)

$\phi_C = 0.038$

T_c: $(50\ mm)(3\ kN) + (50\ mm)(3\ kN)$

$T_c = 300\ N\cdot m$ +2

$T_A = ?$ $T_B = ?$

$\phi_{AC} = \phi_{BC}$ $\phi = \int_0^L \frac{T(x)}{J(x)G(x)}dx = \frac{T\cdot L}{J\cdot G}$ +1

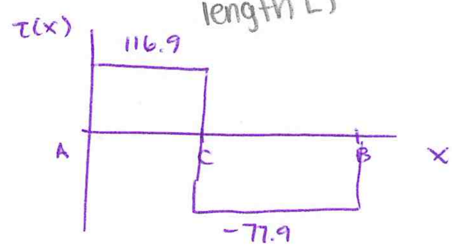
(all values are constant along length L)

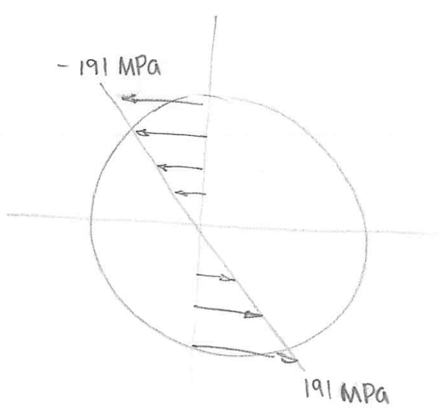
+6

$\phi = 0.0637\ rad$

$\frac{\phi \cdot 180°}{\pi} = 3.647°$

$\boxed{\phi = 3.65°}$





$\tau = \frac{T\rho}{J} = \frac{(300\ N\cdot m)(0.01\ m)}{\frac{\pi}{2}(0.01 m)^4}$

$\tau = 190.99\ MN/m^2$

+2

$x < 400\ mm$

$\tau = 116.88\ MPa$

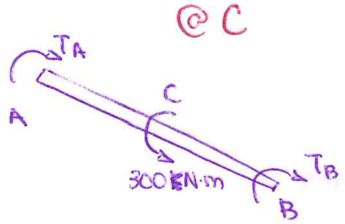
$x > 400\ mm$

$\tau = 77.92\ MPa$

at C $(x = 400 mm)$

$\tau = 194.8\ MPa$

@ C



~~$T_A = T_C(0.40 m) = 120\ N\cdot m$~~

~~$T_B = T_C(0.60 m) = 180\ N\cdot m$~~

+14 (+14)

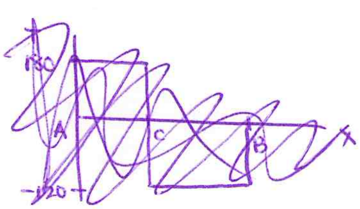
$\phi_{AC} = \frac{T_A \cdot L_A}{JG}$ $\phi_{BC} = \frac{T_B \cdot L_B}{J\cdot G}$

$\phi_{AB} = \phi_{BC}$

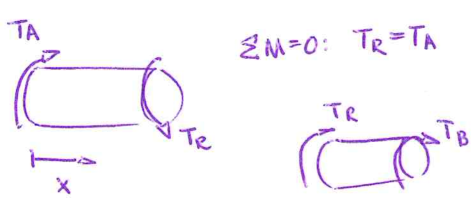
$T_A = 1.5 T_B$

$T_A = 180\ N\cdot m$

$T_B = 120\ N\cdot m$



$T_c = T_A + T_B$



$\Sigma M = 0$: $T_R = T_A$

so $\phi = 0.038\ rad$

$\phi = 2.18°$

Materials Exam #2

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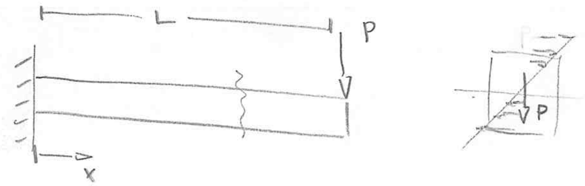
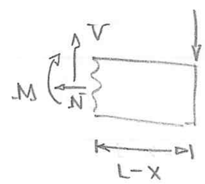
2. $\sigma(y) = -\frac{M_z \cdot y}{I_z}$





$N = 0$

$V = P$

$M = (L-x)P$

Two assumptions:

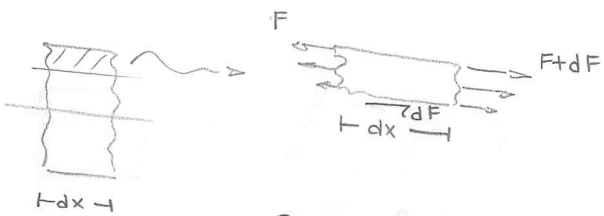
- planar sections remain planar
- planar sections remain perpendicular to the neutral axis, where stress is zero.

+10

strain

- NA does not change length, as strain = 0

$\sigma = \sigma_{max}\left(\frac{y}{c}\right)$



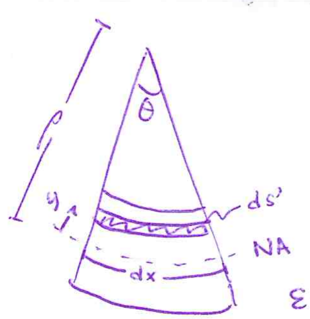
$\int F(x)\,dA = \underbrace{\int F(x) \cdot y\,dx}_{M(x)}$

The shear force on a beam puts one half (above or below the neutral axis) into tension and the other half into compression.

The neutral axis is defined by having a stress equal to zero.

+1

The stress at a particular spot on the beam can be determined by finding the imbalance in the moments from one side to the other of a differential element of the beam.



$\varepsilon = \frac{ds' - dx}{dx}$

$\varepsilon = \frac{(\rho - y)\theta - \rho\theta}{\rho\theta} = \frac{-y}{\rho} = \varepsilon$

+11

So, distribution of strain is linear

$\varepsilon = \varepsilon_{max}\left(\frac{y}{\rho}\right)$

$\underline{E\varepsilon = \sigma}$ so, $\sigma = \frac{-y}{\rho} E \cdot \varepsilon_{max}$

$\sigma = -\sigma_{max}\left(\frac{y}{c}\right)$

$dM = y\sigma b\,dy$

$M = \int_A y\sigma\,dA = \int y\,dF$

$= \frac{\sigma_{max}}{c} I \quad \to \int y^2 dA = I$

$\sigma_{max} = \frac{Mc}{I}$

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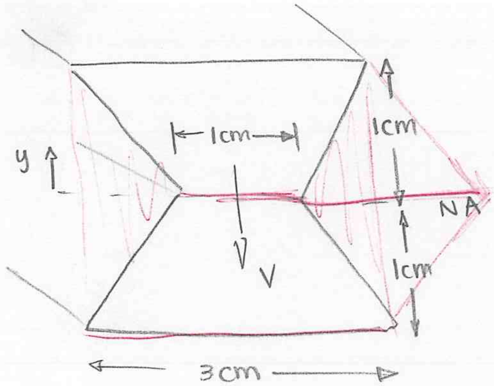
Materials Exam #2

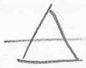
$t = 1 + 2y \qquad dA = t(y)\,dy$

$I = 2\int_0^1 (1+2y)y^2\,dy = \frac{5}{3}\text{cm}^4$

3.



$I = \frac{1}{12}(3\text{ cm})(2\text{ cm})^3 - \left[\int_A y^2\,dA\right]_{\text{triangles}}$



$I = 1/36\,bh^3$

$\to 2\left[\frac{1}{36}(2\text{cm})(1\text{cm})^3 + \frac{1}{2}(1\text{cm})(2\text{cm})\left[\frac{1}{3}(1\text{cm})\right]^2\right]$ ~~4~~

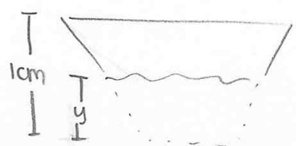
$= 2\text{ cm}^4 + 2(1/6\text{ cm}^4) = 2.33\text{ cm}^4$ or $7/3$ $-$

$= \frac{5}{3}$ +5 LOSER.

$\tau = \frac{V\,Q(y)}{I \cdot t(y)}$

$t(y) = 1\text{ cm} + (y/1\text{cm})(2\text{cm}) = 1\text{ cm} + 2y$ +5

$Q(y) = \bar{y}'A'$: $\quad \bar{y}' = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} \quad A' = \Sigma A_i \quad Q(y) = \Sigma \bar{y}_i A_i = \Sigma \bar{y}_i \cdot t(y) \cdot (1\text{cm}-y)$



$\frac{Q(y)}{t(y)} = \Sigma \bar{y}_i \cdot (1\text{cm}-y)$ τ_{max} @ $y=0$ ~~4~~ +5

$A_i\bar{y}_i(y=0)$: $(1\text{cm})(3\text{cm})\frac{1}{2}(1\text{cm}) - \frac{1}{2}(1\text{cm})(2\text{cm}) \cdot \frac{1}{3}(1\text{cm})$

$= 1.167\text{ cm}^3 \quad \bar{y}_i = \frac{1.167\text{ cm}^3}{(1\text{cm})(2\text{cm}) = A_i}$

$\tau_{max} = \frac{V(0.583\text{cm}^2)}{2.33\text{ cm}^4} = 0.25\,V$

$\boxed{\tau_{max} = 0.25\,V}$

$\bar{y}_i = 0.583\text{ cm}$

$Q = \int_y^1 y\,dA$ $\qquad = \frac{7}{10}V$

$\frac{Q(y)}{t(y)} = (0.583\text{ cm})(1\text{cm}) = 0.583\text{ cm}^2$

along height, $\frac{Q(y)}{t(y)}$ changes...

$$\bar{y}_i = \frac{(3\text{cm})(1\text{cm}-y)(y+\frac{1}{2}(1\text{cm}-y)) - \frac{1}{2}(1\text{cm}-y)(1\text{cm}+2y)(y+\frac{1}{3}(1\text{cm}-y))}{(3\text{cm})(1\text{cm}-y) - \cancel{2}[(1\text{cm}-y)(1\text{cm}+2y)\cdot\cancel{\frac{1}{2}}]}$$

for Q, +3

$$\bar{y}_i = \frac{\frac{1}{2}(3\text{cm})(1\text{cm}+y) - \frac{1}{6}(1\text{cm}+2y)(1\text{cm}+2y)}{(3\text{cm}) - (1\text{cm}+2y)} = \frac{(1.5\text{cm})(1\text{cm}+y) - \frac{1}{6}(1\text{cm}+2y)}{2\text{cm}-2y}$$

$7/6 - 1/2\,y^2 - 2/3\,y^3$

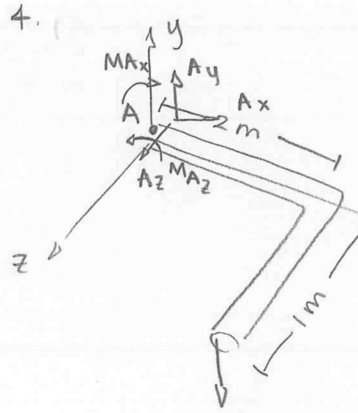
$$\tau(y) = \frac{V}{2.33\text{ cm}^4}\left[\frac{(1.5\text{cm})(1\text{cm}+y) - \frac{1}{6}(1\text{cm}+2y)^2}{\frac{2\text{ cm}-2y}{1+2y}}\right]$$ ~~4~~

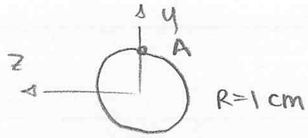
~~14~~ +18

Materials Test #2

Catherine Hovell

4.





$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.01\,m)^4 = 7.854 \times 10^{-9}\,m^4$

$c = 1\,cm$

$A = \pi r^2 = \pi(0.01\,m)^2 = 3.14 \times 10^{-4}\,m^2$

$\overset{+}{\rightarrow} \Sigma F_x: \; A_x = 0$

$+\uparrow \Sigma F_y: \; A_y = 10\,kN$

$\circlearrowleft + \Sigma M_{Ax}: \; M_{Ax} = (1\,m)(10\,kN) = 10\,kN\cdot m$

+7

$\circlearrowleft + \Sigma M_{Az}: \; M_{Az} = (2\,m)(10\,kN) = 20\,kN\cdot m$

$\sigma = \frac{N}{A} \quad \sigma = \frac{Mc}{I} \quad \sigma_x = \frac{(10\,kN\cdot m)(0.01\,m)}{7.854\times10^{-9}\,m^4} \quad \sigma_z = \frac{(20\,kN\cdot m)(0.01\,m)}{7.854\times10^{-9}\,m^4}$

$\tau = \frac{A_y}{A} = \frac{10\,kN}{\pi(0.01\,m)^2} = 31.83\,GPa$

~~26.526 Pa (?)~~

BENDING:

$\cancel{\sigma_x = 12.7\,GPa}$

$\sigma_z = 25.5\,GPa$ +5

SHEAR:

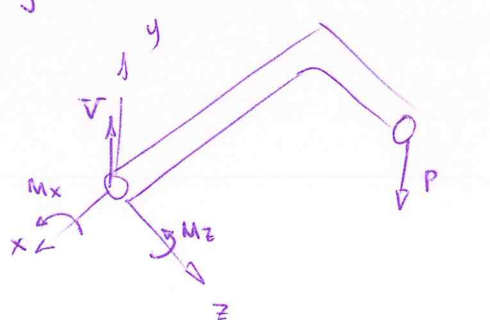
$\tau = 31.8\,MPa$ @ A ~~+~~ $\tau = 0$ (free surface) +1

NORMAL:

$\sigma = 0$ +1

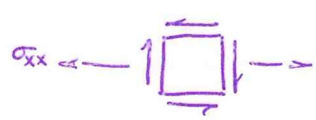
TWIST: $\tau_{xz} = \frac{M_x r}{J} = 6.37\,GPa$

+14



twist?

SHEAR:
$\tau_{xy} = 0$ @ free surface



$\Sigma F_y: \; V = P = 10\,kN \uparrow$

$\Sigma M_x = 0: \; M_x = Pz = 10\,kN\cdot m$

$\Sigma M_z = 0: \; M_z = Px = 20\,kN\cdot m$

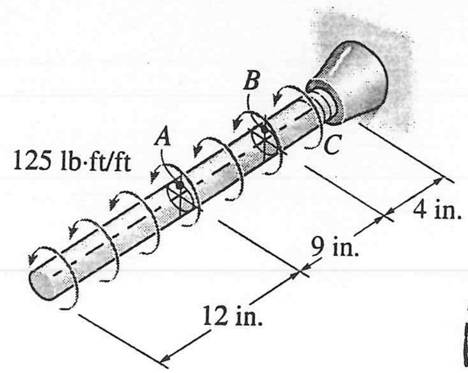
$\sigma_{xx} = 25.5\,GPa$
BENDING

$\sigma = 0$
NORMAL

$\tau = 6.37\,GPa$ TWIST

CE231 Midterm #2
Make-Up Exam
April 2nd, 2003

1. Solve for the angle of twist as a function of position for the rod shown below. The rod is built into a wall on one end, and free on the other. The rod is subjected to a uniform twist per unit length, as shown. (You may ignore the points labeled A, B and C).

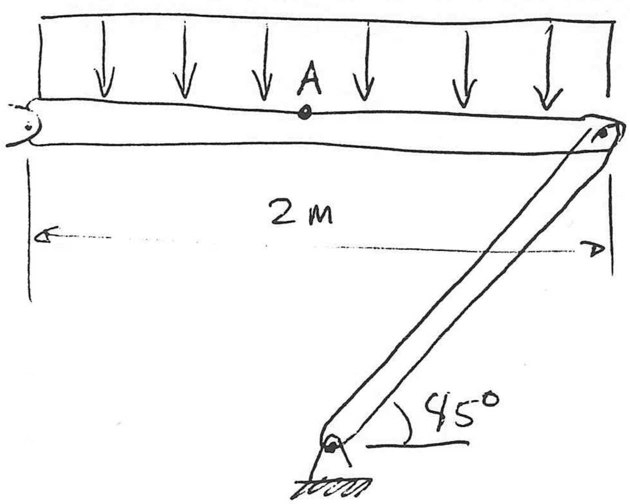


$R = 1$ in

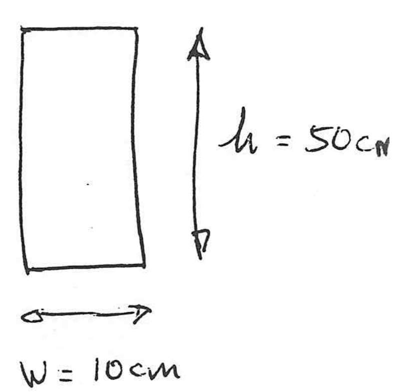
$G = 10$ Msi

2. Solve for the state of stress at the top of the beam, in the center of the span that supports a distributed load, as illustrated below.

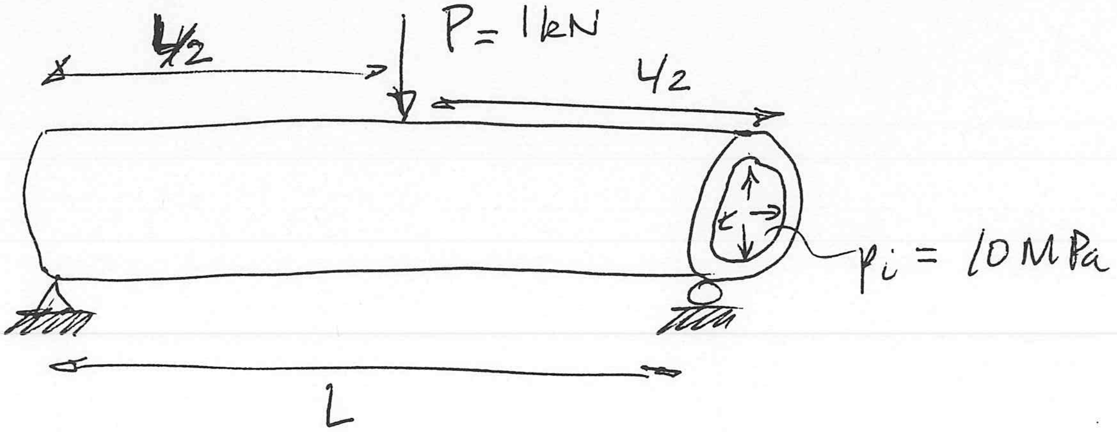
$w(x) = 100$ kN/m



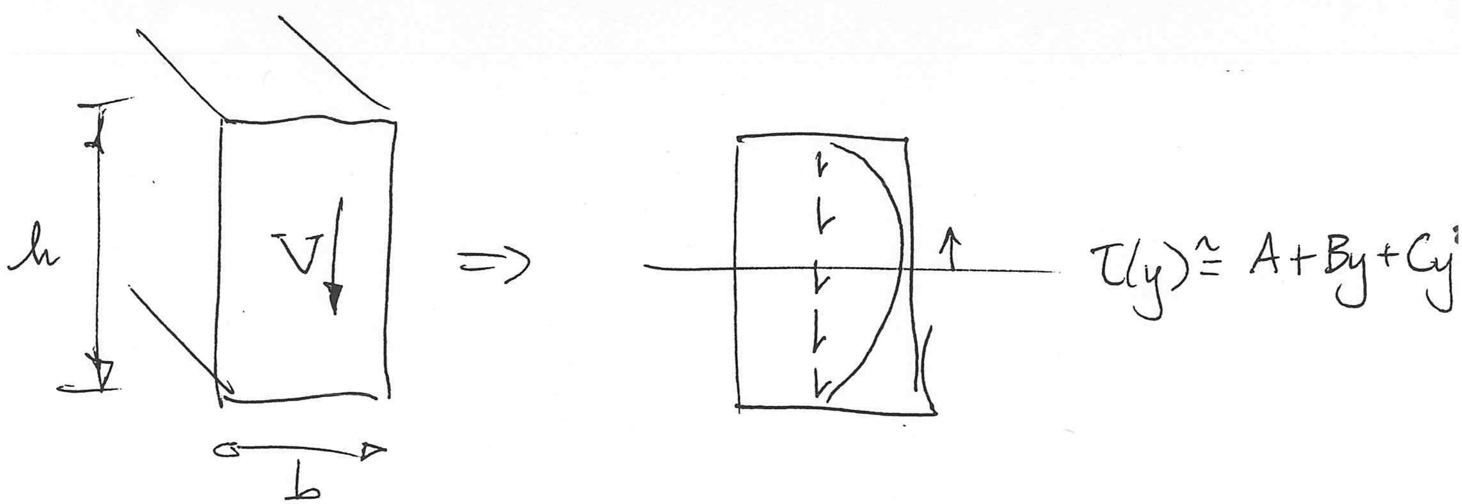
Cross section:



3. A pressurized tube is simply supported and subjected to a point load centered between the supports. The wall thickness is 1 mm and the mean radius of the tube is 50 mm. The internal pressure in the tube is 10 MPa, and the load applied to the center is 1 kN. If maximum allowable tensile stress is 750 MPa, what is the minimum spacing between the supports?

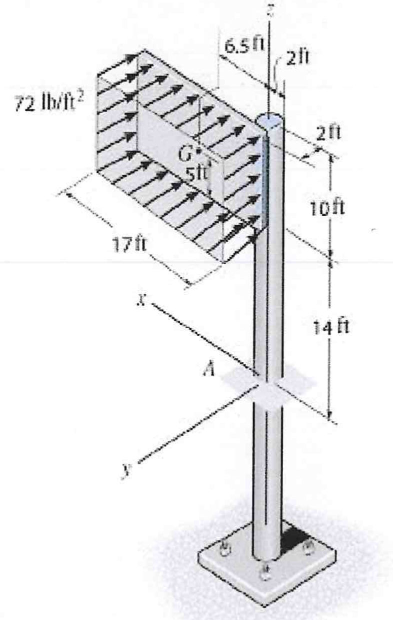


4. Explain why the shear stress is has a parabolic distribution through a beam (in the depth direction) for rectangular cross-sections. Clearly state any assumptions that factor into your explanation, and draw clear figures indicating what your variables refer to.

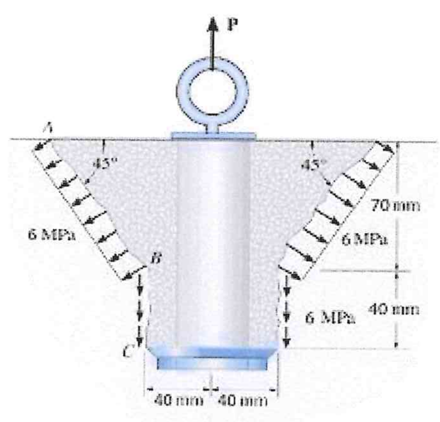


CE 231: Strength of Materials Spring 2003	Homework #1 Due: 1/31/03, 5:00 PM

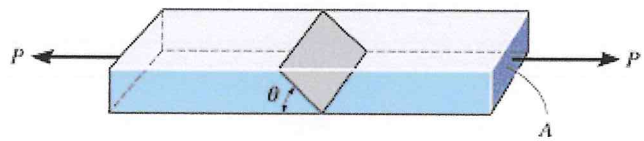
1. The sign has a weight of 1650 lb and a center of gravity at G. If it is subjected to the uniform wind load of 72 lb/ft^2, determine the resultant internal loadings acting on the cross section of the post at A. The post has a weight of 120 lb/ft.



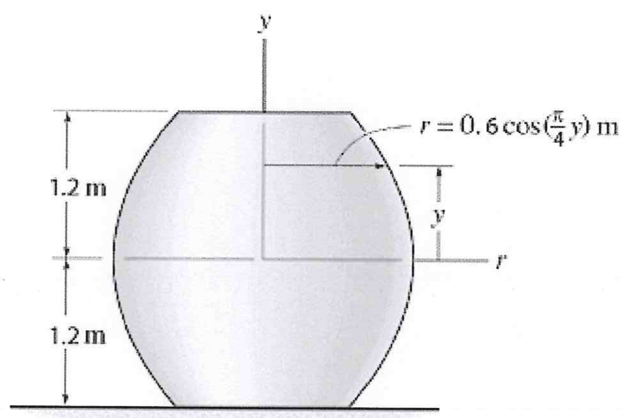
2. The anchor bolt was pulled out of the concrete wall and the failure surface formed part of a frustum and cylinder. This indicates a shear failure occurred along the cylinder BC and tension failure along the frustum AB. If the shear and normal stresses along these surfaces have the magnitudes shown, determine the force **P** that must have been applied to the bolt.



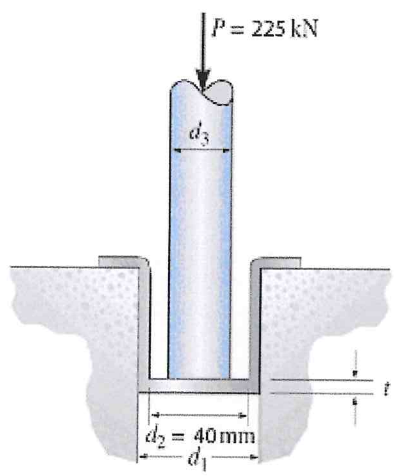
3. The bar has a cross-sectional area A and is subjected to the axial load P. Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ $(0 \le \theta \le 90°)$.



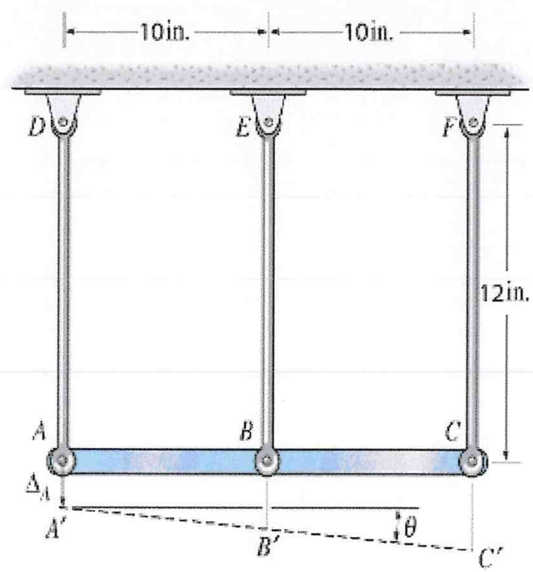
4. The shape has a radius that is defined by $r = 0.6\cos\left(\frac{\pi}{4}y\right)$ m. Determine the average normal stress at the support if the material has a density of $\rho = 4.2$ Mg/m^3.



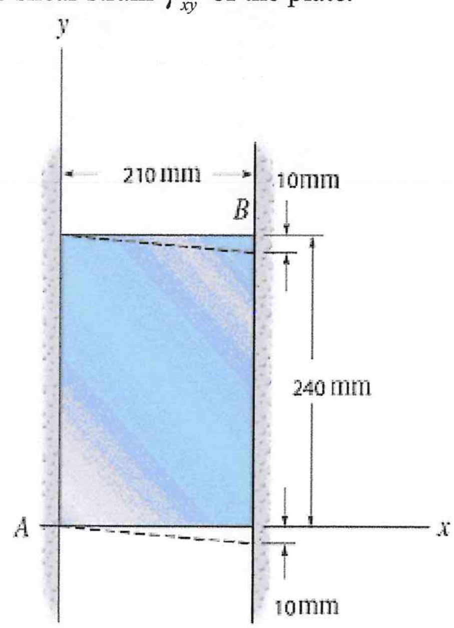
5. Determine the smallest dimensions of the circular shaft and the circular end cap if the load it is required to support is P = 225 kN. The tensile stress, bearing stress, and shear stress is $(\sigma_t)_{allow} = 180$ MPa, $(\sigma_b)_{allow} = 325$ MPa, and $\tau_{allow} = 100$ MPa.



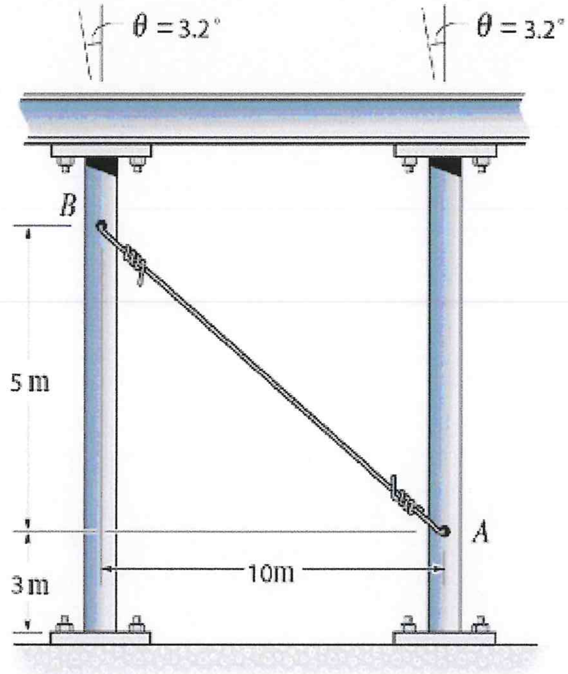
6. The rigid bar ABC is originally in a horizontal position. If loads cause the end A to be displaced downwards $\Delta_A = 0.008$ in. and the bar rotates $\theta = 1.2°$, determine the average normal strain in the rods AD, BE, and CF.



7. The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain γ_{xy} of the plate.



8. The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 3.2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.

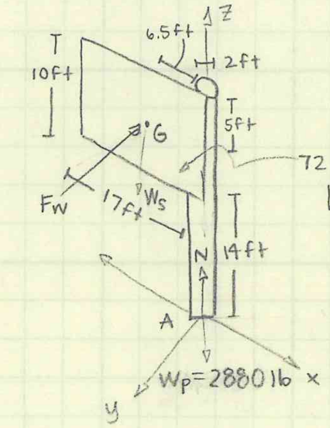


CE 231: Homework #1

26/30.

Catherine Hovell
January 31, 2003

1.



$W_s = 1650$ lb

$W_p = 120$ lb/ft

$\hookrightarrow F_w = 12240$ lb

$\Sigma F_x = 0$ lb

$\Sigma F_y = F_w - V_y = 0$

$V_y = 12240$ lb

$\Sigma F_z = W_s + W_p - N = 0$

$N = 4530$ lb

$\Sigma M_x: F_w \cdot 19\text{ft} - M_{xA} = 0$

$M_{xA} = 232560$ lb$\cdot\cancel{\text{ft}}$

$\Sigma M_y: W_s \cdot 6.5\text{ ft} - M_{yA} = 0$

$M_{yA} = 10725$ lb$\cdot$ft

$\Sigma M_z: F_w \cdot 6.5\text{ ft} - M_z = 0$

$M_z = 79560$ lb$\cdot$ft

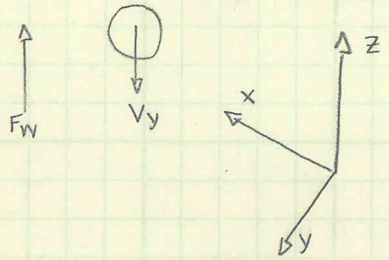
At A: $F_x = 0$ lb
$F_y = V_y = 1.22\times10^4$ lb
$F_z = 4530$ lb
$M_x = 2.33\times10^5$ lb$\cdot$ft
$M_y = 1.07\times10^4$ lb$\cdot$ft
$M_z = -7.96\times10^4$ lb$\cdot$ft

* axis on prob. considered as positive *

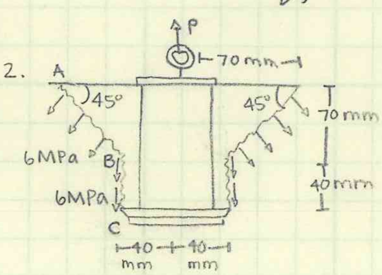
NEED BETTER FBD!

+8.

A (in x-y plane)



2.



$\sigma_{cyl} = 6$ MPa

$\sigma_{frus} = 6 \text{ MPa} \cdot \frac{\sqrt{2}}{2}$

$\sigma = \frac{F}{A}$

$F = P = A\sigma$

$P = \sigma_{cyl} \cdot A_{cyl} + \sigma_{frus} \cdot A_{frus}$

$P = (6\times10^6 \text{Pa})(0.01005 \text{ m}^2) + (3\sqrt{2}\times10^6 \text{ Pa})(0.0342 \text{ m}^2)$

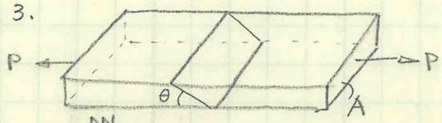
$P = 6.177\times10^6$ N

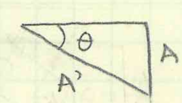
$\boxed{P = 6.18 \text{ MN}}$

$SA_{cyl} = 2\pi \cdot 40\text{mm} \cdot 40\text{mm}$
$= 0.01005 \text{ m}^2$

$SA_{frus} = \pi \cdot 70\text{mm} \cdot \sqrt{2}\,(70\text{mm} + 40\text{mm})$
$= 0.0342 \text{ m}^2$

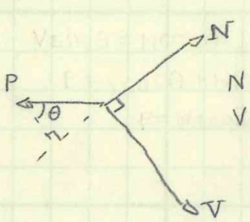
3.





$\boxed{\sigma_N = \frac{P}{A}\sin^2\theta \quad \sigma_V = \frac{P}{A}\sin(2\theta)}$

graphs on attached sheet



$N = P\sin\theta$
$V = P\cos\theta$

$\sin\theta = \frac{A}{A'}$

$A' = \frac{A}{\sin\theta}$

+10.

$A' = A \cdot 1/\sin\theta$

$\sigma_N = \frac{N}{A'}$

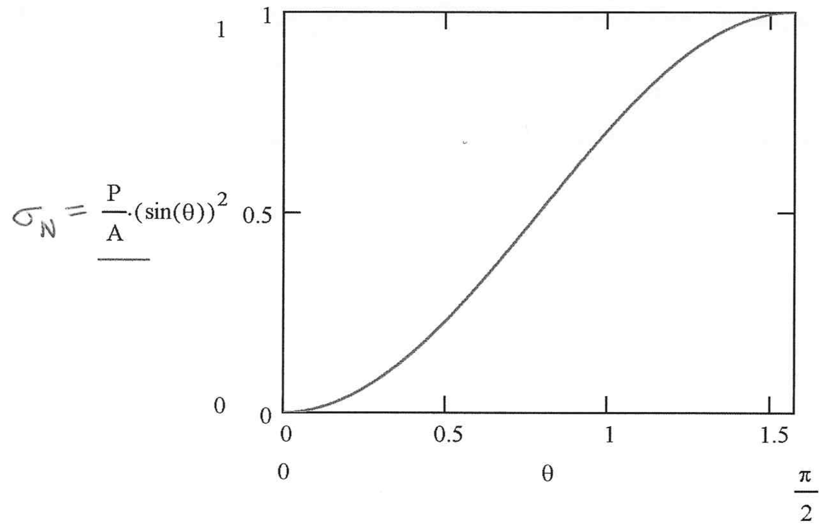
$\sigma_V = \frac{V}{A'}$

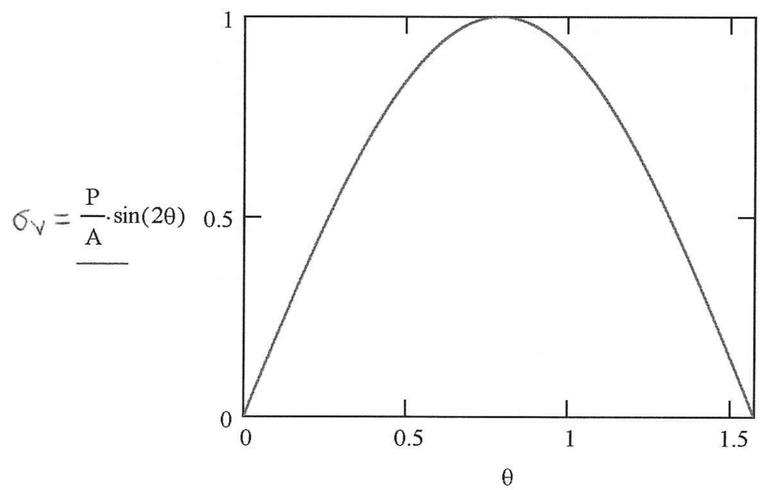
$\sigma_N = \frac{P\sin\theta}{A/\sin\theta}$

$\sigma_V = \frac{P\cos\theta}{A/\sin\theta} = \frac{P}{A}\sin 2\theta$

$\hookrightarrow \sin\theta\cos\theta = \sin 2\theta$

3. Assume constants of: $P := 1 \quad A = 1\,A$

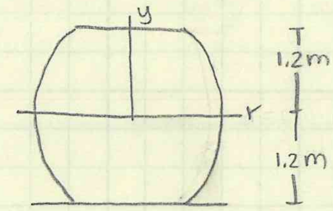




Beautiful!

CE 231: Homework #1

4.



