CE 326 – Design of Concrete Structures (Spring 04)

MWF, 10:00 -10:50 am, Thornton D221

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Text: Design of Concrete Structures, 13th ed. by Nilson, A.H., Darwin, D. and Dolan, C.W.

Course Outline:

1. Introduction (Sect. 1.1-1.2) - 1/2 class
2. Materials (Chapter 2) - 1 class
3. Reliability and Structural Safety (Sect. 1.3-1.8) - 1/2 class
4. Uncracked and Cracked Sections (Sect. 6.1-6.3) - 2 classes
5. Flexural Behavior (Sect. 3.1-3.2) - 1 classes

TEST 1

6. Analysis of Singly R.C. Sections (Sect. 3.3) - 2 classes
7. Design of Singly R.C. Sections (Sect. 3.4-3.6 & 13.1-13.2)- 3 classes
8. Doubly Reinforced Sections (Sect. 3.7-3.8) - 2 classes
9. Deflections and Crack Control (Sections 6.3-6.7) - 1 class
10. Shear Design of R.C. Beams (Sect. 4.1-4.6) - 3 classes
11. Axial Loading (Section 1.9 & 8.1-8.2) - 2 class
12. Axial Load and Bending (Sections 8.3-8.5) - 2 classes
13. Column Design (Section 8.9-8.10) - 1 classes
14. Footings (Section 16.1-16.6) - 3 classes
15. Bond, Development, and Anchorage (Chapter 5) - 2 classes

Attendance:	Not mandatory but will be recorded. Everyone is responsible for material covered in class.
Tests:	3 tests (20% each) and a final exam (30%, no exemptions) will be given. The tentative dates for the tests are ~~2/14, 3/21, & 4/25~~. The tests may be given from 7 pm to 9 pm. 2/23, 3/31
Homework:	Homework assignments will be made and collected. Homework solutions will be made available. Homework will count 10% of the final grade.
Prerequisite:	CE 319 - Structural Mechanics
Office Hours:	1 p.m. to 2 p.m. MW, and, otherwise, anytime I am in the office.
Note:	The Honor System applies to all work in this class. Discussions on homework assignments are encouraged; however, the solutions submitted for grading are to be the individual effort of each student.

Reinforced Concrete – Definitions for Flexural Design

a = depth of rectangular equivalent concrete stress block
A_s = cross-sectional area of tensile reinforcement
A_{sT} = transformed area of steel (representing an equivalent area of concrete)
A_T = transformed area of a flexural section, including concrete compression and equivalent concrete representing a tensile steel section
b = width of flexural member at the extreme compression fiber
b_w = width of the web (tension) of a flexural member
c = depth to the Neutral Axis (N.A.)
C = compressive force of an internal moment couple
d = effective depth, from extreme compression fiber to centroid of the tensile reinforcement
E_c = modulus of elasticity of concrete $\cong 57000\sqrt{f'_c}$ psi
E_s = modulus of elasticity of steel = 29,000,000 psi
f_{ct} = stress in the concrete at the top (compression) edge of a flexural member
f_{cb} = stress in the concrete at the bottom (tension) edge of a flexural member
f_s = stress in the tension reinforcement of a flexural member
f'_c = design compressive strength of concrete
f_r = modulus of rupture, tensile capacity of concrete = $7.5\sqrt{f'_c}$
f_y = yield stress of steel (Grade 60 = 60 ksi, Grade 40 = 40 ksi)
h = depth of a flexural member
h_f = depth of the top (compression) flange of a flexural member
I = moment of inertia of a structural member
I_g = moment of inertia of the gross section of a reinforced member
I_T = transformed moment of inertia of a reinforced member (steel approximated by equivalent concrete area.
l = span length of a flexural member
M_{cr} = cracking moment = moment necessary to crack concrete in tensile region of flexural member
M_n = nominal moment capacity (unfactored) of a flexural member
M_u = required flexural strength = ultimate (factored) moment load on a flexural member
n = modular ratio of steel to concrete = E_s/E_c
P = point load on a structural member
T = tension force of an internal moment couple
w = unit distributed load on a flexural member
z = moment arm (distance) between T and C in an internal moment couple
α = empirical factor relating the average compressive concrete stress capacity in a flexural member to the design compressive strength of the concrete
β = empirical factor relating the depth of cumulative compressive concrete stress to the depth of the neutral axis, *c*.
β_1 = empirical factor that relates to depth of an equivalent rectangular compressive concrete stress block, *a*, to the depth of the neutral axis, *c*.
ε_c = strain in the extreme concrete compression fiber of a flexural member
ε_s = strain in the tensile reinforcing steel of a flexural member
ε_u = limiting (ultimate) compression strain of concrete in a flexural member
ϕ = resistance factor applied to the nominal moment, load, or shear capacity of a member for comparison to required loads.
γ = empirical factor relating α to β_1: $\gamma = 0.85$ for normal strength concretes
ρ = reinforcement ratio = $A_s/(bd)$
ρ_b = balanced reinforcement ratio, such that a flexural member would fail under compression (concrete crushing) and tension (steel yielding) simultaneously if loaded to capacity.

CE 326 **Test 1** **Spring 2004**

February 27, 2004

Name: Catherine Hovell

94/100

I have abided by the provisions of the University of Virginia honor code.

Signature

avg: 80
high: 100

PART I – SHORT ANSWER (CLOSED BOOK & NOTES)

1. Name and describe two time-dependent properties of concrete. (4 points)

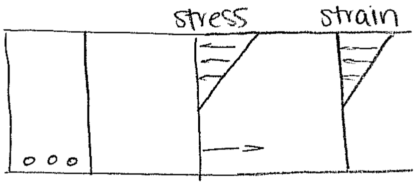
✓ Strength and workability – as the concrete sets, it will be less and less workable (easily formed into the shapes needed). The strength of concrete increases with how long it has set for. Generally, 28 days is considered the point at which the strength has asymptoted enough to assume it is constant.

✓ shrinkage – loss of moisture → deformation

another: creep – time-dependent deformation due to constant load.

2. Describe a balanced failure of a reinforced concrete section. Include a description of the state of strain in the beam in your discussion. (6 points)

If a reinforced concrete beam has a balanced failure, the steel in the tensile side will yield ✓ at the same time as the concrete in compression will crush. ✓



The tensile concrete will have cracked long before this failure occurs.

Strain? $\varepsilon_c = \varepsilon_u$ $\varepsilon_s = \varepsilon_y$ –1

$\varepsilon_c = \varepsilon_u = 0.003$ $\varepsilon_s = \varepsilon_y$

3. Describe the respective purposes of load factors and resistance factors in design of reinforced concrete using the LRFD methodology. (5 points)

Load factors increase the normative load to account ✓ for inaccuracies, special conditions, and general safety. They are always greater than 1. Resistance factors compare the normative moment the beam can carry to the expected applied moment, so that in cases of inaccuracy, etc, the applied value will not exceed the design strength. Resistance factors are always less than 1. ✓

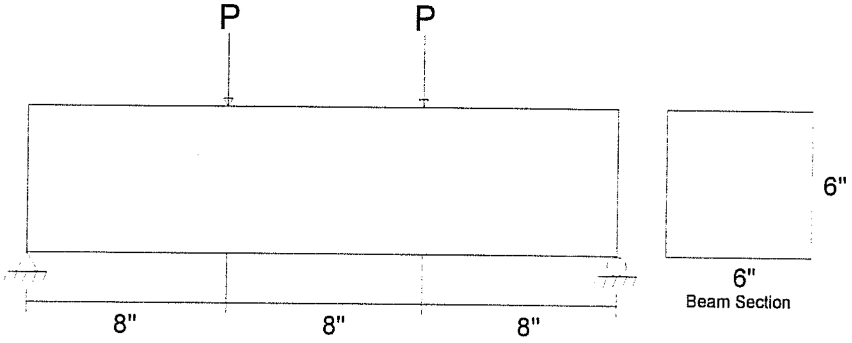
Please turn in PART I when complete, before proceeding to PART II

14/15

Name: Catherine Hovell

PART II – PROBLEMS (OPEN BOOK & NOTES)

1. For the given plain (unreinforced) concrete beam, determine the load "P" that will cause collapse of the beam. Use $f'_c = 4{,}000$ psi. (35 points)



2. A beam is designed for a factored applied load, $M_u = 3{,}000$ in-kips. For the given beam section and material properties, determine: (50 points)

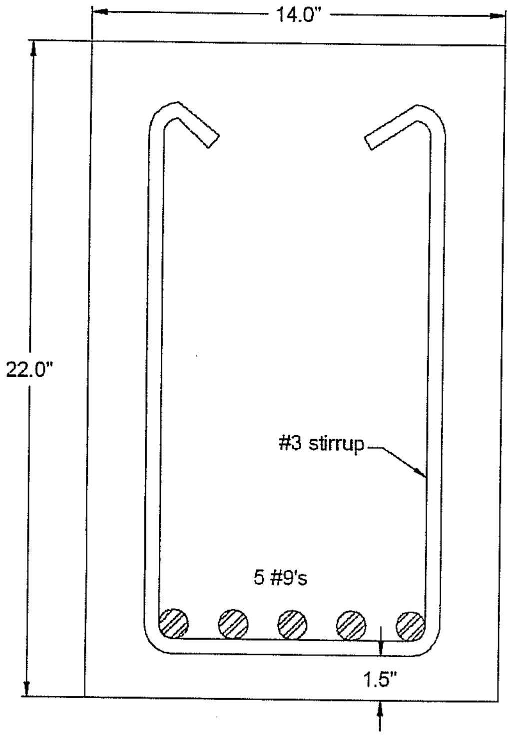
   a. the nominal moment capacity, M_n of the beam.

   b. the design moment, ϕM_n, of the beam.

   c. the strain, ε_s, in the tensile steel at failure.

   d. If all other dimensions are held constant, what is the maximum area of tensile steel, A_s, such that $\phi = 0.90$.

   e. Select new tensile reinforcement to match the recommendation from part d., and calculate the new design moment. Check against M_u.



$f'_c = 3{,}000$ psi

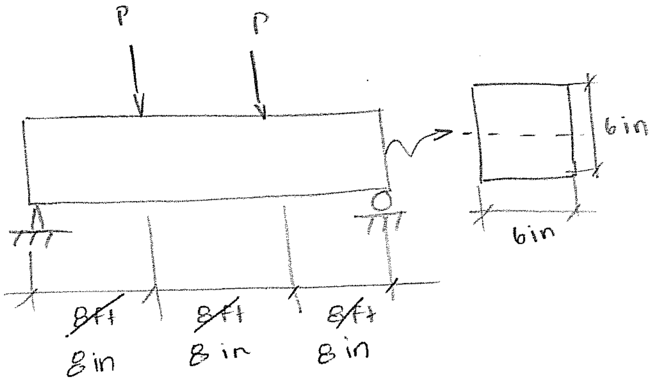
$f_y = 50{,}000$ psi

$E_s = 29{,}000{,}000$ psi

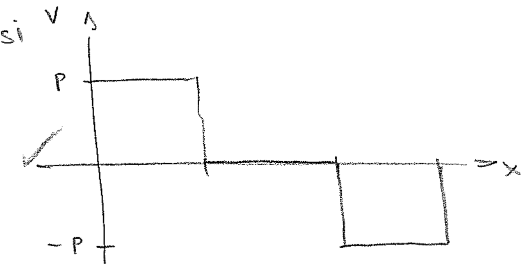
CE 326: TEST #1

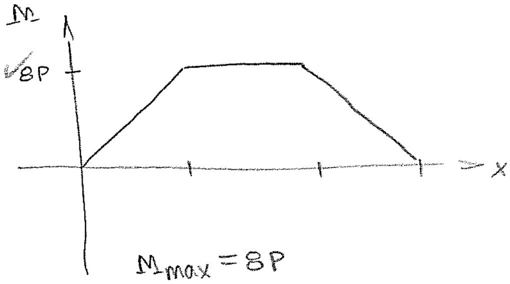
CATHERINE HOVELL
FEBRUARY 27, 2004

1.



$f'_c = 4\text{ksi}$





$M_{max} = 8P$

$f'_c = \frac{Mc}{I}$ ✓

$I = 1/12\, bh^3 = 1/12 (6in)(6in)^3$

$I = 108 in^4$ ✓

$c = 3 in$ ✓

-1

$$f'_c = \frac{(8 \overset{in}{\cancel{ft}}) P (3in)(12 \cancel{in/ft})}{108 in^4} = 4000\ lb/in^2$$

→ compressive strength at top ... bottom will rupture first

$$P = \frac{(4ksi)(108in^4)}{(8ft)(12\cancel{in/ft})(3in)} = 1.5\ kip$$

$$f_r = 7.5\sqrt{f'_c} = 7.5(4000psi)^{1/2} = 474\ psi$$

2.133 kips

$$P = \frac{f_r I}{c(8)} = \frac{(474psi)(108in^4)}{(8 \underset{in}{\cancel{ft}})(\cancel{12in/ft})(3in)} = \cancel{178\ lb}$$

$$\boxed{P = 178\ lb}$$

DUMBASS!

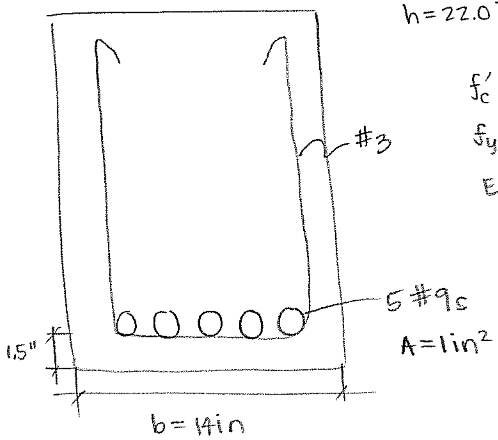
34/35

CE 326: TEST #1

C. HOVELL
FEBRUARY 27, 20

2.

$h = 22.0 \text{ in}$ $\qquad M_u = 3000 \text{ in}\cdot\text{kip}$



$f'_c = 3 \text{ ksi}$

$f_y = 50 \text{ ksi}$

$E = 29000 \text{ ksi}$

$$\varepsilon_y = \frac{f_y}{E} = \frac{50 \text{ ksi}}{29000 \text{ ksi}} = 0.0017$$

$$d = h - (1.5 \text{ in} + 0.5 \text{ in} + \tfrac{1}{2}\cdot \overset{1.128 \text{ in}}{\cancel{1.0}} \text{ in})$$ ⊖1

$$d = 19.5 \text{ in}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5)(1 \text{ in}^2)(50 \text{ ksi})}{0.85(3 \text{ ksi})(14 \text{ in})} = 7.00 \text{ in}$$ ✓

$$c = a/0.85 = 8.24 \text{ in}$$ ✓

$$M_n = 0.85 f'_c b \cdot a\,(d - a/2) = (0.85)(3 \text{ ksi})(14 \text{ in})(7.00 \text{ in})\left[19.5 \text{ in} - \frac{7.00 \text{ in}}{2}\right]$$

$$M_n = 3998.4 \text{ kip}\cdot\text{in}$$

$$\boxed{a.\ M_n = 3998 \text{ kip}\cdot\text{in}}$$ ✓

$$\varepsilon_t = \varepsilon_u\left(\frac{d-c}{c}\right) = 0.003\left[\frac{19.5 \text{ in} - 8.24 \text{ in}}{8.24 \text{ in}}\right]$$

$$\varepsilon_t = 0.0041$$ ✓

$$\phi = 0.483 + 83.3\,(0.0041) = 0.82$$ ✓

used ↓ exactly (0.82453)

Math error

$$M_u = \phi M_n = 0.82\,(3998.4 \text{ kip}\cdot\text{in}) = \cancel{3296.6} \text{ kip in}$$

3279 ⊖1

$$\boxed{b.\ M_u = 3300 \text{ kip in}}$$

46/50

2. (cont'd)

$$\varepsilon_s = \varepsilon_y = \frac{f_y}{E} = \frac{50\text{ ksi}}{29000\text{ ksi}}$$

$$\varepsilon_s = 0.0017$$

Beyond yield

$\boxed{\text{c. } \varepsilon_s = 0.0017}$ X -2

0.0041

For $\phi = 0.90$, $\varepsilon_t = 0.005$

$$0.005 = 0.003\left(\frac{19.5\text{ in} - c}{c}\right)$$

$$c = 7.31\text{ in},\ a = 6.22\text{ in}$$

$$A_s = \frac{a b f_c' \, 0.85}{f_y} = \frac{(6.22\text{in})(14\text{in})(3\text{ksi})(0.85)}{50\text{ ksi}}$$

$$A_s = 4.44\text{ in}^2$$

$\boxed{\text{d. } A_s = 4.44\text{ in}^2}$ ✓

Try using 4 #9s : $A_s = 4.00\text{ in}^2$

$$a = \frac{(4.00\text{ in}^2)(50\text{ ksi})}{0.85(3\text{ksi})(14\text{in})} = 5.60\text{ in}$$

$$M_n = (4.00\text{ in}^2)(50\text{ ksi})\left[d - \frac{5.60\text{ in}}{2}\right]$$

$$d = 22\text{ in} - \underset{\substack{\uparrow \\ \text{stirrup +} \\ \text{cover}}}{(2.0\text{ in})} - \underset{\substack{\uparrow \\ \text{half of} \\ \text{diameter}}}{\tfrac{1}{2}(1\text{ in})} = 19.5\text{ in}$$

$$M_n = 3339.8\text{ kip}\cdot\text{in}$$

$$\phi = 0.9$$

$$M_u = (0.9)(3339.8\text{ kip}\cdot\text{in}) = 3005.8\text{ kip in}$$ ✓ ✓ (close, but there are a lot of safety factors)

$\boxed{\text{e. 4 \#9s achieves the needed } M_u\text{, even using } \phi = 0.90}$ ✓

3 #8 and 2 #9 = 4.37 in^2

CE 326 Test 1 **Spring 2004**

Name: Master

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Signature ______________________

PART I – SHORT ANSWER (CLOSED BOOK & NOTES)

1. Name and describe two time-dependent properties of concrete. (4 points)

Creep - Strain (dimension change) of concrete under sustained stress (load)

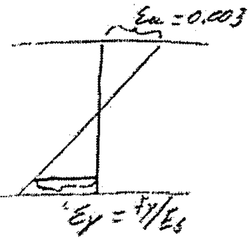
Shrinkage - concrete matrix decreases in volume, due to loss of moisture and and reactions within the cement paste

Strength - concrete gains compressive strength over time as portland cement, and supplemental cementitious materials, hydrate (cure).

2. Describe a balanced failure of a reinforced concrete section. Include a description of the state of strain in the beam in your discussion. (6 points)

A balanced failure occurs when the concrete in compression and the reinforcing steel in tension reach their respective ultimate stresses at the same time when loaded to capacity in flexure.

For simplicity, concrete is assumed to reach a nominal strain capacity, $\varepsilon_u = 0.003$, at the same time steel reaches the yield strain, $\varepsilon_y = f_y/E_s$.



3. Describe the respective purposes of load factors and resistance factors in design of reinforced concrete using the LRFD methodology. (5 points)

Load factors are applied to projected service loads anticipated for a structure or element. Load factors are typically greater than 1.0, and increase as the variability of types of loads increase. Example; Dead loads = 1.2 Live loads = 1.6

Resistance factors are generally less than 1.0, and serve to reduce the estimated capacity of a member to account for uncertainties in materials, construction tolerances, design assumptions, etc.

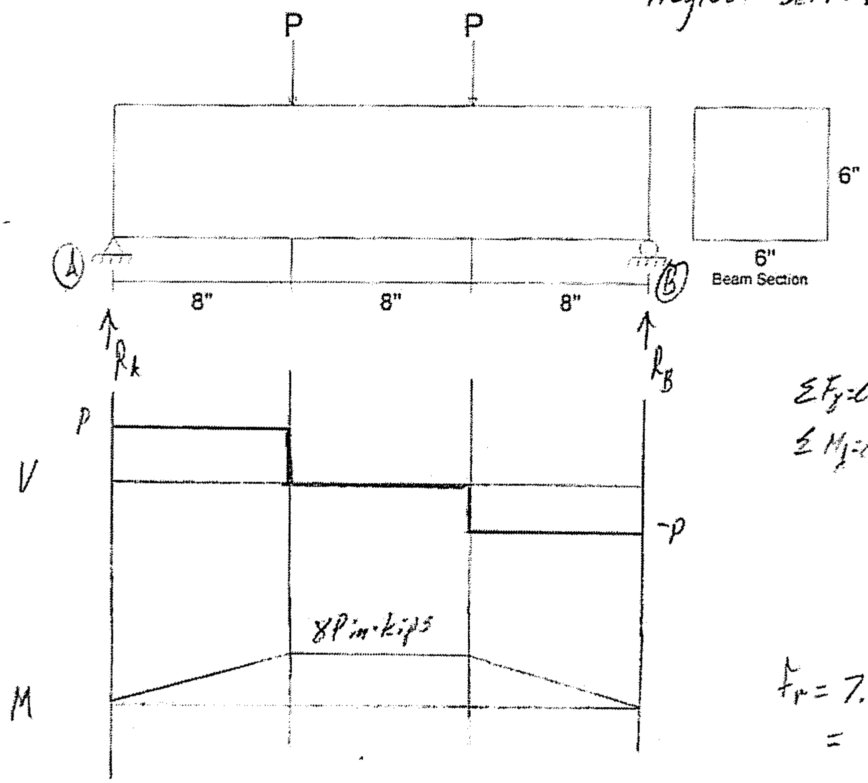
Please turn in PART I when complete, before proceeding to PART II

CE 326 **Test 1** **Spring 2004**

Name: Master

PART II – PROBLEMS (OPEN BOOK & NOTES)

1. For the given plain (unreinforced) concrete beam, determine the load "P" that will cause collapse of the beam. Use f'c = 4,000 psi. (35 points) Neglect self-weight.