$r = 0.6\cos(\pi/4\, y)$ m

$\rho = 4.2\times10^6\ g/m^3 = \frac{m}{V}$

$\sigma = \frac{mg}{A_B}$

$dV = A\,dy = \pi(0.6)^2\cos^2(\pi/4\, y)\,dy$

$A_B = \pi(0.6\cos(\pi/4\cdot 1.2m)\ m)^2$

$= 0.3907\ m^2$

$\int dV = 0.6^2\pi\int\cos^2(\pi/4\, y)\,dy$

$V = 0.36\pi\cdot\frac{1}{2}\int 1+\cos(\pi/2\, y)\,dy$

$V = 0.18\pi\left(y + 2/\pi\sin(\pi/2\, y)\right)\Big|_{-1.2}^{1.2}$

$\cos^2 u = \frac{1+\cos 2u}{2}$

$\rho = 4.2\times10^6\ g/m^3 = \frac{m}{2.042\ m^3}$

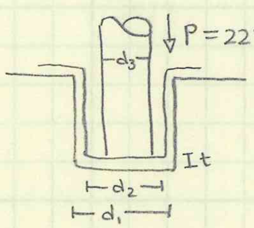
$V = 2.042\ m^3$

$m = 8.576\times10^3$ kg

$\sigma = \frac{8576\ kg\cdot 9.81\ m/s^2}{0.3907\ m^2}$

$\boxed{\sigma = 2.15\times10^5\ Pa}$

5.



$\sigma_t = 180$ MPa

$\sigma_B = 325$ MPa

$\tau = 100$ MPa

$F = P = 225$ kN $\quad d_2 = 40$ mm

$\boxed{\begin{array}{l} d_1 = 56.5\ mm \\ d_2 = 40\ mm \\ d_3 = 29.7\ mm \\ t = 24.1\ mm \end{array}}$

$\sigma_t = \frac{F}{A_1} \qquad A_1 = \frac{F}{\sigma_t} = \frac{225\ kN}{180\times10^6\ Pa} = 0.00125\ m^2$

$\sigma_b = \frac{F}{A_3} \qquad A_3 = \frac{225\ kN}{325\ MPa}$

$A_1 = \pi(\tfrac{1}{2}d_1)^2 - \pi(\tfrac{1}{2}d_2)^2$

$A_3 = 6.92\times10^{-4}\ m^2 = \pi(\tfrac{1}{2}d_3)^2$

$0.00159 = d_1^2 - (0.040\ m)^2$

$d_3 = 0.02969\ m$

$d_1 = 0.0565$ m

$\tau = \frac{F}{c_3\cdot t} \qquad t = \frac{F}{\tau\cdot c_3} = \frac{225\ kN}{100\ MPa\cdot\pi\cdot 29.7\ mm}$

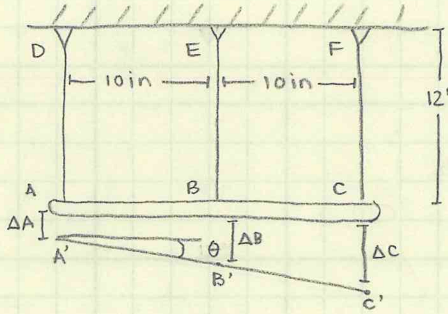
$t = 0.0241$ m

+8.

All correct: but difficult to follow reasoning behind choice of σ_{allow} and area! Picture of failure plane would make huge diff.

CE 231: Homework #1

6.



$\theta = 1.2°$

$\Delta_A = 0.008\text{ in}$

$\varepsilon_{ave} = \dfrac{dx' - dx}{dx}$

so... $\varepsilon_{ave} = \dfrac{AD' - AD}{AD} = \dfrac{\Delta A}{AD}$

find ε_{ave} for AD, BE, CF

$\varepsilon_{ave_A} = \dfrac{0.008\text{ in}}{12\text{ in}}$

$= 6.67 \times 10^{-4}$

$\tan\theta = \dfrac{\Delta B}{AB}$

$\Delta B = 10\text{ in} \cdot \tan 1.2°$

$= 0.209\text{ in}$

$\varepsilon_{ave_B} = \dfrac{0.209\text{ in}}{12\text{ in}}$

$= 0.0175$

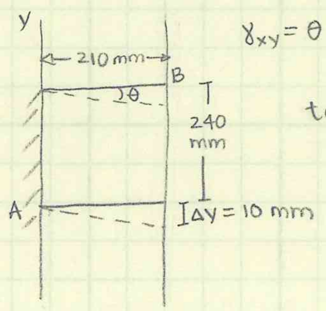
$\tan\theta = \dfrac{\Delta C}{AC}$ $\Delta C = 20\text{ in} \cdot \tan 1.2°$

$\Delta C = 0.4189\text{ in}$

$\varepsilon_{ave_C} = \dfrac{0.4189\text{ in}}{12\text{ in}} = 0.0349$

$$\boxed{\begin{aligned} \varepsilon_{AD} &= 6.67\times10^{-4} \\ \varepsilon_{BE} &= 0.0175 \\ \varepsilon_{CF} &= 0.0349 \end{aligned}}$$

7.



$\gamma_{xy} = \theta$

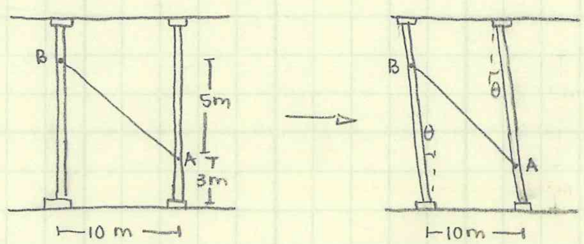
$\tan\theta = \dfrac{10\text{ mm}}{210\text{ mm}}$

$\theta = 2.726° \rightarrow$ radians

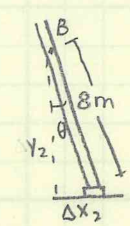
$2.726° \times \dfrac{2\pi}{360°} = 0.04758$

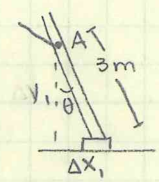
$\boxed{\gamma_{xy} = 0.0476}$

8.



$\theta = 3.2°$

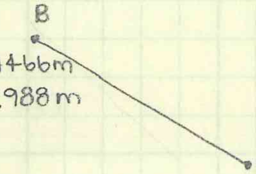




$L_{AB} = \sqrt{125} = 5\sqrt{5}$

$\dfrac{\Delta x_1}{3m} = \sin\theta$ $\dfrac{y_1}{3m} = \cos\theta$

$\dfrac{\Delta x_2}{8m} = \sin\theta$ $\dfrac{y_2}{8m} = \cos\theta$

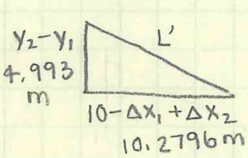


$\Delta x_2 = 0.4466\text{ m}$

$y_2 = 7.988\text{ m}$

$\Delta x_1 = 0.167\text{ m}$

$y_1 = 2.995\text{ m}$



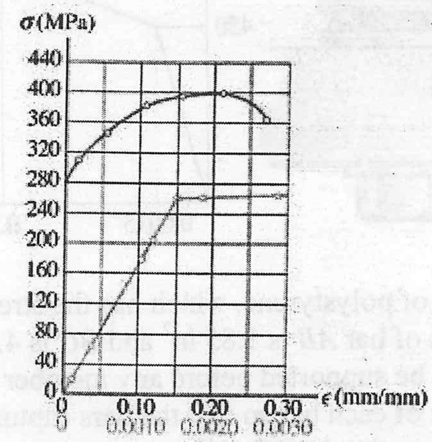
$L' = \sqrt{x'^2 + y'^2} = 11.428\text{ m}$

$\varepsilon = \dfrac{L' - L}{L} = \dfrac{11.428\text{ m} - 5\sqrt{5}\text{ m}}{5\sqrt{5}\text{ m}}$

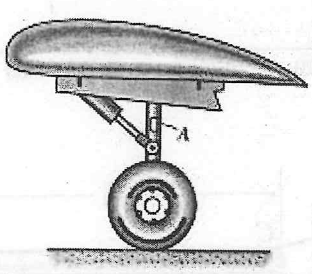
$\boxed{\varepsilon = 0.022}$

CE 231: Strength of Materials Spring 2003	Homework #2 Due: 2/7/03, 5:00 PM

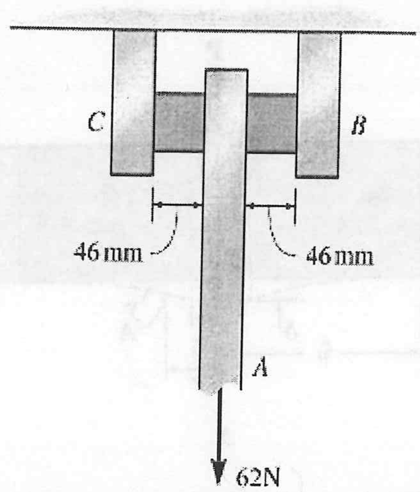
✓ 1. The stress-strain diagram for a bar of steel alloy is shown in the figure. Determine approximately the modulus of elasticity, the proportional limit, the ultimate stress, and the modulus of resilience. If the bar is loaded until it is stressed to 320 MPa, determine the elastic strain recovery and the permanent set or strain in the bar when it is unloaded.



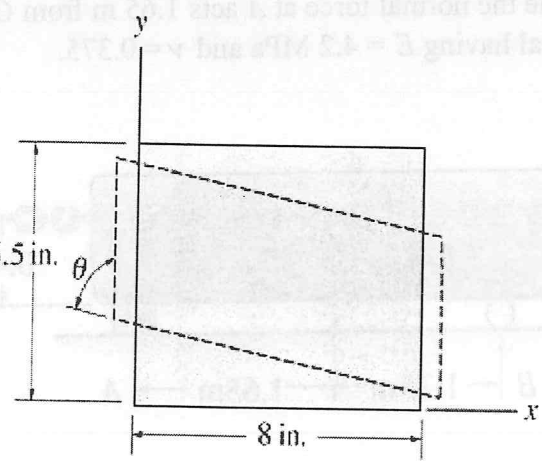
✓ 2. The change in weight in an airplane is determined from reading the strain gauge *A* mounted in the plane's aluminum wheel strut. *Before* the plane is loaded, the strain gauge reading in a strut is $\varepsilon_1 = 0.00250$ in./in., whereas after loading $\varepsilon_2 = 0.00536$ in./in. Determine the change in the force on the strut if the cross-sectional area of the strut is 4.15 in^2. $E_{al} = 10 \times 10^3$ ksi.



✓ 5. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 62 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 34 mm and 18 mm. G_r = 0.24 MPa.



✓ 6. The block is made of titanium Ti-6Al-4V and is subjected to a compression of 0.084 in. along the y-axis and its shape is given a tilit of $\theta = 88.2°$. Determine ε_x, ε_y, and γ_{xy}.

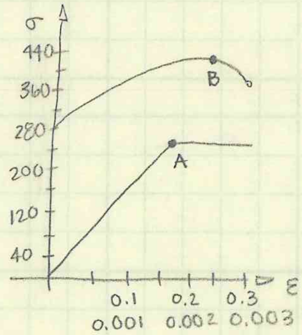


+30/30 Beautiful

Catherine Hovell
February 7, 2003

CE 231: Homework #2

1.



modulus of elasticity:

$$E = \frac{\sigma}{\varepsilon} = \frac{260\text{ MPa}}{0.0015}$$

$\boxed{E = 1.73\times10^{5}\text{ MPa}}$

proportional limit = elastic limit = point A

$\boxed{\sigma_{PL} = \text{elastic limit} = 260\text{ MPa}}$

modulus of resilience

$$u_R = \frac{\frac{1}{2}\sigma_{PL}^2}{E}$$

$$u_R = \frac{1}{2}\frac{(260\text{ MPa})^2}{(1.73\times10^5\text{ MPa})}$$

$\boxed{u_R = 195\text{ kPa}}$

ultimate stress = highest point = point B

$\boxed{\text{ultimate } \sigma = 400\text{ MPa}}$

2.

$\varepsilon_1 = 0.00250$

$\varepsilon_2 = 0.00536$

$A = 4.15\text{ in}^2$

$E_{al} = 10\times10^3\text{ ksi}$

$E\cdot\varepsilon = \sigma \qquad \sigma = \frac{P}{A}$

$P_1 = A\cdot E\cdot \varepsilon_1$

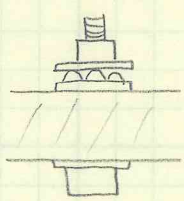
$P_2 = A\cdot E\cdot \varepsilon_2$

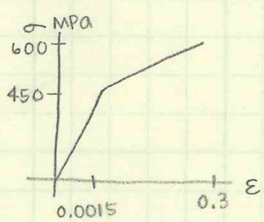
$\Delta P = A\cdot E\,(\varepsilon_2 - \varepsilon_1)$

$\Delta P = 4.15\text{ in}^2\cdot 10\times10^3\text{ ksi}\cdot(0.00536-0.00250)$

$\boxed{\Delta P = 119\text{ kip}}$

3.





$A = 1.85\text{ mm}^2$ each (6x)

$L_o = 4.25\text{ mm}$

$\delta = 0.65\text{ mm}$

$$\varepsilon = \frac{\delta}{L_o} = \frac{0.65\text{ mm}}{4.25\text{ mm}} \qquad \varepsilon = 0.1529$$ ✓

from diagram, at $\varepsilon = 0.15$,
$\sigma \approx 525\text{ MPa}$

a little more detail (e.g. interpolation) would be nice....

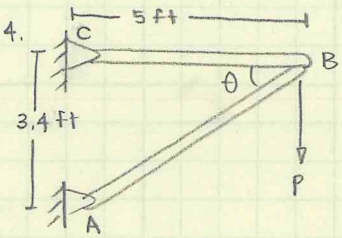
$\sigma = \frac{F}{A} \qquad F = \sigma\cdot A = 525\text{ MPa}\cdot 6\cdot 1.85\text{ mm}^2$

$\boxed{F = 5830\text{ N}}$

+10

CE 231: Homework #2

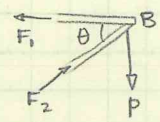
4.



$A_{AB} = 1.85\,in^2$

$A_{BC} = 4.15\,in^2$

$\tan\theta = \frac{3.4}{5} = 0.597$ rad.



$\Sigma F_y = 0 \quad \Sigma F_x = 0$

$P = F_2 \sin\theta$

$F_1 = F_2 \cos\theta$

$\sigma_1 = \frac{F_1}{A_{BC}} \quad \sigma_2 = \frac{F_2}{A_{AB}}$

from graph:

$\sigma_{max\text{-}comp} = 25$ KSI

$\sigma_{max\text{-}tens} = 5$ ksi

$F_1 = 5\,ksi \cdot 4.15\,in^2 \quad F_2 = 25\,ksi \cdot 1.85\,in^2$

$F_1 = 20.8$ kip $\quad F_2 = 46.3$ kip which force limits?

F_1 limits

why do you say this?

$F_2 = 25.09$ kip

$\boxed{\text{a. } P = 14.1 \text{ kip}}$

+10.

$\sigma_{max\,comp} = \frac{F_2}{A'_{AB}}$

$A'_{AB} = \frac{4.5\,kip/\sin\theta}{25\,ksi}$

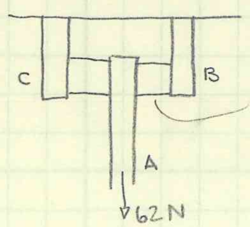
$A'_{AB} = 0.32\,in^2$

$\sigma_{max\,tens} = \frac{F_1}{A'_{BC}} \quad A'_{BC} = \frac{4.5\,kip/\tan\theta}{5\,ksi}$

$A'_{BC} = 1.324\,in^2$

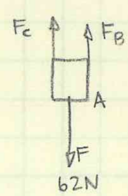
$\boxed{\begin{array}{l}\text{b. } A'_{AB} = 0.32\,in^2 \\ A'_{BC} = 1.32\,in^2\end{array}}$

5.



34 mm × 18 mm × 46 mm

$G_r = 0.24$ MPa



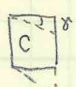
$F_B = F_c = \frac{1}{2}F = 31\,N$

$31\,N = V$

$\tau = \frac{V}{A} = \frac{31\,N}{0.034m \cdot 0.018m}$

$\tau = 50.65\,kPa = G_r \cdot \gamma = 0.24\,MPa \cdot \gamma$

$\gamma = 0.211$

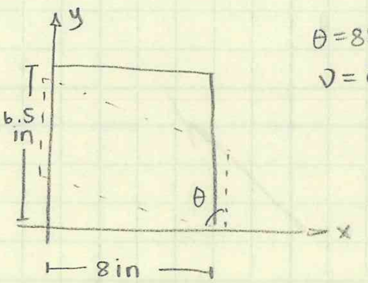


$\gamma = \frac{dy}{w} = \frac{dy}{0.046m}$

$\boxed{dy = 9.71\,mm}$

CE 231: Homework #2

6.



$\theta = 88.2°$ $\quad \delta_y = 0.084 \text{ in}$

$\nu = 0.36$

$$\varepsilon_y = \frac{\delta_y}{L_{y_0}} = \frac{0.084\text{in}}{6.5\text{ in}}$$

$$\varepsilon_y = 0.0129$$

$$\boxed{\begin{array}{l}\varepsilon_x = 0.0047 \\ \varepsilon_y = -0.0129 \\ \gamma_{xy} = 0.031\end{array}}$$

$$\nu = 0.36 = -\frac{\varepsilon_x}{\varepsilon_y}$$

$$\varepsilon_x = -0.36\varepsilon_y$$

$$\varepsilon_x = -0.00465$$

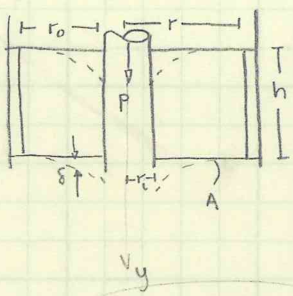
$$\gamma_{xy} = \alpha_{shift}$$

$$\alpha = 90 - \theta = 90 - 88.2°$$

$\gamma_{xy} = 1.8° \longrightarrow$ radians

$$1.8° \cdot 2\pi / 360° = 0.0314$$

7.



$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(P/2\pi hGr\right)$$

assume small γ $\quad \gamma = \tan\gamma$

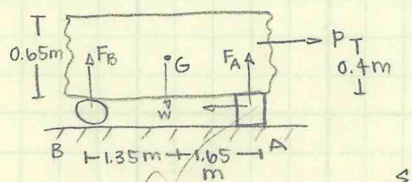
$$\frac{dy}{dr} = \frac{P}{2\pi Ghr} \qquad y = 0 \text{ at } r = r_o$$

$$\int dy = \int \frac{P}{2\pi hG}\cdot\frac{1}{r}\,dr$$

$$y\Big|_0^{y_f} = \frac{P}{2\pi hG}\int_r^{r_o}\frac{1}{r}\,dr = \frac{P}{2\pi hG}\ln\frac{r_o}{r}$$

$$\boxed{\delta = \frac{P}{2\pi hG}\ln\left(\frac{r_o}{r_i}\right)}$$

8.



$m = 845\text{kg}$ $\qquad \nu = \frac{-\varepsilon_y}{\varepsilon_x}$

$\Sigma F_y:\ mg = F_{AY} + F_B \qquad F_{AX} = \mu F_{AY}$

$\Sigma F_x:\ \mu F_{AY} = P$

$\Sigma M_A:\ (3.0\text{m})(F_B) + 0.4\text{m}(P) = mg(1.65\text{m})$

$h = 28\text{mm}$ $\quad \mu = 0.82$

$w = 164\text{ mm}$ $\quad E = 4.2\text{ MPa}$

$l = 178\text{ mm}$ $\quad \nu = 0.375$

$$0.4\text{m}(P) = (845\text{kg})(9.81\text{m/s}^2)(1.65\text{m}) - (3.0\text{m})\left[(845\text{kg})(9.81\text{m/s}^2) - \frac{P}{\mu}\right]$$

$$0.4\text{m}(P) - 3.0\text{m}(P)\left(\frac{1}{0.82}\right) = -11190.8$$

$$P = 3434.3\text{ N} = F_{AX}$$

$$F_{AY} = 4188.2\text{ N} \qquad F_B = 4101.3\text{ N}$$

$$\varepsilon = \frac{\sigma}{E} \qquad \sigma = \frac{F}{A} \qquad G = \frac{E}{2(1+\nu)}$$

$$\tau\ \cancel{\sigma_A} = \frac{F_{AX}}{.164\text{m}\cdot.178\text{m}}$$

$$\tau\ \cancel{\sigma_A} = 117644.9\text{ Pa}$$

$$\gamma = \frac{117644.9\text{ Pa}}{G}$$

$$G = \frac{4.2\text{ MPa}}{2(1+0.375)}$$

$$G = 1.527\text{ MPa}$$

$$\gamma = \frac{117645\text{ Pa}}{1.527\text{ MPa}}$$

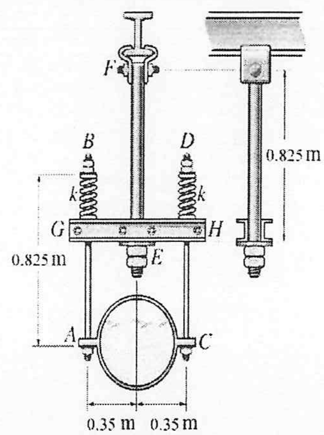
$$\gamma = 0.077 \approx \frac{dx}{0.028\text{m}}$$

$$\boxed{dx = 2.16\text{mm}}$$

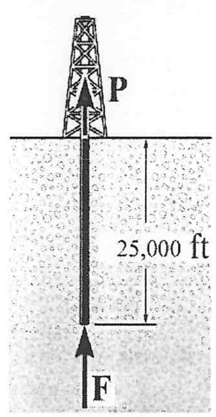
+10

CE 231: Strength of Materials Spring 2003	Homework #3 Due: 2/14/03, 5:00 PM

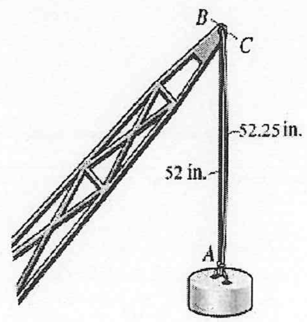
✓ 1. A spring supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 68$ kN/m, three 304 stainless steel rods, *AB* and *CD*, which have a diameter of 5.4 mm and *EF*, which has a diameter of 14 mm, and a rigid beam *GH*. If the pipe and the fluid it carries have a total weight of 4.5 kN, determine the displacement of the pipe when it is attached to the support.



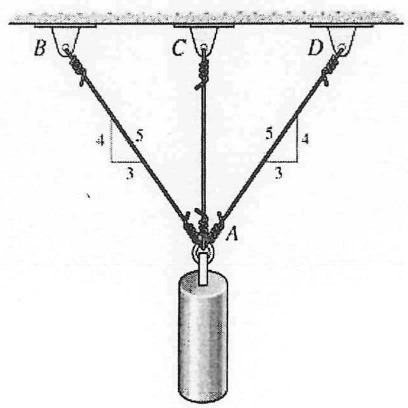
✓ 2. The segments of pipe and couplings used fro drilling an oil well 25,000 ft deep are made of A-36 steel weighing 24 lb/ft. They have an outer diameter of 6.50 in. and an inner diameter of 5.15 in. In order to prevent buckling or sidesway of the pipe due to its own weight, it is partially supported at its tip by the drawworks of the rig. If this force is $P = 319$ kip, determine the force **F** of the ground on the drill pipe and the elongation of the pipe for this condition.



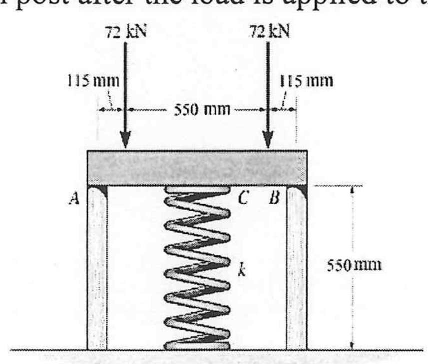
✓ 3. The load of 15 kip is to be supported by the two vertical A-36 steel wires. If, originally, wire AB is 52 in. long, and wire AC is 52.25 in. long, determine the cross-sectional area of AB if the load is to be shared equally between both wires. Wire AC has a cross-sectional area of 0.024 in^2.



✓ 4. The A-36 steel wires AB and AD each have a diameter of 2.3 mm and the unloaded lengths of each wire are $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.15$ m. Determine the required diameter of wire AC so that each wire is subjected to the same force caused by the 160-kg mass suspended from the ring at A.

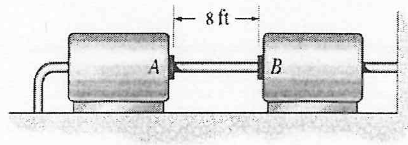


✓ 5. The rigid bar is supported by the two short wooden posts and a spring. If each of the posts has an unloaded length of 550 mm and a cross-sectional area of 830 mm^2, and the spring has a stiffness of $k = 1.6$ MN/m and an unstretched length of 580 mm, determine the force in each post after the load is applied to the bar. $E_w = 12$ GPa.

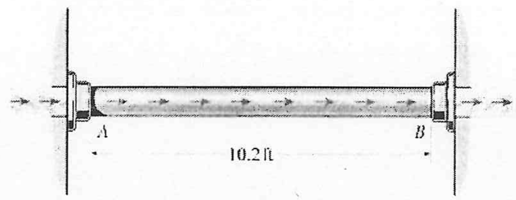


8ft

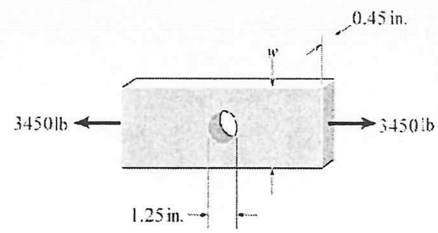
✓6. A ~~7~~-ft-long steam pipe is made of A-36 steel and is connected directly to two turbines A and B as shown. The pipe has an outer diameter of 5 in. and a wall thickness of 0.475 in. The connection was made at $T_1 = 85°F$. If the turbines' points of attachment are assumed to have a stiffness of $k = 90 \times 10^3$ kip/in., determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 260°F$.



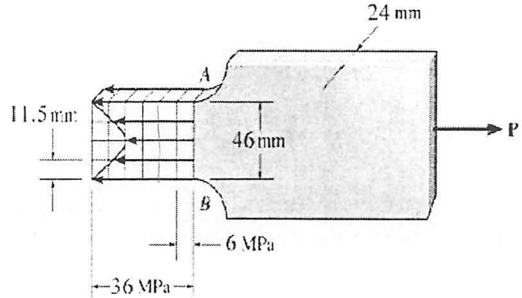
✓ 7. The bronze 86100 pipe has an inner radius of 0.65 in. and a wall thickness of 0.315 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 220°F$ at A to $T_B = 70°F$ at **B**, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 70°F$.



✓ 8. The member is to be made from a steel plate that is 0.45 in. thick. If a 1.25-in. hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 3450 lb. The allowable stress is $\sigma_{allow} = 26$ ksi.



✓ 9. The resulting stress distribution along the section AB of the bar is shown in the figure. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?

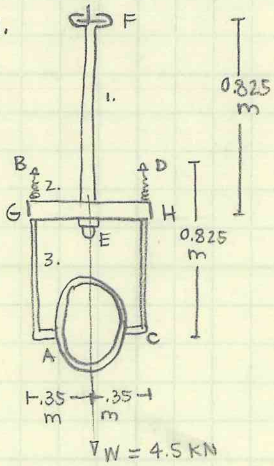


28/30

Catherine Hovell
February 14, 2003

CE 231: Homework #3

1.



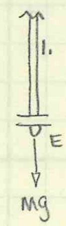
$k = 68 kN/m$

FE: $d = 14mm$

AB, CD: $d = 5.4mm$

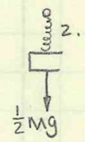
304 stainless steel: $E = 193 GPa$

$\delta_{tot} = \delta_1 + \delta_2 + \delta_3$



$\delta = \frac{F \cdot L_o}{E \cdot A}$ $\delta_1 = \frac{Mg \cdot 0.825m}{193 GPa \cdot \pi (7mm)^2}$

$\delta_1 = 0.125 mm$



$F = kx$

$x = \delta_2 = \frac{\frac{1}{2}Mg}{68kN/m}$

$\delta_2 = 33.09 mm$

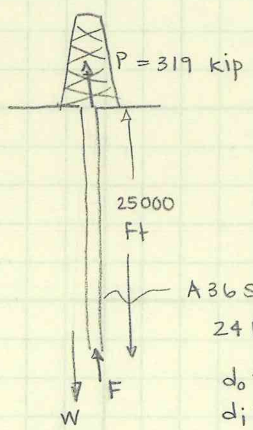
$\delta_{tot} = 0.125mm + 33.09 mm + 0.105 mm$

$\boxed{\delta_{tot} = 33.3 mm}$

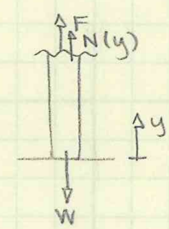
$\delta_3 = \frac{\frac{1}{2}mg \cdot 0.825 m}{193 GPa \cdot \pi (5.4mm)^2}$

$\delta_3 = 0.105 mm$

2.



$W_{pipe} = (25000 ft)(24 lb/ft) = 600 kip$



A36 steel $E = 29 \times 10^3 ksi$

$24 lb/ft \rightarrow 0.024 kip/ft$

$d_o = 6.50 in$

$d_i = 5.15 in$

$A = 12.35 in^2$

$\int N(y) = \int 0.024 y \, ksi - 281 kip$

$N \cdot L = 0.012y^2 - 281y \Big|_0^{25000}$

$N \cdot L = 475{,}000 \, kip \cdot ft$

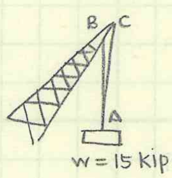
$\delta = \frac{N \cdot L}{EA} = \frac{475000 \, kip \cdot ft}{29 \times 10^3 ksi \cdot 12.4 in^2}$

$\boxed{\delta = 1.33 in}$

$W = P + F$

$F = 600 kip - 319 kip = 281 kip$

3.

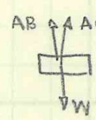


A-36 steel $E = 29 \times 10^3 ksi$

AB: 52 in

AC: 52.25 in

$A_{AC} = 0.024 in^2$



$W = F_{AB} + F_{AC}$

$F_{AB} = F_{AC} = \frac{1}{2}W$

$\delta_{AB} = 0.813 in = \frac{F_{AB} \cdot L_o}{E_{AB} \cdot A_{AB}}$

$\frac{1}{A_{AB}} = \frac{0.813 in \cdot 29 \times 10^3 ksi}{\frac{1}{2}(15 kip)(52.0 in)}$

$\boxed{A_{AB} = 0.017 in^2}$

$\delta_{AB} = \delta_{AC} - 0.25$

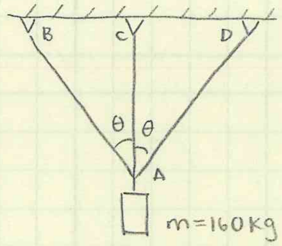
$\delta_{AC} = \frac{F_{AC} L_o}{E_{AC} \cdot A_{AC}} = \frac{\frac{1}{2}(15 kip)(52.25 in)}{29 \times 10^3 ksi \cdot 0.024 in^2}$

$\delta_{AC} = 0.563 in$

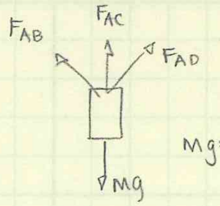
+10

CE 231: Homework #3

4.



$m = 160\,kg$



$\cos\theta = 4/5$

A-36 steel $E = 200\,GPa$

$AB, AD: \; d = 2.3\,mm$

$L_{AC} = 1.60\,m \quad L_{AB} = L_{AD} = 2.15\,m$

$\delta_{AB} = \delta_{AC} = \delta_{AD}$

$\delta_{AB} = \frac{FL}{EA}\cos\theta$

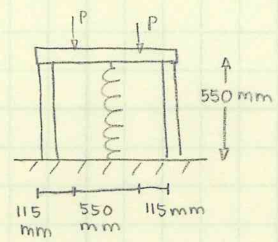
$$\frac{4}{5}\frac{\cancel{F}\cdot L_{AB}}{\cancel{E}\cdot A_{AB}} = \frac{\cancel{F} L_{AC}}{\cancel{E}\cdot A_{AC}}$$

$$\frac{4}{5}\frac{(2.15\,m)}{\pi/4\,(2.3\,mm)^2} = \frac{1.60\,m}{\pi/4\,(d_{AC})^2}$$

$Mg = F_{AC} + \frac{4}{5}(F_{AD} + F_{AB})$

$\boxed{d_{AC} = 2.22\,mm}$

5.



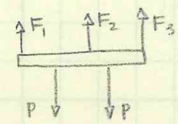
$A = 830\,mm^2$

$K = 1.6\,MN/m$

$x_o = 580\,mm$

$E_w = 12\,GPa$

$P = 72\,kN$



$2P = 2F_1 + F_2$

$F_1 = F_3$

$$\delta_1 = \delta_3 = \frac{F\cdot L_o}{E_w\cdot A} \qquad F_2 = k(0.030\,m + \delta_2) \qquad \delta_1 = \delta_2 = \delta_3$$

$$\frac{2\delta E_w A}{L_o} + k(0.030\,m + \delta) = 2P$$

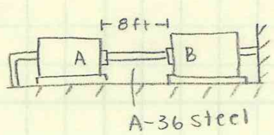
$$\delta\left(\frac{2E_wA}{L_o} + k\right) = 2P - k(0.030\,m)$$

$\delta = 0.0025\,m$

$$\boxed{\begin{aligned} F_1 = F_3 &= 46.0\,kN \\ F_2 &= 52.1\,kN \end{aligned}}$$

+10

6.



$d_o = 5\,in$

$d_i = 4.525\,in$

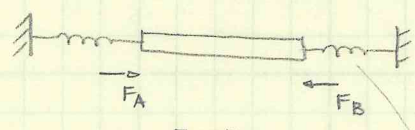
$A = 3.55\,in^2$

$T_o = 85°F$

$T = 260°F$

$k = 90\times10^3\,kip/in$

$\alpha = 6.60\times10^{-6}/°F$



$F_A = F_B$

$F_s = \frac{k\Delta L}{2} = \sigma A$

$$\varepsilon_{tot} = \alpha\Delta T + \frac{\sigma}{E} = \frac{\Delta L}{L_o}$$

$$\Delta L = \left(\alpha\Delta T + \frac{\sigma}{E}\right)\cdot L_o$$

$$\sigma A = \frac{kL_o}{2}\left(\alpha\Delta T + \frac{\sigma}{E}\right)$$

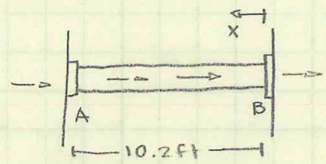
$$\sigma\left(A - \frac{kL_o}{2E}\right) = \frac{kL_o}{2}\alpha\Delta T$$

$\boxed{F = 121.4\,kip \text{ compression}}$

$$F = \frac{\frac{(90\times10^3\,kip/in)\cdot(8\,ft)}{2}\cdot(6.60\times10^{-6}/°F)(260°F - 85°F)}{3.55\,in^2 - \frac{(90\times10^3\,kip/in)(8\,ft)}{2(29\times10^3\,ksi)}}(3.55\,in^2)$$

CE 231: Homework #3

7.



$T_A = 220°F$ $T_B = 70°F$

bronze 86100

$r_i = 0.65$ in

$r_o = 0.965$ in

$A = 1.598 \text{ in}^2$

$\alpha = 9.60 \times 10^{-6}/°F$

$E = 15.0 \times 10^3$ ksi

$$T(x) = 70°F + \frac{(220°F - 70°F)}{2} x \qquad \Delta T(x) = 14.7x$$

$$\varepsilon_{tot}(x) = \varepsilon_{th}(x) + \frac{\sigma(x)}{E}$$

$$\delta(x) = \int \varepsilon_{tot}(x)$$

$$\varepsilon_{th}(x) = \alpha \Delta T$$

$F_A = F_B$

$$\sigma = \frac{F}{A} = E\varepsilon = \frac{E\delta}{L_o}$$

$$\delta_{th} = \int \alpha \Delta T dx = \alpha \frac{14.7}{2} x^2 \Big|_0^{10.2}$$

$$\delta_{th} = 0.00734 \text{ ft} = \frac{FL_o}{EA}$$

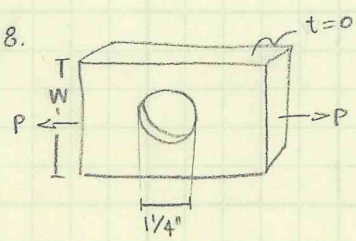
+8

$$F = \frac{0.00734 \text{ in} \cdot 15.0 \times 10^3 \text{ ksi} \cdot 1.598 \text{ in}^2}{10.2 \text{ ft}}$$

$\boxed{F = 1438 \text{ lb} \cdot \text{in}}$

$\boxed{F = 120\ \boxed{\text{lb} \cdot \text{ft}}}$ for force?

8.



r = 0.625 in

P = 3450 lb

$\sigma_{allow} = 26$ ksi

$$\sigma_y = K\left(\frac{r}{w}\right) \frac{P}{tw} \left(\frac{1}{1 - 2r/w}\right)$$

guess K:

w' = 2 in

NO. K = 2.23

$\sigma = 22.8$

w' = 1.9 in

EH... K = 2.3

$\sigma = 27.1$

w' = 1.95 in

K = 2.35

$\sigma = 25.7$

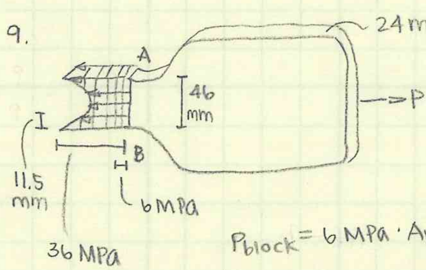
$$26 \text{ ksi} = K \frac{3450 \text{ lb}}{(0.45 \text{ in})w - 2(0.625 \text{ in})(0.45 \text{ in})}$$

$$0.45 \text{ in} \cdot w = \frac{3450 \text{ lb} \cdot 2.35}{26 \text{ ksi}} + 2(0.625 \text{ in})(0.45 \text{ in})$$

w = 1.943 in

$\boxed{w = 1.94 \text{ in}}$

9.



$P_{block} = 6 \text{ MPa} \cdot A_{block}$

$P = 20 \cdot 6 \text{ MPa} \cdot (0.0115 \text{ m})(0.024 \text{ m})$

P = 33120 N

$\boxed{P = 33.1 \text{ kN}}$

$$K = \frac{\sigma_{max}}{\sigma_{Avg.}}$$

$$\sigma_{Avg} = \frac{P}{A} \qquad P = ?$$

A = 46 mm · 24 mm

= 0.001104 m²

σ_{max} (from picture) = 36 MPa

$$K = \frac{36 \text{ MPa}}{P/0.001104 \text{ m}^2}$$

$$K = \frac{36 \times 10^6 \text{ Pa}}{33120 \text{ N}/0.001104 \text{ m}^2}$$

$\boxed{K = 1.2}$

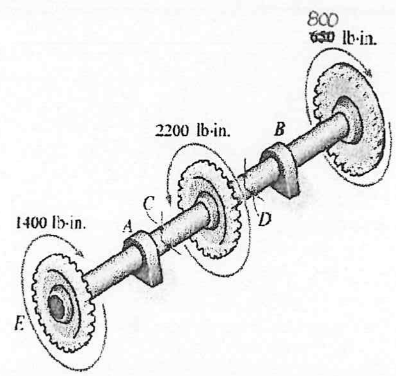
Just kidding!! ~~#10? - It's graded - get it to ME (not Prof. Begley) by~~

~~Wed. @ noon & I'll give you credit. - Only 6/c !~~

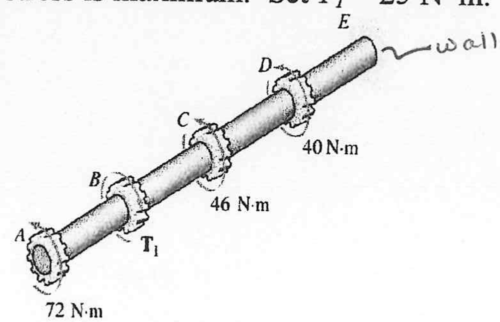
~~[illegible] it's an HONEST mistake! - Kyle~~

CE 231: Strength of Materials Spring 2003	Homework #4 Due: 2/28/03, 5:00 PM

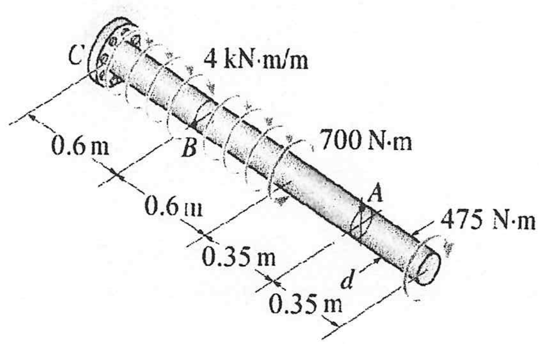
1. The shaft has an outer diameter of 1.50 in. and an inner diameter of 1.15 in. If it is subjected to the applied torques as shown, plot the shear-stress distribution acting along a radial line lying within region *EA* of the shaft. The smooth bearings at *A* and *B* do not resist torque.



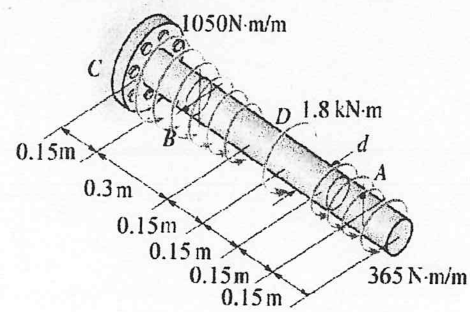
2. The solid aluminum shaft has a diameter of 60 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear stress distribution along a radial line of the shaft where the shear stress is maximum. Set $T_1 = 25$ N*m.



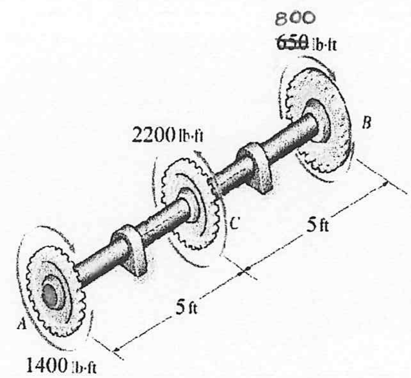
3. The 55-mm diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses in the shaft and specify their locations, measured from the fixed end.



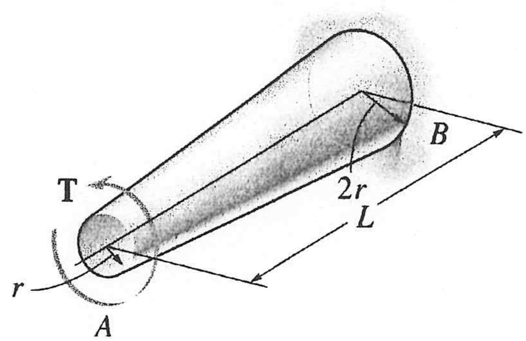
✓ 4. The solid shaft is subjected to the distributed and torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points. The distributed torque from D to C varies from zero to 1050 N*m/m. Take $d = 45$ mm.



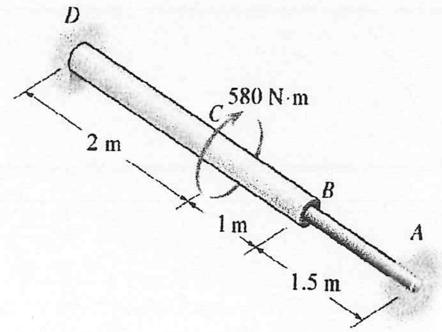
✓ 5. The gears attached to the 304 stainless steel shaft are subjected to the torques shown. Determine the angle of twist of gear C with respect to gear B. The shaft has a diameter of 1.8 in.



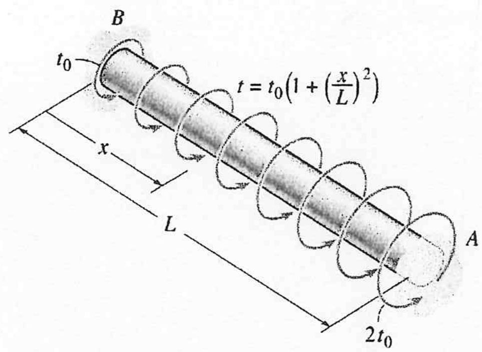
✓ 6. The tapered shaft has a length L and a radius r at end A and $2r$ at end B. If it is fixed at end B and is subjected to a torque T, determine the angle of twist at end A. The shear modulus is G.



✓ 7. A rod is made from two segments: *AB* is A-36 steel and has a diameter of 34 mm and *BD* is C83400 red brass and has a diameter of 56 mm. It is fixed at its ends and subjected to a torque of $T = 580$ N*m. Determine the absolute maximum shear stress in the shaft.



8. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.

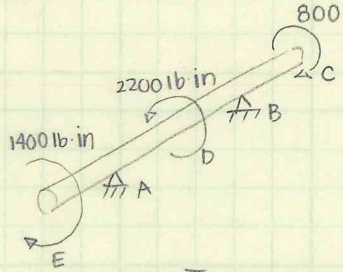


28/30

Catherine Howell
February 28, 2003

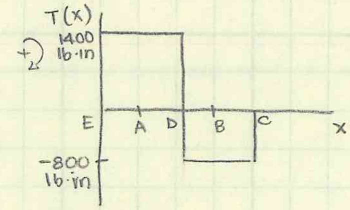
CE 231: Homework # 4

1.



$d_o = 1.50\text{ in}$ $r_o = 0.75\text{ in}$

$d_i = 1.15\text{ in}$ $r_i = 0.575\text{ in}$



$$\tau_{max_{AE}} = \frac{T_{AE} \cdot r_{AE}}{J}$$

$$J = \frac{\pi}{2}(r_o^4 - r_i^4)$$

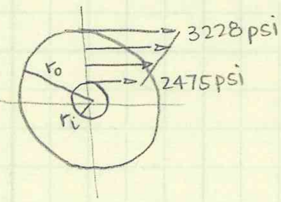
$$J = \frac{\pi}{2}\left[(0.75\text{ in})^4 - (0.575\text{ in})^4\right]$$

$$J = 0.3253\text{ in}^4$$

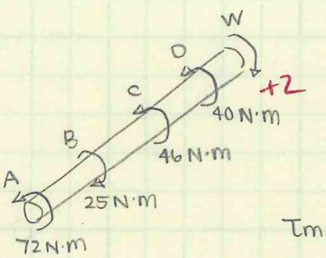
$$\tau_{max_{AE_o}} = \frac{(1400\text{ lb·in})(0.75\text{ in})}{0.3253\text{ in}^4} = 3228\text{ psi}$$

$$\tau_{max_{AE_i}} = \frac{(1400\text{ lb·in})(0.575\text{ in})}{0.3253\text{ in}^4} = 2475\text{ psi}$$

along AE:



2.

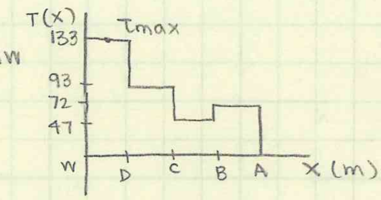


+2

$\Sigma M = 0$ (+)

$72\text{ N·m} - 25\text{ N·m} + 46\text{ N·m} + 40\text{ N·m} = M_W$

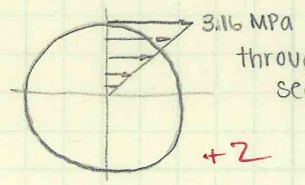
$M_W = 133\text{ N·m}$



$$\tau_{max} = \frac{T_{DW} \cdot r}{J}$$ $J = \frac{\pi}{2} r^4$ +2 $d = 0.06\text{m}$

$$\tau_{max} = \frac{(133\text{ N·m})(0.03\text{ m})}{\pi/2\,(0.03\text{m})^4}$$ +2

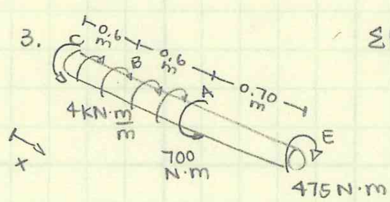
$\boxed{\tau_{max} = 3.16\text{ MPa}}$ +2



through section WD

+2 +10

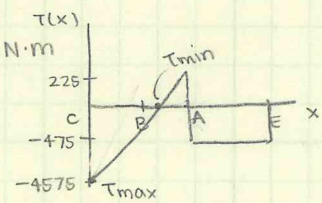
3.



$\Sigma M = 0$ (+)

$M_C = (4\text{kN·m/m})(1.2\text{ m}) - 700\text{ N·m} + 475\text{ N·m}$

$M_C = 4575\text{ N·m}$



$$\tau_{max} = \frac{T_c \cdot r}{J}$$ $r = \frac{1}{2}(0.055)$

$r = 0.0275$

$\boxed{\tau_{max} = 140\text{ MPa @ } x = 0}$

$\tau_{min} = 0$

$4575\text{ N/m} = (4000\text{ N/m})\,x$

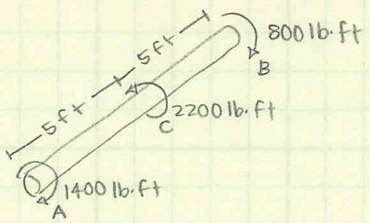
$x = 1.144\text{ m}$ $\boxed{\tau_{min} = 0 \text{ @ } x = 1.14\text{ m}}$

CE 231: Homework #4

4. HAHAHAHA. no.

KYLE MANER IS THE BEST TA EVER. (PS- good thing I'm grading this & not MRB - BOTH our butts would be kicked!!) But thanks! ☺

5.



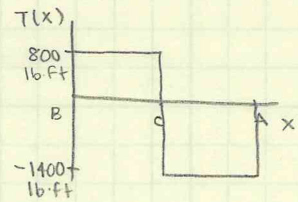
304 stainless steel

$G = 11 \times 10^6\ lb/in^2$

$d = 1.8\,in \quad r = 0.9\,in$

$$\phi_{BC} = \frac{T_{BC} \cdot L}{J \cdot G} \qquad J = \frac{\pi}{2}(0.9\,in)^4$$

$$J = 1.031\,in^4$$

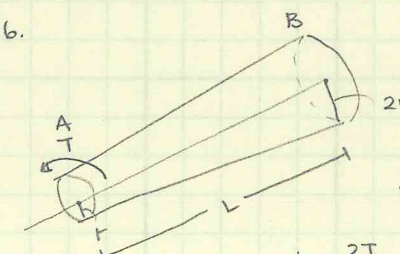


$$\phi_{BC} = \frac{(800\,lb\cdot ft)(5\,ft)(^{12in}/_{ft})^2}{(1.031\,in^4)(11 \times 10^6\,psi)}$$

$$\phi_{BC} = 0.0508\ rad$$

$$\frac{(\phi_{BC})(360°)}{2\pi\ rad} = 2.91° \qquad \boxed{\phi_{BC} = 2.91°}$$

6.



$$\Sigma M = 0$$

$$T = M_B$$

$$\phi = \int \frac{T(x)}{J(x) \cdot G}dx \qquad \phi = \frac{T}{G}\int \frac{dx}{J(x)}$$

+3

$$J(x) = \frac{\pi}{2}\left(2r - \frac{x}{L}r\right)^4$$

+3

$$\phi = \frac{2T}{\pi G} \cdot \frac{1}{r^4}\int \left(2 - \frac{x}{L}\right)^{-4} dx \qquad \phi = \frac{2T}{\pi G \cdot r^4}\left[\frac{-1}{3}\left(2 - \frac{x}{L}\right)^{-3} \cdot \frac{1}{-1/L}\Big|_0^L\right]$$

$$\phi = \frac{2T}{\pi G r^4} \cdot \frac{L}{3}\left[\left(2 - x/L\right)^{-3}\Big|_0^L\right]$$

+2

$$\to \frac{1}{(2-1)^3} - \frac{1}{(2)^3} = \frac{1}{1} - \frac{1}{8} = \frac{7}{8}$$

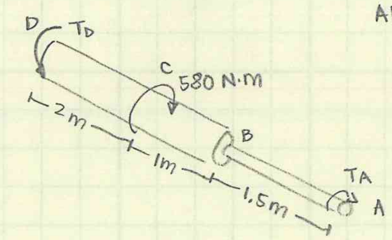
$$\phi = \frac{2TL}{\pi G r^4} \cdot \frac{7}{8} \cdot \frac{1}{3} \qquad \boxed{\phi = \frac{7TL}{12\pi G r^4}}$$ → Thanks to me! HA!

+2

+10

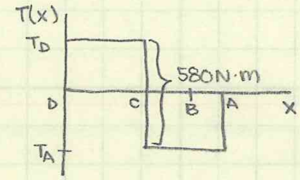
CE 231: Homework #4

7.



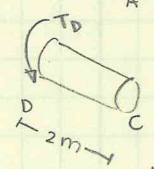
AB: $d = 0.034\,m$
$r_{AB} = 0.017\,m$
$G_{AB} = 75\,GPa$

BD: $d = 0.056\,m$
$r_{BD} = 0.028\,m$
$G_{BD} = 37\,GPa$

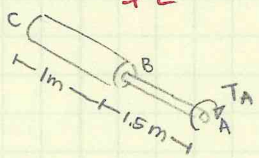


$\phi_{CD} = \phi_{CA}$

$T_A + 580\,N\cdot m = T_D$



+2



$$\frac{T_D(2\,m)}{\frac{\pi}{2}(0.028\,m)^4(37\,GPa)} = \frac{T_A(1.5\,m)}{\frac{\pi}{2}(0.017\,m)^4(75\,GPa)} + \frac{T_A(1\,m)}{\frac{\pi}{2}(0.028\,m)^4(37\,GPa)}$$ +2

$$(5.599\times10^{-5}\,{}^1/_{N\cdot m})T_D = (1.524\times10^{-4}\,{}^1/_{N\cdot m})T_A + (2.799\times10^{-5}\,{}^1/_{N\cdot m})T_A$$

$T_D = 3.22\,T_A$ +2

$580\,N\cdot m = T_A(3.22-1)$ $T_A = 260.9\,N\cdot m$

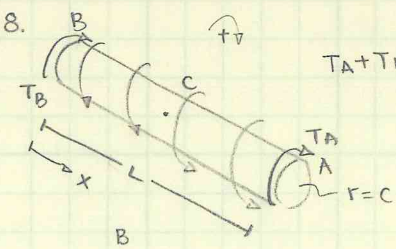
$\underline{T_D = 840.9\,N\cdot m}$

$$\tau_{max} = \frac{T\cdot r}{J}$$ +2

$$\tau_{max_{AC}} = \frac{(260.9\,N/m)(0.017\,m)}{\pi/2\,(0.017\,m)^4}$$ $\boxed{\tau_{max_{AB}} = 33.8\,MPa}$ +8

$$\tau_{max_{CD}} = \frac{(840.9\,N\cdot m)(0.028\,m)}{\frac{\pi}{2}(0.028\,m)^4}$$ $\tau_{max_{CD}} = 24.39\,MPa$

8.



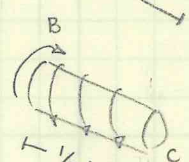
$T_A + T_B = \int t_o(1+(\frac{x}{L})^2)$ $\phi_A = \phi_B = 0$

$\rightarrow t_o\int 1+(\frac{x}{L})^2 dx$

$t_o(x + \frac{x^3}{3L^2}\Big|_0^L) = t_o\frac{4}{3}L$

$T_A + T_B = \frac{4}{3}t_oL$

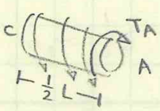
$T_B < T_A$



$\phi_c = \frac{1}{JG}\int T(x)\,dx$

$\int T(x) = T_B - t_o(x + \frac{x^3}{3L^2}\Big|_0^{\frac{1}{2}L})$

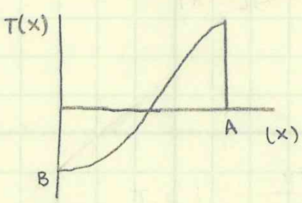
$\phi_c = \frac{1}{JG}(T_B - t_o(^{13}/_{24}L))$



$\phi_c = \frac{1}{JG}\int T(x)\,dx$

$\int T(x)\,dx = T_A - t_o(x + \frac{x^3}{3L^2}\Big|_{\frac{1}{2}L}^{L})$

$\phi_c = \frac{1}{JG}\left[T_A - t_o(\frac{4}{3}L - {}^{13}/_{24}L)\right]$



$T_B - t_oL\,^{13}/_{24} = T_A - t_oL\,^{19}/_{24}$

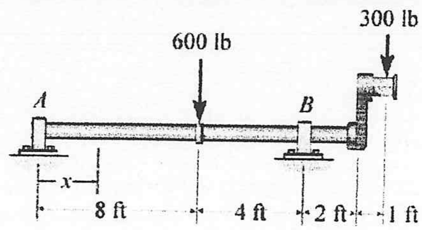
$T_A - T_B = {}^1/_4\,t_oL$ $T_A + T_B = {}^4/_3\,t_oL$

$({}^1/_4\,t_oL + T_B) + T_B = {}^4/_3\,t_oL$

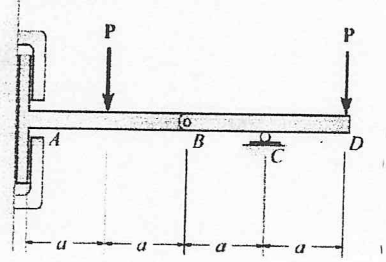
$$\boxed{\begin{aligned} T_B &= \frac{13}{24}t_oL \\ T_A &= \frac{19}{24}t_oL \end{aligned}}$$

CE 231: Strength of Materials Spring 2003	Homework #5 Due: 3/14/03, 5:00 PM

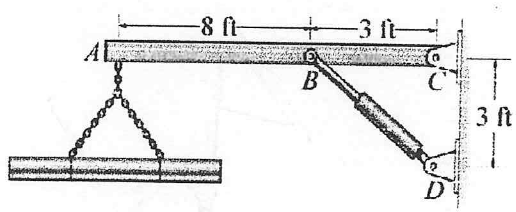
✓ 1. Draw the shear and moment diagrams for the shaft and determine the shear and moment throughout the shaft as a function of x. The bearings at A and B exert only vertical reactions on the shaft.



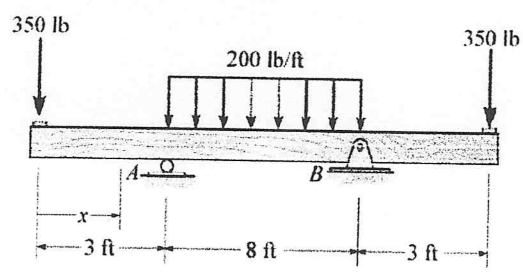
✓ 2. Draw the shear and moment diagrams for the compound beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



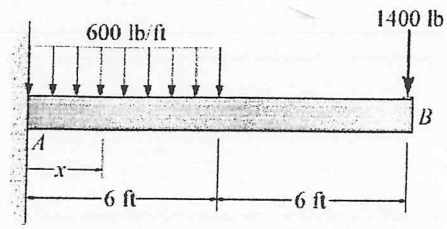
✓ 3. The boom ABC has a weight of 40 lb/ft and is used to lift the load of 2200 lb. Draw the shear and moment diagrams of the boom when it is in the horizontal position shown.



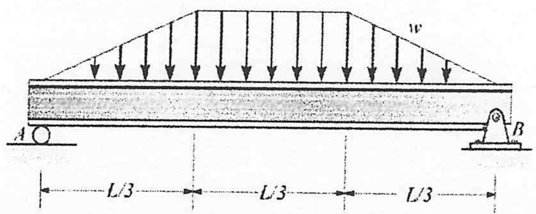
✓ 4. Draw the shear and moment diagrams for the wood beam, and determine the shear and moment throughout the beam as functions of x.



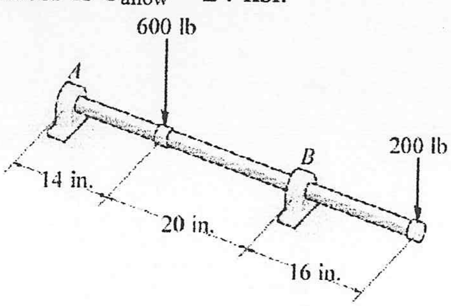
✓ 5. If the beam has a square cross section of 8 in. on each side, determine the absolute maximum bending stress in the beam.



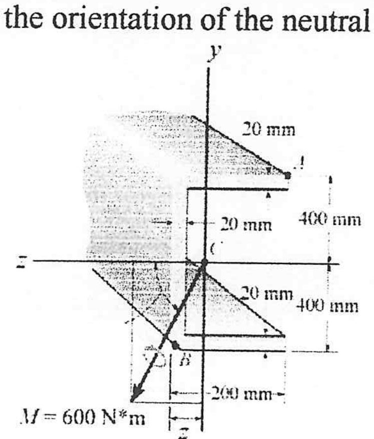
✓ 6. If the beam has a rectangular cross section with a width b and a height h, determine the absolute maximum bending stress in the beam.



✓ 7. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{allow} = 24$ ksi.



8. If the internal moment acting on the cross section of the strut has a magnitude of $M = 600$ N*m and is directed as shown, determine the bending stress at points A and B. The location $\bar{z}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.

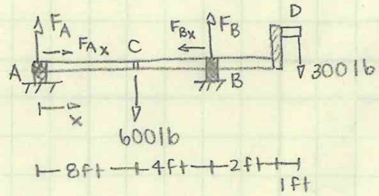


30/30

Catherine Horell
March 14, 2003

CE 231: Homework #5

1.

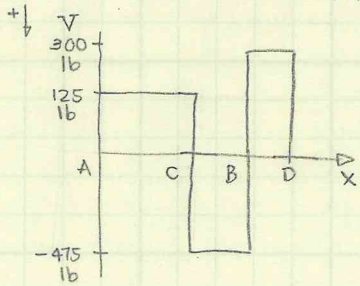


$\Sigma F_x: \quad F_A + F_B = 900\,lb$

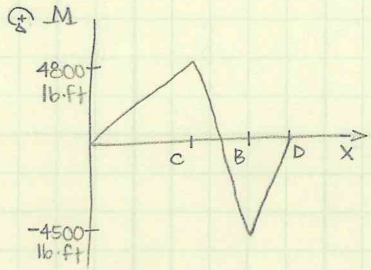
$\Sigma M_A: \quad (600\,lb)(8\,ft) + (300\,lb)(15\,ft) = F_B\,(12\,ft)$

$F_B = 775\,lb$

$F_A = 125\,lb$



$0\,ft < x < 8\,ft$:
$V = 125\,lb$
$8\,ft < x < 12\,ft$:
$V = -475\,lb$
$12\,ft < x < 15\,ft$:
$V = 300\,lb$



$$M = \frac{4800\,lb\cdot ft}{8\,ft}\cdot x$$

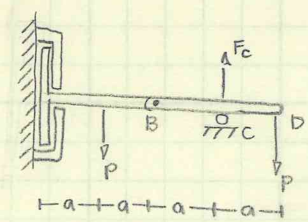
$\rightarrow 600$

$$M = \frac{(-4500-4800)\,lb\cdot ft}{4\,ft}\cdot x + 23100\,lb\cdot ft$$

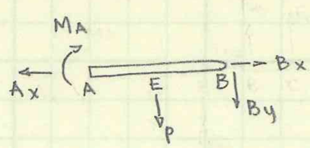
$$M = \frac{(4500\,lb\cdot ft)}{3\,ft}\cdot x - 22500\,lb\cdot ft$$

$0\,ft < x < 8\,ft$:
$M = (600\,lb)\cdot x$
$8\,ft < x < 12\,ft$:
$M = (23100\,lb\cdot ft) - (2325\,lb)\cdot x$
$12\,ft < x < 15\,ft$:
$M = (1500\,lb)\cdot x - (22500\,lb\cdot ft)$

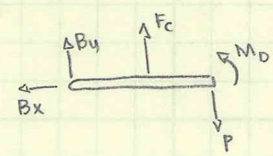
2.

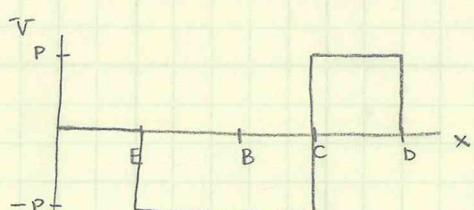


$\Sigma F_y = 0 \quad 2P = F_c$



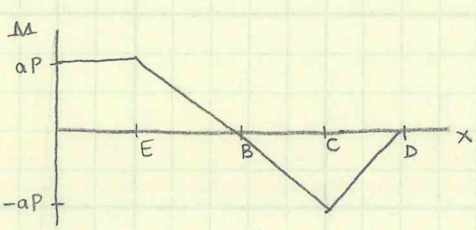
$-B_y = P$
$B_y = P - F_c$





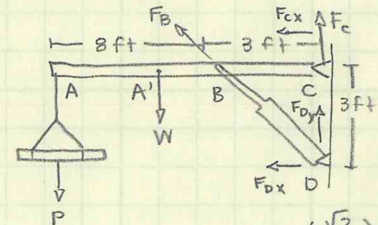
$\Sigma M_A: \quad 3\cdot a\cdot F_c - P\cdot a - 4\cdot a\cdot P = M_A$

$M_A = a\cdot P$



CE 231: Homework #5

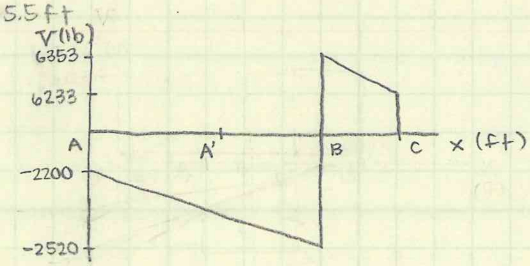
3.



$W = 40\ \text{lb/ft} \cdot 11\ \text{ft}$

$W = 440\ \text{lb}$ at $x = 5.5\ \text{ft}$

$P = 2200\ \text{lb}$

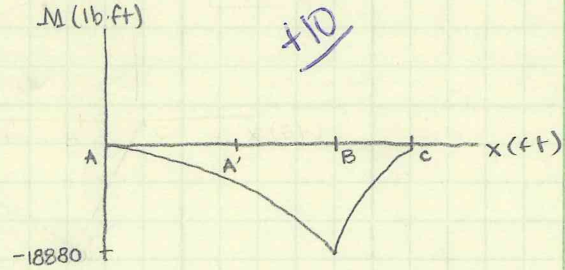


$P + W = F_c + F_B\left(\frac{\sqrt{2}}{2}\right) = 2640\ \text{lb}$

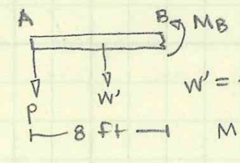
M_C: $(P)(11\ \text{ft}) + (5.5\ \text{ft})(W) = F_B(3\ \text{ft})\frac{\sqrt{2}}{2}$

$F_B = 12549\ \text{lb}$

$F_c = -6233\ \text{lb}$



+10

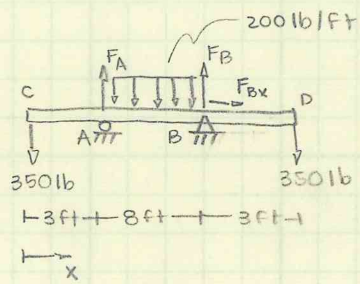


$W' = 40\ \text{lb/ft}\ (8\ \text{ft}) = 320\ \text{lb}$

$M_B = W'(4\ \text{ft}) + P(8\ \text{ft})$

$M_B = 18880\ \text{lb·ft}$

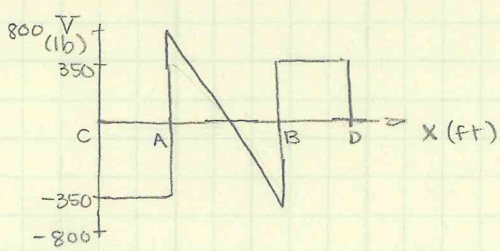
4.



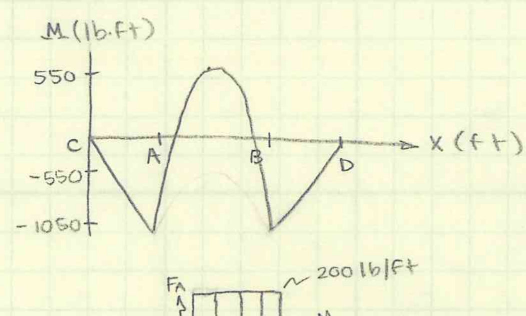
$F_A = F_B$ (symmetry)

$F_A + F_B = 700\ \text{lb} + (200\ \text{lb/ft})(8\ \text{ft}) = 2300\ \text{lb}$

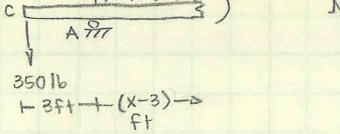
$F_A = F_B = 1150\ \text{lb}$



$0\ \text{ft} < x < 3\ \text{ft}$: $V = -350\ \text{lb}$	$11\ \text{ft} < x < 14\ \text{ft}$: $V = 350\ \text{lb}$
$3\ \text{ft} < x < 11\ \text{ft}$: $V = 612.5\ \text{lb} - 87.5\frac{\text{lb}}{\text{ft}} \cdot x$	



$0\ \text{ft} < x < 3\ \text{ft}$: $M = -350 \cdot x\ \text{lb·ft}$	$11\ \text{ft} < x < 14\ \text{ft}$: $M = 350 \cdot x\ \text{lb·ft} - 4900\ \text{lb·ft}$
$3\ \text{ft} < x < 11\ \text{ft}$: $M = -100x^2 + 1400x - 4350\ \text{ft·lb}$	



$M = (x-3)(1150\ \text{lb}) - 350\ \text{lb}(x\ \text{ft}) - (200\ \text{lb/ft})(x - 3\ \text{ft})(x - 3\ \text{ft})(\tfrac{1}{2})$

$- (100x - 300)(x - 3\ \text{ft})$

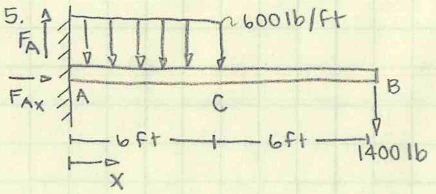
$- (100x^2 - 600x + 900\ \text{ft})$

$M = 1150\ \text{lb} \cdot x - 3450\ \text{lb} - 350\ \text{lb} \cdot x - 100x^2 + 600 \cdot x - 900\ \text{ft}$

$M = -100x^2 + 1400x - 4350$

CE 231: Homework #5

5.

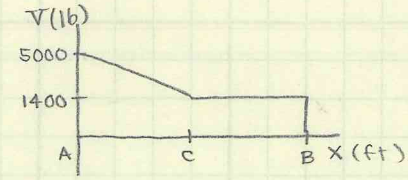


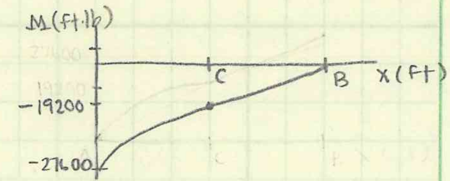
ΣF_y: $1400\ \text{lb} + (600\ \text{lb/ft})(6\ \text{ft}) = F_A$

$F_A = 5000\ \text{lb}$

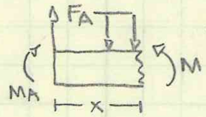
$V_c = 5000\ \text{lb} - 3600\ \text{lb}$

$= 1400\ \text{lb}$





$A = (8\ \text{in})(8\ \text{in})$



$x < 6\ \text{ft}$

$M_A = 0$

$M = F_A \cdot x - (600\ \text{lb/ft})(x)(\tfrac{1}{2}x)$

$M = 5000x - 300x^2$ $\quad M_C = 19200\ \text{ft·lb}$

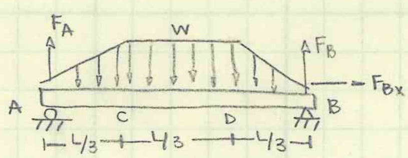
$M_B = (1400\ \text{lb})(12\ \text{ft}) + \tfrac{1}{2}(6\ \text{ft})(5000\ \text{lb} - 1400\ \text{lb})$

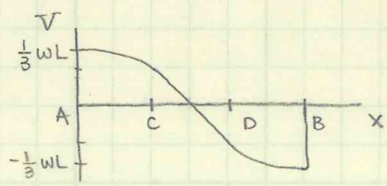
$M_B = 27600\ \text{ft·lb}$ $\quad I = \frac{1}{12} b \cdot h^3$

$$\sigma_{max} = \frac{M_{max} \cdot c}{I} = \frac{(27600\ \text{ft·lb})(12\ \text{in/ft})(4\ \text{in})}{1/12\,(8\ \text{in})(8\ \text{in})^3}$$

$\boxed{\sigma_{MAX} = 3881\ \text{psi}}$

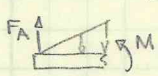
6.





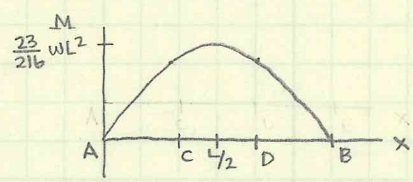
$F_A = F_B = \frac{1}{2}\left[w(L/3) + 2 \cdot \frac{1}{2}(w)(L/3) \right] = \frac{1}{3} \cdot w \cdot L$

M: $0 < x < L/3$

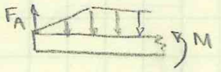


$M = F_A \cdot x - \left(\frac{1}{2} \cdot \frac{x}{L/3} \cdot w \cdot x \right)\left(\frac{1}{3}x \right)$

$M = \frac{1}{3} \cdot w \cdot L \cdot x - \frac{1}{2}\frac{w}{L}x^3$ $\quad M_c = \frac{w}{9}\left(L^2 - \frac{1}{2L^2}\right)$



M: $L/3 < x < 2L/3$



x5

$M = F_A \cdot x - \frac{1}{2} \cdot \frac{L}{3} \cdot w \cdot \left(x - \frac{2L}{9}\right) - w(x - L/3)(1/2)(x - L/3)$

$M = \frac{1}{3} \cdot w \cdot L \cdot x - \frac{L \cdot w}{6}\left(x - \frac{2L}{9}\right) - \frac{w}{2}(x - L/3)^2$ $\quad M_M = 23/216\ wL^2$

$M_D = 5/54\ wL^2$

M_{max} occurs at $x = L/2$ $\quad M = 23/216\ wL^2$

$\sigma_{max} = \frac{M_{max} \cdot c}{I}$ $\quad \sigma_{max} = \frac{23/216\ wL^2 \cdot \frac{1}{2}h}{\frac{1}{12}bh^3} = 23/36 \cdot wL^2 \cdot \frac{1}{bh^2}$

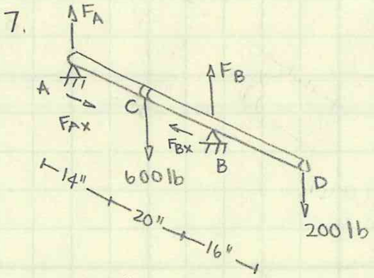
+3

$\boxed{\sigma_{MAX} = \frac{23}{36}\frac{wL^2}{bh^2}}$ +2

+10

CE 231: Homework #5

7.