$\Sigma F_y = 0: \; R_A + R_B = 2P$

$\Sigma M_A = 0: \; P(8'') + P(16'') - R_B(24'') = 0$

$24P = 24R_B$

$R_B = P = R_A$

$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000}$

$= 474 \text{ psi}$

$y = 3''$

$I = \frac{bh^3}{12} = \frac{6(6)^3}{12}$

$= 108 \text{ in}^4$

$f_r = \frac{M_{cr} y}{I}$

$0.474 \text{ ksi} = \frac{\overset{\text{in-kips}}{(8P)}(3\text{ in})}{108 \text{ in}^2}$

$\underline{\underline{P = 2.133 \text{ kips}}}$

2. A beam is designed for a factored applied load, $M_u = 3{,}000$ in-kips. For the given beam section and material properties, determine: (50 points)

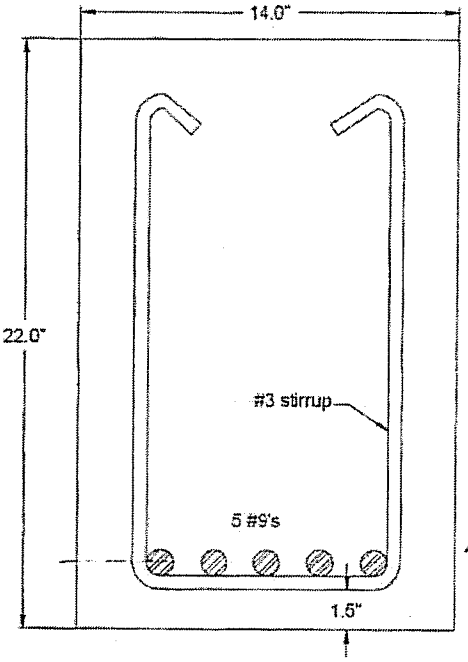
   a. the nominal moment capacity, M_n of the beam.

   b. the design moment, ϕM_n, of the beam.

   c. the strain, ε_s, in the tensile steel at failure.

   d. If all other dimensions are held constant, what is the maximum area of tensile steel, A_s, such that $\phi = 0.90$.

   e. Select new tensile reinforcement to match the recommendation from part d., and calculate the new design moment. Check against M_u.



$f'_c = 3{,}000$ psi

$f_y = 50{,}000$ psi

$E_s = 29{,}000{,}000$ psi

Table A.2

$A_s = 5.00 \text{ in}^2$

$\rho = A_s / bd$

$= \frac{5.00}{(14)(19.6)}$

$= 0.0182$

a) $R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right) = 0.0182(50)\left(1 - 0.59 \frac{0.0182(50)}{3}\right)$

$= 0.747$

$M_n = R b d^2 = 0.747(14)(19.6)^2 = 4018 \text{ in-kips} \underline{\underline{= 335 \text{ ft-kips}}}$

b) $\varepsilon_t = \varepsilon_u \frac{d_t - c}{c}$

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5.00 \text{ in}^2 (50 \text{ ksi})}{0.85(3 \text{ ksi})(14 \text{ in})} = 7.00$

$d = 22 - 1.5 - 0.375 - 1.128/2 = 19.6 \text{ in.}$

$\varepsilon_t = 0.003\left(\frac{19.6 - 8.24}{8.24}\right)$ $\quad c = a/\beta_1 = 7.00/.85 = 8.24$

$= 0.00414 < 0.005 \quad \therefore \phi = 0.483 + 83.3(0.00414) = 0.83$

$\phi M_n = 0.83(335 \text{ ft-kips}) = \underline{\underline{278 \text{ ft-kips}}}$

c.) $\underline{\underline{\varepsilon_s = 0.00414}}$ (see b above)

d.) $\rho(@\varepsilon_t = 0.005) = 0.85 \beta_1 \frac{3^{\text{ksi}}}{50^{\text{ksi}}} \frac{0.003}{0.003 + 0.005} = 0.0163$

$A_s = \rho b d = 0.0163(14 \text{ in})(19.6 \text{ in}) = \underline{4.46 \text{ in}^2}$

e.) Try $2 \#9 + 3 \#8 = 2.00 \text{ in}^2 + 2.37 \text{ in}^2 = 4.37 \text{ in}^2$
all in one row,

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.37 \text{ in}^2 (50 \text{ ksi})}{0.85(3 \text{ ksi})(14 \text{ in})} = 6.12$

$M_n = A_s f_y (d - a/2) = 4.37(50)(19.6 - 6.12/2) = 3614$

$\phi M_n = .9(3614) = \underline{\underline{3253}}^{\text{"k}} > 3000 \text{ in-kips}$
ok M_u

CE 326 **Test 2** **Spring 2004**

March 29, 2004

Name: Catherine Hovell

88

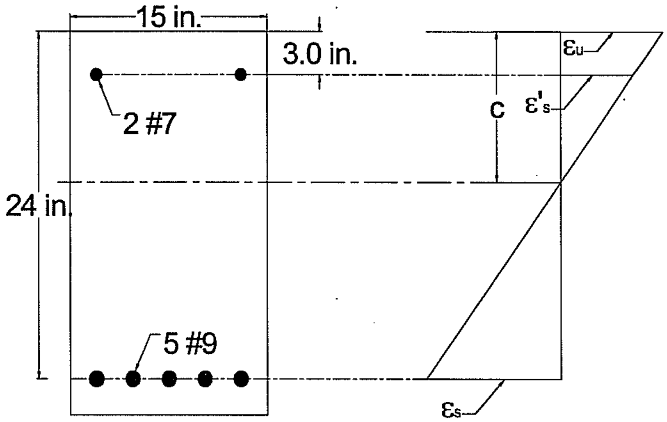
While taking this test, I have abided by the provisions of the University of Virginia honor code.

Signature

PROBLEMS (OPEN BOOK & NOTES)

1. Use the given concrete beam to answer the following questions. (65 points)

$f'_c = 4{,}000$ psi
$f_y = 60{,}000$ psi
E_s=29,000,000 psi.



a) Investigate whether compression steel is necessary for this section to meet ductility requirements of the tensile steel provided.

$\rho < \rho_{max}$?

$$\rho = \frac{5(1.00\,in^2)}{(15\,in)(24\,in)} = 0.0139 \checkmark$$

$\rho_{max} = 0.0206$ ✓

$\rho < \rho_{max}$, so compression steel is unneeded ✓

b) Assuming compression steel is necessary, at the nominal moment capacity, does the compression steel yield?

$f_s, f'_s = f_y$ $\rho_{cy} < \rho$ for comp. to yield ✓

$$\rho_{cy} = 0.85\beta_1 \frac{f'_c}{f_y}\frac{d'}{d}\frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho' = (0.85)^2 \frac{4\,ksi}{60\,ksi} \cdot \frac{3\,in}{24\,in} \cdot \frac{0.003}{0.003-0.00207} + \frac{2(0.6\,in^2)}{(15in)(24\,in}$$

$\rho_{cy} = 0.0228$ ✓

$\rho_{cy} > \rho$

NO. ✓

20/20

c) Determine the nominal moment capacity, M_n, for the beam, as designed.

$A_s f_y = 0.85 \beta_1 f'_c bc + A'_s \varepsilon_u E_s \left(\frac{c-d'}{c} \right)$ solve for c.

1.20 in² ok

3 in

(-1)

$(5.00\,in^2)(60\,ksi) = (0.85)^2 (4\,ksi)(15\,in)c + (\cancel{0.6}\,in^2)(2)(0.003)(29000\,ksi)\left(\frac{c - \cancel{24}\,in}{c} \right)$

(found on calculator) $c = 10.19\,in$

$a = 0.85c = 8.66\,in$

$M_n = 0.85 f'_c ab(d - a/2) + A'_s f'_s (d - d')$ $f'_s = \varepsilon_u E_s \left(\frac{c-d'}{c} \right) = 0.003(29000\,ksi)\left(\frac{10.19\,in - 3\,in}{10.19\,in} \right)$

$f'_s = 61.4\,ksi$ cannot exceed f_y and part b. indic[illegible] $f'_s < f_y$ (-1)

$M_n = 0.85(4\,ksi)(8.66\,in)(15\,in)(24\,in - \frac{8.66\,in}{2}) + (1.2\,in^2)(61.4\,ksi)(24\,in - 3\,in)$

$M_n = 852.7\,kip \cdot ft$

$M_n = 853\,kip\,ft$ ok

537 ft-kip

d) Does the beam meet minimum ductility requirements, as specified in the code specifications (ACI 318-02)?

$As_{min} = \frac{3\sqrt{f'_c}}{f_y} bd \geq \frac{200bd}{f_y}$

$\frac{3\sqrt{4000\,psi}}{60000\,psi}(15\,in)(24\,in) = 1.14\,in^2$

$\frac{200(15\,in)(24\,in)}{60000\,ksi} = 1.2\,in^2$

$< A_s$

$\varepsilon_t = \varepsilon_u \left(\frac{d_t - c}{c} \right) = 0.0095 > 0.004$

$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y}$

check $\varepsilon_t > 0.004$ or $\rho < \bar{\rho}_{max}$ (-4)

$A_s = 1.2\,in^2 + 5.0\,in^2 = 6.2\,in^2$ yes

e) Does the beam meet minimum tensile reinforcement requirements of the code?

$\rho_{min} = \frac{3\sqrt{f'_c}}{f_y} = \frac{3\sqrt{4000\,psi}}{60000\,psi} = 0.00316$ $\rho = 0.0139 > \rho_{min}$

Ok

yes

f) What is the design moment capacity, ϕM_n, of the beam?

$\varepsilon_t = 0.00207 \longrightarrow \phi = 0.65$ ✗ (-4) $\varepsilon_t = 0.0095, \phi = 0.90$

calc's?

if $\varepsilon_t = 0.00207 < 0.004$ beam would have to be redesigned

$M_u = \phi M_n = (0.65)(853\,ft\,kip)$

$M_u = 554\,ft \cdot kip$ 537 ft·kip ⟶ 483 ft·kip

g) In accordance with code requirements, what size ties and tie spacing would you recommend for this double-reinforced beam?

if shear equation for tie spacing can be applied with moments...

$M_u - \phi M_n \leq \frac{\phi A_v f_y d}{s}$ but this equation is normally used with shear, not moment

guess: #3 bar ✓, spaced at 8 in OK

looking for minimum of: $16 d_{bar}$ (-1) = 18.0 in

$48 d_{tie}$ = 18.0 in

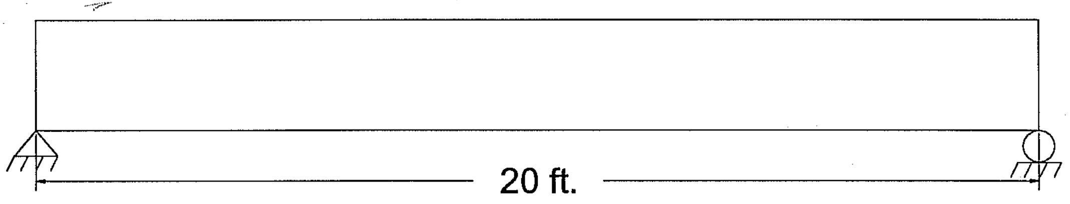
b = 15.0 in

34/45

but shear requirements would override, if more stringent

2. If the cross-section presented in Problem 1 is of a simply-supported beam carrying sustained dead load (including self-weight) w_{DL} = 1.8 kips/ft and applied service live load, w_{LL} = 1.5 kips/ft, determine: (35 points)

$$\delta_{DL} = \frac{5 w_{DL} \ell^4}{384 I_{eff} E} = \frac{5(1.8\text{kip/ft})(20\text{ft})^4(12\text{in})^3}{384(18354\text{in}^4)(3.6\times10^3\text{ksi})} = 0.0981\text{ in}$$



h=26.5

(*hint:* $M_{mid} = wl^2/8$ *and* $d_{mid} = 5wl^4/384EI$)

a. The maximum immediate deflection at mid-span due solely to the dead load.

$$M_{DL} = \frac{(1.8\text{kip/ft})(20\text{ ft})^2}{8} = 90\text{ kip}\cdot\text{ft} ✓$$

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad I_g = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(15\text{in})(26.5\text{in})^3 = 23262\text{in}^4 ✓$$

$$y_t = \tfrac{1}{2}h = 13.25\text{ in} \qquad f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000\text{psi}} = 474.3\text{ psi} ✓$$

$$M_{cr} = \frac{(474.3\text{psi})(23262\text{in}^4)}{13.25\text{ in}} = 69.4\text{ kip}\cdot\text{ft} ✓$$

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$

$$I_{eff} = \left(\frac{69.4\text{kip}\cdot\text{ft}}{90\text{ kip}\cdot\text{ft}}\right)^3(23262\text{in}^4) + \left[1-\left(\frac{69.4\text{k}\cdot\text{f}}{90\text{ k}\cdot\text{f}}\right)^3\right](14197\text{in}^4)$$

$$= 18354\text{in}^4 \quad ok$$

I_{cr}: $\frac{bx^2}{2} + (n-1)A_s'(x-d') = (n)(A_s)(d-x)$ solve for x

$$\frac{15\text{in}}{2}x^2 + (7)(1.2\text{in}^2)(x-3\text{in}) - (8)(5\text{in})(24\text{in}-x) = 0$$

$$x = 8.68\text{in} ✓$$

$$I_{cr} = \tfrac{1}{3}(15\text{in})(8.68\text{in})^3 + (7)(1.2\text{in}^2)(5.68\text{in})^{\cancel{3}\,2} + (8)(5.0\text{in}^2)(24\text{in}-8.68\text{in})^2 \quad (-1)$$

$$I_{cr} = 14197\text{ in}^4$$

a. $\delta_{DL} = 0.098\text{ in}$ ok

b. The maximum total deflection, assuming long-term deflection (at t=5 years) due to the sustained dead load and short-term (immediate) applied live load.

$$\xi = 2.0$$

$$M_{LL} = \frac{(1.5\text{kip/ft})(20\text{ft})^2}{8} = 75\text{ kip}\cdot\text{ft} ✓$$

$$I_{eff} = \left(\frac{69.4\text{k}\cdot\text{f}}{165\text{ k}\cdot\text{f}}\right)^3(23262\text{ in}^4) + \left[1-\left(\frac{69.4\text{k}\cdot\text{f}}{165\text{ k}\cdot\text{f}}\right)^3\right](14197\text{ in}^4) = 14871.8\text{ in}^4 \quad ok$$

$$\delta_{DL+LL} = \frac{5(3.3\text{kip/ft})(20\text{ft})^4(12\text{in})^3}{384(14872\text{in}^4)(3.6\times10^3\text{ksi})} = 0.2219\text{ in} ✓$$

$$\rho' = \frac{1.2\text{ in}^2}{(15\text{in})(24\text{in})} = 0.0033 \qquad \delta_{LL} = \delta_{DL/LL} - \delta_{DL} = 0.124\text{ in} ✓$$

$$\lambda = \frac{2.0}{1+50(0.0033)} = 1.714 ✓$$

$$\delta_{creep/shrink} = (1.714)(0.098\text{ in}) + 0.124\text{ in}$$

$$\delta_{Tot} = 0.292\text{in} ✓$$

34/35

CE 326 **Test 2** **Spring 2004**

Name: Master

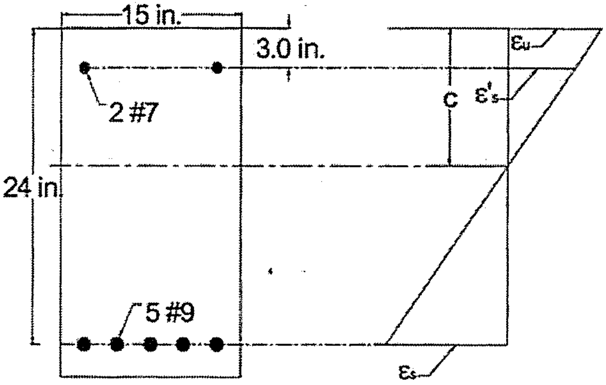
While taking this test, I have abided by the provisions of the University of Virginia honor code.

Signature

PROBLEMS (OPEN BOOK & NOTES)

1. Use the given concrete beam to answer the following questions. (65 points)

$f'_c = 4,000$ psi
$f_y = 60,000$ psi
E_s=29,000,000 psi.



$A_s = 5.00\ in^2 \quad \rho = \frac{A_s}{bd} = 0.0139$

$A'_s = 1.20\ in^2 \quad \rho' = \frac{A'_s}{bd} = 0.0033$

$\rho_{max} = 0.0206$ Table 2.4

$\bar{\rho}_{max} = \rho_{max} + \rho' = 0.0206 + 0.0033 = 0.0239$

a) Investigate whether compression steel is necessary for this section to meet ductility requirements of the tensile steel provided.

$\rho = 0.0139 < \rho_{max} = 0.0206$ ∴ If this section was provided as designed, but without compression steel, it would meet ductility requirements.
∴ Compression steel is not needed.

b) Assuming compression steel is necessary, at the nominal moment capacity, does the compression steel yield?

$\varepsilon_y = \frac{60,000\ psi}{29 \times 10^6\ psi} = 0.00207$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$= 0.85(0.85) \frac{4000\ psi}{60000\ psi} \frac{3\ in}{24\ in} \frac{0.003}{0.003 - 0.00207} + 0.0033$$

$$= 0.0194 + 0.0033 = 0.0227$$

Since $\rho = 0.0139 < \bar{\rho}_{cy} = 0.0227$, the compression steel will not yield.

c) Determine the nominal moment capacity, M_n, for the beam, as designed.

$$A_s f_y = 0.85\beta_1 f'_c b c + A'_s \varepsilon_u E_s \frac{c-d'}{c}$$

$$5.00(60000) = 0.85(0.85)(4000)(15)c + 1.20(0.003)(29\times10^6)\left(\frac{c-3}{c}\right)$$

$$300\,000 = 43350c + 104400\left(\frac{c-3}{c}\right)$$

$$300\,000c = 43350c^2 + 104400(c-3)$$ Solve for c $\Rightarrow -b \pm \sqrt{\frac{b^2-4ac}{2a}}$

$0 = 43350c^2 - 175600c - 313200 \Rightarrow c = 5.77\text{ in}$ $a = 4.90\text{ in}$

$f'_s = \varepsilon_u E_s\left(\frac{c-d'}{c}\right) = 0.003(29\times10^6)\left(\frac{5.77-3.0}{5.77}\right) = 41766\text{ psi} = 41.8\text{ ksi}$ (not yielded)

$$M_n = 0.85\beta_1 f'_c a b(d - a/2) + A'_s f'_s(d-d')$$

$$= 0.85(4)(4.90)(15)\left(24 - \frac{4.90}{2}\right) + 1.20(41.8)(24-3)$$

$= 5385 + 1053 = 6438$ in·kips $= \underline{537\text{ ft·kips}}$

d) Does the beam meet minimum ductility requirements, as specified in the code specifications (ACI 318-02)? Note: 1 layer so $d_t = d$

$\varepsilon_t = \varepsilon_u\left(\frac{d_t-c}{c}\right) = 0.003\left(\frac{24-5.77}{5.77}\right) = 0.0095 > 0.004$ ✓

OR

$\rho = 0.0139 < \bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y} = 0.0206 + 0.0053\left(\frac{41.8}{60}\right) = 0.0229$; Yes, ductility requirements are met.

e) Does the beam meet minimum tensile reinforcement requirements of the code?

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y}$$

$= 1.14\text{ in}^2 \geq \underline{1.20\text{ in}^2}$

Since $A_s = 5.00\text{ in}^2 > 1.20\text{ in}^2$ ✓

Minimum tensile reinforcement is met.

f) What is the design moment capacity, ϕM_n, of the beam?

Since $\varepsilon_t = 0.0095$ (part d. above) $> 0.005 \therefore \phi = 0.90$

$\phi M_n = 0.90(537\text{ ft·kips}) = \underline{483\text{ ft·kips}} = 5794$ in·kips

g) In accordance with code requirements, what size ties and tie spacing would you recommend for this double-reinforced beam?