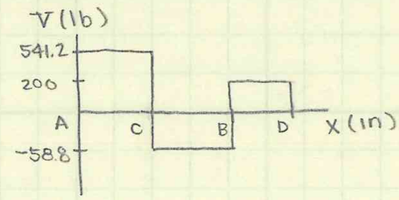
+4

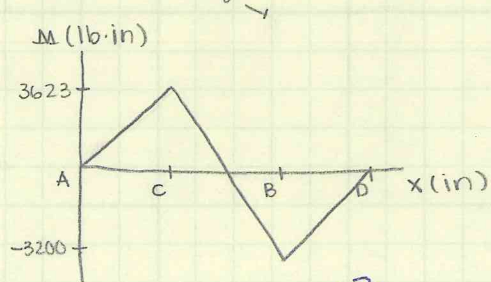
ΣF_y: $F_A + F_B = 800\,\text{lb}$

ΣM_A: $(14\,\text{in})(600\,\text{lb}) + (50\,\text{in})(200\,\text{lb}) = (34\,\text{in})F_B$

$F_B = 541.2\,\text{lb}$

$F_A = 258.8\,\text{lb}$





$M_C = F_A \cdot 14\,\text{in} = 3623.2\,\text{lb}\cdot\text{in}$

$M_B = F_A \cdot 34\,\text{in} - (600\,\text{lb})(20\,\text{in}) = -3200.8\,\text{lb}\cdot\text{in}$

+3

$\sigma_{allow} = 24\,\text{ksi} = \sigma_{max} = \dfrac{M_{max}\cdot c}{I}$ +1

$I_o = \frac{\pi}{4}r^4 \quad c = r$

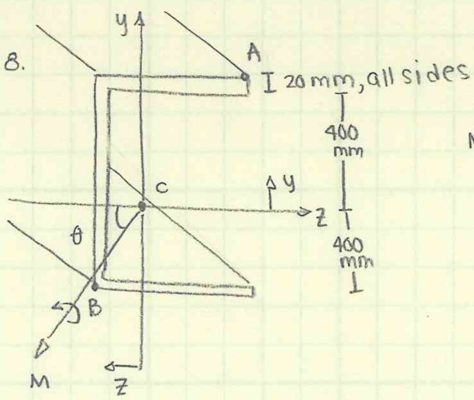
$(24\,\text{ksi})(\frac{\pi}{4})(r)^{\cancel{4}\,3} = (3623\,\text{lb}\cdot\text{in})(\cancel{r})$

$r^3 = \dfrac{3623\,\text{lb}\cdot\text{in}}{\pi/4\,(24\,\text{ksi})(1000)}$

$r = 0.5771\,\text{in} = \frac{1}{2}d$ $\quad \boxed{d = 1.15\,\text{in}}$ +2

+10

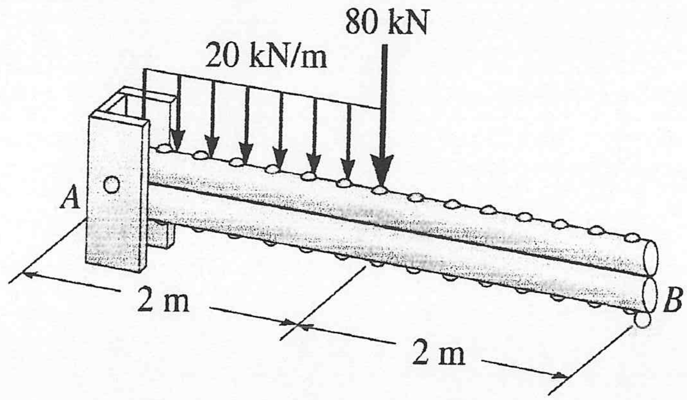
8.



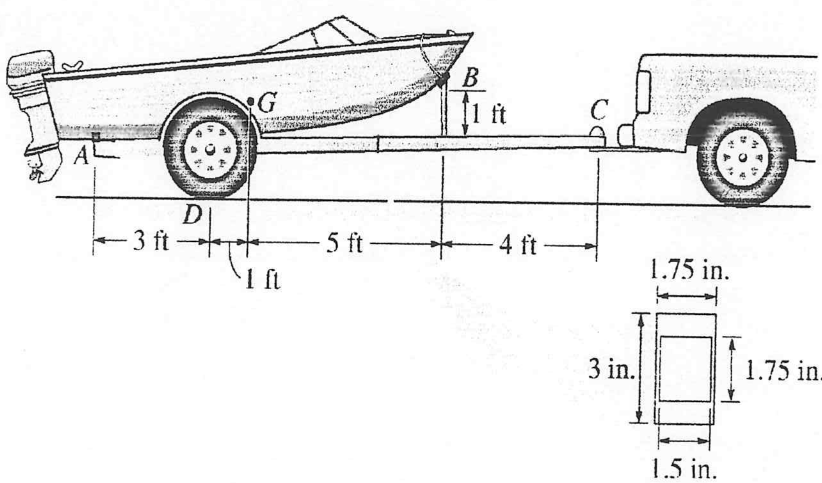
$M = 600\,\text{N}\cdot\text{m}$

CE 231: Strength of Materials	Homework #6
Spring 2003	Due: 3/21/03, 5:00 PM

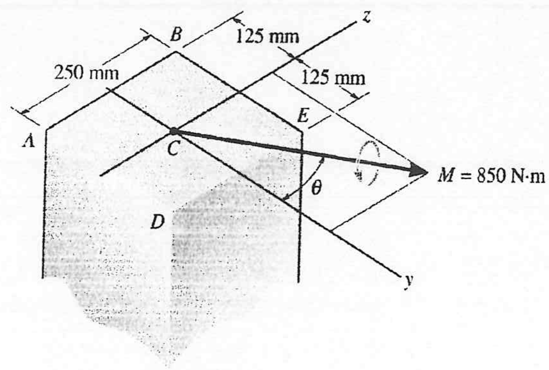
✓ 1. The two steel rods are bolted together along their length and support the loading shown. Assume the support at A is a pin and B is a roller. Determine the required diameter d of each of the rods if the allowable bending stress is $\sigma_{allow} = 130$ MPa.



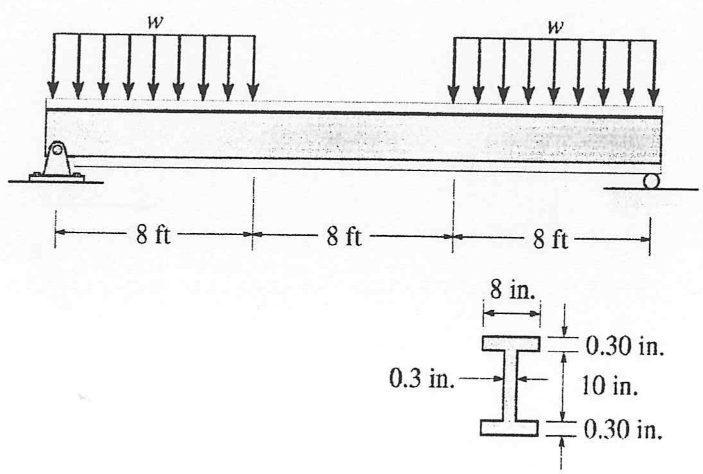
✓ 2. The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam, having the dimensions shown, and pinned at C.



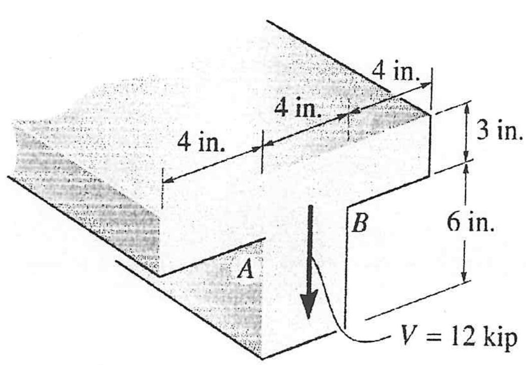
✓ 3. The member has a square cross section and is subjected to a resultant moment of $M =$ 850 N*m as shown. Determine the bending stress at each corner and sketch the stress distribution produced by **M**. Set $\theta = 45$ degrees.



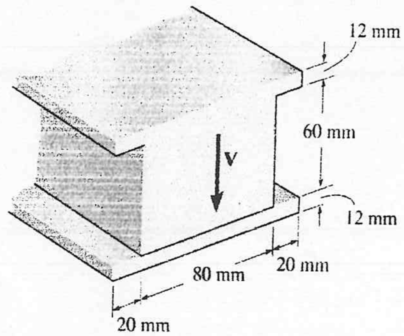
✓ 4. The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w that it can support so that the bending stress does not exceed $\sigma_{max} =$ 22 ksi.



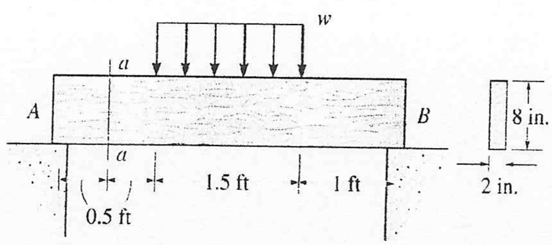
✓ 5. If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the vertical shear force resisted by the flange.



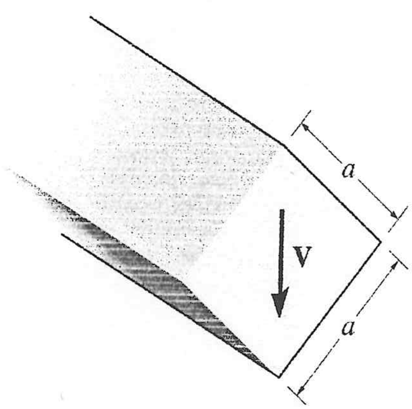
✓ 6. Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.



✓ 7. The supports at A and B exert vertical reactions on the wood beam. If the distributed load is $w = 4$ kip/ft, determine the maximum shear stress in the beam at section a-a.



8. The beam has a square cross section and is subjected to the shear force **V**. Sketch the shear-stress distribution over the cross section and specify the maximum shear stress. Also, from the neutral axis, locate where a crack along the member will first start to appear due to shear.

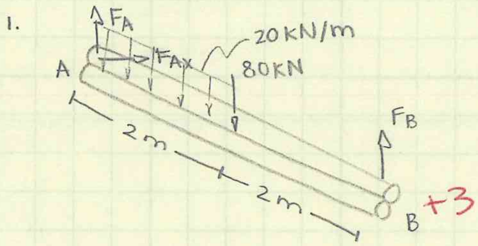


Catherine Hovell
March 21, 2003

22/30

CE 231: Homework #6

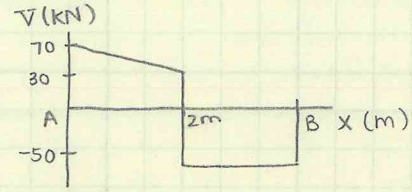
1.

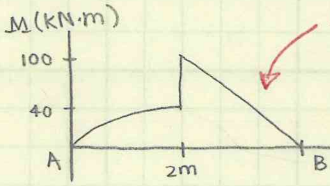


+3

$F_{Ax} = 0$

$\sigma_{allow} = 130\ MPa$





this is wrong!

ΣF_y: $F_B + F_A = 80\ kN + (20\ kN/m)(2m)$

$F_A + F_B = 120\ kN$

ΣM_A: $(20\ kN/m)(2m)(1m) + (80\ kN)(2m) = F_B(4m)$

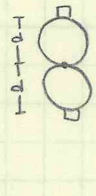
$F_B = 50\ kN \quad F_A = 70\ kN$

$M_{max} = 100\ kN \cdot m\ @\ x = 2m$

$$\sigma_{allow} = 130\ MPa = \frac{M_{max} \cdot d}{I} = \frac{(100\ kN\cdot m)\ d}{5\pi/32\, d^4}$$

$$d^3 = \frac{(1000\ kN\cdot m)}{(130\ MPa)(5\pi/32)} = 0.0157\ m^3$$

$\boxed{d = 0.25\ m}$



$I = I_{\bar{y}} + Ah^2 \qquad A = \pi/4\, d^2$

$h = \frac{1}{2} d$

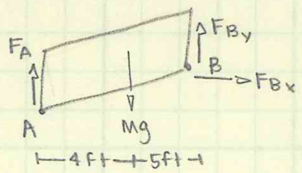
$I_{\bar{y}} = \frac{\pi}{4}\left(\frac{1}{2}d\right)^4$

$I = 2\left[\pi/4\left(d^4/16 + \frac{1}{4}d^4\right)\right]$

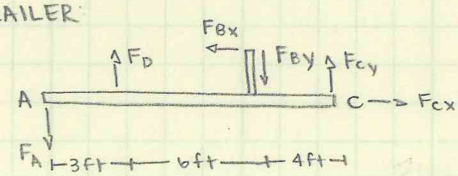
$I = \frac{5\pi}{32} d^4$ +3

+6

2. BOAT:



TRAILER:



ΣF_x: $F_{Cx} = F_{Bx} = 0$

ΣF_y: $F_A + F_B = F_D + F_C$

$Mg = 2300\ lb$

ΣF_x: $F_{Bx} = 0$

ΣF_y: $F_A + F_B = Mg$

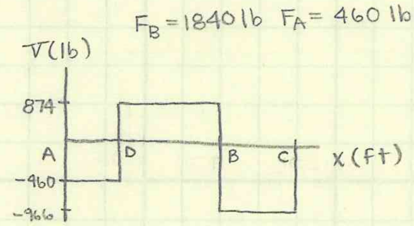
ΣM_A: $Mg(4ft) = F_{By}(5ft)$

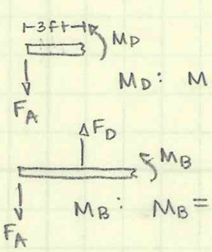
$F_B = 1840\ lb \quad F_A = 460\ lb$

ΣM_C: $(4ft)F_B + (13ft)F_A = (10ft)F_D$

$$F_D = \frac{(4ft)(1840\ lb) + (13ft)(460\ lb)}{10\ ft}$$

$F_D = 1334\ lb \quad F_C = 966\ lb$





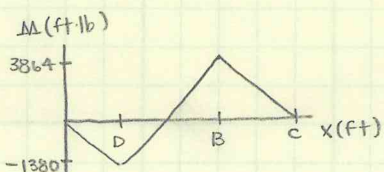
M_D: $M_D = -3ft(F_A)$

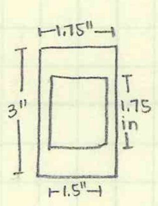
M_B: $M_B = F_D(6ft) - F_A(9ft)$

$$\sigma_{max} = \frac{M_{max} \cdot y}{I}$$

$M_{max} = 3864\ lb \cdot ft$

at $x = 9\ ft$





$I = \frac{1}{12}(1.75in)(3in)^3 - \frac{1}{12}(1.5in)(1.75in)^3$

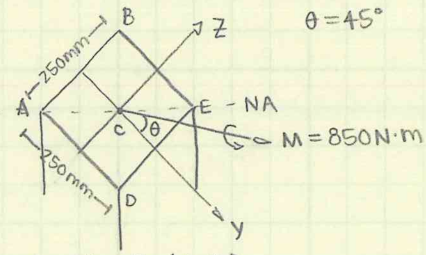
$I = 3.268\ in^4$

$$\sigma_{max} = \frac{(3864\ lb\cdot ft)(12in/1ft)(1.5in)}{3.268\ in^4}$$

$\boxed{\sigma_{max} = 21.3\ ksi}$

CE 231: Homework #6

3.



$\theta = 45°$

$M_y = M_z = 850\,N\cdot m(\sqrt{2}/2)$

$M_y = M_z = 601.0\,N\cdot m$

$\sigma_{tot} = \frac{M_y \cdot z}{I_y} + \left(\frac{-M_z \cdot y}{I_z}\right)$

$I_y = I_z \quad M_y = M_z$

$\sigma_{point} = \frac{M}{I}(z - y)$

if $z = y$, $\sigma_{tot} = 0$

(y, z) C: (0, 0)

A: (−0.125, −0.125)

B: (−0.125, 0.125)

D: (0.125, −0.125)

E: (0.125, 0.125)

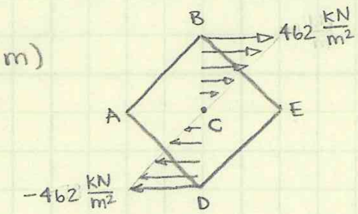
$\sigma_A = \sigma_E = 0$

B: $\frac{601.0\,N\cdot m}{I}(0.125\,m + 0.125\,m)$

$\sigma_B = \frac{150.3\,Nm^2}{I}$

D: $\frac{601.0\,N\cdot m}{I}(-0.125\,m - 0.125\,m)$

$\sigma_D = \frac{-150.3\,N\,m^2}{I}$

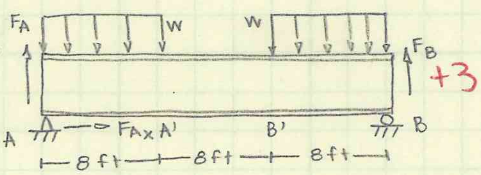


$I = 2\left[\frac{1}{36}(0.25\sqrt{2})(0.25\frac{\sqrt{2}}{2})^3 + (0.25\frac{\sqrt{2}}{2})^2(\frac{1}{3}\cdot 0.25\frac{\sqrt{2}}{2})^2\right]$

$I = 3.26 \times 10^{-4}\,m^4$

$$\boxed{\begin{array}{l}\text{BENDING STRESS:}\\ \sigma_A = \sigma_E = 0\\ \sigma_B = -\sigma_D = 461.6\,kN/m^2\end{array}}$$

4.



+3

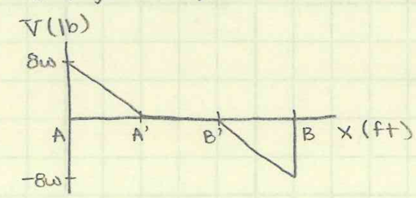
$F_A + F_B = 2w(8ft) \quad F_A = F_B$ (symmetry)

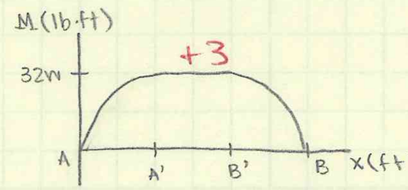
$F_A = F_B = (8ft)(w)$

$F_{Ax} = 0$

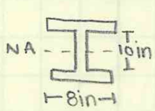
$M_{A'}: \; F_A(8ft) - w(8ft)(4ft) - M = 0$

$M = 32w\,lb\cdot ft = M_{max}$





$\sigma_{max} = 22\,ksi = \frac{M_{max} \cdot c}{I}$



t = 0.3 in, all sections

$I = I_{\bar{y}} + Ad^2$

$I = \frac{1}{12}(0.3\,in)(10\,in)^3 + 2\left[\frac{1}{12}(8\,in)(0.3\,in)^3 + (0.3\,in)(8\,in)(5.15\,in)^2\right]$

$I = 152.34\,in^4$ +2

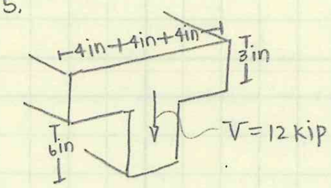
$22{,}000\,lb/in^2 = \frac{(32\cdot w\,lb\cdot ft)(12\,in/1\,ft)(5.15\,in)}{152.34\,in^4}$

$\boxed{w = 1.69\text{ kip/in}}$ +2

+10

CE 231: Homework #6

5.



$$\tau(y) = \frac{V \cdot Q}{I \cdot t}$$

$$I = \frac{1}{12}(4in)(6in)^3 + (4in)(6in)(2.6in)^2 + \frac{1}{12}(12in)(3in)^3 + (3in)(12in)(1.9in)^2$$

$$I = 391.2 \, in^4$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} \qquad \bar{y} = \frac{(3in)(6in)(4in) + (3in)(12in)(7.5in)}{(6in)(4in) + (3in)(12in)}$$

$$\bar{y} = 5.6in = NA$$

$$Q = \bar{y}' A' \qquad \bar{y} = y + \tfrac{1}{2}(3.4in - y)$$

$$A' = (12in)(3.4in - y)$$

$$Q = \left[(12in)(3.4 - y)(y + \tfrac{1}{2}(3.4in - y))\right]$$

$$dF = \tau(y)\, dA$$

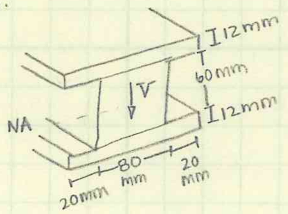
$$F = \int_{0.4}^{3.4} \frac{(12\,kip)}{391.2in^4} \frac{(12\cancel{in})(3.4in - y)\left[y + \frac{1}{2}(3.4in - y)\right]}{12\cancel{in}} dy$$

$$F = 368.1 \, lb/in^3 \int_{0.4}^{3.4} (3.4in - y)(\tfrac{1}{2}y + 1.7in)\, dy$$

$$F = (368.1 \, lb/in^3)(10.8 \, in^3)$$

$$\boxed{F_{flange} = 3.98 \, kip}$$

6.



$$V = 20kN$$

$$\tau(y) = \frac{V \cdot Q}{I \cdot t}$$

$$I = \tfrac{1}{12}(0.120m)(0.084m)^3 - 2\left[\tfrac{1}{12}(0.020m)(0.060m)^3\right]$$

$$I = 5.21 \times 10^{-6} \, m^4 \quad +3$$

MAX SHEAR @ y=0 mm

$$+3 \quad \tau(y) = \frac{V}{I} \cdot \frac{\left[y + \frac{1}{2}(30-y)\right]\left[(30-y)(40\cancel{mm})\right]}{40\cancel{mm}}$$

$$\tau(y) = 0.00384 \, N/mm^4 \, (450mm - \tfrac{1}{2}y^2)$$

$$\tau(y = 0\, mm) = 1.728 \, N/mm^2$$

$$\boxed{\tau_{max} = 1.73 N/mm^2 \text{ at } y = 0\, mm}$$

$0 < y < 0.030m$: $\tau(y) = \frac{V}{I} \frac{\left[y + (30-y)\cdot\frac{1}{2}\right]\left[(30-y)(40\cancel{mm})\right]}{40\cancel{mm}}$

$$\tau(y) = 0.00384 \, N/mm^4 \, (\tfrac{1}{2}y + 15mm)(30mm - y)$$

$\tau(y)_{max}$: $\frac{d\tau}{dy} = 0 \quad y = 0\, mm$

$30mm < y < 42mm$: $\tau(y) = \frac{V}{I} \cdot \frac{\left[y + (42mm - y)\cdot\frac{1}{2}\right](A)}{120\, mm}$

$$\tau(y) = 0.00384 \, N/mm^4 \, (\tfrac{1}{2}y + 21mm)(42mm - y)$$

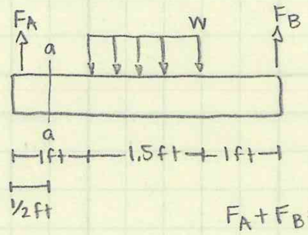
$\tau(y)_{max}$ at $\frac{d\tau}{dx} = 0$, $y = 0$

(not in range; $\tau(y)_{max\ section}$ @ y=30mm

+6

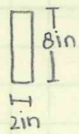
CE 231: Homework #6

7.



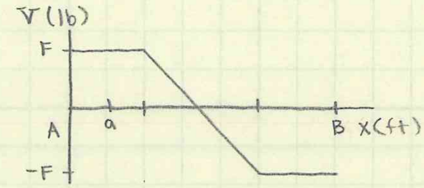
$w = 4000$ lb/ft

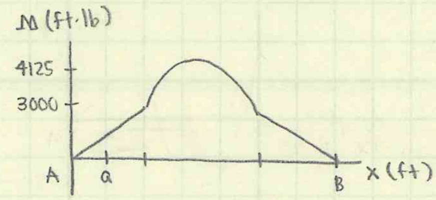
$F_A + F_B = (4000 \text{ lb/ft})(1.5\text{ft})$



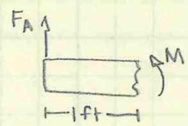
by symmetry, $F_A = F_B$

$\therefore F_A = F_B = 3000 \text{ lb} = F$



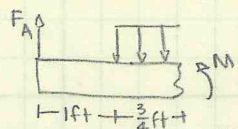


M @ x = 1 ft:



$M = (1\text{ft})(F_A) = 3000 \text{ lb}\cdot\text{ft}$

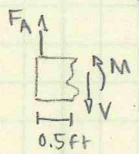
M @ x = 1.75 ft



$M = F_A(1.75\text{ft}) - w(3/4\text{ft})^2(\tfrac{1}{2})$

$M_A = 4125 \text{ lb}\cdot\text{ft}$

M, V @ a-a (x = 0.5ft)



$M = F_A(0.5\text{ft}) = 1500 \text{ lb}\cdot\text{ft}$

$V = F_A = 3000 \text{ lb}\cdot\text{ft}$

$\tau = \frac{V}{I}\cdot\frac{Q}{t}$ $\quad V_a = 3000$ lb

$t_a = 2$in

$I_a = 1/12(2\text{in})(8\text{in})^3 = 85.3\text{in}^4$

$Q = \int_0^{4in} y(2\text{in})dy$

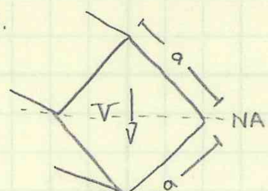
$Q = y^2 \Big|_0^{4in} = 16\text{in}^3$

$$\tau = \frac{(3000\text{ lb})(16\text{ in}^3)}{(85.3\text{in}^4)(2\text{in})}$$

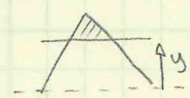
$\tau = 281.25 \text{ lb/in}^2$

$\boxed{\tau = 281 \text{ psi}}$

8.



$\tau = \frac{V}{I}\cdot\frac{Q}{t(y)}$ $\qquad Q = \int y\, t(y)\, dy$



$t(y) = a\sqrt{2}\,(1 - y/a/\sqrt{2})$ $\quad 0 < y < a/\sqrt{2}$

$I = 2\left[1/36(a\sqrt{2})(a\tfrac{\sqrt{2}}{2})^3 + (\tfrac{1}{3}a\tfrac{\sqrt{2}}{2})^2(a\tfrac{\sqrt{2}}{2})^2\right]$

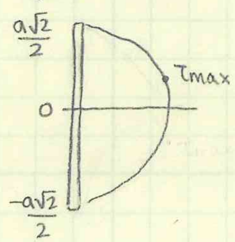
$$Q = a\sqrt{2}\int_y^{a/\sqrt{2}} y(1 - y\sqrt{2}/a)\,dy = a\sqrt{2}\int_y^{a/\sqrt{2}} y - y^2\frac{\sqrt{2}}{a}\,dy$$

$I = 2\left[1/72\, a^4 + 1/36\, a^4\right]$

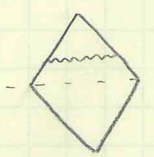
$$\tau = \frac{V}{1/12\, a^4}\cdot\frac{\left(\frac{y^3\sqrt{2}}{3a} - y^2/2 + 1/12\, a^2\right)}{1 - y\sqrt{2}/a} = \frac{V}{a^4}\cdot\frac{(4\sqrt{2}\, y^3/a - 6y^2 + a^2)}{(1 - y\sqrt{2}/a)}$$

$I = 1/12\, a^4$

(side view)



find $d\tau/dy = 0$

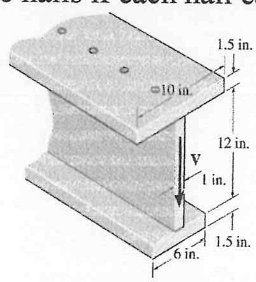


y = 0.178a – FAILURE LOCATION

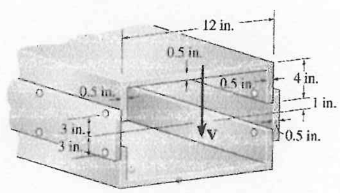
$$\tau_{max} = \frac{V}{a^4}\,\frac{(0.031/a - 0.187 + a^2)}{1 - 0.251/a}$$

CE 231: Strength of Materials | **Homework #7**
Spring 2003 | **Due: 3/28/03, 5:00 PM**

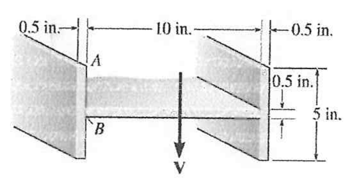
1. The beam is constructed from three boards. Determine the maximum shear V that it can sustain if the allowable shear stress for the wood is $\tau_{allow} = 400$ psi. What is the required spacing s of the nails if each nail can resist a shear force of 400 lb.



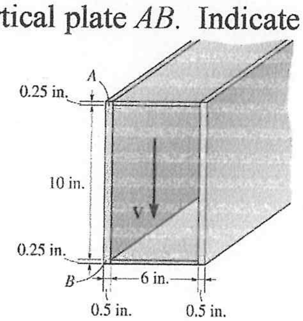
2. The beam is fabricated from two equivalent channels and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If a shear of $V = 50$ kip is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 15 kip.



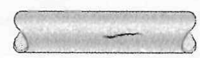
3. The beam supports a vertical shear of $V = 7$ kip. Determine the resultant force developed in segment AB of the beam.



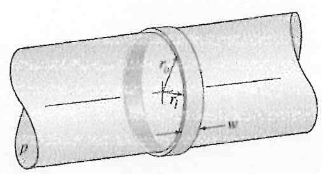
4. The member is subjected to a shear force of $V = 10$ kip. Sketch the shear-flow distribution along the vertical plate AB. Indicate numerical values of all peaks.



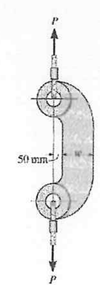
5. The open-ended pipe has a wall thickness of 2 mm and an internal diameter of 40 mm. Calculate the pressure that ice exerted on the interior wall of the pipe to cause it to burst in the manner shown. The maximum stress that the material can support at freezing temperatures is $\sigma_{max} = 360$ MPa. Show the stress acting on a small element of material just before the pipe fails.



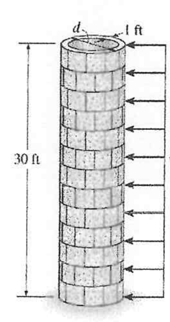
6. The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in internal radius of the ring after this pressure is applied. The modulus of elasticity of the ring is E.



7. The offset link has a width of $w = 200$ mm and a thickness of 40 mm. If the allowable normal stress is $\sigma_{allow} = 75$ MPa, determine the maximum load P that can be applied to the cables.



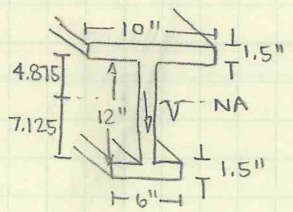
8. The chimney is subjected to the uniform wind pressure of $p = 25$ lb/ft^2. It is to be constructed with 1-ft-thick brick walls. If the bricks and mortar have a specific weight of 145 lb/ft^3, determine the smallest outer diameter d of the chimney so that no tensile stress is developed in the material. The wind loading can be approximated by $w = pd$.



CE231: Homework #7

Catherine Hovell
March 28, 2003

1.



$\tau_{allow} = 400\,psi$

$V_{Nail} = 400\,lb$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$\Sigma A = (10\,in)(1.5\,in) + (1\,in)(12\,in) + (6\,in)(1.5\,in)$

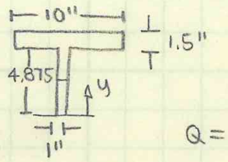
$\Sigma A = 36\,in^2$

$\Sigma \bar{y} A = (9\,in^2)(0.75\,in) + (12\,in^2)(7.5\,in) + (15\,in^2)(14.25\,in) = 310.5\,in^3$

$$\bar{y} = \frac{310.5\,in^3}{36\,in^2} = 8.625\,in$$

$$I = \frac{1}{12}(1\,in)(12\,in)^3 + (12\,in^2)(1.125\,in)^2 + \frac{1}{12}(6\,in)(1.5\,in)^3 + (9\,in^2)(7.875\,in)^2 + \frac{1}{12}(10\,in)(1.5\,in)^3 + (15\,in^2)(5.625\,in)^2$$

$$I = 1196.44\,in^4$$



$Q = \bar{y}'A' = \Sigma \bar{y} A$

$Q = (15\,in^2)(5.625\,in) + (4.875\,in - y)(2.4375\,in + \frac{1}{2}y)$

τ_{max} at Q_{max} $\quad \frac{dQ}{dy} = 0$

$\frac{dQ}{dy} = -y = 0 \quad y = 0$

$Q(y=0) = 96.26\,in^3$

$$\tau = \frac{VQ}{It}$$

$$V = \frac{\tau_{allow} \cdot I \cdot t}{Q}$$

$$V = \frac{(400\,psi)(1196.44\,in^4)(1\,in)}{96.26\,in^3}$$

$V = 4.972\,kip$

$$\boxed{V = 4.97\,kip}$$

$Q(y = 4.875\,in) = (15\,in^2)(5.625\,in)$

$Q_T = 84.375\,in^3$

$$q = \frac{V \cdot Q}{I} = \frac{(4.97\,kip)(84.375\,in^3)}{1196.44\,in^4}$$

$$0.351\,kip/in = \frac{0.400\,kip}{s} \qquad \boxed{s_T = 1.14\,in}$$

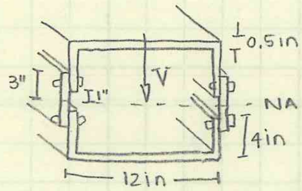
$Q(y = -7.125\,in) = 70.875\,in^3$

$$q = \frac{V \cdot Q}{I} = \frac{(4.97\,kip)(70.875\,in^3)}{1196.44\,in^4}$$

$$0.295\,kip/in = \frac{0.400\,kip}{s} \qquad \boxed{s_B = 1.36\,in}$$

CE 231: Homework #7

2.



$V_{NAIL} = 15\ kip$

$$q = \frac{V \cdot Q}{I}$$

$V = 50\ kip$

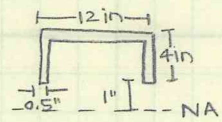
I: $2 \cdot \frac{1}{12}(0.5in)(6in)^3 +$

$4\left[\frac{1}{12}(0.5in)(4in)^3 + (0.5in)(4in)(3in)^2\right] +$

$2\left[\frac{1}{12}(11in)(0.5in)^3 + (0.5in)(11in)(3.75in)^2\right]$

+2 $I = 184.30\ in^4$

$Q = \bar{y}'A'$



$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$ $A' = \Sigma A$

$Q = \Sigma \bar{y} A$

$Q = (0.5in)(4in)(3in)(2)$

$+ (11in)(0.5in)(3.75in)$

$Q = 32.625\ in^3$

+2

$$q = \frac{V \cdot Q}{I} \qquad q = \frac{(50kip)(32.625\ in^3)}{184.30\ in^4}$$

$q = 8.85\ kip/in$

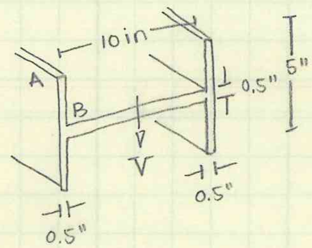
2 bolts, so $s = \frac{2(15\ kip)}{q} = \frac{30\ kip}{8.85\ kip/in}$ +3

$\boxed{s = 3.39 in}$

+7

<u>CE 231: Homework #7</u>

3.



$V = 7\text{ kip}$

$q = \dfrac{V \cdot Q}{I}$

$I = \frac{1}{12} \cdot 2(0.5\text{in})(5\text{in})^3 + \frac{1}{12}(10\text{in})(0.5\text{in})^3$

$I = 10.521\text{ in}^4$

$dF = q\,dx \qquad F = \int q\,dx$

$$F = \int_0^{2.5} \frac{V \cdot Q}{I} = \frac{V}{I}\int_0^{2.5} Q\,dy$$

$$Q = \bar{y}'A' = \left[(0.5\text{ in})(2.5\text{ in} - y)\right]\left[2.5\text{ in} - \tfrac{1}{2}(2.5\text{ in} - y)\right]$$

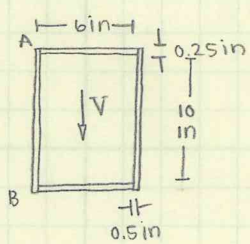
$$Q = (1.25\text{ in}^2)(2.5\text{ in} - y) - \tfrac{1}{4}\text{in}(2.5\text{ in} - y)^2$$

$$F = \frac{7000\text{ lb}}{10.52\text{in}^4}\int_0^{2.5}(1.25\text{ in}^2)(2.5\text{ in} - y) - \tfrac{1}{4}\text{in}(2.5\text{in} - y)^2\,dy$$

$$F = (665.35\text{ lb/in}^4)(2.60\text{ in}^4)$$

$$\boxed{F = 1.73\text{ Kip}}$$

4.



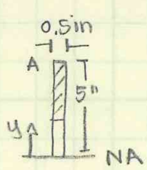
$\tau = \dfrac{V}{I} \cdot \dfrac{Q}{t} \qquad t = 0.5\text{ in}$

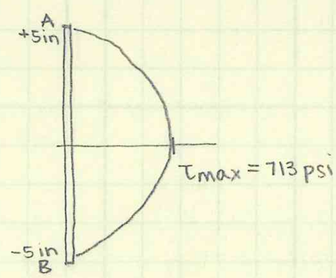
$I = \frac{1}{12}(7\text{in})(10.5\text{in})^3 - \frac{1}{12}(6\text{in})(10\text{ in})^3$

$I = 175.28\text{ in}^4$

V, I, t constant, $Q(y)$

$V = 10\text{ kip}$





$Q = \bar{y}'A' \qquad A' = (5\text{in} - y)(0.5\text{in})$

$\bar{y}' = 5'' - \tfrac{1}{2}(5\text{ in} - y)$

$Q = 2.5\text{in}^2(5\text{in} - y) - 0.25\text{in}(5\text{in} - y)^2$

Q_{max} ($\therefore \tau_{max}$) @ $y = 0$

$Q(y=0) = (2.5\text{in}^2)(5\text{in}) - (0.25\text{in})(5\text{in})^2$

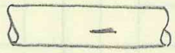
$Q(y=0) = 6.25\text{in}^3$

$$\tau_{max} = \frac{(10\text{ kip})(6.25\text{ in}^3)}{(175.28\text{in}^4)(0.5\text{in})}$$

$$\boxed{\tau_{max} = 713.1\text{ psi}}$$

CE 231: Homework #7

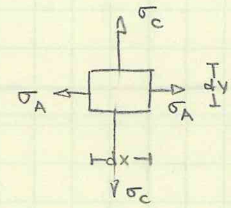
5.



$t = 2\,mm$

$d_i = 40\,mm$

$r_i = 20\,mm$



$\sigma_{max} = 360\,MPa$

$\sigma_C = \frac{R\,p_i}{t}$ $\quad \sigma_C = \frac{(20\,mm)}{(2\,mm)} p_i = 360\,MPa$

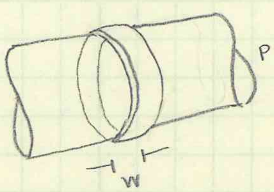
$\boxed{p_i = 36\,MPa}$ +7

$\sigma_C = 360\,MPa$ +2

$\sigma_A = 180\,MPa$ = 0!!!

+9

6.

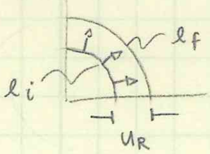


$\sigma = \frac{Rp}{t} = \frac{(r_o + r_i)\,p}{2w}$

$\sigma = E \cdot \varepsilon$

$\varepsilon = \frac{l_f - l_i}{l_i}$

$\varepsilon = \frac{\frac{1}{2}(\cancel{r_o + r_i}) + u_R - \frac{1}{2}(\cancel{r_o + r_i})}{\frac{1}{2}(r_o + r_i)}$



$l_i = \frac{\pi}{2} \cdot \frac{1}{2}(r_o + r_i)$

$l_f = \frac{\pi}{2}\left[\frac{1}{2}(r_o + r_i) + u_R\right]$

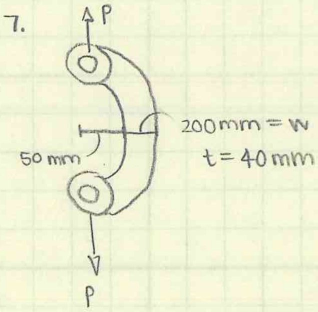
$\varepsilon = \frac{2u_R}{r_o + r_i}$

$\frac{(r_o + r_i)\,p}{2w} = E \cdot \frac{2u_R}{r_o + r_i}$

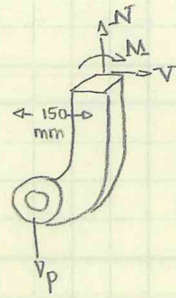
$\boxed{u_R = \frac{(r_o + r_i)^2\,p}{4 \cdot E \cdot w}}$

CE 231: Homework #7

7.



$\sigma_{allow} = 75\ MPa$



$V = 0$

$N = P$

$M = (150\ mm) \cdot P$

$\sigma_{allow} = \sigma_N + \sigma_B$

$\sigma_N = \frac{N}{A} = \frac{P}{(0.20m)(0.04m)} = 125P\ Pa$

$\sigma_B = \frac{Mc}{I} = \frac{(0.150m)P(0.10m)}{1/12(0.04)(0.20)^3} = 562.5P\ Pa$

take point at outside of clamp, not center!

$M = 250mm \cdot P$
$\quad \hookrightarrow d$

$-\frac{Pdc}{I} + \frac{P}{A} = \sigma$

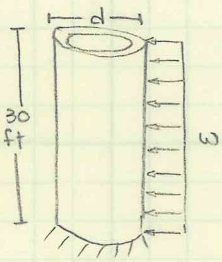
$75\times10^6\ Pa = 125P\ Pa + 562.5P\ Pa$

$P = \frac{75\times10^6\ Pa}{687.5\ Pa}$

$\boxed{P = 109.1\ kN}$

$h = 2in$

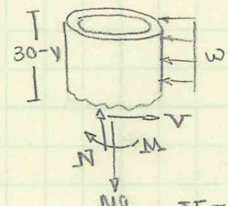
8.



$w = p\cdot d \quad p = 25\ lb/ft^2$

$\gamma = 145\ lb/ft^3$

$\sigma_T = \sigma_N + \sigma_B = 0$



$N = \gamma \cdot A \cdot (30-y)$

$V = p\cdot d\ (30-y)$

$\frac{N}{A} - \frac{M\cdot c}{I} = 0 \quad \frac{N}{A} = \frac{Mc}{I}$

$M = p\cdot d\,(30-y)^2 \cdot \frac{1}{2}$

$\sigma_N = \frac{N}{A}$

$\sigma_B = \frac{Mc}{I}$

$\frac{\gamma \cancel{A}(30ft - \cancel{y})}{\cancel{A}} = \frac{p\cdot d(30ft - \cancel{y})^{\cancel{2}} \cdot \frac{1}{2} \cdot \frac{1}{2}d}{\frac{\pi}{\cancel{4}}\left[(\frac{d}{2})^4 - (\frac{d}{2}-1)^4\right]}$

$I_{disc} = \frac{\pi}{4}\left[(\frac{1}{2}d)^4 - (\frac{1}{2}d - 1)^4\right]$

$\frac{\gamma\cdot\pi}{p\ (30ft)} = \frac{d^2}{(\frac{d}{2})^4 - (\frac{d}{2}-1)^4} = \frac{(145\ lb/ft^3)\pi}{(25\ lb/ft^2)(30ft)}$

stress is max at $y = 0\ ft$

solve for d

$d = 5.647\ ft$

$\boxed{d = 5.65\ ft}$

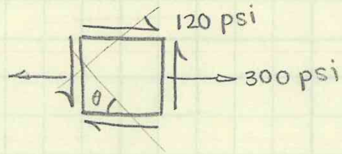
+10

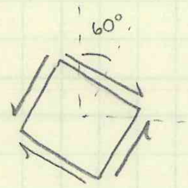
27/30

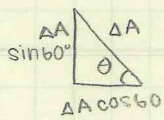
CE 231: Homework #8

Catherine Hovell
April 18, 2003

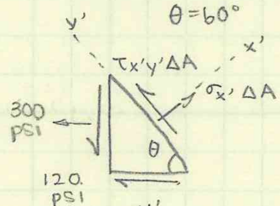
8.

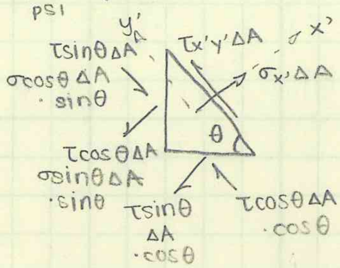


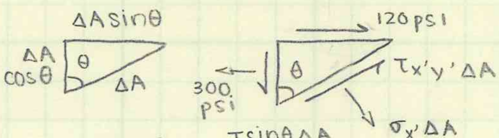


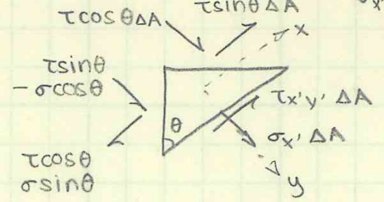


$\theta = 60°$









$+\nearrow \Sigma F_{x'} = 0: \quad \sigma_{x'}\Delta A - (300\text{ psi})\Delta A \sin\theta\sin\theta$

$- (120\text{ psi})\Delta A \sin\theta\cos\theta$

$- (120\text{ psi})\Delta A \sin\theta\cos\theta = \theta$

$\sigma_{x'} = -28.9\text{ ksi}$

$+\nwarrow \Sigma F_{y'} = 0: \quad \tau_{x'y'}\Delta A + (120\text{ psi})\Delta A \cos^2\theta$

$- (120\text{ psi})\Delta A \sin\theta\sin\theta$

$- (300\text{ psi})\Delta A \cos\theta\sin\theta = 0$

$\tau_{x'y'} = -69.9\text{ psi}$

+

Why is everyone doing this twice?

$+\nearrow \Sigma F_x = 0: \quad \tau_{x'y'}\Delta A + (120\text{ psi})\Delta A \sin\theta\sin\theta$

$- (120\text{ psi})\Delta A \cos\theta\cos\theta$

$- (300\text{ psi})\Delta A \sin\theta\cos\theta = 0$

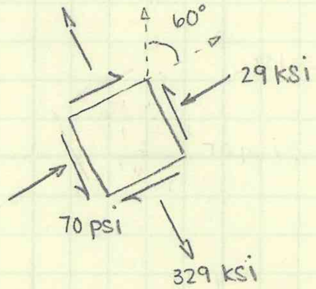
$\tau_{x'y'} = 69.9\text{ psi}$ OK

$+\searrow \Sigma F_y = 0: \quad \sigma_{x'}\Delta A + (120\text{ psi})\Delta A \cos\theta\sin\theta$

$+ (120\text{ psi})\Delta A \sin\theta\cos\theta$

$- (300\text{ psi})\Delta A \cos\theta\cos\theta = 0$

$\sigma_{x'} = 328.9\text{ ksi}$



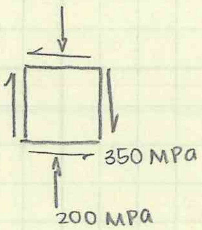
+9

$\tau_{xy} = \tau_{yx} \therefore \tau_{x'y'} = \tau_{y'x'}$

they should be =
not = & opposite!!

CE 231: Homework #8

12.



MAX/MIN NORMAL STRESS: $\tan(2\theta_p) = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$

$\sigma_x = 0$

$\sigma_y = -200\text{ MPa}$

$\tau_{xy} = -350\text{ MPa}$

$$\tan(2\theta_p) = \frac{-350\text{ MPa}}{(0 + 200\text{ MPa})/2} = -3.5$$

$$\theta_p = -37.0°$$

PRINCIPAL STRESS:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-200\text{ MPa}}{2} + \sqrt{\frac{(200\text{ MPa})^2}{4} + (-350\text{ MPa})^2} = 264\text{ MPa}$$

$$\sigma_2 = \frac{-200\text{ MPa}}{2} - \sqrt{\frac{(200\text{ MPa})^2}{4} + (-350\text{ MPa})^2} = -464\text{ MPa}$$

a. $\sigma_1 = 264\text{ MPa}$
$\sigma_2 = -464\text{ MPa}$
$\theta_{p1} = -37.0°$
$\theta_{p2} = \cancel{143}.0°$

+3

53°

MAX IN-PLANE SHEAR STRESS:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\frac{(200\text{ MPa})^2}{4} + (-350\text{ MPa})^2} = 364.0\text{ MPa}$$

AVERAGE NORMAL STRESS:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{avg} = \frac{-200\text{ MPa}}{2} = -100\text{ MPa}$$

b. $\tau_{max} = 364\text{ MPa}$
$\sigma_{avg} = -100\text{ MPa}$
$\theta_{s1} = 7.97°$; $\theta_{s2} = \cancel{188.0}°$

+5

-82°

$$\tan(2\theta_s) = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

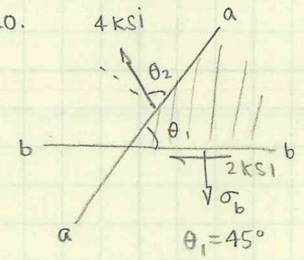
$$\tan(2\theta_s) = \frac{-200\text{ MPa}/2}{-350\text{ MPa}}$$ (should be 45° off θ_p)

$\theta_s = 7.97°$ $(+ -37.0° \approx 45°)$

+8

CE 231: Homework #8

20.



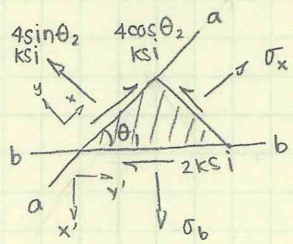
$\theta_1 = 45°$

$\theta_2 = 60°$

$\theta_{trans} = 135°$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\frac{\left[\tau_{x'y'} - \tau_{xy}\cos(2\theta)\right]\cdot 2}{-\sin(2\theta)} + \sigma_y = \sigma_x$$



$\tau_{xy} = 2\text{ ksi}$ +2

$\sigma_y = 3.46\text{ ksi}$ +2

$\sigma_x = ?$

$\tau_{x'y'} = -2\text{ ksi}$

$\sigma_{x'} = \sigma_b$

$$\frac{\left[(-2\text{ ksi}) - (2\text{ ksi})\cos(-270)\right](-2)}{\sin(-270)} + 3.46\text{ ksi} = \sigma_x$$

$$\sigma_x = 7.46\text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$ $\theta = -135°$ +2

$$\sigma_b = \frac{7.46\text{ ksi} + 3.46\text{ ksi}}{2} + \frac{7.46\text{ ksi} - 3.46\text{ ksi}}{2}\cdot\cos(-270) + (2\text{ ksi})\sin(-270)$$

$$\boxed{\sigma_b = 7.46\text{ ksi}}$$ +2

PRINCIPAL STRESSES:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{(7.46\text{ ksi}) + (3.46\text{ ksi})}{2} \pm \sqrt{\left(\frac{7.46\text{ ksi} - 3.46\text{ ksi}}{2}\right)^2 + (2\text{ ksi})^2}$$

$$\boxed{\begin{array}{l}\sigma_1 = 8.29\text{ ksi}\\ \sigma_2 = 2.63\text{ ksi}\end{array}}$$ +2

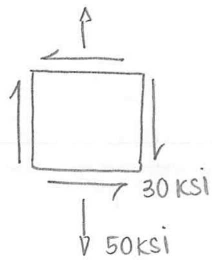
+10

Catherine Hovell
April 25, 2003

28|30

CE 231: HOMEWORK #9

13.



$\sigma_x = 0$

$\sigma_y = 50\text{ ksi}$

$\tau_{xy} = -30\text{ ksi}$

PRINCIPAL STRESSES:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{50\text{ ksi}}{2} + \sqrt{\frac{(-50\text{ ksi})^2}{4} + (-30\text{ ksi})^2}$$

$$\sigma_1 = 64.05\text{ ksi}$$

$$\sigma_2 = \frac{50\text{ ksi}}{2} - \sqrt{\frac{(-50\text{ ksi})^2}{4} + (-30\text{ ksi})^2}$$

$$\sigma_2 = -14.05\text{ ksi}$$

- ORIENTATION:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2(-30\text{ ksi})}{0 - 50\text{ ksi}}\right)$$

$$\theta_p = 25.09°, \ \theta_p = -64.9°$$

$$\boxed{\text{a. } \sigma_1 = 64.1\text{ ksi}, \ \sigma_2 = -14.1\text{ ksi} \quad \theta_{p_1} = 25.1°, \ \theta_{p_2} = -64.9°}$$

- $$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

$$\theta_s = \frac{1}{2}\tan^{-1}\left(\frac{50\text{ ksi}}{2(-30\text{ ksi})}\right)$$

$$\theta_s = -19.9° \text{ or } \theta_s = \theta_p + 45°$$

$$\theta_s = 70.1°$$

MAX SHEAR STRESS / NORMAL:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

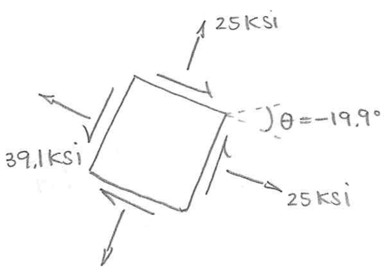
$$\tau_{max} = \sqrt{\frac{(-50\text{ ksi})^2}{4} + (-30\text{ ksi})^2}$$

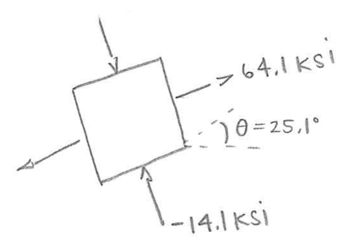
$$\tau_{max} = 39.05\text{ ksi}$$

$$\boxed{\text{b. } \tau_{max} = 39.1\text{ ksi} \quad \sigma_{avg} = 25\text{ ksi} \quad \theta_{s_1} = -19.9°, \ \theta_{s_2} = 70.1°}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50\text{ ksi}}{2}$$

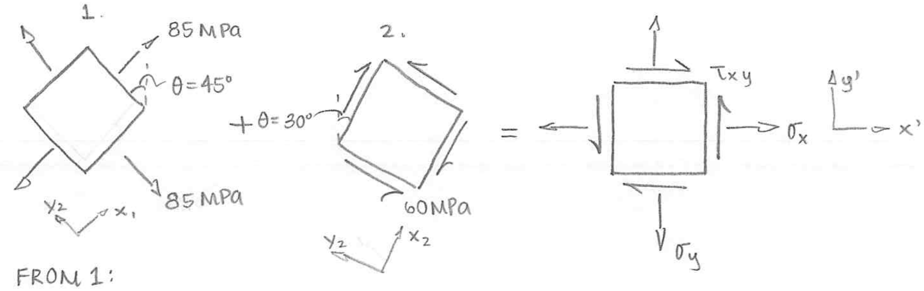
$$\sigma_{avg} = 25\text{ ksi}$$





CE 231: HOMEWORK #9

18.



FROM 1:

$$\sigma_{x_1'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_1 + \tau_{xy}\sin 2\theta_1 \qquad \theta_1 = -45°$$

$$\sigma_{x_1'} = \frac{85\text{ MPa}+85\text{ MPa}}{2} + 0 + 0$$

$$\sigma_{x'} = 85\text{ MPa} \quad +1$$

$$\tau_{x'y_1'} = \frac{\sigma_y-\sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y_1'} = 0\text{ MPa} \quad +1$$

$$\sigma_{y_1'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_1 - \tau_{xy}\sin 2\theta_1$$

$$\sigma_{y_1'} = \frac{85\text{ MPa}+85\text{ MPa}}{2} - 0 - 0 = 85\text{ MPa} \quad +1$$

FROM 2:

$$\sigma_{x_2'} = 0 + 0 + 60\text{ MPa}\cdot\sin[2(-60°)] = -51.96\text{ MPa} \quad +1$$

$$\sigma_{y_2'} = 0 + 0 - 60\text{ MPa}\cdot\sin[2(-60°)] = 51.96\text{ MPa} \quad +1$$

$$\tau_{x'y_2'} = 0 + 60\text{ MPa}\cdot\cos[2(-60°)] = -30\text{ MPa} \quad +$$

$$\sigma_{x'} = \sigma_{x_1'} + \sigma_{x_2'} = 85\text{ MPa} - 51.96\text{ MPa}$$

$$\sigma_{x'} = 33.0\text{ MPa}$$

$$\sigma_{y'} = \sigma_{y_1'} + \sigma_{y_2'} = 85\text{ MPa} + 51.96\text{ MPa}$$

$$\sigma_{y'} = 137.0\text{ MPa}$$

$$\tau_{x'y'} = \tau_{x'y_1'} + \tau_{x'y_2'} = 0\text{ MPa} - 30\text{ MPa}$$

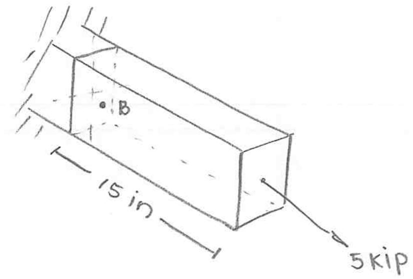
$$\tau_{x'y'} = -30\text{ MPa}$$

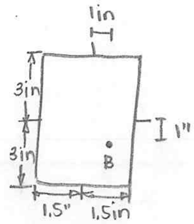
$$\boxed{\begin{aligned}\sigma_{x'} &= 33.0\text{ MPa}\\ \sigma_{y'} &= 137.0\text{ MPa}\\ \tau_{x'y'} &= -30\text{ MPa}\end{aligned}}$$

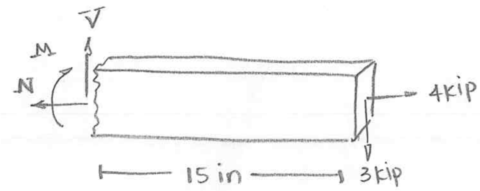
+10

CE 231: HOMEWORK #9

71.







ΣF_y: $V = 3\text{kip}$

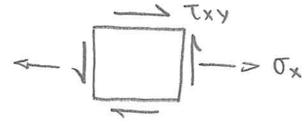
ΣF_x: $N = 4\text{kip}$

ΣM: $M = -45\text{kip}\cdot\text{in}$

$$\sigma = \frac{F}{A}, \quad \sigma = -\frac{My}{I}$$

$$\sigma_x = \frac{4\text{ kip}}{(3\text{in})(6\text{in})} - \frac{(-45\text{ kip}\cdot\text{in})(-1\text{ in})}{1/12\,(3\text{in})(6\text{in})^3}$$

$$\sigma_x = -0.611 \text{ ksi}$$



$$\tau = \frac{VQ}{It} \qquad \tau = \frac{(3\text{kip})(2\text{in})(3\text{in})(-2\text{in})}{1/12\,(3\text{in})(6\text{in})^3(3\text{in})}$$

$$\tau = -0.22 \text{ ksi}$$

PRINCIPAL STRESSES:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-0.611\text{ksi}}{2} + \sqrt{\left(\frac{0.611\text{ksi}}{2}\right)^2 + (-0.22\text{ ksi})^2}$$

$$\sigma_1 = 0.0723 \text{ ksi}$$

$$\sigma_2 = \frac{-0.611\text{ksi}}{2} - \sqrt{\left(\frac{0.611\text{ksi}}{2}\right)^2 + (-0.22\text{ksi})^2}$$

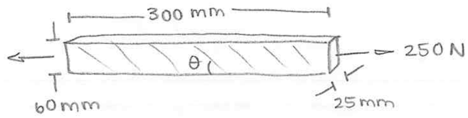
$$\sigma_2 = -0.683 \text{ ksi}$$

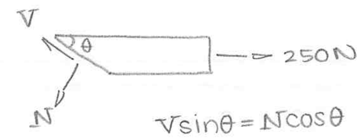
$$\boxed{\begin{array}{l}\text{At Point B:} \\ \sigma_1 = 0.0723\text{ ksi} \\ \sigma_2 = -0.683\text{ksi}\end{array}}$$

+10

CE 231: HOMEWORK #9

72.





$Vsin\theta = N\cos\theta \qquad N = V\tan\theta$

$V\cos\theta + N\sin\theta = 250\,N$

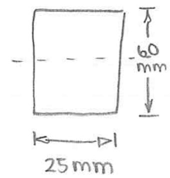
$V(\cos\theta + \tan\theta\sin\theta) = 250\,N$

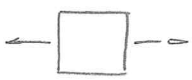
$V = 234.9\,N$

$N = 85.5\,N$

$\theta = 20°$

$\sigma = \frac{P}{A} \qquad \tau = \frac{VQ}{It}$





$\frac{250N}{A}$

at $\theta = 0$

$\sigma_x = 166.7\,kPa$

$I = 1/12(0.025\,m)(0.060\,m)^3$

$I = 4.5\times10^{-7}\,m^4$

$Q = \bar{y}'A' = (30\,mm)\cdot\frac{1}{2}(25\,mm)(30\,mm)$

$Q = 1.125\times10^{-5}\,m^3$

TRANSFORMATION TO $\theta = -20°$:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos2\theta + \tau_{xy}\sin2\theta$$

$$\sigma_{x'} = \frac{166.7\,kPa}{2} + \frac{166.7\,kPa}{2}\cos(-40°) + 0$$

$$\sigma_{x'} = 147.2\ kPa$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos2\theta - \tau_{xy}\sin2\theta$$

$$\sigma_{y'} = \frac{166.7\,kPa}{2} + \frac{-166.7\,kPa}{2}\cos(-40°) + 0$$

$$\sigma_{y'} = 19.5\ kPa$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin2\theta + \tau_{xy}\cos2\theta \qquad \tau_{x'y'} = \frac{-166.7\,kPa}{2}\sin(-40°) + 0$$

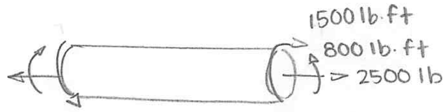
$$\tau_{x'y'} = 53.6\,kPa$$

$$\boxed{\begin{array}{l}\text{AT } \theta = -20°,\\ \sigma_{x'} = 147.2\ kPa\\ \sigma_{y'} = 19.5\ kPa\\ \tau_{x'y'} = 53.6\ kPa\end{array}}$$

+8

CE 231: HOMEWORK #9

95.



$d = 6\text{ in}$

$A = \frac{\pi}{4}(6\text{ in})^2 = 9\pi \text{ in}^2$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(3\text{ in})^4$

$J = 40.5\pi \text{ in}^4$

$\sigma_x = \frac{F}{A} + \frac{My}{I}$ $\qquad \tau = \frac{Tc}{J}$

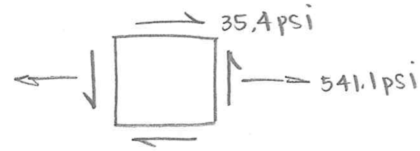
$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(3\text{ in})^4$

$I = 20.25\pi \text{ in}^4$

$$\sigma_x = \frac{2500\text{ lb}}{9\pi \text{ in}^2} + \frac{(800\text{ lb}\cdot\text{ft})(12\text{in}/1\text{ft})(3\text{ in})}{20.25\pi \text{ in}^4}$$

$\sigma_x = 541.1\text{ psi}$ $\qquad \sigma_y = 0$

$$\tau_{xy} = \frac{(1500\text{ lb}\cdot\text{ft})(3\text{in})}{40.5\pi \text{ in}^4}\cdot\frac{12\text{ in}}{1\text{ ft}}$$



$\tau_{xy} = 424.4\text{ psi}$

PRINCIPAL STRESSES / MAX SHEAR STRESS

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{541.1\text{ psi}}{2} + \sqrt{\left(\frac{541.1\text{psi}}{2}\right)^2 + (424.4\text{ psi})^2}$$

$\sigma_1 = 773.9\text{ psi}$

$$\sigma_2 = \frac{541.1\text{psi}}{2} - \sqrt{\left(\frac{541.1\text{psi}}{2}\right)^2 + (424.4\text{psi})^2}$$

$\sigma_2 = -232.8\text{ psi}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tau_{max} = \sqrt{\left(\frac{541.1\text{psi}}{2}\right)^2 + (424.4\text{psi})^2}$$

$\tau_{max} = 503.3\text{ psi}$

At Point A:
$\sigma_1 = 773.9\text{ psi}$
$\sigma_2 = -232.8\text{ psi}$
$\tau_{max} = 503.3\text{ psi}$

Catherine Hovell
May 2, 2003

CE 231: HOMEWORK #10

30/30

6. $\varepsilon_x = 250 \times 10^{-6}$

$\varepsilon_y = 300 \times 10^{-6}$

$\gamma_{xy} = -180 \times 10^{-6}$

$\varepsilon_{xy} = \frac{\gamma_{xy}}{2} = -90 \times 10^{-6}$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^2}$$

$$\varepsilon_{1,2} = \frac{(250+300)\times 10^{-6}}{2} \pm \sqrt{\left(\frac{-50\times 10^{-6}}{2}\right)^2 + (-90\times 10^{-6})^2}$$

$\varepsilon_1 = 3.68 \times 10^{-4}$

$\varepsilon_2 = 1.82 \times 10^{-4}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{-180\times 10^{-6}}{250\times 10^{-6} - 300\times 10^{-6}}\right)$$

$\theta_{p_1} = 37.2°, \ \theta_{p_2} = -52.8°$

$\boxed{\text{a. } \varepsilon_1 = 368\times 10^{-6} \quad \theta_{p_1} = 37.2° \quad \varepsilon_2 = 182\times 10^{-6} \quad \theta_{p_2} = -52.8°}$

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{max} = 2\sqrt{\left(\frac{250\times 10^{-6} - 300\times 10^{-6}}{2}\right)^2 + \left(\frac{-180\times 10^{-6}}{2}\right)^2}$$

$\gamma_{max} = 186.8 \times 10^{-6}$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{250\times 10^{-6} + 300\times 10^{-6}}{2} = 275\times 10^{-6}$$

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right) \qquad \theta_s = \frac{1}{2}\tan^{-1}\left(\frac{300\times 10^{-6} - 250\times 10^{-6}}{-180\times 10^{-6}}\right)$$

$\theta_{s_1} = -7.76°, \ \theta_{s_2} = 82.2°$

$\boxed{\text{b. } \gamma_{max} = 187\times 10^{-6} \quad \varepsilon_{avg} = 275\times 10^{-6} \quad \theta_{s_1} = -7.8°, \ \theta_{s_2} = 82.2°}$

CE 231: HOMEWORK #10

47. $\varepsilon_1 = 630 \times 10^{-6}$ $\quad E_{al} = 10 \times 10^3 \text{ksi}$

$\varepsilon_2 = 350 \times 10^{-6}$ $\quad \nu_{al} = 0.33$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad (630)(10^{-6})(10)(10^3) = \sigma_1 - \tfrac{1}{3}\sigma_2$$

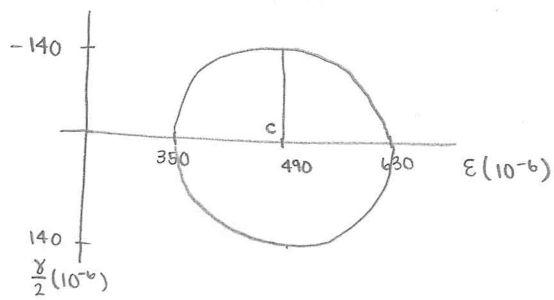
$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \rightsquigarrow (350)(10^{-6})(10)(10^3) = \sigma_2 - \tfrac{1}{3}\sigma_1$$

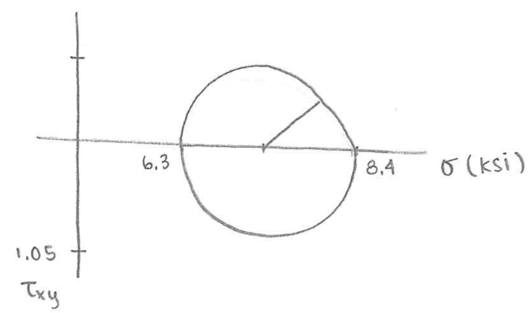
$$\sigma_1 = 6.3\,\text{ksi} + \tfrac{1}{3}\sigma_2$$

$$\sigma_2 = 3.5\,\text{ksi} + \tfrac{1}{3}\sigma_1$$

$$\varepsilon_{avg} = \frac{(630)+(350)}{2}(10^{-6}) = 490(10^{-6})$$

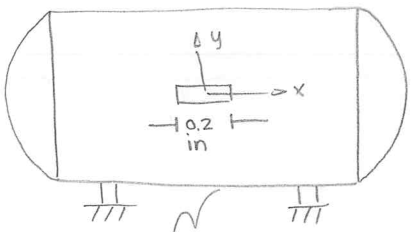
$$\boxed{\begin{aligned}\sigma_1 &= 8.4\,\text{ksi}\\ \sigma_2 &= 6.3\,\text{ksi}\end{aligned}}$$





CE 231: HOMEWORK #10

51.



$d = 30\text{ in}$

$t = 0.5\text{ in}$

$L_o = 0.2\text{ in}$

$\delta = 0.08 \times 10^{-3}\text{ in}$

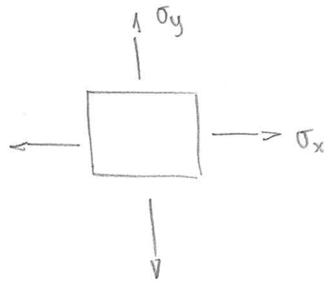
$$\varepsilon_x = \frac{\delta}{L_o} = \frac{0.08 \times 10^{-3}\text{ in}}{0.2\text{ in}}$$

$$\varepsilon_x = 4.0 \times 10^{-4}$$

304 stainless steel:

$E = 28.0 \times 10^3\text{ ksi}$

$\nu = 0.27 = 3/11$



$$\sigma_y = \frac{Pr}{t} \qquad \sigma_x = \frac{Pr}{2t}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_x = \frac{1}{E}\left(\frac{Pr}{2t} - \nu\frac{Pr}{t}\right) = \frac{Pr}{Et}\left(\frac{1}{2} - \nu\right)$$

$$P = \frac{\varepsilon_x E t}{r(1/2 - \nu)} = \frac{(4.0 \times 10^{-4})(28 \times 10^3\text{ ksi})(0.5\text{ in})}{\frac{1}{2}(30\text{ in})(1/2 - 3/11)}$$

$$\boxed{P = 1.64\text{ ksi}}$$

$$\sigma_{max} = \frac{Pr}{t} = \frac{(1.64\text{ ksi})(15\text{ in})}{0.5\text{ in}} = 49.28\text{ ksi}$$

$$\sigma_{mid} = \frac{Pr}{2t} = \frac{(1.64\text{ ksi})(15\text{ in})}{2(0.5\text{ in})} = 24.64\text{ ksi}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{mid}}{2} = \frac{49.28\text{ ksi} - 24.64\text{ ksi}}{2}$$

$$\tau_{max} = 12.32\text{ ksi} = G\gamma_{max} \qquad G = 11 \times 10^3\text{ ksi}$$

$$\boxed{\gamma_{max} = 1.12 \times 10^{-3}}$$

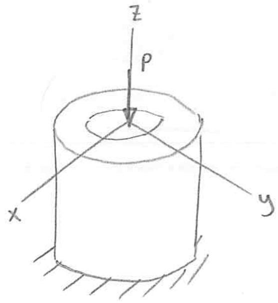
$$\tau_{abs} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{49.28\text{ ksi} - 0}{2}$$

$$\tau_{abs} = 24.64\text{ ksi} = G\gamma_{abs}$$

$$\boxed{\gamma_{abs} = 2.24 \times 10^{-3}}$$

CE 231: HOMEWORK #10

57.



$\nu = 0.3$

because of walls,

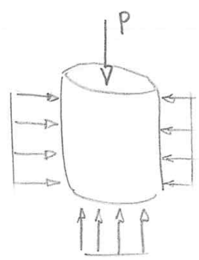
$\delta_x = \delta_y = 0$

$\varepsilon_x = \varepsilon_y = 0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y - \nu\sigma_z\right] = 0$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x - \nu\sigma_z\right] = 0$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\sigma_x - \nu\sigma_y\right]$$



if not confined,

$\sigma_x = \sigma_y = 0$

when confined,

$\sigma_x = \sigma_y > 0$

$\nu = 0.3 = \frac{\varepsilon_{x,y}}{\varepsilon_z}$ due to load P

$\sigma_x = \nu\sigma_y + \nu\sigma_z$

$\sigma_y = \nu\sigma_x + \nu\sigma_z$

$\sigma_x = \nu^2\sigma_x + \nu^2\sigma_z + \nu\sigma_z$

$$\sigma_x = \frac{\sigma_z(\nu^2 + \nu)}{1 - \nu^2}$$

$\sigma_y = \nu^2\sigma_y + \nu^2\sigma_z + \nu\sigma_z$

$$\sigma_y = \frac{\sigma_z(\nu^2 + \nu)}{1 - \nu^2}$$

confined: $$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \frac{\sigma_z\nu^2(\nu+1)}{1-\nu^2} - \frac{\sigma_z\nu^2(\nu+1)}{1-\nu^2}\right] = \frac{\sigma_z}{E}\left[1 - \frac{2\nu^2(1+\nu)}{1-\nu^2}\right]$$

$\rightarrow KE_o$

unconfined: $$\varepsilon_z = \frac{1}{E_o}\sigma_z$$

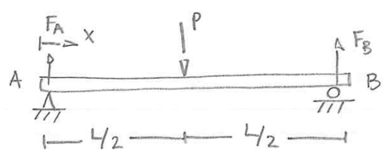
$$\frac{1}{K} = 1 - \frac{2(0.3)^2(1+0.3)}{1-(0.3)^2} = 1.346$$

$$\boxed{K = 1.35}$$

+10

CE 231: HOMEWORK #10

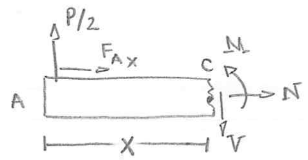
4.