Since tensile reinforcement is #9's use #3 stirrups

Spacing: $16(d_b) = 16(1.128) = 18.0\text{ in}$

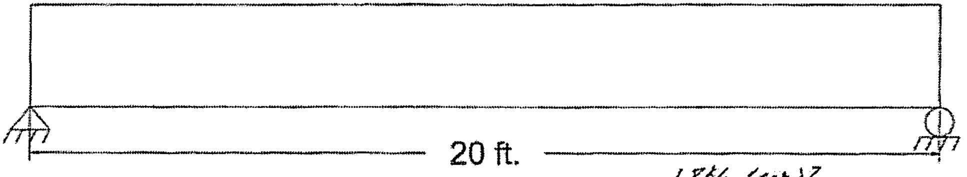
$48(d_{stirrup}) = 48(0.375) = 18.0\text{ in}$

$b = 15 = 15.0\text{ in}$ ✓

Use 15 in o.c. spacing

2. If the cross-section presented in Problem 1 is of a simply-supported beam carrying sustained dead load (including self-weight) $w_{DL} = 1.8$ kips/ft and applied service live load, $w_{LL} = 1.5$ kips/ft, determine: (35 points) Use $h = 26.5$ in

$E_c = 57{,}000\sqrt{f'_c} = 3605$ ksi $\quad f_r = 7.5\sqrt{f'_c} = 474$ psi $\quad n = \frac{29000 \text{ ksi}}{3605 \text{ ksi}} = 8.04 \sim \underline{\underline{8}}$



(hint: $M_{mid}=wl^2/8$ and $d_{mid} = 5wl^4/384EI$)

$M_{DL} = \frac{1.8 \text{ k/ft} (20 \text{ ft})^2}{8} = 90$ ft·kips

$M_{LL} = \frac{1.5 (20 \text{ ft})^2}{8} = 75$ ft·kips

a. The maximum immediate deflection at mid-span due solely to the dead load.

$I_g = \frac{1}{12} bh^3 = \frac{1}{12}(15)(26.5)^3 = 23262 \text{ in}^4$

$M_{cr} = \frac{f_r \cdot I_g}{\bar{y}} = \frac{474(23262)}{26.5/2} = 832 \text{ in·kips} = 69.3 \text{ ft·kips}$

$I_{cr} = \frac{1}{3} b\bar{y}^3 + (n-1)A'_s(\bar{y}-d')^2 + n(A_s)(d-\bar{y})^2$

$= \frac{1}{3}(15)(8.68)^3 + 7(1.2)(8.68-3)^2 + 8(5.0)(24-8.68)^2$

$= 12929 \text{ in}^4$

$\frac{M_{cr}}{M_a} = \frac{69.3}{90} = 0.771$

$I_{cracked}$: $\Sigma M_{\bar{y}} = 0$

$b\bar{y}(\bar{y}/2) + (n-1)A'_s(\bar{y}-d') = n(A_s)(d-\bar{y})$

$7.5\bar{y}^2 + 7(1.2)(\bar{y}-3) = 8(5.0)(24-\bar{y})$

$7.5\bar{y}^2 + 48.4\bar{y} - 985.2 = 0 \Rightarrow \bar{y} = 8.68 \text{ in}$

$I_{eff} = (0.77)^3 23262 + (1-0.77^3) 12929 = 17646 \text{ in}^4$

$\delta_{DL} = \frac{5wl^4}{384EI_e} = \frac{5(1.8 \text{ k/ft})\frac{1 \text{ ft}}{12 \text{ in}}(20 \text{ ft} \times 12 \text{ in/ft})^4}{384(3605 \text{ ksi})(17646 \text{ in}^4)}$

$\boxed{\delta_{DL} = 0.102 \text{ in}}$

b. The maximum total deflection, assuming long-term deflection (at t=5 years) due to the sustained dead load and short-term (immediate) applied live load.

$\xi = 2.0$ for $t = 60$ months {Figure 6.8}

$\lambda = \frac{\xi}{1+50\rho'} = \frac{2.0}{1+50(0.0033)} = 1.717$

$\delta_{cr} = \lambda\, \delta_{DL} = 1.717(0.102) = 0.175 \text{ in}$

For DL+LL: $M_a = 165$ ft·kips $\quad \frac{M_{cr}}{M_a} = \frac{69.3}{165} = 0.42$

$I_{eff} = (0.42)^3(23262) + (1-0.42^3)(12929) = 13694 \text{ in}^4$

$\delta_{DL+LL} = \frac{5(3.3 \text{ k/ft})(\frac{1 \text{ ft}}{12 \text{ in}})(20 \text{ ft} \times 12 \text{ in/ft})^4}{384(3605 \text{ ksi})(13694 \text{ in}^4)} = 0.241 \text{ in}$

$\delta_{LL} = \delta_{DL+LL} - \delta_{DL} = 0.241 - 0.102 = 0.139 \text{ in}$

$\delta_{tot} = \delta_{LL} + \delta_{cr} = 0.139 + 0.175 = \boxed{0.314 \text{ in}}$

96

Name: Catherine Hovell

While taking this test, I have abided by the provisions of the University of Virginia honor code.

Signature

PROBLEMS (TAKE-HOME TEST - OPEN BOOK & NOTES)

1. Design a round spiral column for axial load only, if subject to P_D=180^K and P_L=300^K. Include the design of spirals and sketch the cross-section selected. The column is assumed to be short and have clear cover of 1.5 inches. Use f'_c=3,500 psi and f_y = 50,000 psi. (35 points)

CE 326: TEST #3

CATHERINE HOVELL
APRIL 23, 2004

1.

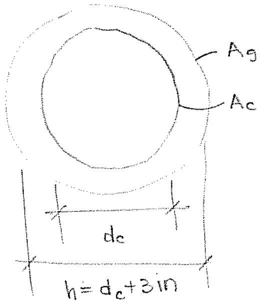
$P_D = 180$ kip

$P_L = 300$ kip

$P = 1.2P_D + 1.6P_L = 1.2(180 \text{ kip}) + 1.6(300 \text{ kip})$

$P_{tot} = 696$ kip ✓

ρ_s & ρ are not the same.



shell strength: $0.85 f'_c (A_g - A_c)$
spiral strength: $2\rho_s A_c f_y$
] should be equal

$\boxed{-1}$

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y} = \frac{4A_{sp}(d_c - d_b)}{s \cdot d_c^2} = \cancel{\frac{A_{st}}{A_g}}$$

$$P_u = \phi(0.85)\left[(0.85)f'_c(A_g - A_{st}) + f_y A_{st}\right]$$

$$A_{st} = 0.45\frac{f'_c}{f_y}A_g\left(\frac{A_g}{A_c} - 1\right)$$

] put together, in terms of d_c

$f'_c = 3.5$ ksi

$f_y = 50$ ksi

$\phi = 0.70$

$A_c = \frac{\pi}{4}d_c^2$

$A_g = \frac{\pi}{4}(d_c + 3\text{in})^2$

$$\frac{P_u}{\phi(0.85)} = 0.85f'_c \cdot \frac{\pi}{4}(d_c+3\text{in})^2 + 0.45 \cdot \frac{\pi}{4}(d_c+3\text{in})^2\left[\frac{(d_c+3\text{in})^2}{d_c^2} - 1\right]\frac{f'_c}{f_y}(f_y - 0.85f'_c)$$

(put in calculator, solve for d_c)

$d_c = 17.56$ in

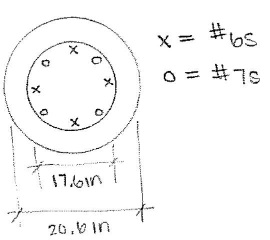
$$A_{st} = 0.45\frac{3.5\text{ ksi}}{50\text{ ksi}} \cdot \frac{\pi}{4}(17.56\text{in} + 3\text{in})^2\left[\frac{(17.56\text{in}+3\text{in})^2}{(17.56\text{in})^2} - 1\right] = 3.88\text{in}^2$$

(check Pn ...

$$P_n = (0.70)(0.85)\left[(0.85)(3.5\text{ksi})\left(\frac{\pi}{4}(17.56\text{in}+3\text{in})^2 + 3.88\text{in}^2\right) + (50\text{ksi})(3.88\text{in}^2)\right] = 696 \text{ kip}$$ ✓

A_{st} made up by 4 #6s ($A = 1.76\text{in}^2$), 4 #7s ($A = 2.4\text{in}^2$) $= 4.16\text{in}^2$ OK

$$\rho_s = \frac{A_{st}}{A_g} = \frac{3.88\text{in}^2}{\pi/4(17.56\text{in}+3\text{in})^2} = 0.0117 = \frac{4A_{sp}(d_c - d_b)}{s \cdot d_c^2}$$



spacing of #3 ties,
s = 4.20in ✗

clear spacing: 1" < s < 3" per code $\boxed{-1}$

use #3 spiral ties, $A_{sp} = \cancel{2}(0.11\text{in}^2)$

$d_b = 0.375$ in

spirals don't have multiple "legs" $\boxed{-1}$

MINIMUM OF:

$$s = \frac{4(\cancel{0.22}\overset{0.11}{}\text{in}^2)(17.56\text{in} - 0.375\text{in})}{(0.0117)(17.56\text{in})^2} = 4.\cancel{2}0\text{ in}$$

2.51 in

$s = 16(0.750\text{in}) = 12$ in

$s = 48(0.375\text{in}) = 18$ in

$s = 20.6$ in

$$\min \rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y} = 0.45\left(\frac{(20.56)^2\pi/4}{(17.56)^2\pi/4} - 1\right)\frac{3.5}{60} = 0.45\left(\frac{332}{242} - 1\right)\frac{3.5}{60} = .0097$$

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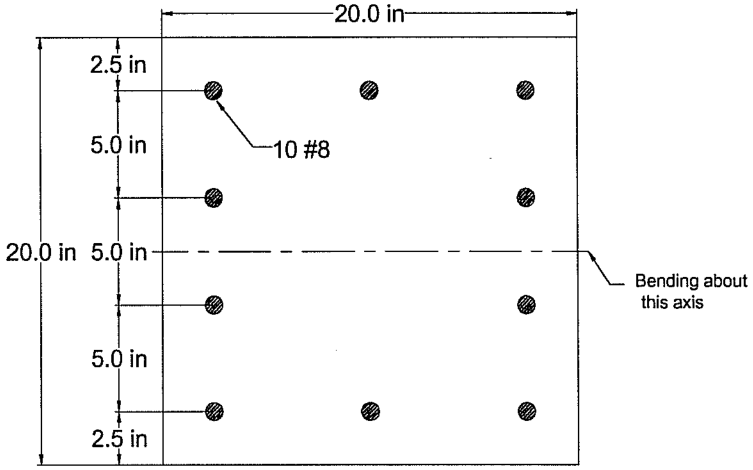
2. Given the concrete short column shown, determine the axial load capacity, P_b, moment capacity, M_b, and eccentricity, e_b at the balanced condition. (35 points)

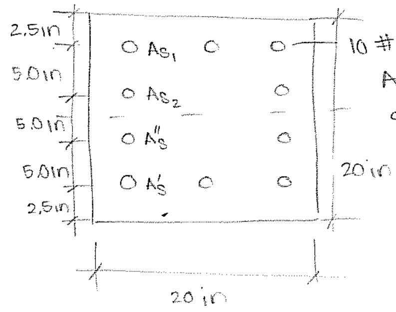
$f'_c =$ 4,000 psi
$f_y =$ 60,000 psi
$E_s =$ 29,000,000 psi



CE 326: TEST #3

C. HOVELL
APRIL 23, 2004

2.



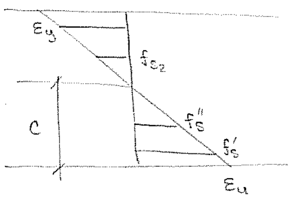
10 #8s
$A_8 = 0.79 in^2$
$d_8 = 1.00 in$

$f_c' = 4$ ksi
$f_y = 60$ ksi
$E_s = 29000$ ksi

$$\varepsilon_s = \varepsilon_y = \frac{60 \text{ ksi}}{29000 \text{ ksi}} = 0.00207$$ ✓

$$C_b = \frac{\varepsilon_u}{\varepsilon_s + \varepsilon_u} \cdot d = \frac{0.003 \cdot 17.5 in}{0.00207 + 0.003} = 10.36 in$$ ✓

✓
$a = 0.85c = 8.80 in$ ($\beta = 0.85$, as $f_c' = 4$ ksi)



$f_{s_1} = f_y = 60$ ksi ✓

$$f_s' = \varepsilon_u \frac{c-d'}{c} \cdot E_s = (0.003)(29000 \text{ ksi}) \frac{10.36 in - 2.5 in}{10.36 in} = 66 \text{ ksi} > f_y$$ ✓

$$\frac{f_y}{h-c-d'} = \frac{f_{s_2}}{h-c-d'-5.0 in} \qquad f_{s_2} = \frac{(60 \text{ ksi})(2.14 in)}{7.14 in} = \underline{18.0 \text{ ksi}} < f_y$$ ✓

$$f_s'' = \frac{(c-d'-5.0 in)}{c} \varepsilon_u E_s = \frac{(2.86 in)(0.003)}{10.36 in}(29000 \text{ ksi}) = \underline{24.0 \text{ ksi}} < f$$ ✓

$$P_b = 0.85 f_c' (ab - 5A_8) + \cancel{A_s' f_s'} + A_s'' f_s'' - \cancel{A_{s_1} f_{s_1}} - A_{s_2} f_{s_2}$$
(cancel each other)

$$P_b = 0.85(4 \text{ ksi})\left[(8.80 in)(20 in) - 5(0.79 in^2)\right] + 2(0.79 in^2)(24.0 \text{ ksi}) - 2(0.79 in^2)(18.0 \text{ ksi})$$

$$\boxed{P_b = 594.5 \text{ kip}}$$ ✓

$$M_b = 0.85 f_c' (ab - 5A_8)(1/2)(h-a) + A_s' f_s' (h/2 - d') + A_s'' f_s'' (h/2 - d'') + A_{s_1} f_{s_1} (d - h/2) + A_{s_2} f_{s_2} (d')$$

$$= 0.85(4 \text{ ksi})\left[(8.80 in)(20 in) - 5(0.79 in^2)\right](1/2)(20 in - 8.80 in) \qquad = 3275.8 \text{ in} \cdot \text{kip}$$

$$+ 3(0.79 in^2)(60 \text{ ksi})\left(\frac{20 in}{2} - 2.5 in\right) + 2(0.79 in^2)(24.0 \text{ ksi})\left(\frac{20 in}{2} - 7.5 in\right) \qquad = 1161.3 \text{ in} \cdot \text{kip}$$

$$+ 3(0.79 in^2)(60 \text{ ksi})\left(17.5 in - \frac{20 in}{2}\right) + 2(0.79 in^2)(18.0 \text{ ksi})(2.5 in) \qquad = 1137.6 \text{ in} \cdot \text{kip}$$

$$\boxed{M_b = 464.6 \text{ ft} \cdot \text{kip}}$$ ✓

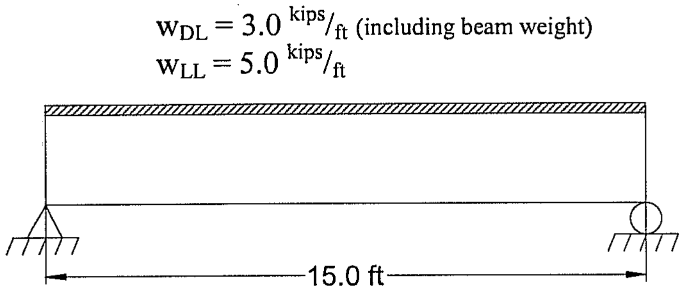
$$e = \frac{M_b}{P_b} \cdot 12 \text{ in/ft} = \frac{464.6 \text{ ft} \cdot \text{kip}}{594.5 \text{ kip}} \cdot \frac{12 in}{1 ft}$$ ✓

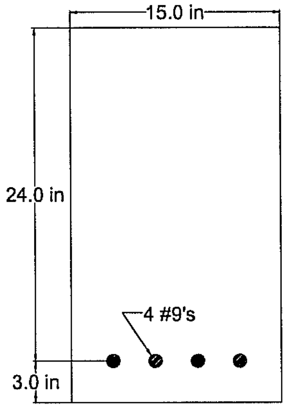
$$\boxed{e = 15.4 in} \times$$ 9.38 Math error (-1)

DUMBASS.

34

3. Design shear reinforcement by selecting stirrup size and maximum spacing at the critical section of the simply supported beam for the section and loadings shown.
Use f'_c=3,000 psi and f_y = 60,000 psi. (30 points)

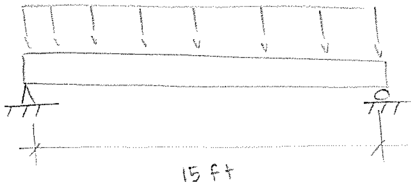




CE 326: TEST #3

C.HOVELL
APRIL 23, 2004

3.

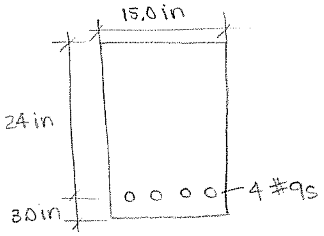


$w_{DL} = 3.0$ kip/ft

$w_{LL} = 5.0$ kip/ft

$f'_c = 3$ ksi

$f_y = 60$ ksi



$w = 1.2w_{DL} + 1.6w_{LL} = 1.2(3.0 \text{ kip/ft}) + 1.6(5.0 \text{ kip/ft})$

$w = 11.6$ kip/ft ✓

shear max occurs at supports; $V = \frac{1}{2}$ total load

$V_{max} = \frac{1}{2}(11.6 \text{ kip/ft})(15 \text{ ft})$

$= 87$ kip ✓

$V_{u/d} = V(x=d)$, $d = 24$ in $V_{u/d} = 63.8$ kip ✓

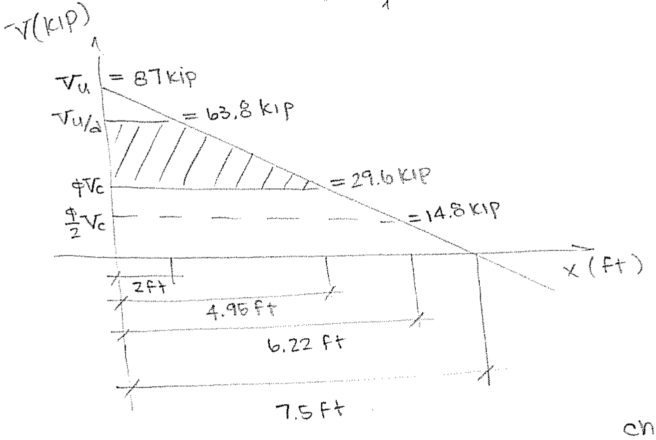
$V_c = 2\sqrt{f'_c}\,bd = 2\sqrt{3000 \text{psi}}\,(15 \text{in})(24 \text{in}) = 39.4$ kip ✓

$\phi V_c = (0.75)(39.4 \text{ kip}) = 29.6$ kip ✓

$$S_d = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{(0.75)(0.22 \text{in}^2)(60 \text{ksi})(24 \text{in})}{\boxed{87 \text{kip}} - 29.6 \text{ kip}}$$

use V_u at d

$S_d = 4.14$ in ✓



check minimums:

- $s \leq d/2 = \frac{24 \text{in}}{2} = 12$ in
- $s \leq 24$ in
- $s \leq \frac{A_v f_y}{50b} = \frac{(0.22 \text{in}^2)(60000 \text{ psi})}{50(15.0 \text{in})} = 17.6$ in

choose smallest, $S_d = 4.14$ in

$V_s \leq 4\sqrt{f'_c}\,bd$ to use these minimums:

$V_s = \frac{V_u}{\phi} - V_c = \frac{87 \text{kip}}{0.75} - 39.4 \text{kip}$

$= 76.6$ kip

$4\sqrt{3000 \text{psi}}\,(15 \text{in})(24.0 \text{in}) = 78.9$ kip

$V_s \leq 4\sqrt{f'_c}\,bd$ ✓

critical section at $x = d = 24$ in from face of support; $V = 63.8$ kip. Stirrups (#3 bars) must be spaced at $s = 4.14$ in. ✓ → 4.00 in until $\frac{1}{2}\phi V_c$ ($x = 6.22$ ft, $V = 14.8$ kip).

CE 326 **Test 3** **Spring 2004**

Name: Master

While taking this test, I have abided by the provisions of the University of Virginia honor code.