$0 \le x < L/2$

$F_A + F_B = P$

$F_A = F_B = \frac{1}{2}P$



ΣF_y: $V = P/2$

ΣF_x: $F_{Ax} = N$

ΣM_c: $M = \frac{P}{2} \cdot x$

$$M(x) = EI \frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$\frac{d^2v}{dx^2} = \frac{P}{EI} \cdot \frac{x}{2}$$

$$\frac{dv}{dx} = \frac{P}{2EI} \int x\,dx = \frac{P}{2EI}\frac{x^2}{2} + C_1$$

$$v = \frac{P}{4EI} \int x^2 + C_1 dx$$

$$v(x) = \frac{Px^3}{12EI} + C_1x + C_2$$

at $x = 0$, $\delta = v = 0$; $C_2 = 0$

at $x = L/2$, $v' = 0$; $C_2 = -\frac{P(L/2)^2}{4EI} = \frac{-PL^2}{16EI}$

$$\boxed{v(x) = \frac{P}{EI}\left[\frac{x^3}{12} - \frac{L^2}{16}\right]}$$

$$v'(x) = \frac{Px^2}{4EI} - \frac{PL^2}{16EI} \quad \text{at A, } x = 0$$

$$v'(x) = -\frac{PL^2}{16EI}$$

$$\boxed{\text{slope at A: } v = -\frac{PL^2}{16EI}}$$

v_{max} occurs at $x = L/2$

$$v(L/2) = \frac{P}{EI}\left[\frac{1}{12}(L/2)^3 - \frac{L^2}{16}\right]$$

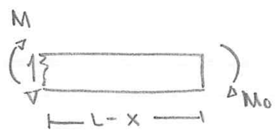
$$= \frac{P}{EI}\left[\frac{L^3}{96} - \frac{L^2}{16}\right]$$

+10

$$\boxed{v_{max} = \frac{PL^2}{EI}\left[L/96 - 1/16\right]}$$

CE 231: HOMEWORK #10

13.



$\Sigma F_x = 0; \Sigma F_y = 0$

$\Sigma M: -M_x = M_o$

$-M_o = EI \dfrac{d^2 v}{dx^2}$

$$\frac{dv}{dx} = \int \frac{-M_o}{EI}\,dx = \frac{-M_o}{EI}x + C_1 \qquad \theta = \frac{-M_o}{EI}x$$

$$v = \int \frac{-M_o x}{EI} + C_1\,dx = \frac{-M_o x^2}{2EI} + C_1 x + C_2$$

at $x=0$, $v=0$, $v'=0$

so, $C_1 = C_2 = 0$

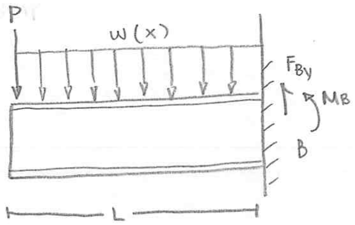
$\boxed{v(x) = \dfrac{M_o}{2EI}x^2}$ downwards

Max slope, deflection occurs at $x=L$

$$\boxed{\begin{array}{l} v_{max} = \dfrac{M_o L^2}{2EI} \text{ down} \\ \theta_{max} = \dfrac{-M_o L}{EI} \end{array}}$$

CE 231: HOMEWORK #10

77.



$w(x) = 5$ kip/ft

$P = 8$ kip

$L = 15$ ft

W24 × 104 beam → $I_{NA} = 3100 \text{ in}^4$

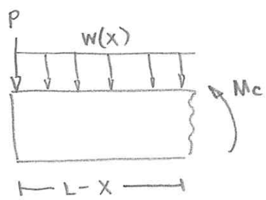
A-36 steel

$E = 29 \times 10^3$ ksi

$\nu = 0.32$

ΣF_y: $F_{By} = w(x) \cdot L + P$

ΣM: $-M_B = w(x) \cdot L \cdot \frac{1}{2} L + P \cdot L$



$$-M_c = P(L-x) + \frac{1}{2} w (L-x)^2$$

$$EI \frac{d^2 v}{dx^2} = M_c = P(x-L) - \frac{w}{2}(L-x)^2$$

$$EI \frac{dv}{dx} = \frac{P}{2}(x-L)^2 + \frac{w}{6}(L-x)^3 + C_1$$

$$EI\, v = \frac{P}{6}(x-L)^3 - \frac{w}{24}(L-x)^4 + C_1 x + C_2$$

$\frac{dv}{dx} = 0$ at $x = 0$

$$C_1 = \frac{-P}{2}(-L)^2 - \frac{w}{6}(L)^3$$

$v = 0$ at $x = 0$

$$C_2 = \frac{-P}{6}(-L)^3 + \frac{w}{24}(L)^4$$

$$v(x) = \frac{1}{EI}\left[\frac{P}{6}(x-L)^3 - \frac{w}{24}(L-x)^4 - \frac{Px}{2}(L)^2 - \frac{wx}{6}(L)^3 - \frac{P}{6}(-L)^3 + \frac{w}{24}(L)^4\right]$$

at $x = 15$ ft:

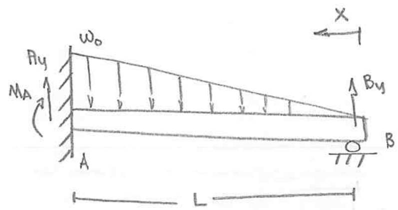
$$v = \frac{1}{EI}\left[\frac{-8\text{kip}}{2}(15\text{ft})^3 - \frac{5\text{kip/ft}}{6}(15\text{ft})^4 - \frac{8\text{kip}}{6}(-15\text{ft})^3 + \frac{5\text{kip/ft}}{24}(15\text{ft})^4\right] \cdot \left(\frac{12\text{ in}}{1\text{ ft}}\right)^3$$

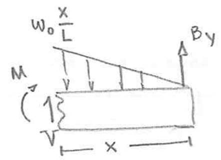
$$v = \frac{-7.023 \times 10^7 \text{ in}^3}{(29 \times 10^3 \text{ksi})(3100 \text{in}^4)} = -0.781 \text{ in}$$

$$\boxed{v_A = 0.781 \text{ in down}}$$

CE 231: HOMEWORK #10

95.





$\Sigma F_y: V + B_y = \frac{1}{2} w_o \frac{x^2}{L}$

$\Sigma M: \quad M = B_y \cdot x - \frac{1}{6} w_o \frac{x^3}{L}$

$$M = \frac{d^2v}{dx^2} EI = B_y x - \frac{1}{6} w_o \frac{x^3}{L}$$

$$EI \frac{dv}{dx} = \frac{1}{2} B_y x^2 - \frac{1}{24} w_o \frac{x^4}{L} + C_1$$

$$EI v = \frac{1}{6} B_y x^3 - \frac{1}{120} w_o \frac{x^5}{L} + C_1 x + C_2$$

CONSTANTS:

$C_2 = 0$ because $v = 0$ at $x = 0$

at $x = L$, $v = 0$, $\theta = 0$

$$\begin{cases} 0 = \frac{1}{2} B_y L^2 - \frac{1}{24} w_o \frac{L^4}{L} + C_1 \\ 0 = \frac{1}{6} B_y L^3 - \frac{1}{120} w_o \frac{L^5}{L} + C_1 \cdot L \end{cases}$$

Two equations, two unknowns...

$$C_1 = \frac{w_o L^3}{24} - \frac{B_y L^2}{2}$$

$$C_1 = \frac{w_o L^3}{120} - \frac{B_y L^2}{6}$$

$B_y = \frac{1}{10} w_o L$

$C_1 = -\frac{1}{120} w_o L^3$

USING EQUILIBRIUM:

$A_y + B_y = \frac{1}{2} w_o L$

$A_y = \frac{2}{5} w_o L$

$M_A = B_y \cdot L - \frac{1}{6} w_o L^2$

$M_A = \frac{-1}{15} w_o L^2$

+10

$$\boxed{\begin{array}{l} A_y = 2/5\, w_o L \\ M_A = 1/15\, w_o L^2 \circlearrowleft \\ A_x = 0 \\ B_y = 1/10\, w_o L \end{array}}$$

CE 231: Strength of Materials | **Mini-Project**
Spring 2003 | **Due: Monday, 4/21/03, 5:00 PM**

The following schematic is an example of micro-electro-mechanical (MEMS) device that is currently being developed by a colleague in the ECE Department and being tested in the Advanced Materials and Structures Laboratory in Civil Engineering. The goal is to determine whether or not the fabrication procedure introduces residual stresses in the device. Measuring the deflection of the device under mechanical load provides one piece of evidence that can be used towards this end. Your assignment is to model the response of the device to an applied load. The deflection of the load point (i.e. where the load is applied) needs to be determined for loading at any location on the center span of the device, i.e. 100 μm < x < 230 μm. (Put another way, we need to know the load-displacement relationship for any point in the center section). The device is made of gold. A preliminary test revealed the load-displacement behavior shown in the schematic on the next page.

You may use the following relationships to develop expressions for the deflection as a function of position along the beam, which is denoted as $v(x)$;

$$EIw'' = EI\frac{d^2v}{dx^2} = M(x), \qquad EI\frac{d^3v}{dx^3} = V(x), \qquad EI\frac{d^4v}{dx^4} = -w(x)$$

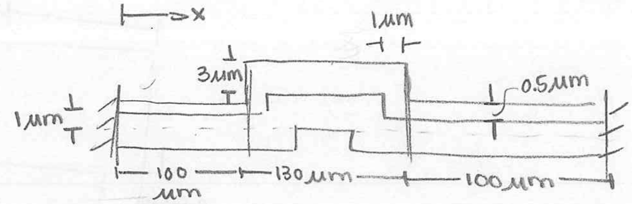
The first of these expressions comes from relating the curvature of the beam to the strain distribution; the second two follow naturally from the first via equilibrium, as discussed in class.

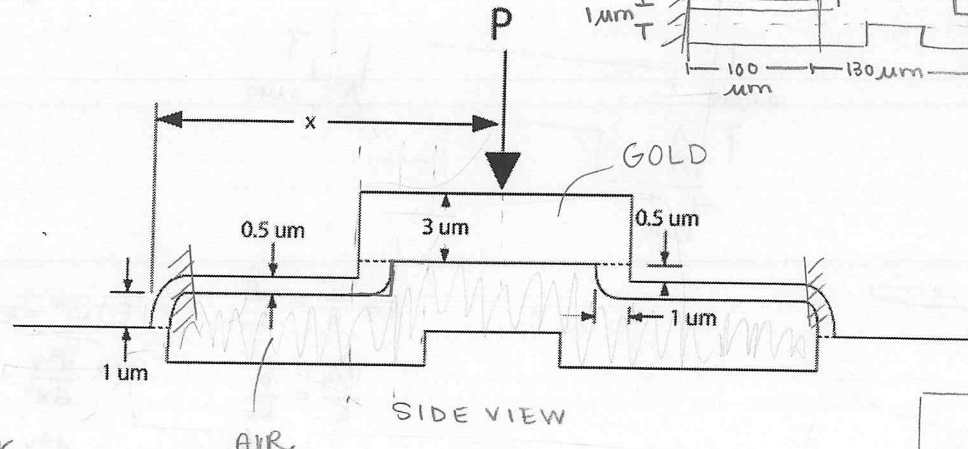
While the deflection of beams has not been covered yet (it's in Chapter 12 if you wish to get a leg up), it can be solved for simply by twice integrating an equation for the resultant moment as a function of position. The integration constants are determine from the boundary conditions (i.e. the deflection and its derivatives at various locations along the beam). ***IMPORTANT: there is no single answer to this!*** This will involve your engineering judgment as to how the physical reality can be approximated. Feel free to discuss it with us (Prof. Begley and Kyle), and more importantly, with each other. Feel free to come up with multiple models/assumptions, and check them against experiment.

On that note, there will be a sign-up sheet outside Prof. Begley's door for groups to observe (and possibly conduct) experiments on the devices you are modeling. It is not required, but all are welcome (provided you sign up in advance). To get the most out of your lab experience, you should have (at the very least), prelimary models of the beam worked out to give you an idea of the forces, displacements and response you will be looking for.

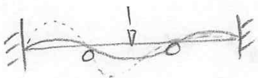
NOTE:

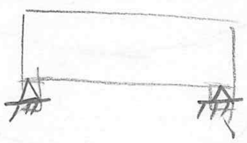
Unfortunately, this will be the only mini-project, due to time constraints. This project will comprise 6.5% of the final course grade, or roughly the equivalent of two homework assignments, which we anticipate will be worth ~ 10x2.35 = 23.5%.

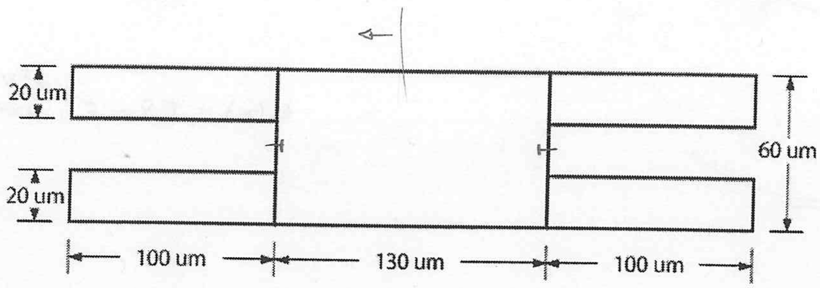




SIDE VIEW



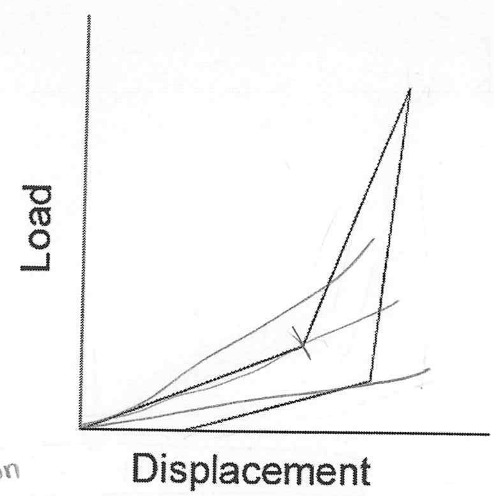




TOP VIEW

less than 1/2 way

• two pieces?



2 paragraph writeup
+ a graph
- load displacement curve (exp.)
- load from our eq. in terms of x, sub in some value

- 2 microns deep
- oscillates at 40 → 35

1st loading w/ oscillation

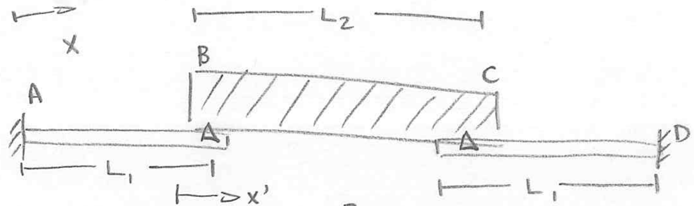
or 40?

$\frac{1}{12}(20)(0.5)^3 = \frac{1}{48}$

$\frac{1}{12}(60)(3)^3 = \frac{27}{2}$

$\frac{\cancel{27}}{2 \cdot 48}$

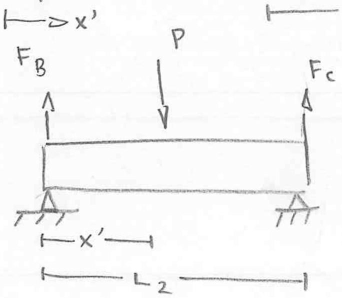
$27 \cdot 24 = \cancel{654}\ 648$ or $\cancel{327}\ 324$



ASSUMING THICK SECTION IS RIGID.

$L_1 = 100\ \mu m$

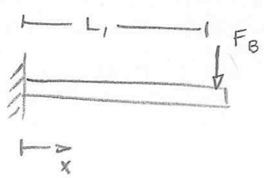
$L_2 = 130\ \mu m$

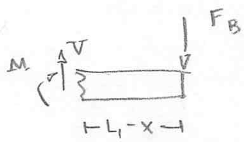


$F_B + F_c = P$

$F_c(L_2) = P(x')$

$F_B + \dfrac{Px'}{L_2} = P \qquad \underline{F_B = P\left(1 - \dfrac{x'}{L_2}\right)}$





$M = F_B(L_1 - x) = EI\,w''$

$w'' = \dfrac{F_B(L_1 - x)}{EI}$

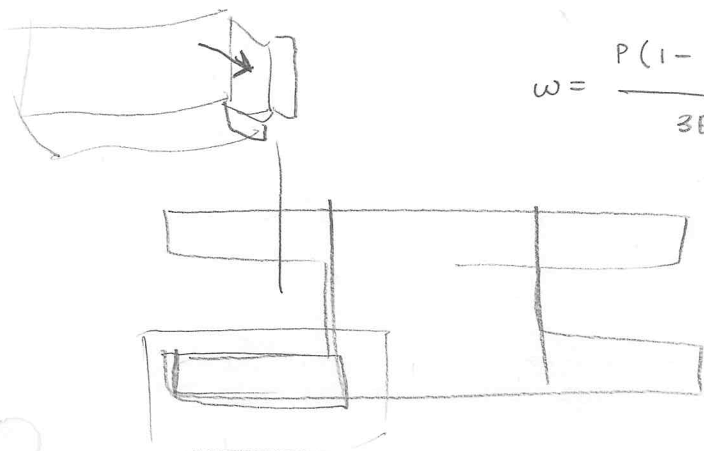
$w' = \dfrac{F_B L_1 x}{EI} - \dfrac{F_B x^2}{2EI} + c \qquad w'(x=0)=0,\ C=0$

$w = \dfrac{F_B L_1 x^2}{2EI} - \dfrac{F_B x^3}{6EI} + \cancel{cx} + \cancel{D} \qquad w(x=0)=0,\ D=0$

$w = \dfrac{F_B L_1 x^2}{2EI} - \dfrac{F_B x^3}{6EI}$ at $x = L_1$...

$w = \dfrac{F_B L_1^3}{3EI}$

$\dfrac{21}{49}$



$w = \dfrac{P\left(1 - \dfrac{x'}{L_2}\right)L_1^3}{3EI} = \boxed{\dfrac{PL_1^3}{3EI}\left[1 - \dfrac{(x - L_1)}{L_2}\right] = w(x)}$

$\frac{1}{12}(20\,\mu m)(0.5\,\mu m)^3 = I$

Assume:

- rigid body
- pins
- wall (cantelever)
- center y-dir.

53/60

CE 231: STRENGTH OF MATERIALS
MINI-PROJECT

James Hayne
Catherine Hovell

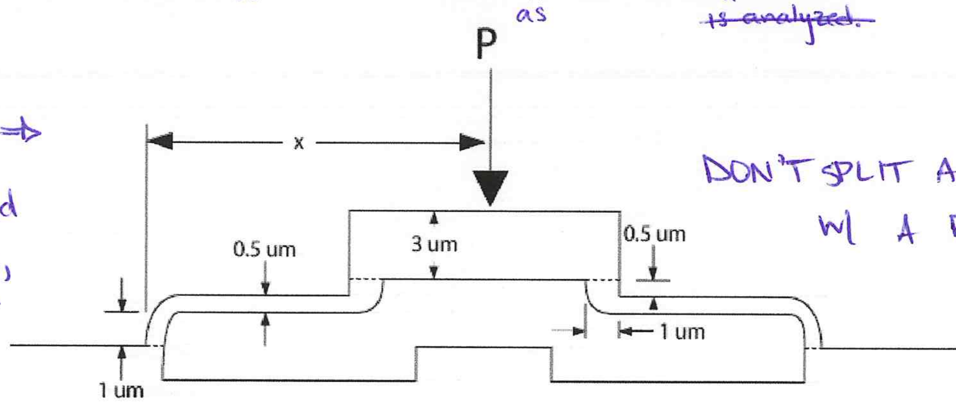
April 23, 2003

regardless such as

~~There are~~ multiple factors ~~that~~ determine the amount of deflection of a beam regardless of ~~any~~ size, such as ~~The most important are the~~ placement and amount of applied load, the material of the beam ~~is made of~~, and the geometry of the beam. ~~In the situation analyzed here, there is a free-standing film of gold (modulus of elasticity, E, of 78 GPa) with the cross-sectional geometry shown in Figure 1.~~

your opinion

a ~~gold film, E~78 GPa,~~ as ~~is analyzed.~~



actually, condense → A freestanding gold film, E~78 GPa, is suspended over a gold substrate w/ a cross-sectional geometry as shown in Figure 1.

DON'T SPLIT A PARAGRAPH W/ A FIGURE!!!

Figure 1: Thin Gold film, side view

thin & film is somewhat redundant.

i.e. - omit needless words.

~~In Figure 1, it should be noted that within the lower block is air, and the rest of the object is made of gold.~~ ~~The structure is composed of a freestanding gold film suspended above a gold substrate.~~ ~~Of the three key factors of deformation in the film, it is already known that the material is gold. Next, the geometry should be considered.~~

The geometry of the film leads to various simplifications

~~As seen in Figure 1, the shape of the film is not one that leads to an obvious guess of how a load would deflect the film. There are many different ways to simplify this geometry, each of~~ which can be compared to ~~tested against~~ experimental data and considered for accuracy. The simplification chosen here ~~The method detailed in this analysis~~ made three key assumptions: (1) ~~first,~~ the vertical sections of gold at either end are ~~could be~~ considered to be insignificant, and a comparable model would have either end & the ends are fixed into a solid wall; (2) ~~Secondly,~~ the height of the top horizontal section is so great in comparison with that of the smaller pieces on the sides (3 μm vs. 0.5 μm) that it can be considered to be a rigid beam – all deflection across the film comes in the thin sections on either side. This assumption can be justified by realizing that deflection in a beam is inversely related to the moment of inertia, I, of the beam. For a rectangular section,

we do not "guess" as engineers - we estimate using good judgement

w/ your 3 key assumptions - state them all briefly ping... ; (1) ... ; (2) ... ; ...

$$I_{rect} = \frac{1}{12} \cdot b \cdot h^3$$

where b is the width of the section (the depth into the page, in this case), and h is the height. So, in comparing the simple deflection of two beams of heights 3 μm and 0.5 μm, it can be seen that the larger

Then, if you want, have a paragraph devoted to detailing each assumption.

section deflects not just 1/6th of the amount of the smaller one, but $1/6^3$, or 1/216th times less, assuming equal depths into the page. However, looking at the film from the top, as shown in Figure 2, it should be noted that the two sections do not have the same depth.

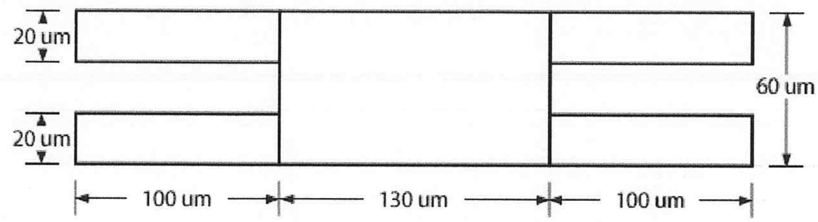


Figure 2: Top view of gold film

To clarify, the thinner section of the film is from x = 0 μm to x = 100 μm, and the thicker section is from x = 100 μm to x = 230 μm. From Figure 2, it can be seen that the thicker section has a value for depth of b = 60 μm, whereas for the thinner section, b = 20 μm (although the load is distributed across two "legs" of the film on each side, only one is considered at a time). Taking the different values for b into consideration as well as the different heights of the sections, it can be seen that the thicker section has a moment of inertia around 650 times as large as the thinner section. Remembering the inverse relationship between deflection and moment of inertia, this justifies the assumption that the top section of the film can be thought to be rigid.

– this isn't right, the way you did it, $I = 2\left[\frac{(20)(.5)^3}{12}\right]$ or $I = \frac{40(.5)^3}{12}$

The last assumption considers the intersection between the thin and thick parts of the film. In Figure 1, it is shown that the larger section rests on the smaller sections, overlapping by 1 μm on each side. In considering that the entire section is 130 μm, this overlap is very small. The third assumption made is that the larger section is held up by the ends of the smaller sections as if resting on a pin. Any upward reaction forces at the pins at either end of the larger beam due to the applied load will result in downward forces on the ends of the smaller beams and cause deflection. ~~Keeping these three assumptions in mind, the film can be redrawn, as shown in Figure 3.~~

Apply the given assumptions, the film can be drawn as shown in Figure 3.

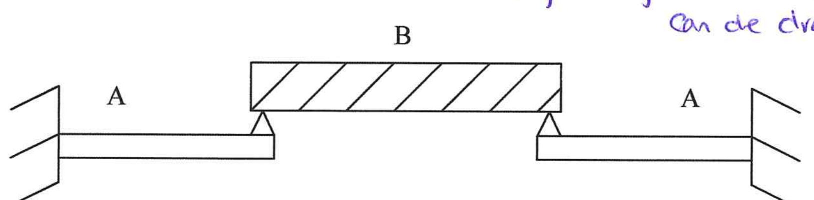


Figure 3: Simplified version of the film.

To determine the

~~Now, the expected~~ deflection in one of the smaller sections (labeled A on Figure 3) can be calculated as a function of where the load is applied within the larger section (labeled B). This is done by finding the reaction forces at either of the pins when a load, P, is applied, as a function of distance from the pin. This can be written as

reword. (too wordy)

I could be wrong but why shouldn't the force @ each pin simply be P/4? – nevermind, that's assuming load is in th

& center this

$$F_{pin} = \frac{P}{2}\cdot\left(1-\frac{x_{pin}}{L_B}\right)$$

I know I'm picky (we should know where I've learned this from) but get away from MathCad. Use it for calculations, fine, but for a technical paper, use the equation editor.

where x_{pin} is the distance from the pin to the location of the applied load, and L_B is the entire length of section B, or 130 μm. This force, F_{pin}, is equal in magnitude to the applied force on section A that is then responsible for any deflection in the film.

The deflection of section A can be calculated using the expressions for the deflection of a beam as a function of the position along the beam, and by knowing boundary conditions. The relationships are

$$E\cdot I\cdot v'' = E\cdot I\cdot\frac{d^2v}{dx^2} = M(x), E\cdot I\cdot\frac{d^3v}{dx^3} = V(x), E\cdot I\cdot\frac{d^4v}{dx^4} = -w(x)$$

and it is known that at the wall (x = 0), the deflection, v, and the slope, v', both equal zero. This leads to the general equation for the deflection of the beam of

unless grouping 2 together

$$v = \frac{F_{pin}\cdot L_A^{\,3}}{3\cdot E\cdot I}$$

Substituting in the F_{pin} found above, as well as changing the x_{pin} to $(x - L_A)$, since x is measured from the wall results in a specific equation for the deflection of the film in terms of the magnitude and placement of the applied load:

$$v(x) = \delta(x) = \frac{P\cdot L_A^{\,2}}{3\cdot E\cdot I}\cdot\left[1-\frac{(x-L_A)}{L_B}\right]$$

BAD math somewhere this is untless!!

It was initially stated that the location and magnitude of the load were important in determining deflection within the film, and this is shown in that equation. The problem is that, because of the small scale of the apparatus, the placement of the load is not always exactly known. The goal is to indent at the middle, or x = 165 μm, but this is not always possible. For one, the location of indentation is only as close to the middle as the tester can make it by eyeing the location on a computer screen. As well, due to the accuracy limitations within the apparatus, the actual location of indentation may not occur at the point specified by the experimenter. For these reasons, the value of x is not precisely known. It was noted, upon running this trial, that the location of the indentation was at a point less than halfway across the entire film; x < 165 μm.

The data gathered in the laboratory test was plotted as load vs. displacement, and is shown in Figure 3. Also included in this graph are two theoretical load-displacement relationships. As marked, one uses a value of x = 165 μm, and one uses x = 160 μm. A perfect theoretical relationship would end up mirroring the section of the experimental load-displacement curve from 300 to 1300 nm.

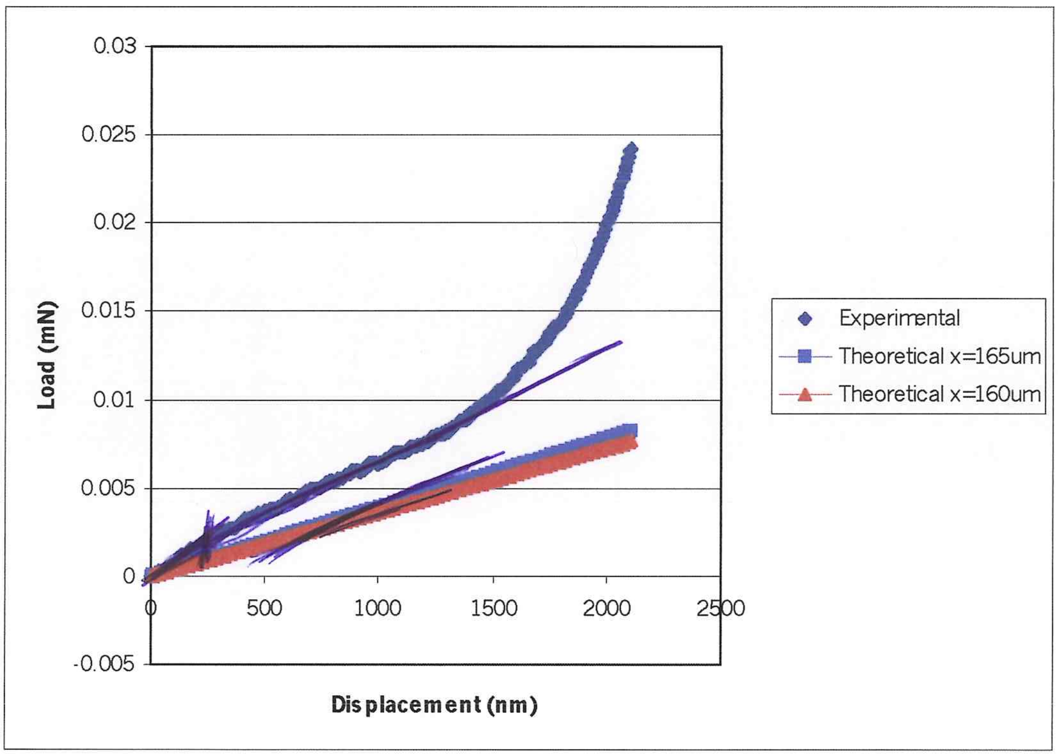


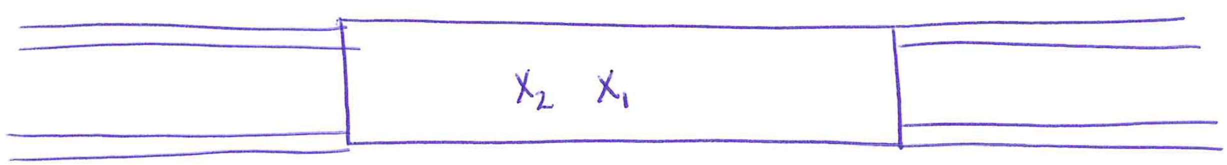
Figure 4: Load vs. Displacement, experimental and theoretical

The theoretical and experimental curves do not line up exactly because of the different behavior of the gold at small deflection values. This variation could be caused by many things; the most likely cause is in limitations or uncertainties in the testing apparatus, such as tip calibration and the possibility that at first contact, the tip bites into the gold film, indenting the surface but not actually causing deflection of gold. This would result in a different relationship between load and displacement as opposed to when an increase in load causes pure deflection as opposed to deflection and penetration into the surface. An upper limit to the load-displacement relationship should be considered because there is a point at which increasing the load will not increase the displacement.

An accurate theoretical relationship should have a slope equal to that of the experimental data within the range mentioned above. In Figure 4, two theoretical plots are made: one is at the center point, or the intended location of indentation. As noted, the actual location of the indent was at an x-value less than that, and so a second plot was made at x = 160 μm. The slopes of these two plots can be compared to the experimental data. If the theoretical curve were set with a different y-intercept, the theoretical plots should line up well with the experimental data. However, practically, the load needed for zero displacement is zero, so the theoretical line must pass through the origin to be correct at boundary conditions.

Developing a relationship between load and displacement will depend on the specifics of the experiment run. The general equation, given above, for δ(x), can be used to write an equation for load as a function of displacement for any given x. Therefore, if experimental data is plotted, theoretical graphs can be compared to the data with various values for x. The theoretical relationship that best approximates the slope of the experimental data is the one that incorporates the most accurate value for x.

1st of all heres where we wanted to indent (X_1)



after indentation, the microscope showed the crosshairs @ X_2 → indentation can be anywhere from $X_2 \leq X \leq X_1$ so indentation could have been done @ X_1.

- equations for your 2 theoretical lines as well as linear portion of experimental line could be useful.

C. Hovell

Strength of Materials January 16, 2003

Three keys to Solid Mechanics

1. Equilibrium

sum of forces, sum of moments

no violations of physical laws in your work!

2. Material

3. Strains and displacements

statically indeterminant problems

→ more variables than equations

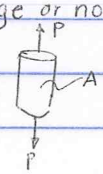
Stress, a formal, philosophical discussion of stress January 21, 2003

$$\text{stress} = \frac{\text{load}}{\text{area}} = \sigma$$

(physical)

(emotional stress: $\frac{\#\text{ things to do}}{\text{amt. of time}}$)

average or nominal stress:



non-uniform stress at the top

St. Venant's Principle:

stresses/strains far from load point/geometry change are

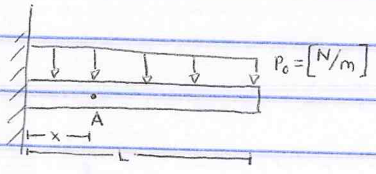
uniform = σ_{ave}

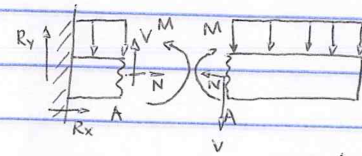
C. Hovell

MMM... STRENGTH OF MATERIALS

January 21, 2003

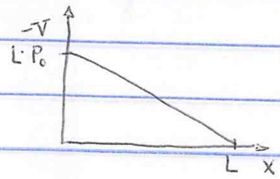
Equilibrium and Internal forces





$\Sigma F_x = \Sigma F_y = \Sigma M = 0$

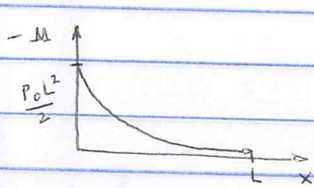
$N = 0$ (resultant force - tension and compresion spread out and eventually cancel)



$-V = P_o(L-x)$

$\boxed{V = -P_o(L-x)}$

(all that is needed for a shear diagram)



$\Sigma M_A = 0 = -M - \frac{1}{2} P_o (L-x)^2$

$\boxed{M = -\frac{1}{2} P_o (L-x)^2}$

M is integration of V!

negative signs just say that we guessed wrong.

$\boxed{\frac{dM}{dx} = -V(x)}$

an equilibrium statement

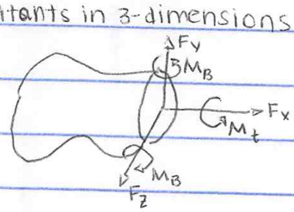
Three Keys:

1. Equilibrium
2. Material
3. Strains and displacements

"The Method of Sections"

1. cut at point, draw free body force diagram
2. applied equilibrium

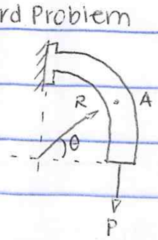
Resultants in 3-dimensions

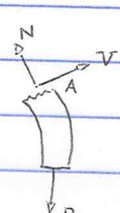


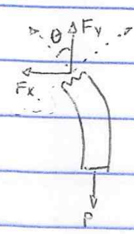
M_t = torsional moment (anti-plane shear)

M_B = bending moments (in-plane)

Hard Problem







$F_x = 0 \quad F_y = P$

$V = \cancel{\frac{P}{\sin\theta}} \quad \cancel{N = \frac{P}{\cos\theta}}$

$V = P\sin\theta \quad N = P\cos\theta$

check: $@\theta = 0°$, $V = 0$, $N = P$

C. Hovell

Materials are cool

January 23, 2003

Average / Nominal Stress

$\sigma_{ave} = \frac{P}{A}$ $[N/m^2]$ or $[lb/in^2]$ $\to$ psi

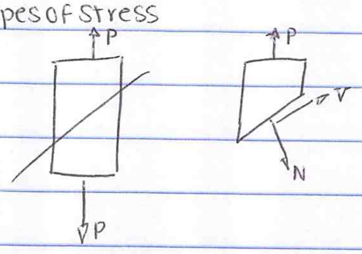
$\sigma_{atm} = 14.7$ psi

$\sigma = \lim_{A \to 0} \frac{dF}{dA}$ stress can act at a point!

$\sigma = f(x)$ ... distribution

$F = t \cdot w \cdot \sigma_{ave}$

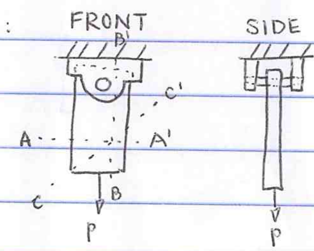
Types of Stress



shear stress $\tau = \frac{V}{A}$

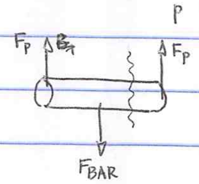
normal / direct stress $\sigma = P/A$

Example:



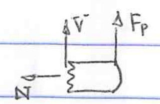
① what is the stress in the pin?

② what are the average stresses in the bar (a) in the plane A-A' (b) B-B' (c) C-C'



$F_p = \frac{1}{2} F_B$

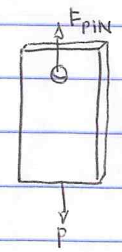
$F_p = P/2$



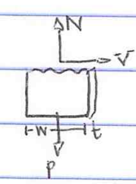
$V = -P/2$ $N = 0$

① $\tau = \frac{P}{2A}$ $\sigma = 0$, so pin will shear if F is too great

"double shear pin"



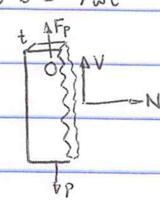
$F_{pin} = P$



$N = P$ $V = 0$

$F = t \cdot w \cdot \sigma_{ave}$

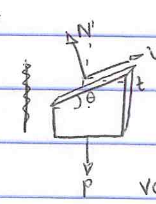
$\tau = 0$ $\sigma = P/wt$



$N = 0$

$V = P - F_p = 0$

$\sigma = 0$ $\tau = 0$



$V\cos\theta = N\sin\theta$ $N\cos\theta = P - V\sin\theta$

$V\cos\theta \cdot \tan\theta = P - V\sin\theta$

$V\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = P$ $V = P/\sqrt{2} = N$

$\tau = \frac{P}{\sqrt{2}\, t \cdot w \cdot \sqrt{2}}$ $\sigma = \frac{P}{\sqrt{2}\, t \cdot w \cdot \sqrt{2}}$

$w = w\sqrt{2}$

② $\boxed{\tau = \frac{P}{2t \cdot w} \quad \sigma = \frac{P}{2t \cdot w}}$

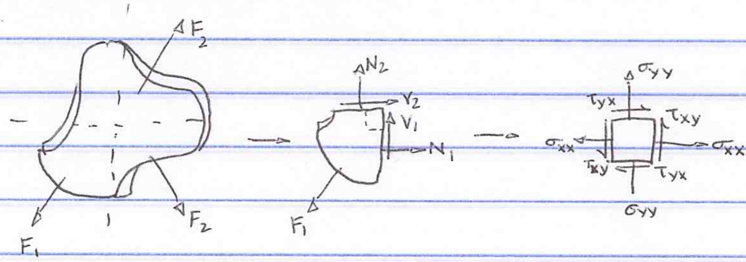
	σ	τ
A-A'	$P/\omega t$	0
B-B'	0	0
C-C'	$P/2\omega t$	$P/2\omega t$

C.Hovell

Materials

January 23, 2003

Two-dimensional states of stress



notation:

1: direction of normal of face

2: direction of force

$\tau_{xy} = \tau_{yx}$ because if they weren't equal, block would rotate!

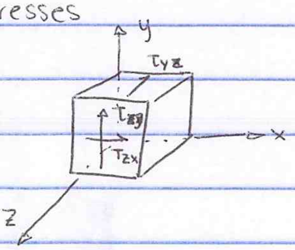
$\sigma_{xx}, \sigma_{yy}, \tau_{xy}$

C. Hovell

More Stuff

January 28, 2003

3D Stresses



$\tau_{zy} = \tau_{yz}$

$\tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

Deformation (and strain)

- change in length

  displacement = $L_{new} - L_{orig}$

- % change in length = strain

$\boxed{\text{strain: } (L_{new} - L_{orig}) / L_{orig} \cdot 100}$

$\varepsilon = \frac{\delta}{L_o}$

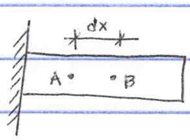
unitless! dimensionless!

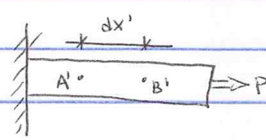
ex. $\varepsilon = 0.002$ or $\varepsilon = 0.002 \frac{mm}{mm}$

→ allowable strain of steel

very small number - once you see change, it'll fail

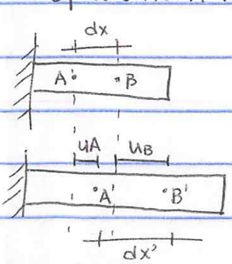
structural strains are small





$\varepsilon_{ave} = \frac{dx' - dx}{dx}$ strain at a point: $\lim_{dx \to 0} \frac{dx' - dx}{dx}$

strain-displacement relationship



$dx = x_B - x_A$

$dx' = dx + du$ du is difference in displacements of A-B = $u_B - u_A$

so... $\varepsilon = \frac{du}{dx}$ ... derivitive of displacement

$\boxed{\varepsilon(x) = \frac{d}{dx} u(x)}$

if everything moves the same distance,

there is NO STRAIN!

C. Hovell

WORK STINKS

January 28, 2003

Stress, strain, Displacement

$\varepsilon = \frac{du}{dx}$ so $u(x) = Ax + B$

uniform strain

$\Delta L = \varepsilon L_0$ $\left(\frac{L}{L_0}\right) = 1 + \varepsilon$

NOTE:

$\varepsilon < 0$ → compressive strain

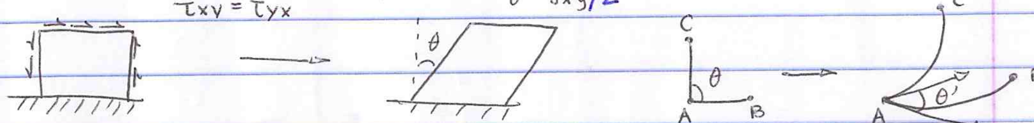
$\varepsilon > 0$ → tensile strain

$\varepsilon < 1$, because this only works for SMALL deformation!

Shear strain

$\tau_{xy} = \tau_{yx}$

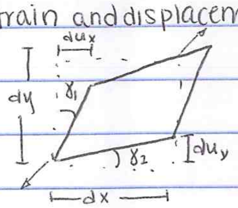
$\theta = \gamma_{xy}/2$



$\gamma = \lim_{\substack{|AB|, \\ |AC| \to 0}} (\theta - \theta')$

(see text: 2.2)

shear strain and displacements



"dat box been sheared."

shear strain = change in angle of the box

... but there are two! ADD THEM.

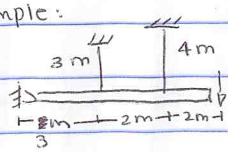
$\gamma_T = \gamma_1 + \gamma_2$

$\gamma_1 = \tan^{-1}\left(\frac{du_x}{dy}\right)$ $\gamma_2 = \tan^{-1}\left(\frac{du_y}{dx}\right)$

for small γs... $= \frac{du_x}{dy}$ $= \frac{du_y}{dx}$

so. $\boxed{\gamma_T = \frac{du_x}{dy} + \frac{du_y}{dx}}$

Example:

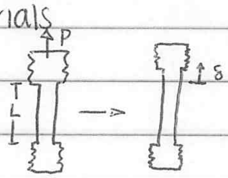


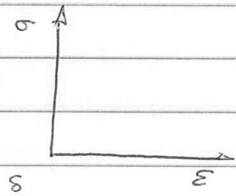
what is δ_{end} if $\varepsilon_{max} = 0.002$?

C. Hovell

Damn. More snow. January 30, 2003

Materials

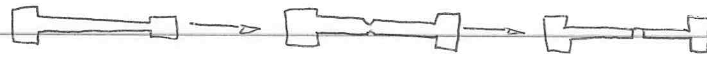




micron = 1 μm

$\sigma = P/A \quad \varepsilon = \frac{\delta}{L}$

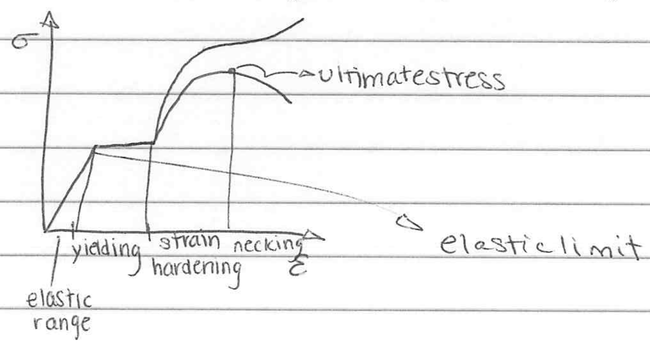
failure of ductile materials

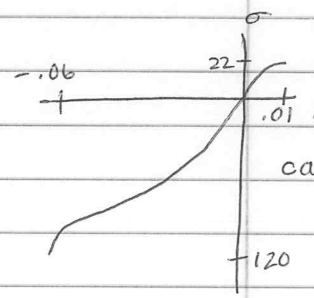


"necking fracture"

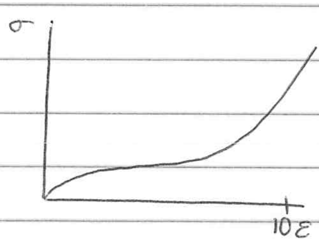
purely a function of material

add tiny bits of impurities, change properties drastically

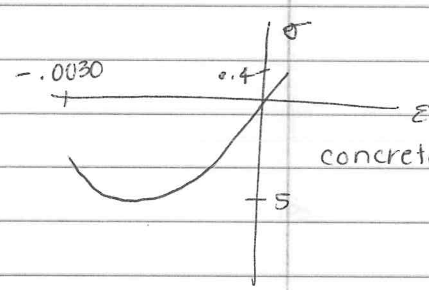




cast iron



natural ~~[illegible]~~ rubber!



concrete

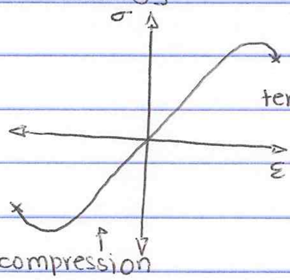
C. Hovell

I'm so sleepy February 4, 2003

Different materials

- metals (stiff)
- plastic / rubber (compliant)
- concrete (compliant) (!!)

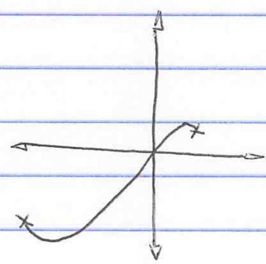
σ - ε terminology



Isotropic material

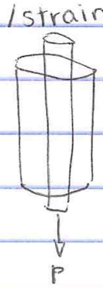
$\sigma(\varepsilon) = -\sigma(-\varepsilon)$

include some metals, some polymers, etc.



anisotropic - eg. concrete

stress / strain measures



d_0 = original diameter

d = current diamerer

Stress

$\sigma_{true} = \frac{P}{A} = \frac{4P}{\pi d^2}$

$\sigma_{eng} = \frac{P}{A_0} = \frac{4P}{\pi d_0^2}$ non-conservative

much easier to use

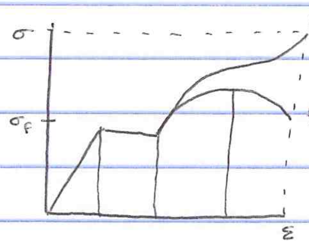
Strain

$\varepsilon_{eng} = \frac{\delta}{L_0}$

$\varepsilon_{true} =$ L_f is final length

$\hookrightarrow \ln\left(\frac{\ell_f}{\ell_0}\right) = \ln(1+\varepsilon_{eng})$

$= \varepsilon_{eng} - \frac{1}{2}\varepsilon_{eng}^2 + \frac{1}{3}\varepsilon_{eng}^3$



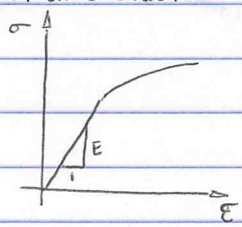
so... concrete... stiff, compliant or purple?

C. Hovell

<u>There's something In My Eye!</u>

February 4, 2003

Small strains: Elastic



$E = \frac{\Delta\sigma}{\Delta\varepsilon}$ (in linear range)

= modulus of elasticity

= Young's Modulus

$E_{steel} = 30 \times 10^6$ psi

$E_{concrete} = 8 \times 10^6$ psi

$\boxed{\sigma = E\varepsilon}$ Hooke's Law

<u>steel</u> vs. <u>concrete</u>

metal is insentive (ε) to composition

$\varepsilon_{conc.}$ <u>is</u> sensitive

Transverse Deformation (small strains)

aluminum is 3x more compliant than steel

DON'T MY PROFESSORS KNOW YET? I DO NOT DO WELL WITH EXAMPLES! HONESTLY. GEEZ.

Poisson's Ratio

$\nu = -\frac{\varepsilon_{trans}}{\varepsilon_{long}}$ property of a material

cork has a negative poisson's ratio – ex. wine bottle corks

Shear Deformation

$\boxed{\tau = G\gamma}$ G = shear modulus, elastic property

if there's stress, ~~therefore~~ there's strain, and visa versa

not really needed...

$\boxed{G = \frac{E}{2(1+\nu)}}$ two constants needed: E, ν

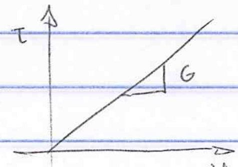
stiffness, ratio

C. Hovell

Hehe-Stump the Professor!

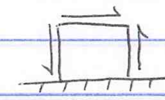
February 6, 2003

Small strains: elastic



$\tau = G \cdot \gamma = \frac{V}{A}$

test by shearing:



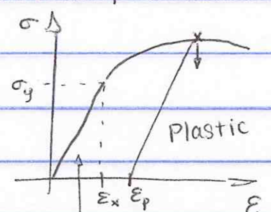
BAD PLAN.

instead:

$\gamma = \frac{dx}{L}$

$\tau = \frac{T}{2\pi R^2 t}$

Large strains: plastic behavior

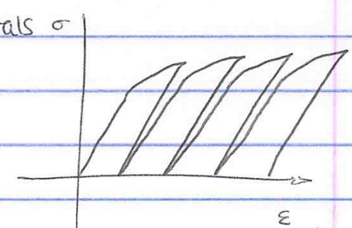


at x, then unload

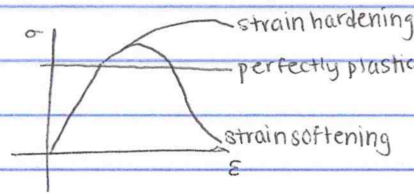
unloads parallel to the elastic section

at zero load, $\varepsilon_p = \varepsilon_{permanet} =$

cyclic loading in metals

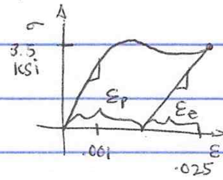


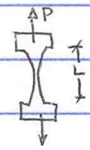
strain hardening



you can change the yield stress, you can't change the modulus (much)

Example like homework:





if strain to 0.024, what is Δl?
↳ then unload

$\Delta l = l_o \cdot \varepsilon_p$

ε_e = elastic strain

$\varepsilon_{tot} = \varepsilon_e + \varepsilon_p = 0.024$

$\varepsilon_e = \varepsilon @ 3.5\ ksi = \frac{\sigma_{ap}}{E}$

$E = 0.5$ Msi (million psi)

$\varepsilon_p = 0.024 - \frac{3.5\ ksi}{0.5\ Msi}$

$\varepsilon_p = 0.0166$

$\Delta l = 10'' \cdot 0.0166$

small E - must be polymer

$\Delta l = 0.17''$

C. Hovell

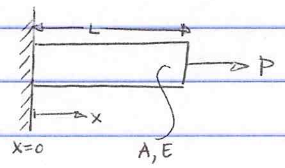
H- ... Yay, Exam.

February 11, 2003

Exam: 7-9 PM wednesday, February 19, 2003

chapters 1-4, no torsion

Last topic - axial loads



- what are stresses along beam?
- what are strains?
- what are displacements?

stress: $\sigma = \frac{P}{A}$ at all points along L

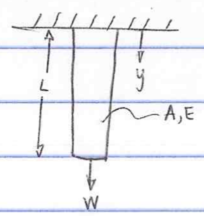
strains: $\varepsilon(x) = \frac{\sigma(x)}{E}$ direct strain $= \frac{P}{AE}$

ε = derivative of displacement with respect to $x = \frac{du}{dx}$

no shear strain cause there's no shear stress

displacement: $\delta(x=L) = L\,\varepsilon(x=L) = \frac{PL}{AE}$ only for uniform strain!

→ second example:



$\sigma_{top} = \frac{\rho L A}{A} = \rho \cdot L$ — weight density

$\sigma_{bottom} = 0$

stress distribution $\sigma(y) = \rho(L-y)$

strain distribution $\varepsilon(y) = \frac{\rho}{E}(L-y)$

displacement distribution:

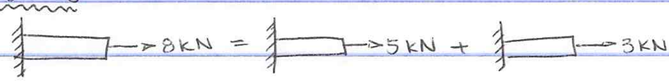
$\int \varepsilon(y)\,dy = u(y)$ $\quad u(y) = \frac{\rho L y}{E} - \frac{\rho y^2}{2E} + \cancel{c} \quad c=0$

or: $\boxed{u(y) = \frac{\rho L^2}{E}\left[(y/L) - \frac{1}{2}(y/L)^2\right]}$

if all weight is at bottom (L=y), displacement is twice as large!

Superposition

if everything is linear, solutions can be added



EVERYTHING MUST BE LINEAR.

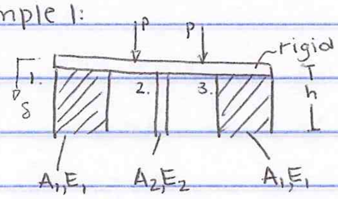
- material response must be linear
  $\sigma = E\varepsilon$ (no plasticity)
- strain displacements must be linear
  $\varepsilon = \frac{du}{dx}$

C. Hovell

Yay. Work on Homework

February 13, 2003

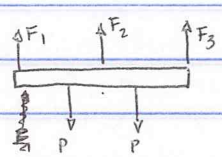
Example 1:



concrete: $A_1 = 0.25\ m^2$ $E_1 = 8\ GPa$

steel: $A_2 = 0.01\ m^2$ $E_2 = 200\ GPa$

what is the % of load carried by each



$\Sigma F_y = 0$

$F_1 + F_2 + F_3 = 2P$

$\Sigma M = 0$

$F_1 l - F_3 l + Px - Px = 0$

$F_1 = F_3$

STATICALLY INDETERMINANT

$\sigma_1 = E_1 \varepsilon_1 \quad \sigma_2 = E_2 \varepsilon_2 \quad \sigma_3 = E_1 \varepsilon_3$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{\delta}{H}$ solve for δ first!

so: $2F_1 + F_2 = 2P \rightarrow \frac{\delta}{H} = \frac{2P}{2A_1E_1 + A_2E_2}$

$F_1 = F_2 = F_3 = \frac{2}{3}P$ all support the same load!

Thermal Stress

$\varepsilon_{total} = \varepsilon_{mech} + \varepsilon_{therm}$

stress is related only to mechanical stress

strain (thermal) is possible without stress

$\boxed{\varepsilon_{th} = \alpha(T - T_0)}$ → reference temperature → current temperature → coefficient of thermal expansion

$\varepsilon_{th} = \alpha \Delta T \quad \delta_{th} = \alpha \Delta T L$

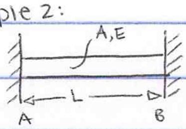
as long as the strain (temp!) is spacially uniform

$\alpha_{ceramic} = 5 \times 10^{-6}\ 1/°C$ (concrete)

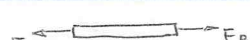
$\alpha_{steel} = 12 \times 10^{-6}\ 1/°C$

$\alpha_{aluminum} = 25 \times 10^{-6}\ 1/°C$

Example 2:



$\varepsilon_{th} = \alpha \Delta T$



$F_A = F_B = F$

$\sigma = \frac{F}{A} = \text{constant}$

$\varepsilon(x)_{tot} = \frac{\sigma}{E} + \alpha \Delta T = 0 \qquad \delta = \int_0^L \varepsilon_{tot}\, dx = 0$

$\sigma = -E\alpha\Delta T$ (-) shows: temp up, compression

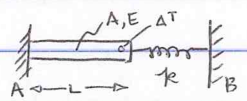
temp down, tension

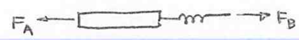
C. Hovell

Whee – Temperature

February 13, 2003

Example 3:





$F_A = F_B$ stress is uniform across bar

$$\varepsilon_{tot} = \varepsilon_{mech} + \varepsilon_{therm} = \frac{\sigma}{E} + \alpha \Delta T = \text{constant}$$

$$\delta_{end} = \int_0^L \varepsilon_{tot}(x)\,dx = \left(\frac{\sigma}{E} + \alpha \Delta T\right) \cdot L$$

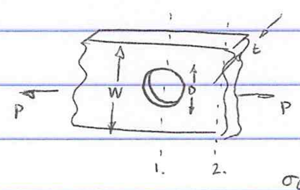
$$F = k \cdot \delta \qquad F = \sigma \cdot A = k \alpha \Delta T \cdot L + \frac{k \sigma L}{E}$$

$$\sigma = \frac{k \cdot \alpha \cdot \Delta T \cdot L}{A - kL/E}$$

Stress concentrations

all geomerry changes (shapes with sharp changes) lead to higher stresses.

the sharper, the worse

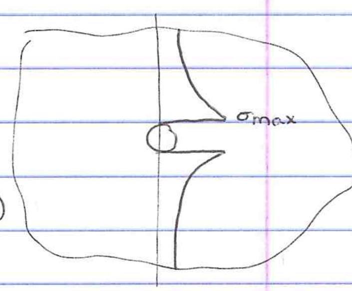


"putting holes in things is not a good thing!"

$$\sigma_{avg_2} = \frac{P}{wt}$$

$$\sigma_{avg_1} = \frac{P}{(w-d)t}$$

$$\sigma_{net} = \sigma_{net}^{hole} = \frac{P}{wt(1-\frac{d}{w})} = \frac{\sigma_{remote}}{(1 - d/w)}$$



K = stress concentration factor $= \frac{\sigma_{max}^{defect}}{\sigma_{ave}} = f\left(\frac{d}{w}\right)$

→ LOOK IT UP.

dimensioning / critical loads

$$\sigma_{yield}: \sigma_y = \sigma_{max} = K \sigma_{ave}^{defect} = K\left(\frac{r}{w}\right) \cdot \frac{P}{(w-2r) \cdot t} = K\left(\frac{r}{w}\right) \cdot \frac{P}{tw} \cdot \left(\frac{1}{1 - 2r/w}\right)$$

→ $K\left(\frac{r}{w}\right)$

$\sigma_{max} > \sigma_y$ drop $\frac{r}{w}$

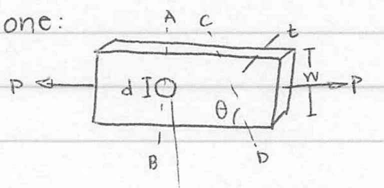
$\sigma_{max} < \sigma_y$ increase $\frac{r}{w}$

C. Hovell

No Homework! February 20, 2002

Exam Review (no, we don't have them back yet)

one:



stress concentration
3x normal stress

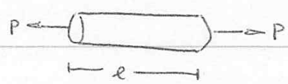
AB:

$A = t(w-d)$ $\sigma = P/t(w-d)$

→ of course, I realize this NOW.

NOTE THIS FOR FINAL!

Two:

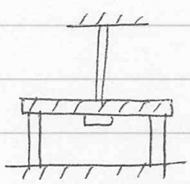


$d_0 = 1\text{ cm}$

$$G = \frac{E}{2(1+\nu)}$$

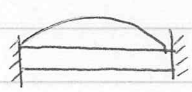
rod elongates and gets bigger!

Three:



AH, DISCREPANCIES. I DON'T CARE!

Four:



$$\Delta T(x) = 150°C\left(x/L - (x/L)^2\right)$$

$R = AE\alpha\Delta T \cdot 1/6$

C. Hovell

Homework!

February 25, 2003

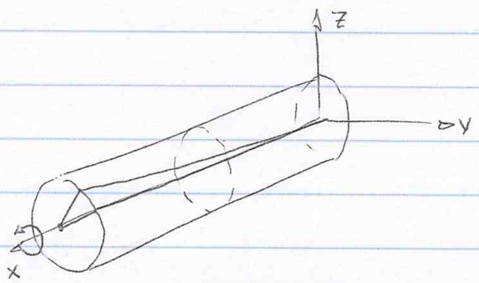
YAY! TORSION PROBLEMS USE POLAR COORDINATES. AIN'T THAT GRAND.

Shear deformation

along length, shear is uniform

along a slice, deformation depends on R

$\varepsilon_{ave} = const$, then $\sigma_{avg} = const.$



twist is like displacement

$\varepsilon = \frac{du}{dx}$

$\gamma = \frac{d\phi}{dx} \cdot \rho$

radial location rho

$\boxed{\gamma = \rho \frac{d\phi}{dx}}$ 3. strain/displacement

shear strain down center of the shaft is zero! hollow shafts are good!

$\cancel{\tau = G\gamma}$ $\boxed{\tau = G\gamma}$ 2. materials

1. equilibrium - how do τ and twist T relate?

shear stress-strain...

$\tau = \frac{T}{2\pi R^2 \cdot t}$ T = force t = thickness

$T = \tau_{max} \cdot 2\pi \left. \frac{\rho^4}{4c} \right|_0^c$

$\frac{d\phi}{dx} = \frac{T(x)}{J(x)G(x)}$

$\boxed{T = \tau_{max} \left(\frac{\pi}{2}\right) c^3}$

$T = \tau_{max} \cdot \frac{1}{c} \underbrace{\int \rho^2 \, dA}_{\text{polar moment of inertia, } J}$

$\boxed{\phi(x) = \frac{TL}{JG}}$ for constant T

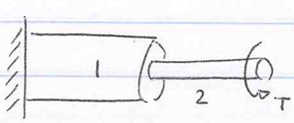
$\left(\delta = \frac{FL}{EA} ?! \right)$

think tapered bar!

$J_{circ} = \frac{\pi c^4}{2}$

$J_{hollow} = \frac{\pi}{2}(c_o^4 - c_i^4)$

YAY. HOMEWORK IS NOW (SUPPOSEDLY) DOABLE. DAMMIT - HE'S STILL GOT AN EXAMPLE!



Find ϕ_1, then ϕ_2, add - SUPERPOSITION!

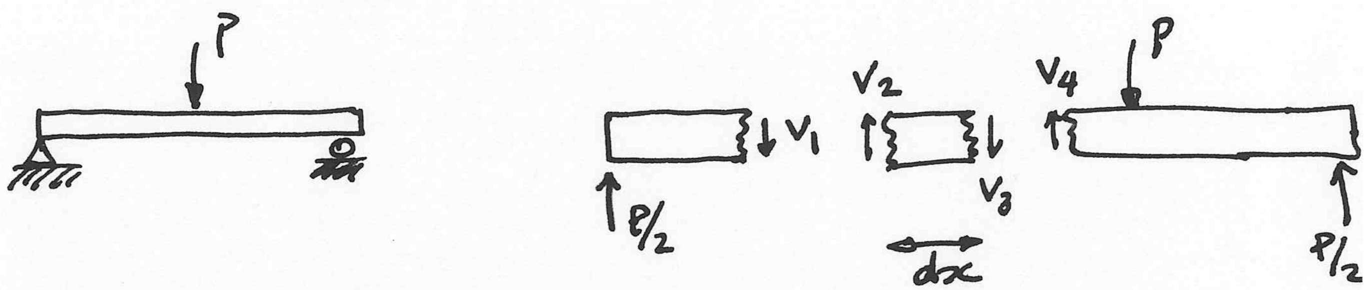
$\phi_{tot} = \sum_{i=1}^{N} \frac{T_i L_i}{J_i G_i}$ yay.

Equilibrium's Differential Element Meets Confusion at the Sign Convention

In class, I created confusion and controversy when I tried to apply the sign convention used for positive internal shear and moment to the differential element (a.k.a. 'little piece') used to derive the equilibrium equations. A quagmire of a debate ensued as to which way I should draw the shear forces to obtain the correct equilibrium equations.

The short answer is this: the sign convention is not relevant when doing a sum of forces (i.e. deriving equilibrium). For equilibrium purposes, I define a positive direction of force - forces going that direction are positive, those going opposite are negative. The sign convention is only useful for distinguishing which direction *balanced* forces are aligned – i.e. the resultant forces/moments at a cut! The sign convention should not be applied when considering a single force – it physically points in a single direction, whose sign is dictated by the positive force/displacement direction.

For example, consider two cuts in a simply-supported beam with a center load, as follows:



The resultant force parallel to the surface denoted as V_1 is a shear force, and obviously physically points in the downward direction; I define this as negative. The resultant force on the mating surface, denoted as V_2, is an equal in magnitude and opposite reaction force and hence is positive. (If V_2 was not equal and opposite, this point would be in motion as the sum of forces at this point would not equal zero.) The internal forces V_1 and V_2 indicate that "internal shear" at this point is positive, by convention. This does not alter the fact that the resultant force acting on the left side of the cut is a negative force, and that acting on the right side of the cut is a positive force.

Equilibrium on the little piece of width *dx* says that the resultant force V_3 should be equal and opposite to V_2, so it acts downwards – i.e. it is a negative force. At the second cut (i.e. mating surfaces with resultants V_3 and V_4), the internal shear is still positive, because it meets the positive sign convention for shear at a point.

So, when drawing the forces acting on a little piece – i.e. a differential element, the internal resultant forces should be drawn consistent with equilibrium (i.e. off-setting), and their sign should be different (to ensure that sum of forces is zero.) When deciding on the sign used to draw on a shear-moment diagram, use the convention given in the book and covered in class – it doesn't affect the sign of *individual resultant forces used to sum forces.*

C. Hovell

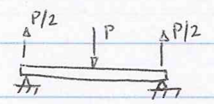
Kyle's Gone Crazy

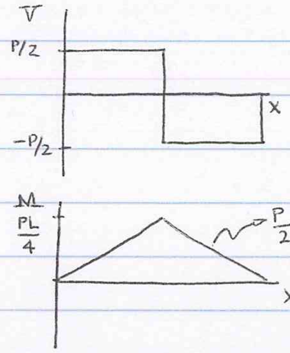
March 11, 2003

Beam Theory (cornerstone of strength of materials)

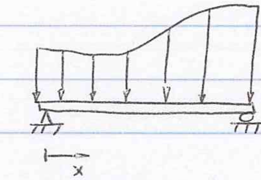
· sign conventions – be consistent

ex.

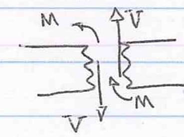




$\frac{P}{2}$ = slope



$\omega(x) = (+)$ acting downwards



$V(x) = (+)$ internal shear

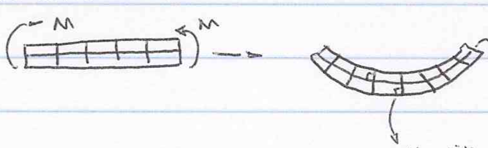
$M(x) = (+)$ internal moment

from ex #2: $\boxed{\frac{dV}{dx} = -w(x)}$ $\boxed{\left(\frac{dM}{dx}\right) = V(x)}$

Always true – based in equilibrium

IMP. PROOF – KNOW!

Beam Deformation



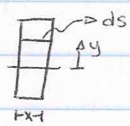
line doesn't change length

NEUTRAL AXIS

all still at right angles

- plane sections remain planar (straight)
- plane sections are ⊥ to the neutral axis
- neutral axis doesn't stretch

Before:

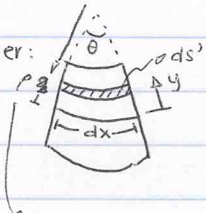


BERNOULLI-EULER BEAM THEORY

valid when $L/h > 12$

otherwise, $L/h < 8$ means short, shear-dominated beam

After:



$\varepsilon_x = \frac{ds' - dx}{dx}$ $\varepsilon = \frac{(\rho - y)\theta - \rho\theta}{\rho\theta} = \boxed{-\frac{y}{\rho} = \varepsilon}$

if it is flat, $\rho = \infty$, $\varepsilon = 0$

deformation variable (relates to displacement)

$\varepsilon = -\varepsilon_{max}\left(\frac{y}{c}\right)$ where c is the y value of the top from the neutral axis

→ goes through the centroid!

$\boxed{\sigma(x,y) = -E \cdot \varepsilon_{max}(x) \cdot \left(\frac{y}{c}\right)}$

C. Hovell

It's Thursday! March 13, 2003

Homework help?

Stress vs. Moment

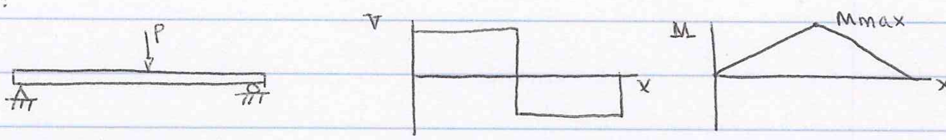
$dM = y\,\sigma\, b\, dy$ b = thickness

$(M_R)_z = \int_{AREA} y\,\sigma\, dy$ c = distance to outside from neutral axis

$(M_R)_z = \frac{\sigma_{max}}{c} \cdot I$ $I = \int_{AREA} y^2\, dA$

$$\boxed{\sigma_{max} = \frac{M_R \cdot c}{I}}$$

example:



$\sigma(x,y) = \frac{M(x) \cdot c}{I}$ FLEXURE FORMULA

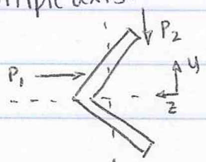
Moment of Inertia (I)

- rectangular beam $\boxed{I = \frac{1}{12} b h^3}$
- circular beam $\boxed{I = \pi/4\, r^4}$ subtract out inner rings, if necessary

Other cases:

~~Flexu~~ Flexure Formula is only good for principal axis of inertia → ~~aligned axis~~ aligned axis of symmetry and perpendicular to it

forces on multiple axis:

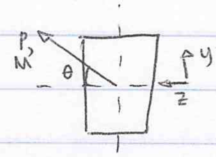


$\sigma_1 = \frac{M_z y}{I_z}$

$\sigma_2 = \frac{M_y z}{I_y}$

$\sigma_{Total} = \frac{M_y z}{I_y} + \left(-\frac{M_z y}{I_z}\right)$ y is (−)

unsymmetric bending (comprised of two parts)



Principal axis: through center of mass, along symmetric line

$M_z = M\cos\theta$

$M_y = M\sin\theta$

neutral axis: stress = 0 − no movement along axis

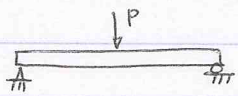
$y(z) = \frac{M_y I_z}{M_z I_y} \cdot z$ $\boxed{\frac{y}{z} = \frac{M_y I_z}{M_z I_y} = \tan\alpha}$ α = orientation of neutral axis off z-axis

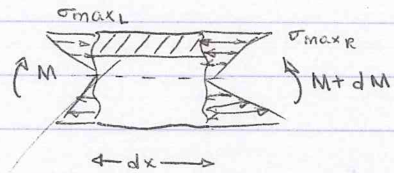
C. Hovell

Damn - I thought class was over ;)

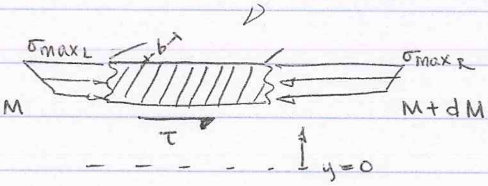
March 13, 2003

Transverse Shear





σ_{max} on each side could be different



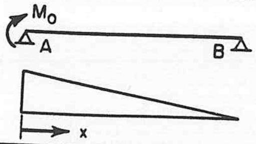
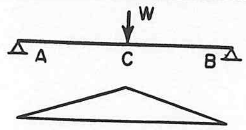
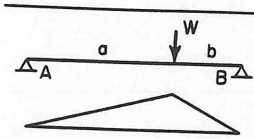
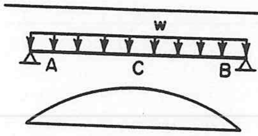
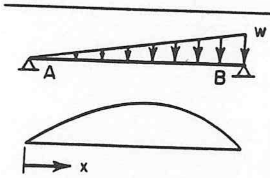
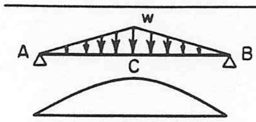
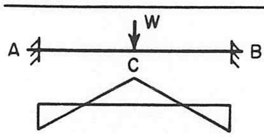
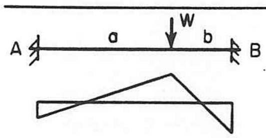
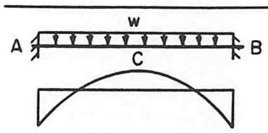
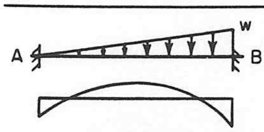
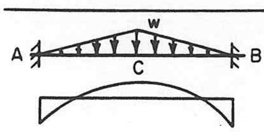
$$\Sigma F_x : \int \sigma_L \, dA - \int \sigma_R \, dA + \tau \cdot b \, dx = 0$$

$$\frac{dM}{I} \int y \, dA = \tau \cdot b \, dx$$

$$\tau = \frac{1}{I \cdot b} \underbrace{\left(\frac{dM}{dx} \right)}_{V} \underbrace{\int_{piece} y \, dA}_{Q}$$

$$\boxed{\tau = \frac{V \cdot Q}{I \cdot b}}$$

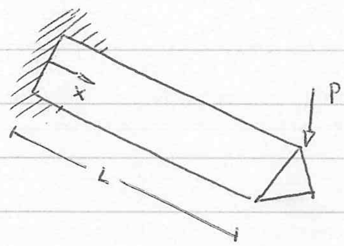
Table 4. Formulas for Beams

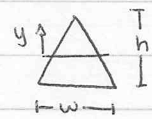
Structure	Shear	Moment	Slope	Deflection
Simply supported beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_oL}{3EI}$ $\theta_B = -\frac{M_oL}{6EI}$	$y_{max} = 0.062\frac{M_oL^2}{EI}$ at $x = 0.422L$
	$S_A = \frac{W}{2}$	$M_C = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$y_C = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = -\frac{Wa}{L}$	$M_a = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6L}(L+b)$ $\theta_B = -\frac{Wab}{6L}(L+a)$	$y_a = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{wL}{2}$	$M_C = \frac{wL^2}{8}$	$\theta_A = -\theta_B = \frac{wL^3}{24EI}$	$y_C = \frac{5wL^4}{384EI}$
	$S_A = \frac{wL}{6}$ $S_B = -\frac{wL}{3}$	$M_{max} = 0.064\ wL^2$ at $x = 0.577L$	$\theta_A = \frac{7wL^3}{360EI}$ $\theta_B = -\frac{8wL^3}{360EI}$	$y_{max} = 0.00652\frac{wL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{wL}{4}$	$M_C = \frac{wL^2}{12}$	$\theta_A = -\theta_B = \frac{5wL^3}{192EI}$	$y_C = \frac{wL^4}{120EI}$
Fixed beam				
	$S_A = \frac{W}{2}$	$M_C = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$y_C = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = -\frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$y_a = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{wL}{2}$	$M_A = M_B = -\frac{wL^2}{12}$	$\theta_A = \theta_B = 0$	$y_C = \frac{wL^4}{384EI}$
	$S_A = \frac{3wL}{20}$ $S_B = -\frac{7wL}{20}$	$M_A = -\frac{wL^2}{30}$ $M_B = -\frac{wL^2}{20}$	$\theta_A = \theta_B = 0$	$y_{max} = 0.00131\frac{wL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{wL}{4}$	$M_A = M_B = -\frac{5wL^2}{96EI}$	$\theta_A = \theta_B = 0$	$y_C = \frac{0.7wL^4}{384EI}$

C. Hovell

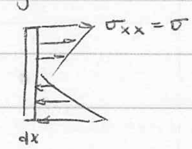
Just One Problem

March 18, 2003



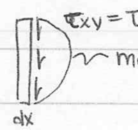


Normal (bending) stress



what are the maximum stresses in the beam?

shear stress



Assumptions:

- plane sections remain planar
- plane sections are $\perp$ to the neutral axis

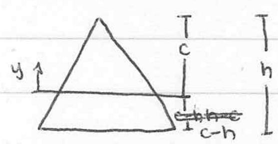
} if true, neutral axis is at centroid

$\boxed{\varepsilon = \frac{-y}{\rho}}$ y = distance from neutral axis BERNOULLI-EULER!