Signature ______________________

PROBLEMS (TAKE-HOME TEST - OPEN BOOK & NOTES)

1. Design a round spiral column for axial load only, if subject to P_D=180^K and P_L=300^K. Include the design of spirals and sketch the cross-section selected. The column is assumed to be short and have clear cover of 1.5 inches. Use f'$_c$=3,500 psi and f$_y$ = 50,000 psi. (35 points)

$P_u = 1.2(180^k) + 1.6(300^k) = 696^k$ ② assume $\rho_g = 2\%$ $\Rightarrow A_{st} = 0.02 A_g$

$P_u = \phi .85 [.85 f'_c (A_g - A_{st}) + f_y A_{st}]$

$696^k = 0.70(.85)[.85(3.5^{ksi})(A_g - 0.02A_g) + 50^{ksi}(.02A_g)]$

$696^k = 0.595 [2.975(.98A_g) + 1A_g]$

$696^k = 2.33 A_g$

$A_g = 299 in^2$ $\pi r^2 = 299$ $r = 9.75$ $d = 19.5 \Rightarrow$ use $\underline{d = 20''}$

$A_g = 314.2 in^2$

$696 = 0.70(.85)[.85(3.5)(314.2 - A_{st}) + 50 A_{st}]$

$1169.7 = 2.975(314.2 - A_{st}) + 50A_{st}$

$1169.7 = 934.7 - 2.975A_{st} + 50A_{st}$ Must use at least 6 bars

$235.0 = 47.0 A_{st}$ Try 6 #9's = 6 in^2

$A_{st} = 5.0 in^2$ $\rho = 1.6\%$ $\rho = 1.9\%$

Spirals: Assume #3 spiral

$Min \rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y}$

$= 0.45\left(\frac{314}{227} - 1\right)\frac{3.5}{50}$

$= 0.012$

$d_c = 20 - 2(1.5) = 17 in$

$A_c = 227 in^2$

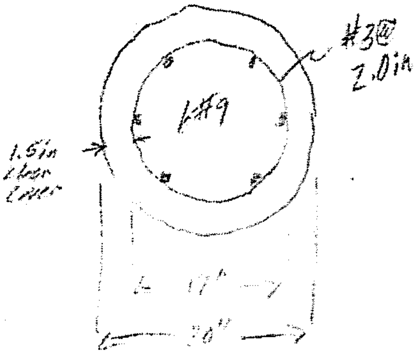
$\rho_s = \frac{4A_{sp}(D_c - d_b)}{s D_c^2}$

$0.012 = \frac{4(0.11)(17 - .375)}{s(17'')^2}$

$s = 2.1 in \Rightarrow$ use $\underline{\underline{2.0 in}}$

Check clr. space

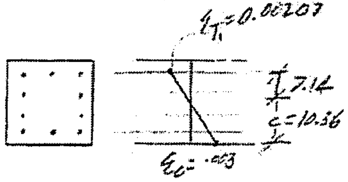
$s_c = 2.0'' - 0.375 = 1.625 in$ $> 1'' \& < 3''$ ok

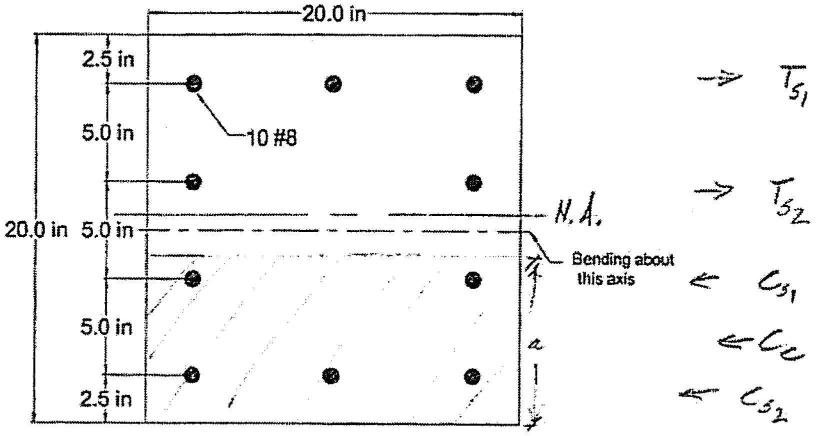


2. Given the concrete short column shown, determine the axial load capacity, P_b, moment capacity, M_b, and eccentricity, e_b at the balanced condition. (35 points)

$f'_c =$ 4,000 psi
$f_y =$ 60,000 psi
$E_s =$ 29,000,000 psi

$\varepsilon_y = \frac{60 ksi}{29000 ksi} = 0.00207$





$c_b = \frac{.003}{.003+.00207} \times 17.5 = 10.36$ in

$a_b = 8.81$ in

Tension
$\varepsilon_{s_1} = 0.00207 \quad f_{s_1} = 60$ ksi
$\varepsilon_{s_2} = \frac{7.14-5}{7.14}(.00207) = 0.00062 \quad f_{s_2} = 18.0$ ksi

Compression
$\varepsilon_{s_1} = \frac{10.36-7.5}{10.36}(.003) = 0.00083 \quad f_{s_1} = 24.0$ ksi
$\varepsilon_{s_2} = \frac{10.36-2.5}{10.36}(0.003) = 0.00228 > 0.00207 \quad f_{s_2} = 60$ ksi

$\Sigma F = 0$

$$P_n = \underbrace{(-60)(3)(.79) + (-18)(2)(.79)}_{T_s} + \underbrace{(24)(2)(.79) + (60)(3)(.79)}_{C_s} + \underbrace{.85(4^{ksi})[(8.81 in)(20) - 5(.79)]}_{C_c}$$

$\underline{\underline{P_{n_b} = 595.1^K}}$

$\Sigma M @ centroid = 0$

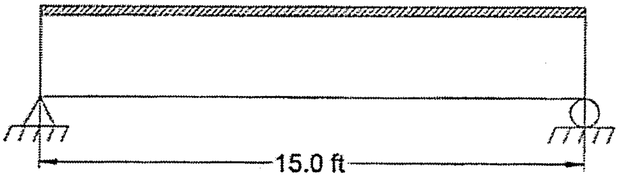
$$(595.1)e = 2[60(3)(.79)(7.5)] + 18.0(2)(.79)(2.5) + 24.0(2)(.79)(2.5) + .85(4)(8.81)(20)(10 - 8.81/2)$$

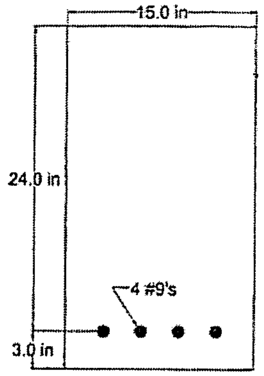
$\underline{\underline{M_{nb}}} = 595.1 e = 5651$ in·kips $= \underline{\underline{470.9 \text{ ft·kips}}}$

$\underline{\underline{e_b = 9.50 \text{ in}}}$

3. Design shear reinforcement by selecting stirrup size and maximum spacing at the critical section of the simply supported beam for the section and loadings shown.
Use f_c=3,000 psi and f_y= 60,000 psi. (30 points)

$w_{DL} = 3.0\ ^{kips}/_{ft}$ (including beam weight)
$w_{LL} = 5.0\ ^{kips}/_{ft}$





$w_u = 1.2(3.0) + 1.6(5.0) = 11.6\ ^{kips}/_{ft}$

$V_u = 11.6 \times {}^{15ft}/_2 = 87$ kips

$V_u @ d = 87 - 11.6 \times {}^{24}/_{12} = 63.8$ kips

$\phi V_c = 0.75 \times 2\sqrt{3000}\,(15)(24)/1000 = \overset{\phi}{0.75}\,(\overset{V_c}{39.4}) = 29.6$ kips

check adequacy of section:

$(8+2)\sqrt{3000}\,(15)(24)/1000 = 197.2 > V_n = \frac{63.8}{.85} = 75.1^k$ ✓OK

since $V_n > \frac{1}{2}V_c$, stirrups are necessary.

Try #3, two-legged stirrups (area = 0.11 in² per leg)

$A_v = 0.22\ in^2$

$S_d = \frac{A_v f_y d}{(V_u/\phi) - V_c} = \frac{0.22\,(60)(24)}{(63.8/.75 - 39.4)} = 6.94$ in

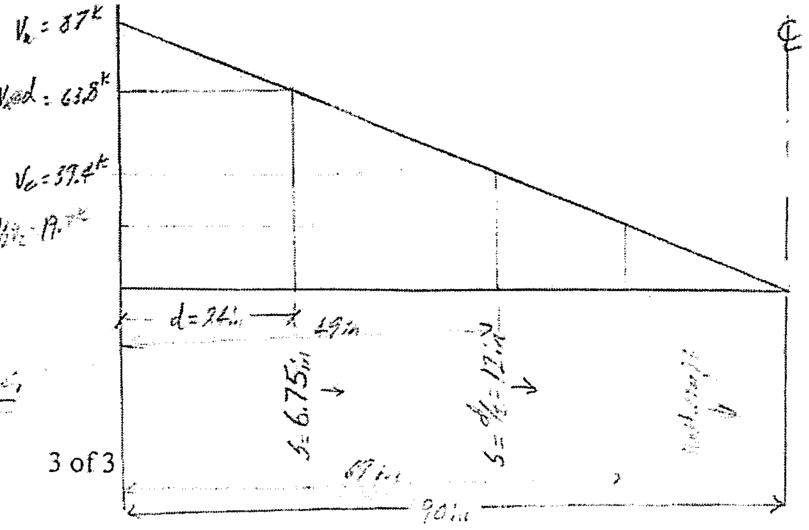
$S \leq d/2 = 24/2 = 12$ in

$S \leq 24$ in

$S \leq \frac{0.22 \times 60000}{50 \times 15} = 17.6$ in

Critical section @ d from face of support $x = 2.0'$ $V = 63.8^k$

min. stirrups required are #3's at 6.75 in c/c

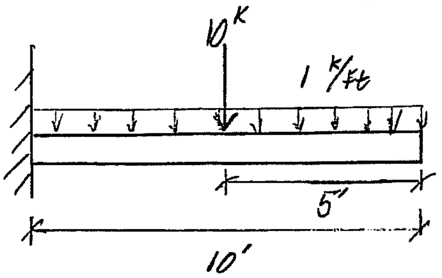


Homework #1 | CE 326 | Spring 04

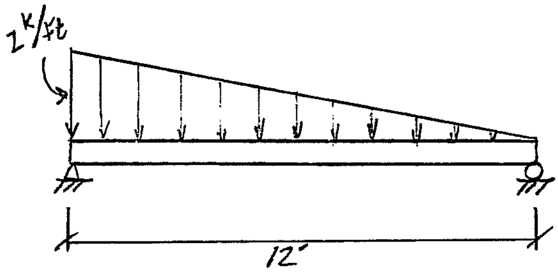
Due 1/30/04

Draw the shear and moment diagrams for the following beams:

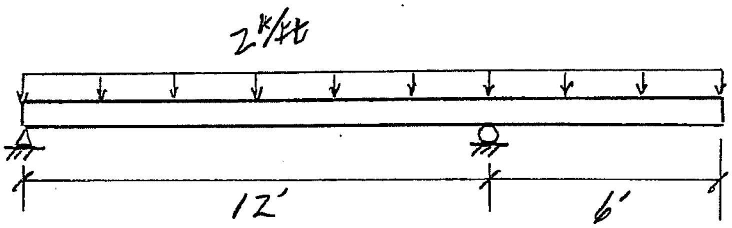
1)



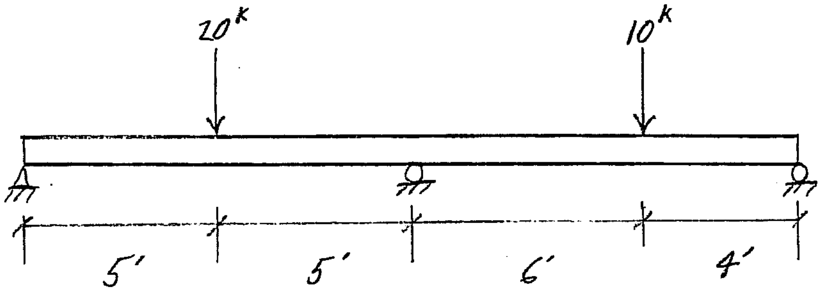
2)



3)



4)

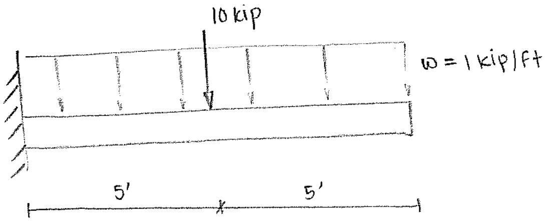


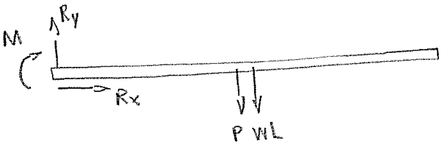
CE 326: HOMEWORK #1

$\frac{36}{30}$

Catherine Hovell
January 30, 2004

1.





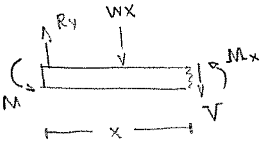
ΣF_x: $R_x = 0$

ΣF_y: $R_y = P + wL = 10\,kip + (1\,kip/ft)(10\,ft)$

$R_y = 20\,kip$

ΣM: $-M = (10\,kip)(5\,ft) + (1\,kip/ft)(10\,ft)(5\,ft)$

$M = 100\,kip \cdot ft$ ↻



$V = R_y - wx$

$M_x = M + wx\frac{1}{2}x - R_y x$

$V = 20\,kip - (1\,kip/ft)x$

$-M = 100\,kip \cdot ft + \frac{1}{2}\,kip/ft \cdot x^2 - 20\,kip \cdot x$

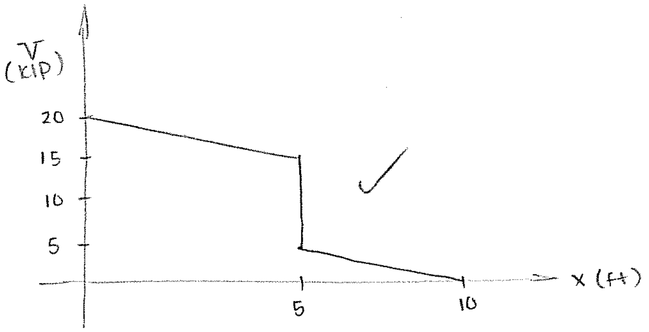
$0 < x < 5\,ft$

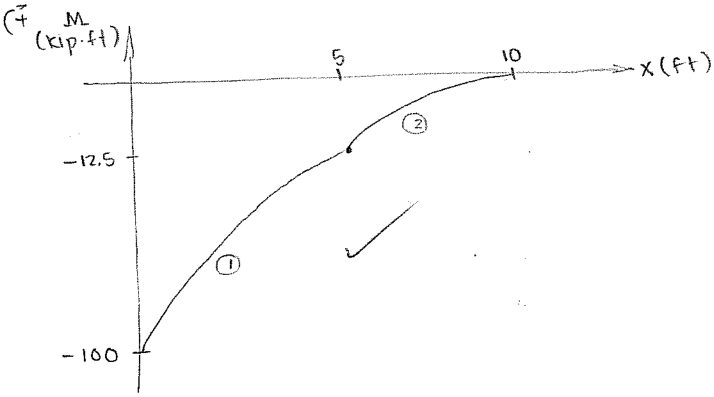
$-M = 100\,kip \cdot ft + \frac{1}{2}\,kip/ft \cdot x^2 - 20\,kip \cdot x + 10\,kip(x-$

$-M = \frac{1}{2}\,kip/ft \cdot x^2 - 10\,kip \cdot x + 50\,kip \cdot ft$

$V = 10\,kip - (1\,kip/ft)\,x$

$5\,ft < x \leq 10\,ft$



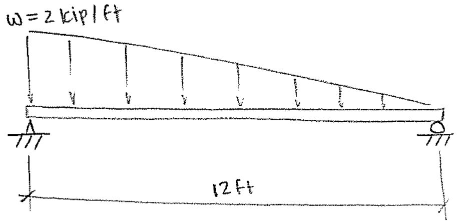


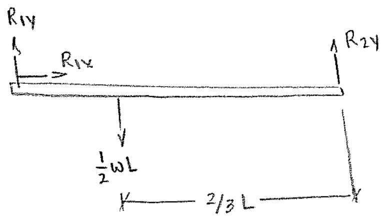
①: $M = (20\,kip)x - (\frac{1}{2}\,kip/ft)x^2 - 100\,kip\,ft$

②: $M = (10\,kip)x - (\frac{1}{2}\,kip/ft)x^2 - 50\,kip \cdot ft$

CE 326: HOMEWORK #1

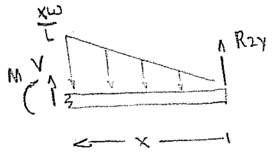
2.





$\Sigma F_x: R_{1x} = 0$

$\Sigma F_y: R_{1y} + R_{2y} = \frac{1}{2}\omega L$



$\Sigma M_1: \frac{1}{2}\omega L \cdot \frac{1}{3}\cancel{L} = R_{2y} \cdot \cancel{L}$

$R_{2y} = \frac{1}{6}\omega L = \frac{1}{6}(2\text{ kip/ft})(12\text{ ft}) = 4\text{ kip}$

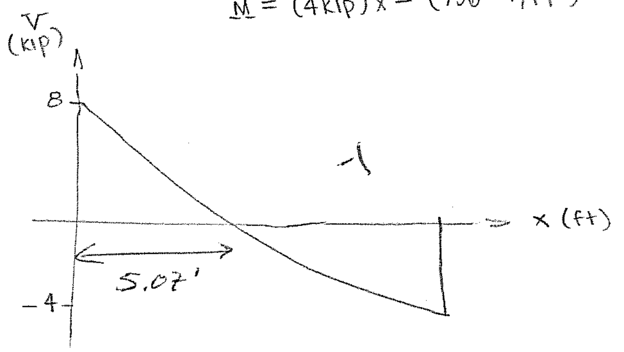
$R_{1y} = \frac{1}{2}(2\text{ kip/ft})(12\text{ ft}) - 4\text{ kip} = 8\text{ kip}$

$$V = R_{2y} - \frac{x\omega}{L}\cdot\frac{1}{2}x = 4\text{ kip} - \frac{(2\text{ kip/ft})}{2(12\text{ ft})}x^2$$

$$V = 4\text{ kip} - (1/12\text{ kip/ft}^2)x^2$$

$$M = R_{2y}\cdot x - \frac{1}{2}x\omega\frac{1}{L}\cdot x\cdot\frac{1}{3}x = (4\text{ kip})x - \frac{2\text{ kip/ft}}{6(12\text{ ft})}x^3$$

$$M = (4\text{ kip})x - (1/36\text{ kip/ft}^2)x^3$$



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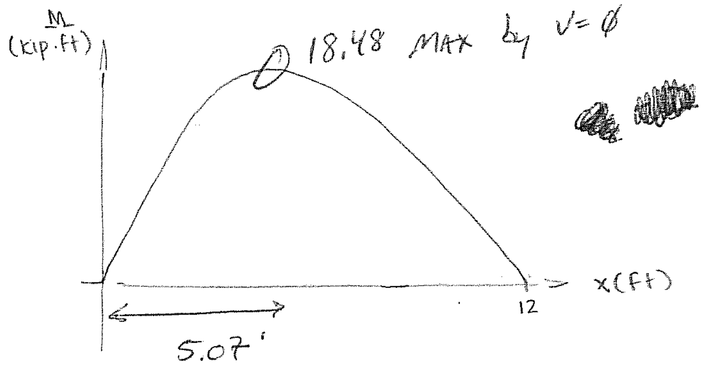
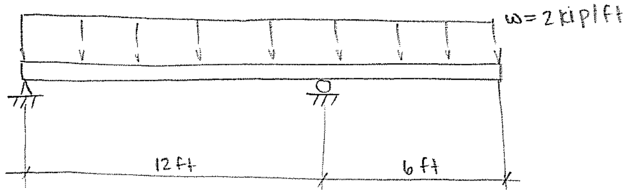


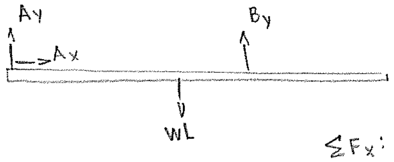
18.48 MAX by V = 0

-1

CE 326: HOMEWORK #1

3.



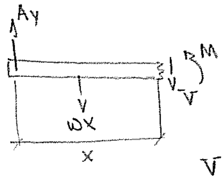


$\Sigma F_x: A_x = 0$

$\Sigma F_y: A_y + B_y = \omega L = (2\ kip/ft)(18\ ft) = 36k$

$\Sigma M_A: \omega L \cdot \frac{1}{2} L = B_y(12\ ft) = (2\ kip/ft)(18\ ft)^2 \cdot \frac{1}{2}$

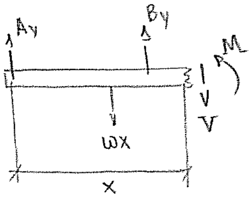
$B_y = 27\ kip \quad A_y = 9\ kip$



$V = \omega x - A_y = (2\ kip/ft)x - 9\ kip$

$M = A_y x - \omega x \cdot \frac{1}{2} x = (9\ kip)x - (1\ kip/ft)x^2$

$0 < x < 12\ ft$



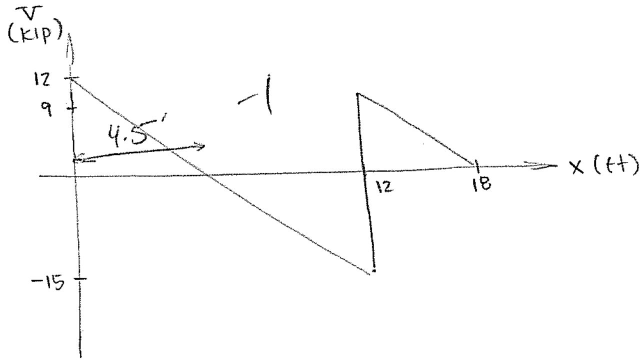
$V = \omega x - A_y - B_y = (2\ kip/ft)x - 36\ kip$

$M = A_y x + B_y(x - 12\ ft) - \omega x \cdot \frac{1}{2} x$

$M = (36\ kip)x + (27\ kip)x - (27\ kip)(12\ ft) - (1\ kip/ft)x^2$

$M = (63\ kip)x - (1\ kip/ft)x^2 - 324\ kip \cdot ft$

$12\ ft < x < 18\ ft$



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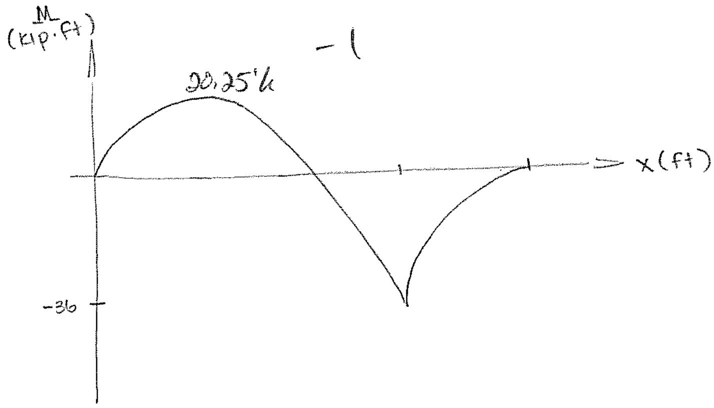

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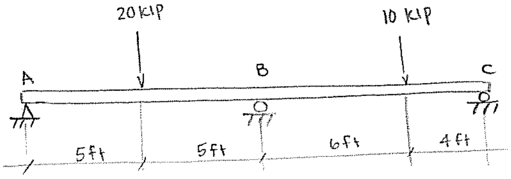
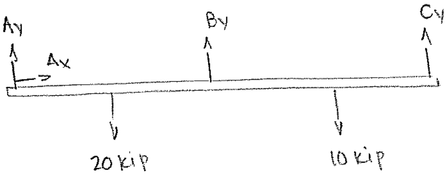
CE 326: HOMEWORK #1

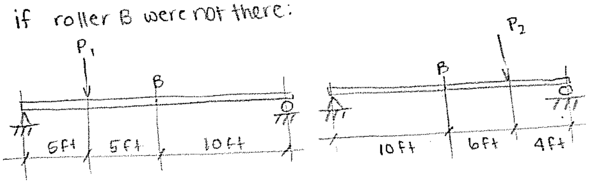
4.





ΣF_x: $A_x = 0$

if roller B were not there:



ΣF_y: $A_y + B_y + C_y = 30 \text{ kip}$

ΣM_A: $(20 \text{ kip})(5 \text{ ft}) + (10 \text{ kip})(16 \text{ ft}) = (10 \text{ ft})B_y + (20 \text{ ft})C_y$

$B_y + 2C_y = 26 \text{ kip}$

From P1: $$V_{B_1} = \frac{-P_1 (5 \text{ ft})(10 \text{ ft})}{6EI(20 \text{ ft})}\left[(20 \text{ ft})^2 - (5 \text{ ft})^2 - (10 \text{ ft})^2\right]$$

$$V_{B_1} = \frac{-2292 \text{ ft}^3 \cdot \text{kip}}{EI}$$

from P2: $$V_{B_2} = \frac{-P_2 (4 \text{ ft})(10 \text{ ft})}{6EI(20 \text{ ft})}\left[(20 \text{ ft})^2 - (4 \text{ ft})^2 - (10 \text{ ft})^2\right]$$

$$V_{B_2} = \frac{-947 \text{ ft}^3 \text{kip}}{EI}$$

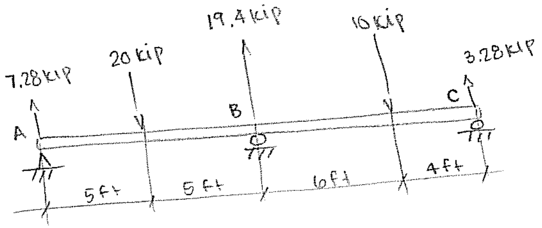
but, $V_B = 0$, F_{By} must cancel ↑

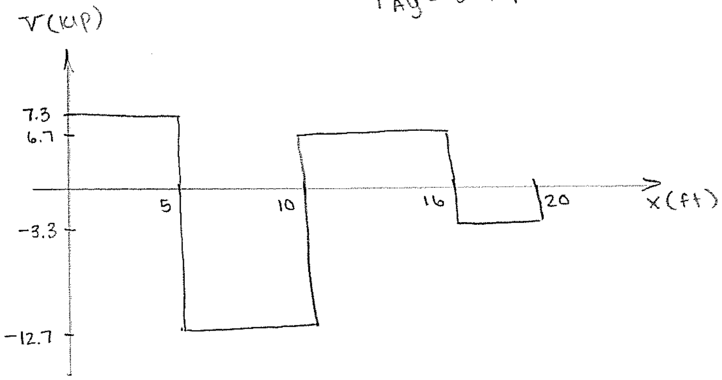
$$V_B = \frac{+F_{By}(20 \text{ ft})^3}{48 \cancel{EI}} = \frac{-1}{\cancel{EI}}\left(2292 \text{ ft}^3 \text{kip} + 947 \text{ ft}^3 \text{kip}\right)$$

$$F_{By} = 19.4 \text{ kip}$$

so, $F_{cy} = 13 \text{ kip} - \frac{F_{By}}{2} = 3.28 \text{ kip}$

$F_{Ay} = 30 \text{ kip} - F_{By} - F_{cy} = 7.28 \text{ kip}$





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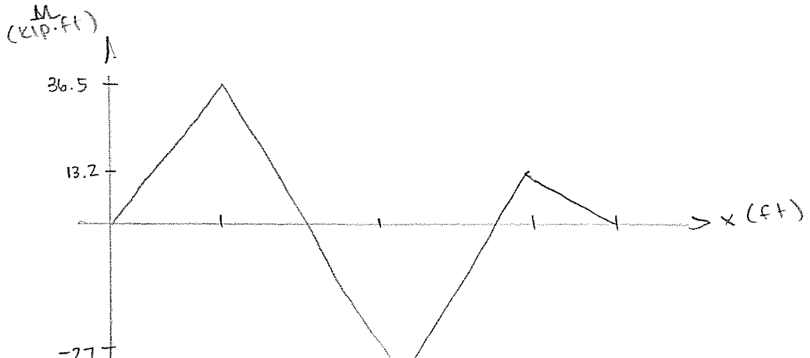
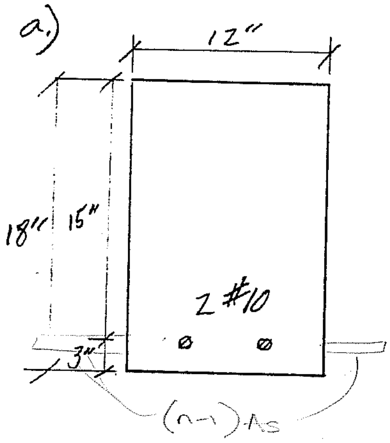


Homework 2 CE 326 Spring '04

1) Determine the cracking moment for the following sections:

a.)



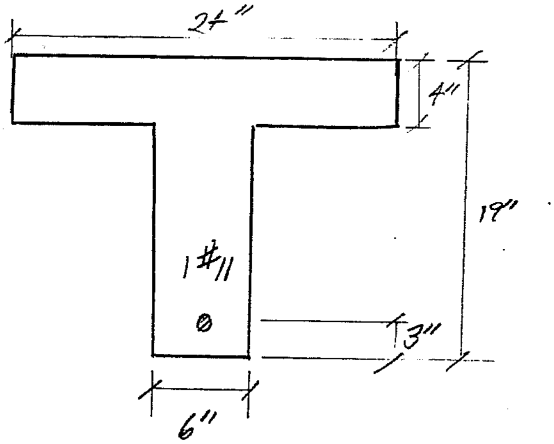
$f_{ys} = 60\,000$ psi

$f'_c = 3$ ksi

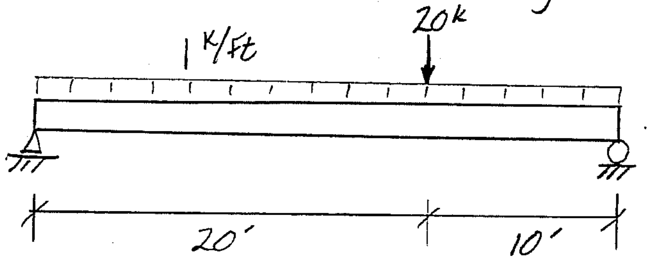
$E_s \sim 29000$ ~~ksi~~ ksi

$E_c \sim 57000\sqrt{f'_c}$

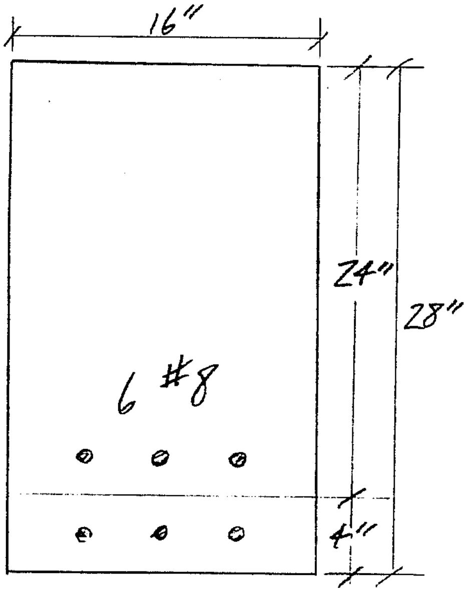
$n = E_s/E_c = 9$



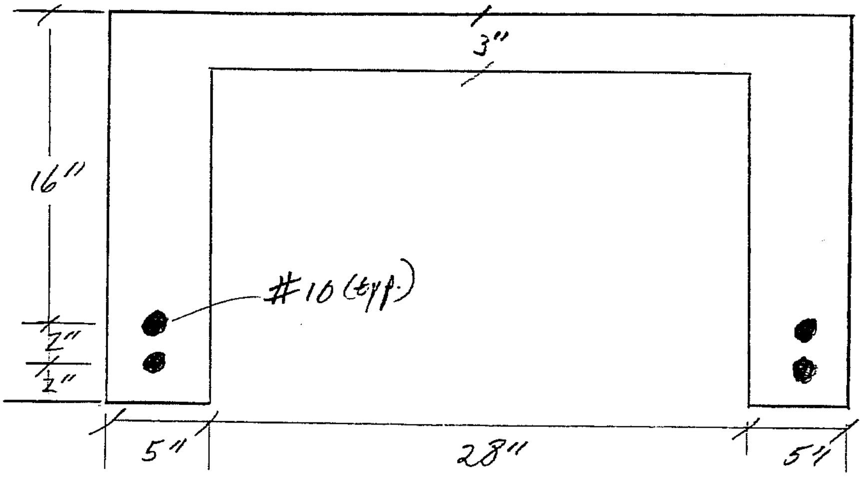
2) Assuming the given section has cracked, compute the bending stresses in the steel and concrete at the section of maximum moment. Check the assumption of cracking for $f'_c = 3$ ksi.



$n = 10$



3.) Determine the flexural stresses in a beam with the following cross-section: $M = 150'^{k}$ $n = 9$

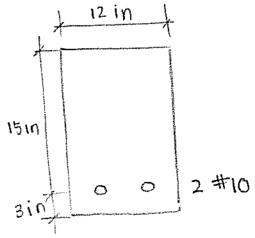


CE 326: HOMEWORK #2

$\frac{24}{30}$

CATHERINE HOVELL
FEBRUARY 6, 2004

1.



$f_c' = 3$ ksi

$n = E_s/E_c = \frac{29000 \text{ ksi}}{57000\sqrt{f_c'}} = 9.2$

$n = 9$

$A_s = (2)(9)(1.27 \text{ in}^2)$

$A_s = 22.86 \text{ in}^2$

#10: $d = 1\frac{1}{4}$ in
$A = 1.27 \text{ in}^2$

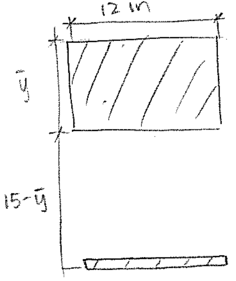
$M_{cr} = \frac{f_r I}{c}$

$c = \bar{y}$ to edge (T)
(bottom)

$f_r = 7.5\sqrt{f_c'}$

$f_r = 7.5(3000 \text{ ksi})^{1/2}$

$f_r = 0.411$ ksi



$\bar{y}(12 \text{ in})\frac{1}{2}\bar{y} = (15-\bar{y})(22.86 \text{ in}^2)$

$\bar{y} = 5.\cancel{8}9$ in

Assume not cracked.

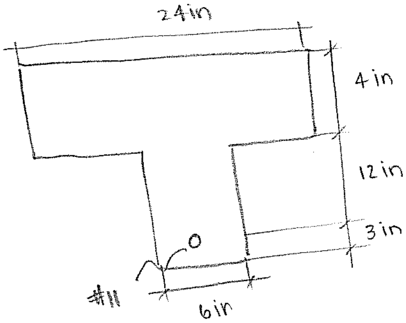
-1

$I = 1/3(5.89 \text{ in})^3(12 \text{ in}) + (22.86 \text{ in}^2)(9.11 \text{ in})^2$

$I = 2715 \text{ in}^4$

$M_{cr} = \frac{(0.411 \text{ ksi})(2715 \text{ in}^4)}{12.11 \text{ in}} = 92.1$ kip in

$\boxed{a.\ M_c = 92.1 \text{ kip} \cdot \text{in}}$

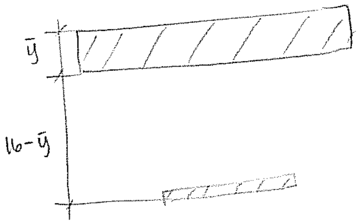


#11: $A = 1.56 \text{ in}^2$
$d = 1\frac{3}{8}$ in

$n = 9$

$A_s = (1.56 \text{ in}^2)(1)(9) = 14.0 \text{ in}^2$

Assume $\bar{y} < 4$ in



$\bar{y}(24 \text{ in})\frac{1}{2}\bar{y} = (14.0 \text{ in}^2)(16 \text{ in} - \bar{y})$

$\bar{y} = 3.78\cancel{\text{in}}$ -1

$I = 1/3(24 \text{ in})(3.78 \text{ in})^3 + (14.0 \text{ in}^2)(12.22 \text{ in})^2$

$I = 252\cancel{3} \text{ in}^4$ -1

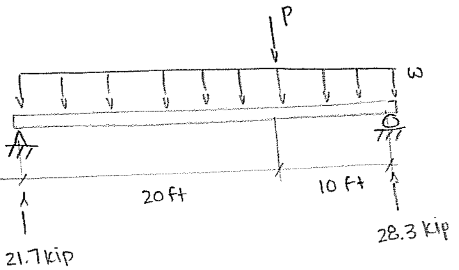
$M_c = \frac{f_r I}{c_T} = \frac{(0.411 \text{ ksi})(2523 \text{ in}^4)}{15.22 \text{ in}} = 68.1$ kip·in

$\boxed{b\ M_c = 68.1 \text{ kip} \cdot \text{in}}$

CE 326: HOMEWORK #2

C.HOVELL

2.

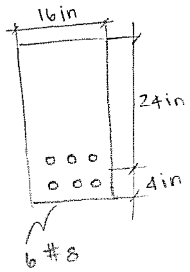


$w = 1\,kip/ft$

$P = 20\,kip$

$f'_c = 3\,ksi$

$n = 10$

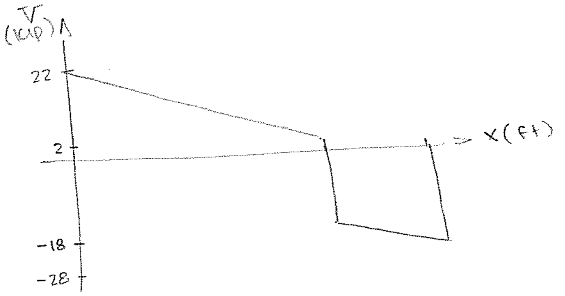


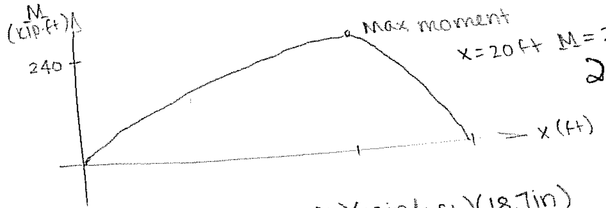
#8: $d = 1\,in$

$A = 0.79\,in^2$

$A_s = (6)(10)(0.79\,in^2)$

$A_s = 47.4\,in^2$

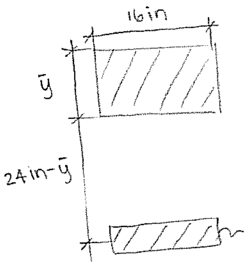




Max moment

$x = 20\,ft$ $M = 240\,kip\cdot ft$

234'k



$\frac{1}{2}(16\,in)(\bar{y})^2 = (47.4\,in^2)(24\,in - y)$

$\bar{y} = 9.3\,in$

$I = \frac{1}{3}\left[(16\,in)(9.3\,in)^3 + (14.7\,in)^2(47.4\,in^2)\right]$

$I = 7704\,in^4$ −1

$f_r = 7.50\sqrt{f'_c} = 7.5(3000\,psi)^{1/2} = 0.411\,ksi$

$f_{c_b} > f_r$ rupture has occurred assumption is correct ✓

$$f_{c_b} = \frac{M c_b}{I} = \frac{(240\,kip\cdot ft)(12\,in/1\,ft)(18.7\,in)}{7704\,in^4} = 7.0\ ksi\ (T)$$

$$f_{c_T} = \frac{M c_T}{I} = \frac{(240\,kip\cdot ft)(12\,in/ft)(9.3\,in)}{7704\,in^4} = 3.48\ ksi\ (c)$$

$$f_s = \frac{M c_s}{I}\cdot n = \frac{(240\,kip\cdot ft)(12\,in/ft)(14.7\,in)}{7704\,in^4}(10) = 55.0\,ksi\ (T) < f_y \text{ for 60-grade steel}$$

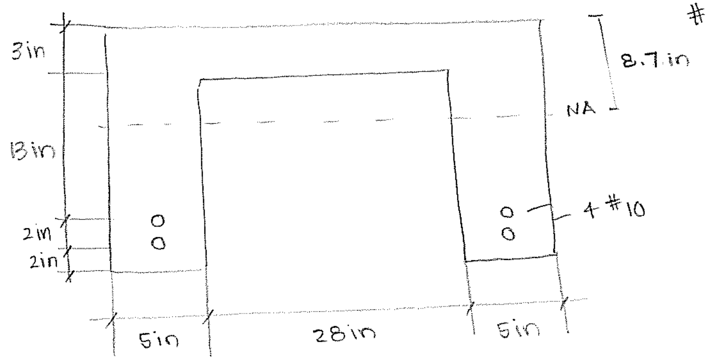
$f_{c_T} = 3.5\,ksi\ (c)$ at $x = 20\,ft$

$f_s = 55.0\,ksi\ (T)$

CE 326: HOMEWORK #2

C. HOVELL

3.