ρ = curvature

$\boxed{\sigma = \frac{-M(x)\cdot y}{I}}$ $M(x) > 0$

solve for the neutral axis:



$\Sigma F_x: \int \sigma \, dA$

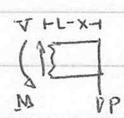
$NA \equiv \int y \, dA = 0$ $\int y \, dA$ = centroid

for a triangle, $c = 2/3 h$

moment of inertia:

$\boxed{I_\Delta = 1/36 \, w h^3}$ $I = \int y^2 \, dA = \int y^2 \, t(y) \, dy$ *use for problem 8

stress distributions: $\sigma(x,y) = \frac{-M(x)\, y}{I}$



$V = P$ $M = P(L-x)$

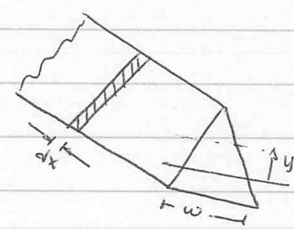
$$\boxed{\sigma(x,y) = \frac{P(L-x)\cdot y}{wh^3} \cdot 36}$$

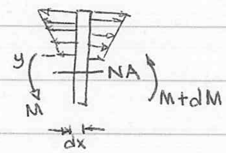
C. Hovell

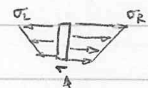
Wow-Lots of Notes

March 18, 2003

Back to transverse shear...







balances inequality in stresses

what is shear stress at y?

$\Sigma F_x: \int \sigma_R dA - \int \sigma_L dA - \underline{\underline{\tau}} \cdot w(y)\, dx$

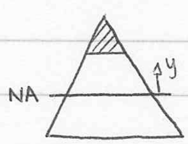
solve!

$$\tau = \frac{dM}{dx} \cdot \frac{1}{I t(y)} \int y\, t(y)\, dy$$

so, for this particular case:

$$\boxed{\tau = \frac{V}{I \cdot t(y)} \cdot Q}$$

$$Q = \int_y^h y\, t(y)\, dy$$



integrate over smaller top piece

$\underline{Q(y):}$ $\int y\, dA$ ← not centroid formula because limits of integration are different!

$$Q = \int_y^{2/3h} y\left(\frac{2}{3} - \frac{y}{h}\right) dy \qquad Q(y) = wh^2\left[4/27 - \left(\frac{y}{h}\right)^2 + \left(y/h\right)^3\right]$$

check $Q(2/3h) = 0$ ✓

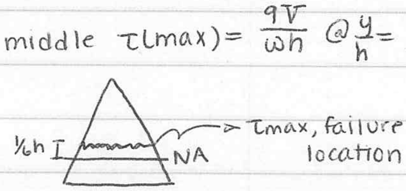
$$\tau(y) = \frac{V}{I} \cdot \frac{Q(y)}{t(y)}$$

$$\tau(y) = \frac{36V}{wh^{\cancel{3}}} \cdot \frac{\cancel{w}h^{\cancel{2}}}{\cancel{w}} \cdot \frac{\left(4/27 - (y/h)^2 + (y/h)^3\right)}{\left(2/3 - y/h\right)}$$

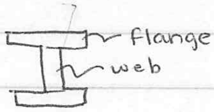
$$\boxed{\tau = \frac{36V}{wh} \cdot \frac{\left[4/27 - (y/h)^2 + (y/h)^3\right]}{2/3 - y/h}}$$

τ_{max} somewhere near the middle $\tau(max) = \frac{9V}{wh}$ @ $\frac{y}{h} = 1/6$

$\frac{d\tau}{dx} = 0$



Stuff:

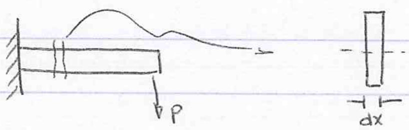


$$V_{flange} = \int_{flange} \tau\, dA$$

C. Hovell

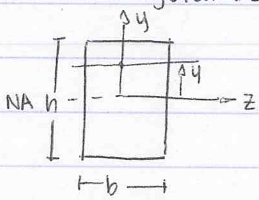
Kyle's Still Sick ☹ — March 19, 2003

Transverse shear stresses



$$\tau(y) = \frac{V_{tot}\, Q}{I \cdot t}$$

shear in rectangular beams

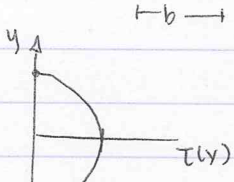


$$\tau(y) = \frac{V \cdot Q}{I \cdot b}$$

$Q = \bar{y}' \bar{A}'$ (uh... from book...)

$\bar{y}'$ is location of centroid of section $= y + \frac{1}{2}\left(\frac{h}{2} - y\right)$

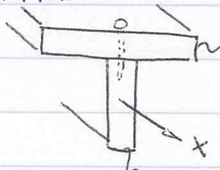
$\bar{A}'$ is area of section $= \left(\frac{h}{2} - y\right) \cdot b$



$$\tau(y) = \frac{V}{Ib}\left[\left(y + \tfrac{1}{2}\left(\tfrac{h}{2} - y\right)\right) \cdot \left(\tfrac{h}{2} - y\right) \cdot b\right]$$

find max at $\frac{d\tau}{dy} = 0$ $\quad \tau_{max} = \frac{3V}{2A}$

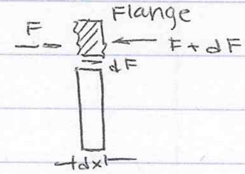
Shear Flow



flange

web

what is the force supported by nails joining sections?



Flange

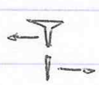
Shear flow:

$$\boxed{q = \frac{dF}{dx} = \frac{VQ}{I}}$$

(force / unit length)

keeps blocks from sliding relative to each other (in x-dir.)

$V_{max_{nail}} = 40\,lb$



9" apart

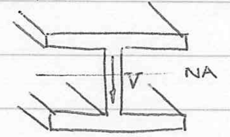
$$q = \frac{40\,lb}{9\,in}$$

C.Hovell

<u>Yay - Test Next Week</u>

March 25, 2003

Shear Flow in a thin-walled member



how is the force distributed?

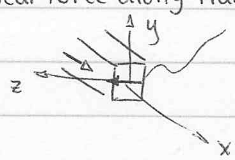
bending stress $\sigma = \frac{-My}{I}$ large I, small σ :)

flanges increase I

web holds shear, flange holds bending

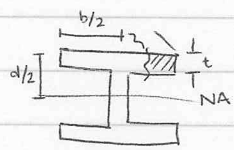
$q = \frac{dF}{dx}$

shear force along flange face heading towards center (on top flange)



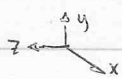
vertical shear thought to equal zero

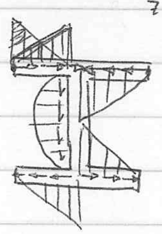
because it carries such a small percentage



$q(s) = \frac{V}{I}\left[\frac{d}{2}\left(\frac{b}{2} - s\right)\cdot t\right]$

• force / unit length acting in z-direction to balance force/unit length in x-direction

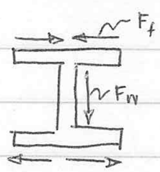




is the force created on the bottom by the moment imbalance the same as the top? No - opposite direction

• sections ⊥ to direction of shear have linearly distributed shear flow •

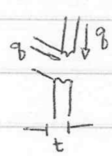
• sections ∥ to shear (V) are parabolic in shear flow •



$F_f = \int_0^{b/2} q(s)\,ds$

$F_w = V$

$q = \tau(y) \cdot t$

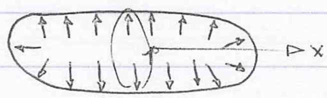


since $\tau(y)$ is parabolic, so must q ...

C. Hovell

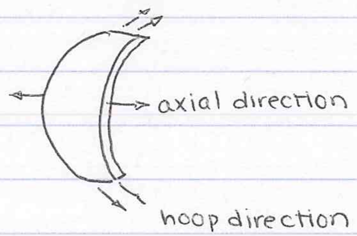
<u>No More Beams!</u> March 25, 2003

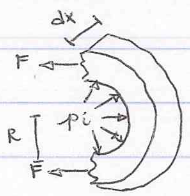
Pressure vessels



stress is ... highest on the inside wall

however, thin walls, so assume constant stress through thickness of tank





force caused by pressure:

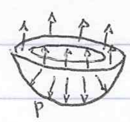
$p_i R \cdot d\theta \cdot dx$

$$\Sigma F = -2F + \int_{-\frac{\pi}{2}}^{\pi/2} p_i R \cos\theta \cdot d\theta \cdot dx$$

$F = p_i R dx$ $\quad \sigma = \frac{F}{A}$ $\quad A = dx \cdot t$ $\quad t$ = wall thickness

$$\boxed{\sigma_c = \frac{R}{t} p_i}$$ circumferential stress

axial stress...



$$\boxed{\sigma_{Axial} = \frac{p_i R}{2t}}$$

assuming t is small

$$\sigma_A = \frac{1}{2}\sigma_c$$

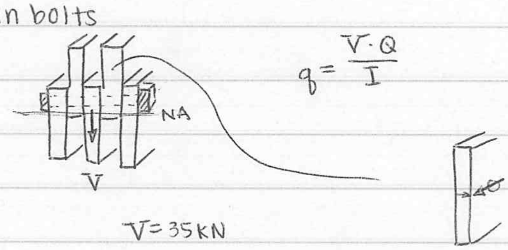
Combined Loadings

- Bending $\sigma = \frac{Mc}{I}$
- Shear $\tau = \frac{VQ}{It}$
- axial $\sigma = \frac{P}{A}$
- torsional $\tau = \frac{T \cdot c}{J}$

C. Hovell

Argh - No Office Hours! 	March 27, 2003

Shear in bolts



$q = \frac{V \cdot Q}{I}$

$V = 35\,kN$

NA @ 22 cm from bottom

what Q do you use?

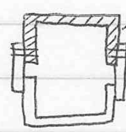
$Q = \bar{y}' A'$

Area for Q is what is removed by the cut

$F_{\substack{each\\bolt}} = \frac{q}{4} \cdot s$ → space between nails

Homework #2

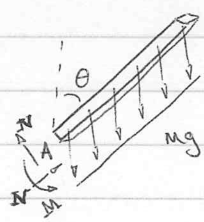
· 2 bolts, 2 locations of shear



use for A value

$\frac{q}{2}$

Combined Stresses



what is θ_{max} such that there is no tensile stress at A?

shear stress: $\tau = \frac{VQ}{It}$

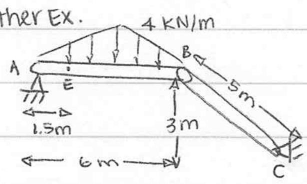
normal stress: $\sigma = \frac{N}{A}$

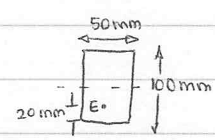
bending stress: $\sigma = \frac{Mc}{I}$

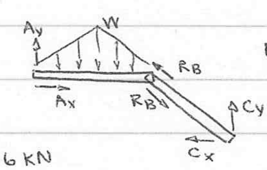
Total stress: $\frac{-N}{A} + \frac{Mc}{I} = 0$

point at which compressive stress is balanced by tensial stress

Another Ex.







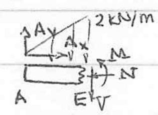
$R_B = 10\,kN$

$A_y = 6\,kN$

$A_x = 8\,kN$

$C_x = 8\,kN$

$C_y = 6\,kN$



$V = 4.5\,kN$

$N = 8\,kN$

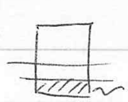
$M = 8.25\,kN \cdot m$

$\sigma_A = \frac{-N}{A}$ 	$\sigma_B = \frac{M \cdot (30\,mm)}{I}$

$\sigma_E = \frac{-N}{A} - \frac{My}{I}$ ← (-) sign gets cancelled out

$\sigma_E = 57.8\,MPa$

$\tau_E = \frac{V \cdot Q}{I \cdot t}$



use for Q

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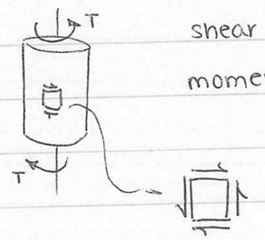
A Review?!

April 1, 2003

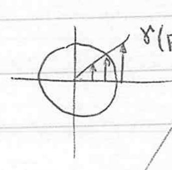
Torsion (5.1–5.5)



shear stress

moment is out of plane (as opposed to bending shear stress)

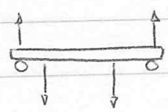
$\llcorner$ moment is in plane



$\gamma(\rho) = \gamma_{max} \frac{\rho}{c}$

ρ = radius

[No bending?



in the middle, $V(x) = 0$

$M(x) = $ constant] ~ slight aside

$\boxed{\tau(\rho) = \frac{T \cdot \rho}{J}}$

ρ = radius

$J = \int_A \rho^2 dA = \frac{\pi}{2} c^4$ for a circle

τ_{max} at $\rho = c$ (outer radius)

$\boxed{\phi(x) = \frac{T \cdot L}{J \cdot G}}$ for constant T

$(T(x) \rightarrow T \cdot L)$

Bending (6.1–6.5)

$\frac{dV}{dx} = -\omega(x)$ $\quad$ $\frac{dM}{dx} = V(x)$ $\quad$ $\varepsilon = \frac{-y}{\rho} = \varepsilon_{max}\left(\frac{y}{c}\right)$ bending strain

Two assumptions

1. ~~sm~~ planar sections remain plane
2. planar sections are $\perp$ to the neutral axis

so – strain varies linearly through ANY beam

$\sigma(y) = E \cdot \varepsilon_{max}\left(\frac{y}{c}\right)$ $\quad$ $\boxed{\sigma_{max} = \frac{M_{max} \cdot C}{I}}$ $\quad$ c = distance from N.A. to outermost fiber

Shear (7.1–7.5)

Combined Loadings (8.1–8.2)

$\boxed{\tau = \frac{V \cdot Q}{I \cdot b}}$ $\quad$ $q = \tau \cdot b = \frac{V \cdot Q}{I}$

$\rightarrow$ pressure vessels

$\sigma_1 = \frac{pr}{t}$ $\quad$ $\sigma_2 = \frac{pr}{2t} \rightarrow$ axial

$\llcorner \rightarrow$ circumferential

$V = \int \tau(y) b \cdot dy$

Strength of Materials Review

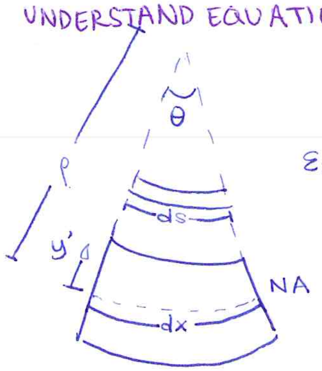
$Q = \bar{y}'A'$ $\quad \bar{y}' = \frac{\Sigma \bar{y} A'}{\Sigma A'}$

- shear / moment
- bending
- torsion
- shear flow *
- combined load
- pressure vessels

$Q = \Sigma \bar{y}' A'$

$\sigma = \frac{N}{A} + \frac{Mc}{I}$ (normal + bending)

UNDERSTAND EQUATIONS + WHERE THEY COME FROM.



$$\varepsilon_x = \frac{ds - dx}{dx} = \frac{(\rho - y)\theta - \rho\theta}{\rho\theta}$$

$\frac{-y}{\rho} = \varepsilon$ strain varies linearly along axis of beam

$$\varepsilon = -\varepsilon_{max}\left(\frac{y}{\rho}\right)$$

$E \cdot \varepsilon = \sigma$ $\quad \sigma = -\sigma_{max}\left(\frac{y}{\rho}\right) = \frac{-y}{\rho} E \cdot \varepsilon_{max}$

$dM = y \sigma b\, dy$

$M = \int y \sigma\, dA = \sigma_{max} \int y^2 dA \cdot \frac{1}{c}$

$\int y^2 dA = I$

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

C. Hovell

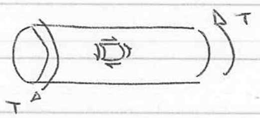
Damn Begley in Cali

April 8, 2003

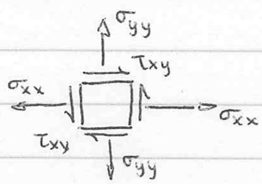
Stress Transformation

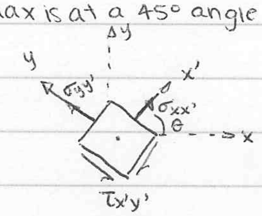
to find orientation of max normal, shear stress

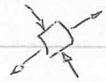
- brittle - fail due to normal
- ductile - fail due to shear

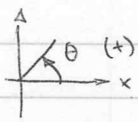


max is at a 45° angle



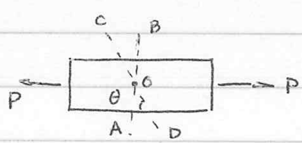






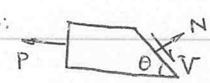
(+) direction of angle

★ given stress state in one coordinate system (x,y), what is stress in another coordinate system (x', y')?



AB: $\sigma = \frac{P}{A}$ $\tau = \frac{V}{A} = 0$

CD:

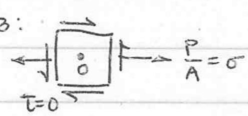


$N = P\sin\theta$ $\sigma = \frac{P}{A}\sin^2\theta$

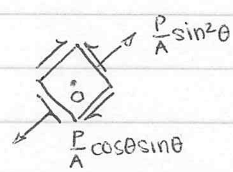
$V = P\cos\theta$ $\tau = \frac{P}{A}\cos\theta\sin\theta$

At POINT O, Average stress state

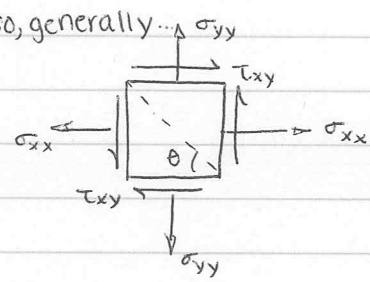
AB:



CD:

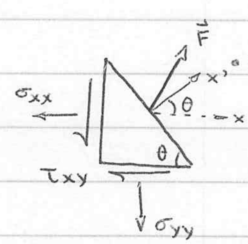


So, generally...



Sliced element:

CAUCHY'S TETRAHEDRON



what is the force/unit area acting on diagonal?

$$\begin{cases} F_x = \sigma_x dy dz + \tau_{xy} dx dz \\ F_y = \sigma_{yy} dx dz + \tau_{xy} dy dz \end{cases}$$

C. Hovell

Test = EW. Project = EWWW. April 8, 2003

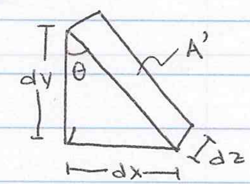
continuing on stress transformations ...

$$F_x = \sigma_x\, dy\, dz + \tau_{xy}\, dx\, dz \qquad F_y = \sigma_{yy}\, dx\, dz + \tau_{xy}\, dy\, dz$$

move to x', y' axis, from x, y:

$$\boxed{F_{x'} = F_x \cos\theta + F_y \sin\theta} \qquad \boxed{F_{y'} = F_y \cos\theta - F_x \sin\theta}$$

area transformation: $A' = \dfrac{dz\,dy}{\cos\theta} = \dfrac{dz\,dx}{\sin\theta}$



$$\left[\begin{aligned}
\sigma_{x'x'} &= \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\
\tau_{x'y'} &= \frac{-(\sigma_{xx}-\sigma_{yy})}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \\
\sigma_{y'y'} &= \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \tau_{xy}\sin 2\theta
\end{aligned}\right]$$

EWWW.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

C. Hovell

Neat. Tuesday. April 15, 2003

Stress transformations

· use to find stress in a new coordinate system

$$\begin{cases} \sigma_{x'} = \dfrac{(\sigma_x + \sigma_y)}{2} + \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{x'y'} = -\left(\dfrac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \\ \sigma_{y'} = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) - \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \end{cases}$$

Principal stress

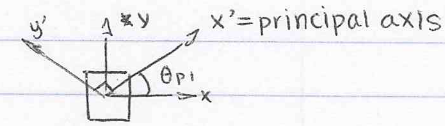
· Maximum Normal Stress

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$ two solutions, 90° apart – θ_{p1}, θ_{p2}

(shear stress)

$\tau_{x'y'}$ in $x'_p, y'_p = 0$ ALWAYS.

(for principal stresses)



θ_p = orientation off x, y of max normal stress

θ_s (plane of maximum shear) is always 45° off θ_p

· $\sigma_1\ \sigma_2$ ... Plane stress Max:

$$\boxed{\sigma_1, \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

In A given coordinate system

who cares

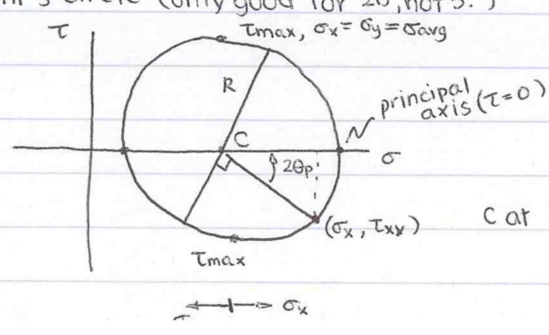
Max Shear Stress

· even though shear stress is zero on principal axis (max normal stress), it doesn't work the other way

· however, $\sigma_x = \sigma_y$ $\tan 2\theta_s = \dfrac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$ (or, $^{-1}/\tan\theta_p$)

$\theta_s = \theta_p \pm 45°$

~~Mohr~~ Mohr's circle (only good for 2D, not 3!)



ROTATIONS ARE 2X WHAT THEY REALLY ARE!

(pretend that's a good circle!)

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$

C at $\left(\dfrac{\sigma_x + \sigma_y}{2}\right) = \sigma_{avg}$

$$R = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

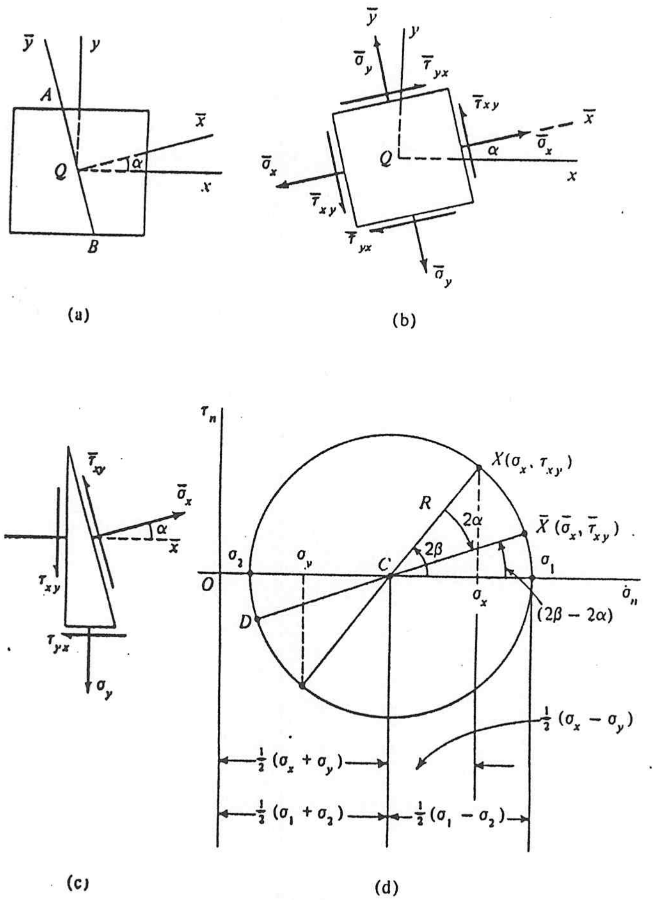


Fig. 3.14 Mohr's representation of plane stress.

$$\boxed{\begin{aligned} \bar{\sigma}_x &= \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha, \\ \bar{\tau}_{xy} &= \tau_{xy} (\cos^2\alpha - \sin^2\alpha) - (\sigma_x - \sigma_y) \sin\alpha \cos\alpha. \end{aligned}} \quad \begin{aligned} &(3.5.3) \\ &(3.5.4) \end{aligned}$$

: formulas can be given a convenient form by substituting

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}, \qquad \sin^2\alpha = \frac{1 - \cos 2\alpha}{2},$$

$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha, \qquad 2 \sin\alpha \cos\alpha = \sin 2\alpha.$$

After some rearrangement of terms, we obtain

$$\bar{\sigma}_x = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y) \cos 2\alpha + \tau_{xy} \sin 2\alpha, \tag{3.5.5}$$

$$\bar{\tau}_{xy} = \tau_{xy} \cos 2\alpha - \tfrac{1}{2}(\sigma_x - \sigma_y) \sin 2\alpha. \tag{3.5.6}$$

The points with coordinates (σ_x, τ_{xy}), $(\sigma_x, 0)$, $(\sigma_y, 0)$, and $[(\sigma_x + \sigma_y)/2, 0]$ are plotted in the stress representation σ_n, τ_n-plane of Fig. 3.14(d) with normal stress as abscissa and shear stress as ordinate. Here we permit τ_n to have negative values, thus differing from the discussion in Sec. 3.4. Let 2β be the angle shown in Fig. 3.14(d). Then

$$\boxed{\tfrac{1}{2}(\sigma_x - \sigma_y) = R \cos 2\beta, \qquad \tau_{xy} = R \sin 2\beta} \tag{3.5.7}$$

where

$$\boxed{R = [\tfrac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2]^{1/2}} \tag{3.5.8}$$

and

$$\boxed{\tan 2\beta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}.} \tag{3.5.9}$$

Equations (5) and (6) then take the form

$$\bar{\sigma}_x = \tfrac{1}{2}(\sigma_x + \sigma_y) + R (\cos 2\beta \cos 2\alpha + \sin 2\beta \sin 2\alpha),$$

$$\bar{\tau}_{xy} = R (\sin 2\beta \cos 2\alpha - \cos 2\beta \sin 2\alpha),$$

which by use of trigonometric addition formulas are given the simple form

$$\boxed{\begin{aligned} \bar{\sigma}_x &= \tfrac{1}{2}(\sigma_x + \sigma_y) + R \cos (2\beta - 2\alpha), \\ \bar{\tau}_{xy} &= R \sin (2\beta - 2\alpha). \end{aligned}} \tag{3.5.10}$$

For a given state of stress, σ_x, σ_y, R, and β are definite numbers. The stresses $\bar{\sigma}_x, \bar{\tau}_{xy}$ on planes at various angles α in Fig. 3.14(a) are then given by Eqs. (10). If the points with coordinates $(\bar{\sigma}_x, \bar{\tau}_{xy})$ are plotted in the stress

representation plane for various values of α, all such points will lie on the circle with center at $[\frac{1}{2}(\sigma_x + \sigma_y), 0]$ and radius R in Fig. 3.14(d). Equations (10) are parametric equations of the circle in terms of the parameter α. Its Cartesian equation is found, by eliminating the trigonometric terms, to be

$$[\bar{\sigma}_x - \tfrac{1}{2}(\sigma_x + \sigma_y)]^2 + \bar{\tau}_{xy}^2 = R^2, \tag{3.5.11}$$

where R is given by).

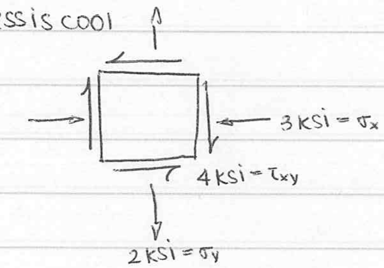
KEEP THIS SHEET.

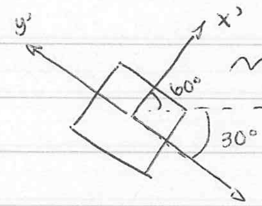
C. Hovell

My Baloney Has A First Name...

April 17, 2003

Stress is cool

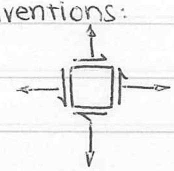




positive rotations = good fun = +

(x, y)

sign conventions:



all σ, τ are positive

★ positive shear stress causes a counter-clockwise rotation

Let's use Mohr's circle some ... mohr...

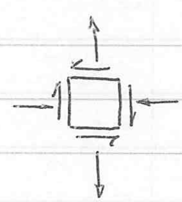
OH MY GOD. I THINK I AM SO MUCH FUNNIER THAN I ACTUALLY AM. BUT THAT WAS FUNNY.

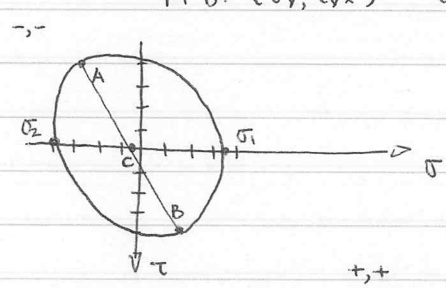
- calculate some points:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\frac{(\sigma_x + \sigma_y)^2}{4} + \tau_{xy}^2}$$

PT A: (σ_x, τ_{xy})

PT B: (σ_y, τ_{yx}) $\quad \tau_{yx} = -\tau_{xy}$





(pts from example above)

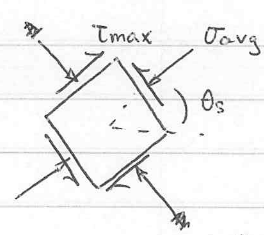
ah, I'm so good at drawing circles

principal stresses are at max, min σ values

solve by finding radius

$$\sigma_1 = \sigma_{avg} + R \qquad \sigma_2 = \sigma_{avg} - R$$

- pay attention to which face σ_1, σ_2 goes on!



$\theta_s = \theta_p + 45°$

C. Hovell

It's H-O-M-E-R! April 22, 2003

Strain transformations

- very similar to stress transformations
- ONE BIG DIFFERENCE:

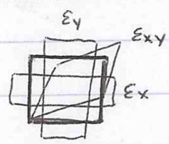
$$\boxed{\varepsilon_{xy} = \frac{\gamma_{xy}}{2}}$$

VERY IMPORTANT CHANGE!

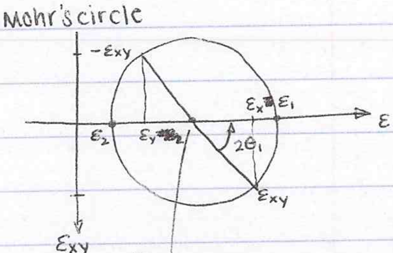
γ_{xy} = engineering shear strain

ε_{xy} = tensor shear strain

$$\boxed{\begin{aligned} \varepsilon_{x'} &= \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta \\ \varepsilon_{x'y'} &= -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \varepsilon_{xy}\cos 2\theta \end{aligned}}$$



Mohr's circle



$$\varepsilon = \frac{\delta}{L_0}$$

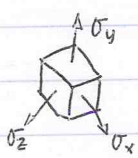
(just to keep in mind)

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^2}$$

Planar elasticity problems

REMEMBER POISON'S RATIO! $\nu = -\dfrac{\varepsilon_y}{\varepsilon_x}$



how does block change?

use SUPERPOSITION.

$$\boxed{\begin{aligned} \gamma_{xy} &= 2\varepsilon_{xy} = \frac{\tau_{xy}}{G} \\ \gamma_{yz} &= \frac{\tau_{yz}}{G} \quad \gamma_{zx} = \frac{\tau_{zx}}{G} \end{aligned}}$$

Generalized Hooke's Law

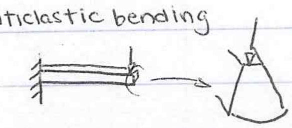
$$\boxed{\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\ \varepsilon_y &= \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} \\ \varepsilon_z &= \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \end{aligned}}$$

WOW. LOTS OF EQUATIONS. NO DERIVATIONS.

Plane strain

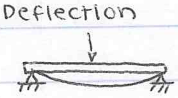
$$\sigma_z = -\nu(\sigma_x + \sigma_y)$$

★ anticlastic bending



beam face bends, too

Beam Deflection



positive curvature (concave up)

$$\varepsilon_x = \frac{-y}{\rho}$$

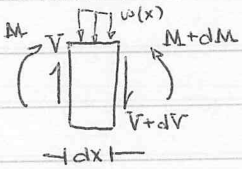
$$\boxed{M(x) = EI\frac{d^2v}{dx^2}}$$

C. Hovell

<u>No Kyle. No Begley.</u>

April 24, 2003

Beam deflection



$\frac{dV}{dx} = -\omega(x)$

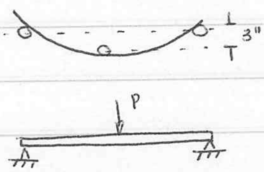
$\frac{dM}{dx} = V(x)$

slope $= EI\frac{dv}{dx}$ of deflection

$M(x) = EI\frac{d^2v}{dx^2}$ v = deflection

$V(x) = EI\frac{d^3v}{dx^3}$

Example:



what is the max bending stress?

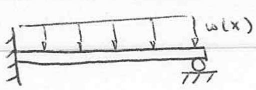
$\sigma = \frac{Mc}{I}$

max deflection = 3"

max moment, at center $= \frac{P}{2}\cdot\frac{L}{2} = \frac{PL}{4} = M_{max}$

$$\omega(x) = \frac{P}{EI}\left[\frac{x^3}{12} - \frac{L^2x}{16}\right]$$

Indeterminant situations

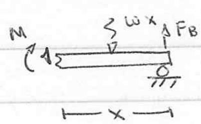


$\delta = 0$ $\delta = 0$

slope = 0

solve using v'', v', v at wall and roller

three boundary conditions: $\delta(0)$, $\delta(L)$, slope (L)



or, use superposition

C. Hovell

<u>Last Day of Class!</u>

April 29, 2003

WE TALKED ABOUT BUCKLING, A TOPIC (GASP) NOT ON THE FINAL. YAY.

Euler buckling:

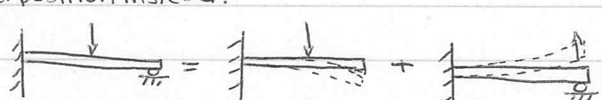
$$P_{CR} = EI\left(\frac{\pi}{L}\right)^2$$

Ah, yes... the final...

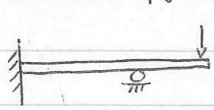
- Review: Saturday, May 3, 11 AM - 1 PM
- Material - previous and new
  - previous:
    - stress concentrations
    - pressure vessels
    - shear stresses, formulae
    - shear flow in thin-walled members
      bolted connections
    - stuff we got wrong
  - new:
    - deflection - statically indeterminant
      - curvature $M(x) = EI\frac{d^2v}{dx^2}$
      - <u>superposition</u> (below)
    - stress transformation
      - strain transformations
      - Mohr's circle
      - principal (max) stresses
    - Hooke's law in multiple dimensions
      - $\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$
- six problems
  three new, three old
- cheat sheet - reasonable equation additions from the last one
  beam deflection

Examples

- don't use integration to solve statically indeterminant deflection problems
  use superposition instead!



- review two problems: pg 643



also one like this one...