#10: d = 1.25 in
A = 1.27 in²

M = 150 ft·kip
n = 9

$$\bar{y} = \frac{(5\text{in})(20\text{in})(10\text{in})(2) + (28\text{in})(3\text{in})(18.5\text{in}) + (8)(2)(1.27\text{in}^2)[(2\text{in})+(4\text{in})]}{(5\text{in})(20\text{in})(2) + (28\text{in})(3\text{in}) + (1.27\text{in}^2)(4)(8)}$$

$$\bar{y} = 11.\cancel{3}\text{in} \qquad -1$$

$$I = \tfrac{1}{3}(38\text{in})(8.70\text{in})^3 - \tfrac{1}{3}(28\text{in})(5.70\text{in})^3 + \tfrac{1}{3}(10\text{in})(11.3\text{in})^3 + (2)(8)(1.27\text{in}^2)[(9.3\text{in})^2 + (7.3\text{in})^2]$$

$$I = 1426\cancel{3}\text{in}^4 \qquad -1$$

$$f_{c_b} = \frac{(150\text{ft·kip})(12\text{in/ft})(11.3\text{in})}{14263\text{in}^4} = 1.43\text{ ksi (T)}$$

$$f_{c_T} = \frac{(150\text{ft·kip})(12\text{in/ft})(8.70\text{in})}{14263\text{in}^4} = 1.10\text{ ksi (C)}$$

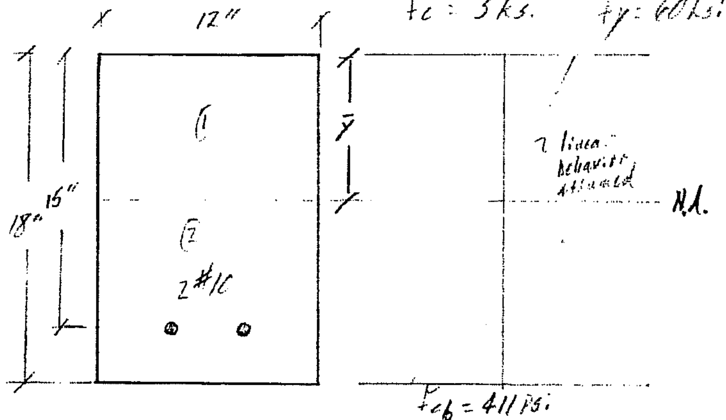
$$f_{s_1} = \frac{(150\text{ft·kip})(12\text{in/ft})(7.3\text{in})}{14263\text{in}^4} = 0.92\text{ ksi (T)}$$

$$f_{s_2} = \frac{(150\text{ft·kip})(12\text{in/ft})(9.3\text{in})}{14263\text{in}^4} = 1.17\text{ ksi (T)}$$

$$\boxed{\begin{array}{l} f_{c_B} = 1.43\text{ ksi (T)} \\ f_{c_T} = 1.10\text{ ksi (C)} \\ f_{s_{top}} = 0.92\text{ ksi (T)} \\ f_{s_{bot}} = 1.17\text{ ksi (T)} \end{array}}$$

1) a) Determine cracking moment, Mcr, of the section:

$f'_c = 3 ksi$ $f_y = 60 ksi$ $n = E_s/E_c = \frac{29 \times 10^6}{57000\sqrt{f'_c}} = 9.29 \approx \underline{9}$



$A_s = 2(1.27) = 2.54 in^2$

$A_{ST} = (n-1)A_s = 20.32 in^2$

$A_T = b \cdot h + (n-1)A_s$

$= 12(18) + 8(2.54)$

$= \underline{236 in^2}$

Assume section has just cracked ⇒ $f_{cb} = f_r = 7.5\sqrt{f'_c} = 411 psi = \underline{0.411 ksi}$

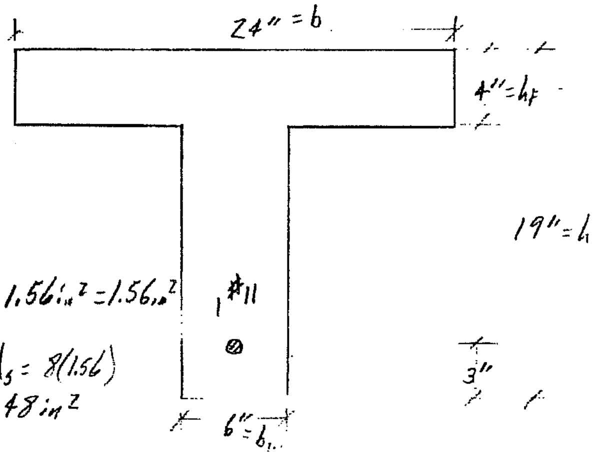
$$\bar{y} = \frac{(b)h(h/2) + A_{ST}(d)}{A_T} = \frac{(12)(18)(9) + (20.32)(15)}{236} = \underline{9.53 in} = c$$

$$I_T = \Sigma\left(\frac{bh^3}{3} + A_{ST}(d-c)^2\right) = \overset{①}{\frac{12''(9.53'')^3}{3}} + \overset{②}{\frac{12''(18''-9.53'')^3}{3}} + 20.32 in^2 (15''-9.53'')^2$$

$$= 3462 + 2431 + 608 = \underline{6501 in^4}$$

Recall $f = \frac{My}{I} \Rightarrow M_{CR} = \frac{(f_{cb})(I_T)}{(h-c)} = \frac{0.411 ksi (6501 in^4)}{(18in - 9.53in)} = 315 in \cdot kips = \underline{\underline{26.3 ft \cdot k}}$

b)



$f'_c = 3 ksi$

$f_y = 60 ksi$

$n = 9$

$A_s = 1 bar \times 1.56 in^2 = 1.56 in^2$

$A_{ST} = (n-1)A_s = 8(1.56)$

$= 12.48 in^2$

$A_T = bh - (b-b_w)(h-h_f) + (n-1)A_s$

$= 24(19) - (24-6)(19-4) + 8(1.56)$

$= \underline{198.5 in^2}$

$d = h - 3'' = 16''$

$$\bar{y} = \frac{(b)(h)(h/2) - (b-b_w)(h-h_f)(h_f + \frac{h-h_f}{2}) + A_{ST}(d)}{A_T} = \frac{(24in)(19in)(\frac{19in}{2}) - (24in-6in)(19in-4in)(4in + \frac{19in-4in}{2}) + 12.48in^2(16in)}{198.5 in^2}$$

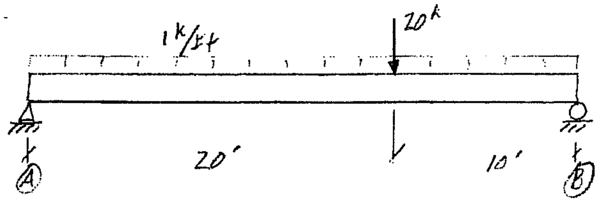
$$\bar{y} = \frac{4332 in^3 - 3105 in^3 + 200 in^3}{198.5 in^2} = \underline{7.19 in} = c$$

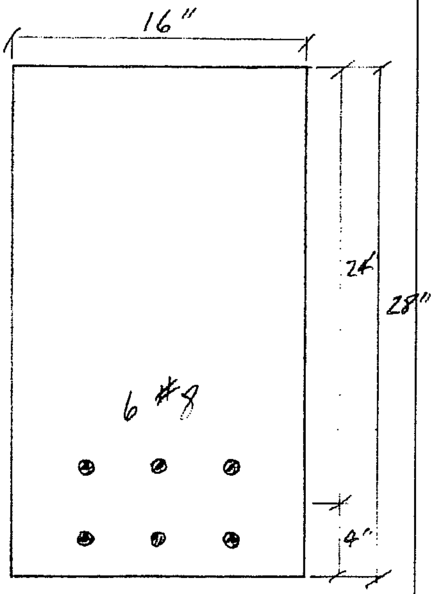
$$I_T = \Sigma\frac{bh^3}{3} + A_{ST}(d-c)^2 = \frac{24in(7.19in)^3}{3} - \frac{(24in-6in)(7.19in-4in)^3}{3} + \frac{(6in)(19in-7.19in)^3}{3} + 12.48in(16in-7.19in)^2$$

$$I_T = 2974 in^4 - 194.8 in^4 + 3294.4 in^4 + 968 in^4 = \underline{7042 in^4}$$

$$M_{CR} = \frac{f_r(I_T)}{(h-c)} = \frac{0.411 \frac{kip}{in^2}(7042 in^4)}{(19in - 7.19in)} = 245 in \cdot kips = \underline{20.4 ft \cdot kips}$$

2)





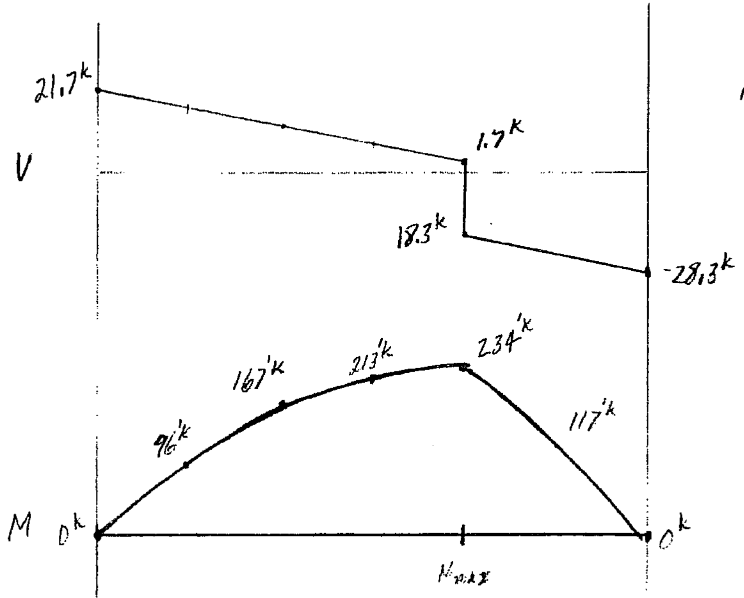
$\Sigma F_y = 0: \; -1(30) + {}^-20 + R_A + R_B = 0$

$R_A + R_B = 50^k$

$\Sigma M_A = 0: \; 30ft(1^k/ft)(\frac{30ft}{2}) + 20^k(20ft) - R_B(30ft) = 0$

$R_B = (450 + 400)/30 = 28.3^k$

$R_A = 50^k - R_B = 21.7^k$



Mmax where V crosses 0 @ 20ft

$M_{max} = \left(\frac{21.7^k + 1.7^k}{2}\right)(20ft) = \underline{\underline{234}}\ k.ft$

Assume cracked and find f_{ct}, f_s, and f_{cb} (check)

$f'_c = 3ksi$

$n = 10$

$A_{ST} = n(A_s) = 10(6 \times 0.79)$
$= \underline{47.4 in^2}$

$$\Sigma M @ N.A. = 0: \; \overset{C}{b\bar{y}(\bar{y}/2)} = \overset{T}{A_{ST}(d - \bar{y})}$$

$$16(\bar{y})(\frac{\bar{y}}{2}) = 47.4(24 - \bar{y})$$

$$8\bar{y}^2 + 47.4\bar{y} - 1137.6 = 0 \Rightarrow \underline{\bar{y} = 9.32 in}$$

$$I_T = \frac{1}{3}(16)(9.32)^3 + 47.4(24 - 9.32)^2 =$$

$$4318 + 10215 = 14533 \ in^4$$

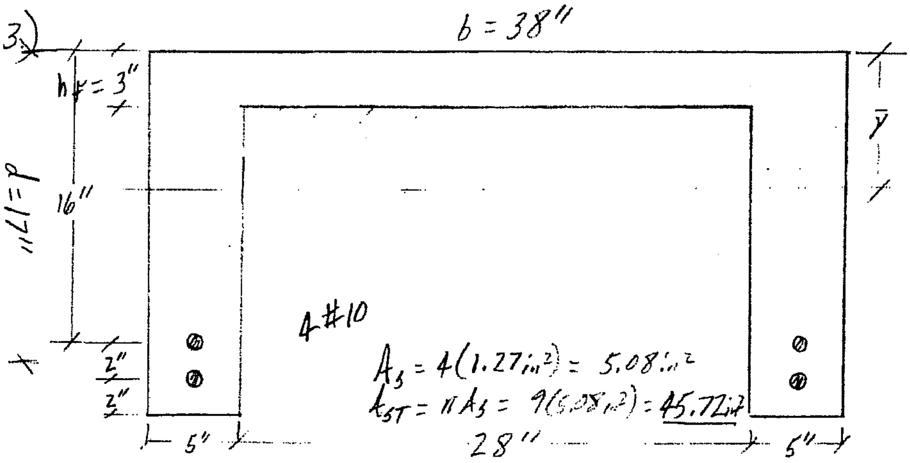
$f = \frac{My}{I}$:

$$f_{cb} = \frac{(234 k \cdot ft)(12 in/ft)(28 in - 9.32 in)}{14533 in^4} = \underline{\underline{3.61 ksi}} \; --> f_r = 0.411 ksi \; \checkmark \text{cracked}$$

$$f_{ct} = \frac{(234 k \cdot ft)(12 in/ft)(9.32 in)}{14533 in^4} = \underline{\underline{1.80 ksi}} < f'_c = 3.0 ksi \; \checkmark \text{not failed}$$

$$f_s = (10)\frac{(234 k \cdot ft)(12 in/ft)(24 in - 9.32 in)}{14533} = \underline{\underline{28.4 ksi}} < f_y = 60 ksi \; \checkmark \text{not yielded}$$

3)



M = 150'k

n = 9

Find: f_{ct}
f_s
f_{cb}

$A_s = 4(1.27 in^2) = 5.08 in^2$

$A_{sT} = n A_s = 9(5.08 in^2) = \underline{45.72 in^2}$

Assume section is cracked to start and assume NA is in flange ($\bar{y} < 3"$)

$\Sigma M @ NA = 0: \quad C = T$

$$b(\bar{y})(\bar{y}/2) = A_{sT}(d - \bar{y})$$

$$38 in(\bar{y})\left(\frac{\bar{y}}{2}\right) = 45.72 in^2 (17 - \bar{y})$$

$$19\bar{y}^2 + 45.72\bar{y} - 777.24 = 0 \Rightarrow \bar{y} = 5.30 in > h_f = 3 in$$

No Good

$\Sigma M @ NA = 0: \quad C = T$

$$b(\bar{y})\left(\frac{\bar{y}}{2}\right) - (b - 2b_w)(\bar{y} - 3)\left(3 + \frac{\bar{y} - 3}{2}\right) = A_{sT}(d - \bar{y})$$

$$38(\bar{y})(\bar{y}/2) - (38 - 10)(\bar{y} - 3)\left(3 + \frac{\bar{y} - 3}{2}\right) = 45.72(17 - \bar{y})$$

$$19\bar{y}^2 - 28(\bar{y} - 3)\left(3 + \frac{\bar{y}}{2} - \frac{3}{2}\right) = 777.24 - 45.72\bar{y}$$

$$19\bar{y}^2 + (-28\bar{y} + 84)(1.5 + \bar{y}/2) = 777.24 - 45.72\bar{y}$$

$$19\bar{y}^2 + -42\bar{y} - 14\bar{y}^2 + 126 + 42\bar{y} = 777.24 - 45.72\bar{y}$$

$$5\bar{y}^2 + 45.72\bar{y} - 651.24 = 0 \Rightarrow \underline{\underline{\bar{y} = 7.72 in}} > h_f \quad ✓$$

$$I_T = \Sigma \frac{bh^3}{3} + A_{sT}(d - c)$$

$$= \tfrac{1}{3}(38 in)(7.72 in)^3 - \tfrac{1}{3}(28 in)(7.72 - 3 in)^3 + 45.72 in^2 (17 - 7.72)^2$$

$$5828 in^4 \quad - \quad 981 \quad + \quad 3937$$

$$= 8784 in^4$$

$$f_{cb} = \frac{(150'^k)(12^{in}/ft)(20" - 7.72")}{8784 in^4} \qquad f_{ct} = \frac{(150'^k)(12^{in}/ft)(7.72")}{8784 in^4} \qquad f_s = (9)\frac{(150'^k)(12^{in}/ft)(17 - 7.72 in)}{8784 in^4}$$

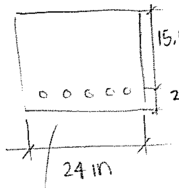
$f_{cb} = \underline{2.52 ksi} > f_r = 0.411 ksi$ ✓ $\qquad f_{ct} = \underline{1.58 ksi} < f'_c$ ✓ $\qquad f_s = \underline{19.0 ksi} < f_y$ ✓

CE 326: HOMEWORK #3 — 3.1, 3.4, 3.5

$\frac{21}{30}$

CATHERINE HOVELL
FEBRUARY 16, 2004

3.1



$f'_c = 4000$ psi

$f_y = 60000$ psi

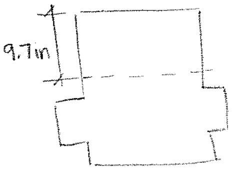
$f_{r_{conc}} = 475$ psi

$E_s = 29000$ ksi
$E_c = 3600$ ksi } $n = 8$

$0.45 f'_c$, $0.40 f_y$ = max

$0.45(4000\text{psi}) = 1800\text{psi}$; $0.40(60000\text{psi}) = 24000\text{psi}$

$A_s = 5$ #11
$= (5)(1.56\text{in}^2) = 7.8\text{in}^2$



$$\bar{y} = \frac{(18\text{in})(24\text{in})(9\text{in}) + (7.8\text{in}^2)(7)(2.5\text{in})}{(18\text{in})(24\text{in}) + (7.8\text{in}^2)(7)} = 8.2\cancel{7}\text{ in (from bottom)}$$

ΣM @ NA → $\bar{y}$

-2

$$I = \tfrac{1}{3}(24\text{in})\left[(9.7\text{in})^3 + (8.3\text{in})^3\right] + (7.8\text{in}^2)(5.8\text{in})^2$$

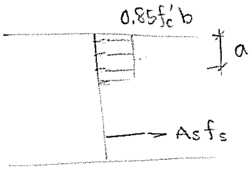
$$I = 12140\text{ in}^4$$

$$M = \frac{f I}{c} \qquad M_c = \frac{(1800\text{psi})(12140\text{in}^4)}{\cancel{9.7\text{in}}} = 188\text{ ft·kip (conc) limits}$$

-1

$$M_s = \frac{(24\text{ksi})(12140\text{in}^4)}{5.8\text{in}} = 4186\text{ ft·kip (steel)}$$

$\boxed{a.\ M_{max} = 188\text{ ft·kip}}$



$$a = \frac{A_s f_y}{0.85 f'_c b} \qquad a = \frac{(7.8\text{in}^2)(60\text{ksi})}{0.85(4\text{ksi})(24\text{in})} = 5.74\text{in}$$

$$M_n = 0.85 f'_c a b (d - a/2) = 0.85(4\text{ksi})(5.74\text{in})(24\text{in})\left(15.5\text{in} - \frac{5.74\text{in}}{2}\right)$$

$M_n = 493$ ft·kip ratio: $\frac{493\text{ ft·kip}}{188\text{ ft·kip}} = 2.62$

✓ $\boxed{\begin{array}{l} b.\ M_n = 493\text{ ft·kip} \\ 2.62 M_{service} = M_n \end{array}}$

$$M_{rupture} = \frac{f_r I}{c} = \frac{(475\text{psi})(12140\text{in}^4)}{8.27\text{in}} = 697\text{ kip·in}$$

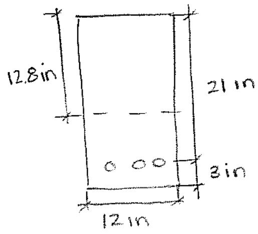
$= 58.1$ kip·ft

which is < M_{max}

$\boxed{\text{c. yes, it will show flexural cracking}}$ ✓

CE 326: HOMEWORK # 3 – 3.4, 3.5

3.4



#10 bars = $3.81\,in^2$

$f_y = 60\,ksi$

$f'_c = 4\,ksi$

n = 8 (assumed)

$$\bar{y} = \frac{(12in)(24in)(12in) + (3.81in^2)(7)(3in)}{(12in)(24in) + (3.81in^2)(7)}$$

$\bar{y} = 11.2$ in up, 12.8in down ✓

$$I = \tfrac{1}{3}(12in)\left[(12.8in)^3 + (11.2in)^3\right] + (3.81in^2)(7)(8.2in)^2$$

$I = 15800\,in^4$ ✓

(a.) I'm not sure how to calculate I_g that isn't the same as I_{ut}.

-2 ↳ ignore steel

$$M = \frac{f'_c I}{c} = \frac{(4ksi)(15800in^4)}{11.2in} = 5643\ kip\cdot in = 470\ kip\cdot ft$$

$\boxed{b.\ M = 470\ \cancel{kip}\cdot ft}$ -1

Ratio?

$0.45f'_c = 1.8\,ksi$ | $0.4f_y = 24\,ksi$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.81in^2)(24\,ksi)}{0.85(1.8ksi)(12in)} = 4.98\ in$$

$$M_n = 0.85 f'_c ab\,(d - a/2) = 0.85(1.8ksi)(4.98in)(12in)(21in - \tfrac{4.98in}{2})$$

$M_n = 141.0\ \cancel{kip}\cdot ft$ -1

$$M_c = \frac{(18ksi)(15800in^4)}{11.2in} = 211.6\ ft\cdot kip$$

$$M_s = \frac{(24ksi)(15800in^4)}{8.2in} = 3850\ ft\cdot kip$$

$\boxed{c.\ M = 21\cancel{2}\ ft\ kip}$

-2

$$\varepsilon_t = 0.003\left(\frac{8.2in - 11.2in}{11.2in}\right) = -8.04\times10^{-4}$$

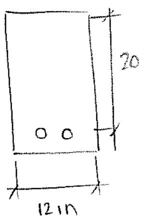
/ $\phi = 0.9$ $M_u = \phi M_n$

$$\boxed{\begin{array}{l} d.\ M_n = 141.0\ ft\cdot kip \\ M_u = 127\ ft\cdot kip \end{array}}$$

$$\frac{M_u}{M} = \frac{127\ ft\cdot kip}{212\ ft\cdot kip}$$ $\boxed{e.\ R = 0.60}$

CE 326: HOMEWORK #3 - 3.5

3.5



2 #8, 2 #10, 3 #10

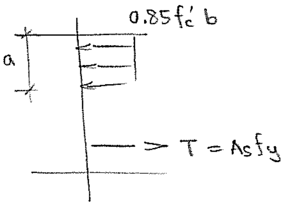
$f_y = 60 \text{ ksi}$

$f'_c = 5 \text{ ksi}$

$A_{s_1} = (2)(0.79 \text{ in}^2) = 1.58 \text{ in}^2$

$A_{s_2} = (2)(1.27 \text{ in}^2) = 2.54 \text{ in}^2$

$A_{s_3} = (3)(1.27 \text{ in}^2) = 3.81 \text{ in}^2$



$$a_1 = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1.58 \text{ in}^2)(60 \text{ ksi})}{0.85(5 \text{ ksi})(12 \text{ in})} = 1.86 \text{ in}$$

$$M_n = A_s f_y (d - a/2) = (1.58 \text{ in}^2)(60 \text{ ksi})(20 \text{ in} - \frac{1.86 \text{ in}}{2})$$

$$\boxed{a. M_n = 151 \text{ ft} \cdot \text{kip}}$$

$$a_2 = \frac{(2.54 \text{ in}^2)(60 \text{ ksi})}{0.85(5 \text{ ksi})(12 \text{ in})} = 2.99 \text{ in}$$

$$M_n = (2.54 \text{ in}^2)(60 \text{ ksi})(20 \text{ in} - 2.99 \text{ in}/2)$$

$$\boxed{b. M_n = 235 \text{ ft} \cdot \text{kip}}$$

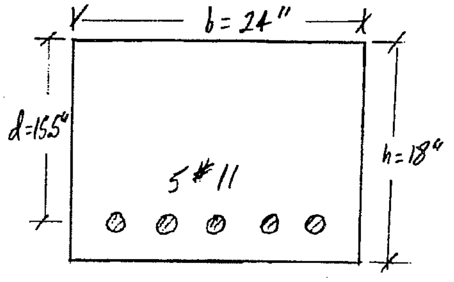
$$a_3 = \frac{(3.81 \text{ in}^2)(60 \text{ ksi})}{0.85(5 \text{ ksi})(12 \text{ in})} = 4.48 \text{ in}$$

$$M_n = (3.81 \text{ in}^2)(60 \text{ ksi})(20 \text{ in} - 4.48 \text{ in}/2) = 4060 \text{ in} \cdot \text{kip}$$

$$\boxed{c. M_n = 338 \text{ ft} \cdot \text{kip}}$$

Problem 3.1: $f'_c = 4000$ psi $\quad E_c = 3{,}600{,}000$ psi
$f_y = 60{,}000$ psi $\quad E_s = 29{,}000{,}000$ psi
$f_r = 475$ psi

$$n = \frac{29{,}000{,}000}{3{,}600{,}000} = 8$$



a) Find: M such that $f_{ct} \le 0.45 f'_c$ and $f_s \le 0.40 f_y$

Note: Specified stress limits keep both concrete and steel in linear range. Assume uncracked.

$A_s = 5(1.56 in^2) = 7.80 in^2$ (Table A.1) $\qquad A_{sT} = (n-1)A_s = (8-1)(7.8) = 54.6 in^2$

$A_T = b \cdot h + (n-1)A_s = (24in)(18in) + (8-1)(7.8in^2) = 487 in^2$

$$\bar{y} = \frac{b(h)(h/2) + A_{sT}(d)}{A_T} = \frac{(24in)(18in)(18in/2) + 54.6in^2(15.5in)}{487 in^2}$$

$\bar{y} = \underline{9.72 in}$

$$I_T = \Sigma\left(\frac{bh^3}{3}\right) + A_{sT}(d-c)^2 = \frac{(24in)(9.72in)^3}{3} + \frac{(24in)(18in - 9.72in)^3}{3} + 54.6in^2(15.5in - 9.72in)^2$$

$I_T = 13712 in^4$

$$f = \frac{My}{I} \Rightarrow M = \frac{fI}{y}: \; ^{(C)}M_c = \frac{f_{ct} I_T}{\bar{y}} = \frac{0.45(4 ksi)(13712 in^4)}{9.72 in} = 2539 \text{ in-kips} = \underline{\underline{212}} \text{ ft-kips}$$

$$^{(T)}M_s = \frac{f_s I_T}{d - \bar{y}(n)} = \frac{0.40(60 ksi)(13712 in^4)}{(15.5in - 9.72in)(8)} = 7117 \text{ in-kips} = 593 \text{ ft-kips}$$

∴ Compression controls, must check cracking: $f_{cb} = \frac{M(h-\bar{y})}{I_T} = \frac{(2539 in\text{-}kips)(18 - 9.72in)}{13712 in^4}$

$f_{cb} = 1.53 ksi > f_r = 0.475 ksi \Rightarrow$ Cracked!
Must reevaluate!

Assuming cracked section: $A_{sT} = n(A_s) = 8(7.8) = 62.4 in^2$

$\Sigma M @ NA = 0: \; b(\bar{y})(\bar{y}/2) = A_{sT}(d - \bar{y})$

$(24in)(\bar{y})(\frac{\bar{y}}{2}) = 62.4 in^2(15.5in - \bar{y})$

$12\bar{y}^2 = 967.2 - 62.4\bar{y}$

$12\bar{y}^2 + 62.4\bar{y} - 967.2 = 0 \qquad \bar{y} = 6.75 in$

$I_T = \Sigma \frac{bh^3}{3} + A_{sT}(d-\bar{y})^2 = \frac{1}{3}(24in)(6.75in)^3 + 62.4in^2(15.5in - 6.75in)^2$

$= 2460 in^4 + 4778 = 7238 in^4$

$$M = \frac{fI}{\bar{y}}: \; ^{(C)}M_c = \frac{f_{ct} I_T}{\bar{y}} = \frac{0.45(4 ksi)(7238 in^4)}{6.75 in} = 1930 \text{ in-kips} = \underline{161 \text{ ft-kips}} \leftarrow \text{compression controls}$$

$$^{(T)}M_s = \frac{f_s I_T}{(d - \bar{y})n} = \frac{0.40(60 ksi)(7238 in^4)}{(15.5in - 6.75in)(8)} = 2482 \text{ in-kips} = 207 \text{ ft-kips}$$

Problem 3.1 (cont'd)

b) Find M_n: From notes ⇒ $M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)$

$\rho = A_s/bd = 7.80\,in^2/(24in)(15.5in) = 0.0210$

$M_n = 0.0210(60ksi)(24in)(15.5in)^2 \left(1 - 0.59 \frac{(0.0210(60ksi))}{4ksi}\right)$

$= 5915\ in \cdot kips = \underline{\underline{493}}\ ft \cdot kips$

Ratio of nominal flexural strength to service load moment ⇒ $\frac{161\ ft \cdot kips}{493\ ft \cdot kips} = \underline{\underline{0.33}}$

c) Determine whether beam will show flexural cracking before reaching service load.

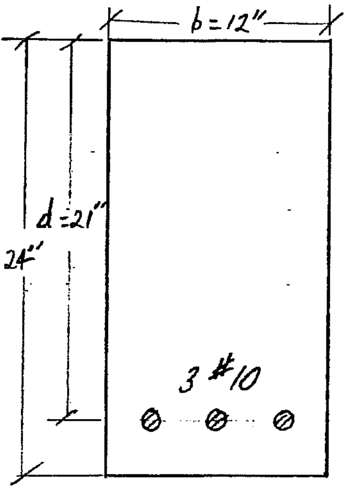
$f_{cb} = \frac{M(h-\bar{y})}{I_T} = \frac{(1930\ in \cdot kips)(18in - 6.75in)}{7238\ in^4} = 3.00\ ksi > f_r = 0.475ksi$

$\therefore$ Yes, the section will crack

Problem 3.4: $f'_c = 4000\,psi$

$f_y = 60{,}000\,psi$

$$n = \frac{29{,}000{,}000}{57000\sqrt{4000}} = \underline{8}$$



$$A_s = 3(1.27 in^2) = 3.81 in^2$$

a.) $f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 0.474\ ksi$

$$I_g = \frac{bh^3}{12} = \frac{(12in)(24in)^3}{12} = 13824 in^4$$

$$f_r = \frac{M_{CR}(\frac{h}{2})}{I_g}$$

$$M_{CR} = \frac{f_r(I_g)}{(h/2)} = \frac{0.474 ksi (13824 in^4)}{(24 in/2)} = 546\ in\cdot kips = \underline{\underline{45.5}}\ ft\cdot kips$$

b.) $A_{ST} = (n-1)(A_s) = (8-1)(3.81 in^2) = 26.67 in^2$

$$A_T = (12in)(24in) + 26.67 in^2 = 314.67 in^2$$

$$\bar{y} = \frac{(12in)(24in)(12in) + 26.67in^2(21in)}{314.67 in^2} = \underline{12.76}\ in$$

$$I_{uT} = \Sigma\frac{bh^3}{3} + A_{ST}(d-\bar{y})^2 = \frac{(12in)(12.76in)^3}{3} + \frac{(12in)(24-12.76in)^3}{3} + 26.67in^2(21in-12.76in)^2$$

$$= 15801\ in^4$$

$$M_{CR} = \frac{f_r(I_{uT})}{(h-\bar{y})} = \frac{0.474 ksi (15801 in^4)}{(24in - 12.76in)} = 666\ in\cdot kips = \underline{\underline{55.5}}\ ft\cdot kips$$

c.) assume cracked, thus concrete below NA has no tensile strength

$$A_{ST} = n(A_s) = 8(3.81) = 30.48 in^2$$

$\Sigma M @ NA = 0$: $b\bar{y}(\bar{y}/2) = A_{ST}(d-\bar{y})$

$$(12in)(\bar{y})(\bar{y}/2) = 30.48in^2(21in - \bar{y})$$

$$6\bar{y}^2 + 30.48\bar{y} - 640.08 = 0 \quad \Rightarrow \quad \bar{y} = 8.10 in$$

$$I_T = \Sigma\frac{bh^3}{3} + A_{ST}(d-\bar{y})^2 = \frac{(12in)(8.10in)^3}{3} + 30.48in^2(21in - 8.10in)^2$$

$$= 2126 in^4 + 5072 = 7198 in^4$$

(C) $M_c = \frac{f_{ct} I_T}{\bar{y}} = \frac{0.45(4ksi)(7198in^4)}{8.10in} = 1600\ in\cdot kips = \underline{\underline{133\ ft\cdot kips}}$

(T) $M_s = \frac{f_s I_T}{(d-\bar{y})n} = \frac{0.40(60ksi)(7198in^4)}{(21in - 8.10in)\ 8} = 1674\ in\cdot kips = 139\ ft\cdot kips$

Compression controls

Problem 3.4 (cont'd)

d.) Find M_n: $\rho = A_s/bd = 3.81in^2/(12in)(21in) = 0.015$

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$= 0.015(60ksi)(12in)(21in)^2\left(1-0.59\frac{0.015(60ksi)}{4ksi}\right)$$

$$= 0.9ksi\ (5292 in^3)\ (1-0.59(0.225))$$

$$= 4131\ in\cdot kips = \underline{344}\ ft\cdot kips$$ (nominal flexural strength)

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.81(60)}{0.85(4)(12)} = 5.60 in \qquad c = a/\beta_1 = 6.59 in$$

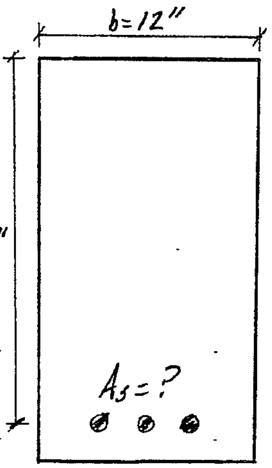
$\varepsilon_t = \varepsilon_u\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{21in - 6.59in}{6.59in}\right) = 0.0066 > 0.005$ ∴ Under reinforced $\phi = 0.90$

Design Moment: $\phi M_n = 0.90(344\ ft\cdot kips) = 310\ ft\cdot kips$

e.) Ratio of design moment to service load moment

310 ft·kips/133 ft·kips = $\underline{\underline{2.33}}$

Problem 3.5



$f'_c = 5000$ psi $\qquad n = 29000000/5700\sqrt{5000} = 7$

$f_y = 60000$ psi

Find M_n for:

a) $A_s = 2$ No. 8 bars $= 2(0.79\,in^2) = 1.58\,in^2$ (Table A-1)

$\rho = A_s/bd = \frac{1.58\,in^2}{(12in)(20in)} = 0.00658$

$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85(0.80)\frac{5ksi}{60ksi}\left(\frac{.003}{.003+.004}\right)$

$\rho_{max} = 0.02429$ ✓

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1.58in^2(60ksi)}{0.85(5ksi)(12in)} = 1.86$ in

Depth to N.A., $c = a/\beta_1$; $\beta_1 = 0.80$ for $f'_c = 5000$ (Table 3.1)

$c = 1.86\,in/0.80 = 2.33$ in

$M_n = A_s f_y(d - a/2) = 1.58in^2(60ksi)(20in - \frac{1.86}{2}in)$

$= 1807$ in·kips $=$ $\underline{\underline{151}}$ ft·kips

b.) $A_s = 2$ No. 10 bars $= 2(1.27in^2) = 2.54in^2$

$\rho = A_s/bd = 2.54in^2/(12in)(20in) = 0.01058 \quad < \rho_{max} = 0.02429$ ✓

$a = \frac{2.54in^2(60ksi)}{0.85(5ksi)(12in)} = 2.99$ in $\qquad c = 2.99/.80 = 3.74$ in

$M_n = 2.54in^2(60ksi)(20in - \frac{2.99in}{2}) = 2820$ in·kips $=$ $\underline{\underline{235}}$ ft·kips

c) $A_s = 3$ No. 10 bars $= 3(1.27in^2) = 3.81in^2$

$\rho = A_s/bd = 3.81in^2/(12in)(20in) = 0.01588 \quad < \rho_{max} = 0.02429$ ✓

$a = \frac{3.81in^2(60ksi)}{0.85(5ksi)(12in)} = 4.48$ in $\qquad c = 4.48/.80 = 5.60$ in

$M_n = 3.81in^2(60ksi)(20in - \frac{4.48in}{2}) = 4060$ in·kips $=$ $\underline{\underline{338}}$ ft·kips

$\underline{OR}$ $M_n = \rho f_y bd^2\left(1 - 0.59\frac{\rho f_y}{f'_c}\right) = 0.01588(60ksi)(12in)(20in)^2\left(1 - 0.59\frac{0.01588(60ksi)}{5ksi}\right) = 4059$ in·kips

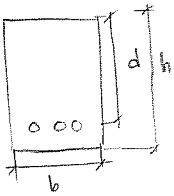
$= \underline{\underline{338}}$ ft·kips

CE 326: HOMEWORK #4 – 3.6

CATHERINE HOVELL
FEBRUARY 18, 2004

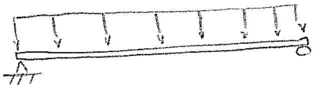
$\frac{26}{30}$

3.6



$d \sim 1.5b$

$\rho = 0.5\rho_{max}$



$L = 24\ ft$

$LL = 1500\ lb/ft$ $\quad DL = 150\ lb/ft^3 = 150hb\ lb/ft$

$1.2DL + 1.6LL = w(x)$

$f_y = 60\ ksi\ /\ f'_c = 4\ ksi$ $\qquad \omega(x) = 200\ lb/in + 0.104\ lb/in^3 \cdot bd$

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85(0.85)\frac{4\ ksi}{60\ ksi}\frac{0.003}{0.003+0.004} = 0.0206$$ — part A is on second page!

$$M_u(b,d) = \frac{w\ell^2}{8} = 1080\frac{lb}{in}bd + 2074000\ lb \cdot in = \phi\rho f_y bd^2\left(1 - 0.59\frac{\rho f_y}{f'_c}\right)$$

-1

$\phi \neq 0.9$

$$(\cancel{0.9})(0.0206)(60\ ksi)(bd^2)\left[1 - 0.59(0.0206)\frac{60\ ksi}{4\ ksi}\right] = 0.911\ ksi \cdot bd^2$$

$$(0.911\ ksi)(bd^2) = 1080\frac{lb}{in}(bd) + 2.07 \times 10^6\ lb \cdot in$$

$1.5b \sim d$ $\qquad b = 12\ in, d = 14.4\ in$ × $\qquad b = 10\ in, d = 15.7\ in$ ✓

$A_s = \rho bd = (0.0206)(10\ in)(15.7\ in) = 3.23\ in^2$ $\rightsquigarrow$ use 4 #8

$A_s = 3.16\ in^2,\ D = 0.79\ in$

find h: cover = 1.5 in
stirrup = 0.5 in
$\frac{1}{2}$ bar = 0.4 in

$h = d + 2.4\ in = 18.0\ in$

$\rightsquigarrow h = 18\ in$

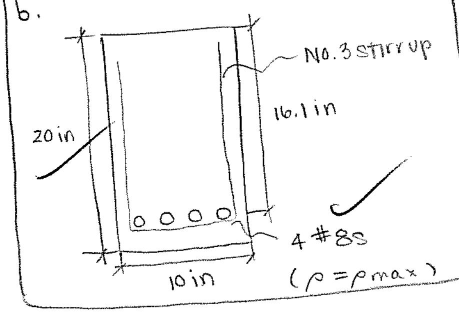
$d = 16.1\ in,\ A_s = (0.0206)(16.1\ in)(10\ in) = \underline{\underline{3.32\ in}}$

$$M_u = \frac{[(24\ ft)(12\ in)]^2}{8}\left[(200\ lb/in) + (0.104\ lb/in^3)(10\ in)(16.1\ in)\right] = 187\ kip \cdot ft$$

$$M_n = (0.0206)(60\ ksi)(10\ in)(16.1\ in)^2\left[1 - 0.59(0.0206)\frac{60\ ksi}{4\ ksi}\right] = 218\ kip \cdot ft$$

$\phi M_n = 196\ kip \cdot ft > M_u = 187\ kip \cdot ft$ ✓

b.



✓

CE 326: HOMEWORK #4 - 3.6

-1

3.6 (part A)

$w(x) = 200\ \text{lb/in} + 0.104\ \text{lb/in}^3 \cdot b\cancel{d}\ h$ $\quad f_y = 60\ \text{ksi},\ f'_c = 4\ \text{ksi}$

$\rho_{max} = 0.0206 \qquad \frac{1}{2}\rho_{max} = 0.0103$ ✓

$M_u = 1080\ \text{lb/in} \cdot bd + 2074000\ \text{lb}\cdot\text{in} = (0.9)(0.0103)(60\ \text{ksi})bd^2\left[1 - 0.59(0.0103)\frac{60\ \text{ksi}}{4\ \text{ksi}}\right]$

$= 0.505\ \text{ksi}\ (bd^2)$ ✓

$b = 12\ \text{in}\quad d = 19.\overset{?}{6}\ \text{in}$ ✓ $\quad b = 14\ \text{in}\quad d = 18.2\ \text{in}$ X

↳ ~~Check your math~~?

$A_s = \rho b d = (0.0103)(12\ \text{in})(19.6\ \text{in}) = 2.42\ \text{in}^2$ $\quad$ use 4#7s $\quad A_s = 2.40\ \text{in}^2,\ D = 0.60\ \text{in}$

h: 0.5" cover
1.5" stirrup
0.3" $\frac{1}{2}$ bar

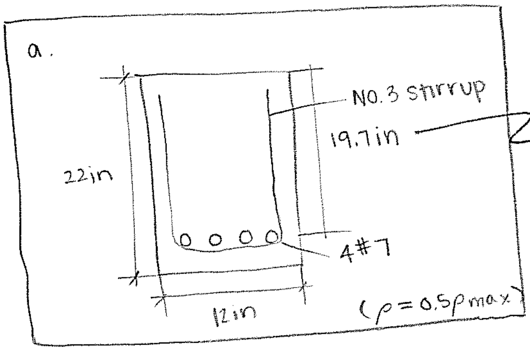
$h = 19.6\ \text{in} + 2.3\ \text{in} = 21.9\ \text{in}$

$h = 22\ \text{in},\ d = 19.7\ \text{in} \qquad A_s = 2.43\ \text{in}^2$ ✓

$$M_u = \frac{[(24\text{in})(12\text{in})]^2}{8}\left[(0.104\ \text{lb/in}^3)(12\text{in})(19.7\text{in}) + 200\ \text{lb/in}\right] = 194\ \text{kip}\cdot\text{ft}$$

$$M_n = (0.0103)(60\ \text{ksi})(12\text{in})(19.7)^2\left[1 - 0.59(0.0103)\frac{60\ \text{ksi}}{4\ \text{ksi}}\right] = 218\ \text{kip}\cdot\text{ft}$$

$$\phi M_n = 196.2\ \text{kip}\cdot\text{ft} > M_u = 194\ \text{kip}\cdot\text{ft}$$ ✓

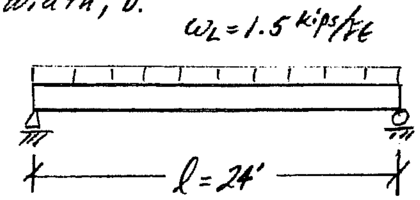


→ Readjust after you pick As actual -1

Check
$\phi = 0.9$ -1
$A_{s_{min}}$

Problem 3.6 Design a singly-reinforced beam where the effective depth, d is approximately 1.5 times the width, b.

$f'_c = 4000$ psi
$f_y = 60000$ psi
unit wt of beam = 0.150 kips/ft^3
assume #3 stirrups



Find b, d, h, & steel reinforcement for a) $\rho = 0.50\,\rho_{max}$ b) $\rho = \rho_{max}$

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85(0.85)\frac{4ksi}{60ksi}\frac{0.003}{0.003+0.004} = \underline{0.0206}$$

$$M_{LL} = \frac{\omega_{LL}(l)^2}{8} = \frac{1.5\,^{k}/_{ft}(24ft)^2}{8} = 108\ ft\cdot kips = 1296\ in\cdot kips$$

$$\omega_{DL} = 0.150\,^{k}/_{ft^3}(b)(h)(1) = 0.150\,bh\ \ ^{k}/_{ft}$$

$$M_{DL} = \frac{\omega_{DL}(l)^2}{8} = \frac{0.150bh\,(24ft)^2}{8} = 10.8\,bh\ (b+h\ in\ feet)\cdot\frac{1ft^2}{144in^2} = 0.075\,bh\ (b+h\ in\ inches)$$

$$M_u = 1.2\,(0.075\,bh)(12\,^{in}/_{ft}) + 1.6\,(1296\ in\cdot kips)$$

$$= 1.08\,bh + 2074\ \ in\cdot kips$$

If $d \cong 1.5b$ and $h \cong d + 2.5$:

$$M_u = 1.08(b)(1.5b + 2.5) + 2074$$

$$M_u = 1.62b^2 + 2.7b + 2074$$

---

a) $\phi M_n = \phi\rho f_y bd^2\left(1 - 0.59\frac{\rho f_y}{f'_c}\right)$, but Let $R = \rho f_y\left(1 - 0.59\frac{\rho f_y}{f'_c}\right)$

$\rho = \frac{1}{2}\rho_{max}$: $R = 0.5(0.0206)(60ksi)\left(1 - 0.59\frac{(0.5)(0.0206)(60ksi)}{4ksi}\right)$

$R = 0.562$ where $\rho = 0.0103$

Let $M_u = \phi M_n = \phi R b d^2$ and recall $d \cong 1.5b$

$$1.62b^2 + 2.7b + 2074 = 0.90(0.562)(b)(1.5b)^2$$

$$0 = 1.14b^3 - 1.62b^2 - 2.7b - 2074$$

Solve for $b = 12.77$in $\Rightarrow$ round up to $\underline{b = 14\,in}$ $d = 1.5b = 21$in

$h = d + cover + stirrup + \frac{1}{2}bar = 21 + 1.5 + 0.375 + 0.5 = 23.4 \Rightarrow$ round to $\underline{h = 24in}$

$\therefore$ Choose section $b = 14''$, $h = 24''$ and $d = h - cover - stirrup - \frac{1}{2}bar =$ $\underline{d = 21.6\,in}$

For $\rho = 0.0103$, $A_{s_{req'}} = \rho bd = 0.0103\,(14in)(21.6in) = 3.11\,in^2$

4 #8's gives $A_{s_{actual}} = 3.16\,in^2$ and fit into one row if $b = 14$in.

Check capacity:

$$M_u = 1.08(14in)(24in) + 2074 = \underline{2437\ in\cdot kips}$$

$$\phi M_n = \phi R b d^2 = 0.90(0.562)(14in)(21.6in)^2 = \underline{3304\ in\cdot kips}$$ ✓

check ϕ, based on ε_t:

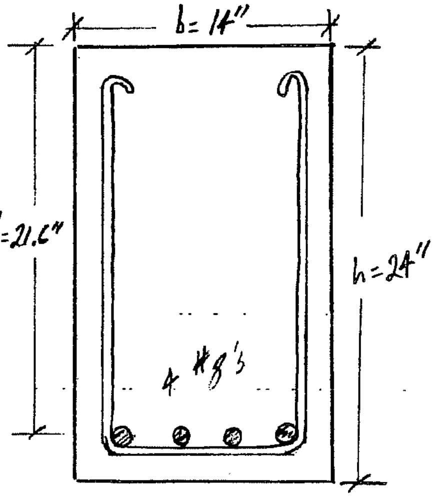
$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.16in^2(60ksi)}{0.85(4ksi)(14in)} = 3.98in \qquad c = a/\beta_1 = 4.7in$$

$$d_t = d = 21.6in$$

$$\varepsilon_t = \varepsilon_u\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{21.6in - 4.7in}{4.7in}\right) = 0.011 > 0.005 \quad \therefore \phi = 0.90$$ ✓

Check $A_{s,min}$: $A_s = 3.16 > \frac{200bd}{f_y} = 1.008in^2$ ✓

$> \frac{3\sqrt{f'_c}bd}{f_y} = 0.96in^2$ ✓ OK



b) $\rho = \rho_{max} = 0.0206$

$$R = 0.0206(60ksi)\left(1 - 0.59\frac{0.0206(60ksi)}{4ksi}\right) = 1.011$$

Let $M_u = \phi M_n = \phi R b d^2$

$1.62b^2 + 2.7b + 2074 = 0.90(1.011)(b)(1.5b)^2$

$0 = 2.047b^3 - 1.62b^2 - 2.7b - 2074$

solving for $b = 10.36 \Rightarrow$ round up to $\underline{b = 12in}$

$d \cong 1.5b = 18"$ $\quad h = d + cover + stirrup + \frac{1}{2}bar$

$h = 18 + 1.5 + .375 + .5 = 20.375in \Rightarrow$ round up to $\underline{h = 22in}$

$\therefore$ Choose a section $b = 12"$, $h = 22"$ and $d = h - cover - stirrup - \frac{1}{2}bar \Rightarrow d = 19.6in$

For $\rho = 0.0206$, $\quad A_{s_{req}} = \rho b d = (0.0206)(12in)(19.6in) = 4.85in^2$

6 #8's $= \underline{4.74in^2}$, but only 4 #8's fit in one row of 12in beam

$d = h - cover - stirrup - bar - \frac{1}{2}space = 22 - 1.5 - 0.375 - 1.0 - 0.5 = \underline{18.65in}$

$A_{s_{reqd}} = \rho b d = 0.0206(12in)(18.65in) = 4.61in^2 < 4.74in^2$ ✓OK

$M_u = 1.08(12in)(22in) + 2074 = \underline{2359\ in\cdot kips}$

$\phi M_n = \phi R b d^2 = 0.90(1.011)(12in)(18.65in^2) = \underline{3798\ in\cdot kips}$ ✓OK

Check ϕ based on ε_t:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(4.74in^2)(60ksi)}{0.85(4ksi)(12in)} = 6.97in \qquad c = a/\beta_1 = 8.2in$$

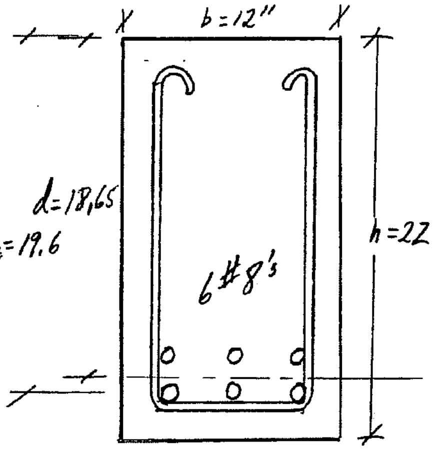
$d_t = 19.6in$ (as when there was only one layer)

$$\varepsilon_t = \varepsilon_u \left(\frac{d_t - c}{c}\right) = 0.003 \left(\frac{19.6 - 8.2}{8.2}\right) = 0.004$$

From p. 82 $\phi = 0.483 + 83.3\,\varepsilon_t = 0.82$

Recalculate ϕM_n and check

$$\phi M_n = 0.82\,(1.011)(12in)(18.65in)^2 = \underline{3460}\text{ in·kips} > M_u = 2359\text{ in·kips} \checkmark$$



Comment: Note that when $\rho = \frac{1}{2}\rho_{max}$, there is less steel, and a larger beam section is required for a comparable capacity.

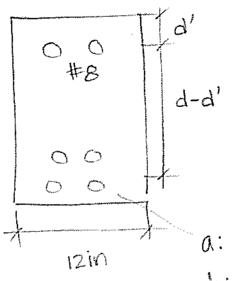
However, the when there is more steel, the strain at failure is not as great, and a more conservative strength reduction factor must be used.

CE 326: HOMEWORK #5 — Problem 3.9

$\frac{29}{30}$

CATHERINE HOVELL
MARCH 3, 2004

3.9



$d' = 2.5\text{ in}$
$d = 18\text{ in}$

$f_c' = 4\text{ ksi}$
$f_y = 60\text{ ksi}$

a: 3 #10
b: 4 #10 } two layers
c: 6 #9 }

$$M_u = \phi\left[A_{s2} f_y (d - a/2) + A_s' f_y (d - d')\right]$$

check yield: $\rho > \rho_{cy}$

$$\rho = \frac{A_s}{bd} \quad \rho_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$\varepsilon_y = \frac{f_y}{E} = \frac{60\text{ ksi}}{29000\text{ ksi}} = 0.00207$$

A: $A_s' = 2(0.79\text{ in}^2) = 1.58\text{ in}^2$

$A_s = 3(1.27\text{ in}^2) = 3.81\text{ in}^2$

$$\rho_{cy} = (0.85)^2 \frac{4\text{ ksi}}{60\text{ ksi}} \cdot \frac{2.5\text{ in}}{18\text{ in}} \cdot \frac{0.003}{0.003 - 0.00207} + \frac{1.58\text{ in}^2}{(12\text{ in})(18\text{ in})}$$

$\rho_{cy} = 0.0289 > \rho$ NO YIELD.

$$\rho = \frac{3.81\text{ in}^2}{(12\text{ in})(18\text{ in})} = 0.0176$$

$\rho_{max} = 0.0206 > \rho$ ✓ Treat as singly rein.

$$a = \frac{A_s f_y}{.85 f_c' b} \quad c = \frac{a}{.85}$$

$$A_s f_y = 0.85\beta_1 f_c' bc + A_s' \varepsilon_u E_s \left(\frac{c - d'}{c}\right)$$

$$(3.81\text{ in}^2)(60\text{ ksi}) = (0.85)^2(4\text{ ksi})(12\text{ in})[c] + (1.58\text{ in}^2)(0.003)(29000\text{ ksi})\left[\frac{c - 2.5\text{ in}}{c}\right]$$

solve for c: $c = \cancel{4}.73\text{ in},\ a = 0.85c = 4.02\text{ in}$

$$M_n = 0.85(4\text{ ksi})(4.02\text{ in})(12\text{ in})\left[18\text{ in} - \frac{4.02\text{ in}}{2}\right] + (1.58\text{ in}^2)(0.003)(2900\cancel{0\text{ ksi}})\left[\frac{4.73\text{ in} - 2.5\text{ in}}{4.73\text{ in}}\right](18\text{ in} - 2.5\text{ in})$$

$M_n = 285.4\text{ kip}\cdot\text{ft}$ $\phi = 0.90$ $M_u = 256.9\text{ kip}\cdot\text{ft}$

$\boxed{a.\ M_u = 257\text{ kip}\cdot\text{ft}}$ *

B: $A_s' = 1.58\text{ in}^2,\ \rho' = 0.0073$

$A_s = 4(1.27\text{ in}^2) = 5.08\text{ in}^2$

$\rho_{cy} = 0.0289 > \rho$: NO YIELD.

$$\rho = \frac{5.08\text{ in}^2}{(12\text{ in})(18\text{ in})} = 0.0235$$

$$(5.08\text{ in}^2)(60\text{ ksi}) = (0.85)^2(4\text{ ksi})(12\text{ in})[c] + (1.58\text{ in}^2)(0.003)(29000\text{ ksi})\left[\frac{c - 2.5\text{ in}}{c}\right]$$

$C = 6.05$ $c = 6.\cancel{38}\text{ in},\ a = 5.42\text{ in}$

$$M_n = (0.85)(4\text{ ksi})(5.42\text{ in})(12\text{ in})\left[18\text{ in} - \frac{5.42\text{ in}}{2}\right] + (1.58\text{ in}^2)(0.003)(29000\text{ ksi})\left[\frac{6.38\text{ in} - 2.5\text{ in}}{6.38\text{ in}}\right](15.5\text{i}$$

$M_n = 377.5\text{ kip}\cdot\text{ft}$ $\phi = 0.9$

$\boxed{b.\ M_u = 340\text{ kip}\cdot\text{ft}}$ ✓

I saw your note on a) at the end.

CE 326: HOMEWORK #5

C. HOVELL

3.9 (cont'd)

c: $A_s' = 1.58 in^2$, $\rho' = 0.0073$ $\rho_{cy} = 0.0289$

$A_s = 6(1.0 in^2) = 6.0 in^2$

$$\rho = \frac{6.0 in^2}{(12in)(18in)} = 0.0278 \text{ NO YIELD.}$$

$$(6.0in^2)(60ksi) = (0.85)^2 (4ksi)(12in)[c] + (1.58in^2)(0.003)(29000ksi)\left[\frac{c-2.5in}{c}\right]$$

$$c = 7.70 in^2, \quad a = 6.55in$$

$$M_n = (0.85)(4ksi)(6.55in)(12in)\left[18in - \frac{6.55in}{2}\right] + (1.58in^2)(0.003)(29000ksi)\left[\frac{7.70in^2 - 2.5in}{7.70in^2}\right](15.5in$$

$M_n = 437.6$ kip·ft

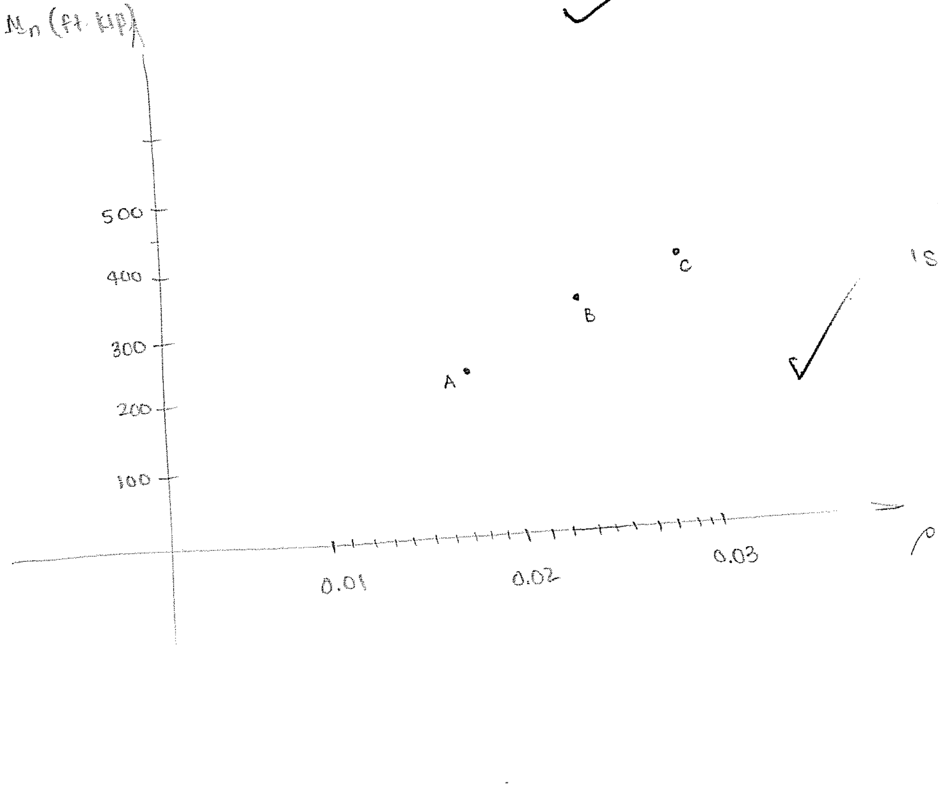
$\boxed{c.\ M_u = 39\cancel{4} \text{ kip·ft}}$ $\phi = 0.86$ -1

* A note on A: $\rho < \rho_{max}$, so compression steel is unneeded.

$$M_n = A_s f_y (d - a/2) \qquad a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(3.81in^2)(60ksi)}{0.85(4ksi)(12in)} = 5.60in$$

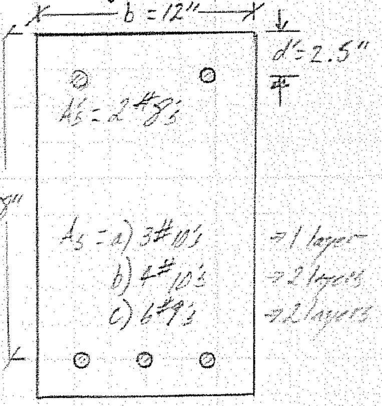
$$M_n = (3.81in^2)(60ksi)\left[18in - \frac{5.60in}{2}\right] = 289.5 \text{ kip·ft}$$

$\boxed{a.\ M_u = 261 \text{ kip·ft}}$ without considering compression steel, concrete can hold more moment!



As more tensile steel is added, a greater moment can be resiste with a constant amoun of compressive steel.

Problem 3.9) Given $f'_c = 4ksi$ $f_y = 60ksi$



For all: $A'_s = 1.58 in^2$ $\rho' = \frac{1.58}{12 \times 18} = 0.0073$

$\rho_{max} = 0.0206$ (Table A.4) $\bar{\rho}_{max} = 0.0206 + 0.0073 = 0.0279$

a) $A_s = 3.81 in^2$ $\rho = \frac{3.81}{12 \times 18} = 0.0176$

since $\rho < \rho_{max}$, we can analyze the section as singly reinforced and ignore compression steel.

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.81 in^2 (60ksi)}{0.85(4ksi)(12in)} = 5.60 in$ $c = 5.60/.85 = 6.59 in.$

$M_n = 0.85 f'_c a b (d - a/2) = 0.85(4ksi)(5.60in)(12in)(18in - 5.60/2) = 3473 in \cdot kips = \underline{289 ft \cdot kips}$

$\varepsilon_t = \varepsilon_u \left(\frac{d-c}{c}\right) = 0.003\left(\frac{18in - 6.59in}{6.59in}\right) = 0.0052 > 0.005 \therefore \phi = 0.90$

$\boxed{\phi M_n = 260 ft \cdot kips}$

<u>Alternatively</u>, if compression steel is considered, we need to determine stress in compression steel. Start by estimating a.

$a = \frac{(A_s - A'_s) f_y}{0.85 (f'_c) b} = \frac{(3.81 - 1.58)(60)}{0.85 (4)(12)} = 3.28 in$ $c = 3.28/.85 = 3.86 in$

$f'_s = \varepsilon_u E_s \left(\frac{c - d'}{c}\right) = 0.003(29 \times 10^6)\left(\frac{3.86 - 2.5}{3.86}\right) = 30653 \cong 31 ksi$

To account for $f'_s < f_y$, recalculate a.

$a = \frac{(A_s - \frac{f'_s}{f_y} A'_s) f_y}{0.85 (f'_c) b} = \frac{(3.81 - \frac{31}{60}(1.58)) 60}{0.85 (4)(12)} = 4.40 in$ $c = 5.18 in$

$f'_s = 0.003(29 \times 10^6)\left(\frac{5.18 - 2.5}{5.18}\right) = 45011 \cong 45 ksi$

Iterating, we close in on a and f'_s:

$a = 3.86 in$ $c = 4.54 in$ $f'_s = 39 ksi$
$a = 4.09 in$ $c = 4.81 in$ $f'_s = 42 ksi$
$a = 3.98 in$ $c = 4.68 in$ $f'_s = 40.5 ksi$
$a = 4.03 in$ $c = 4.75 in$ $f'_s = 41 ksi$ ⇒ close enough.

$M_n = 0.85(4ksi)(4.03in)(12in)\left(18in - \frac{4.03in}{2}\right) + 1.58in^2(41ksi)(18in - 2.5in)$

$= 2628 + 1004 = 3632 in \cdot kips = \underline{302 ft \cdot kips}$

$\varepsilon_t = \varepsilon_u \left(\frac{d-c}{c}\right) = 0.003\left(\frac{18in - 4.75in}{4.75in}\right) = 0.0084 > 0.005 \therefore \phi = 0.90$

$\boxed{\phi M_n = 272 ft \cdot kips}$

b) $A_s = 5.08 \text{ in}^2$ $\quad \rho = 0.0235 > \rho_{max} = 0.0206$ $\quad \therefore$ comp. steel needed

$\rho = 0.0235 < \bar{\rho}_{max} = 0.0279$ $\quad \therefore$ section will have adequate ductility

$$A_s f_y = 0.85 \beta_1 f'_c b c + A'_s \varepsilon_u E_s \frac{c-d'}{c}$$

$$5.08 \text{ in}^2 (60000 \text{ psi}) = 0.85(.85)(4000 \text{ psi})(12 \text{ in}) c + 1.58 \text{ in}^2 (0.003)(29 \times 10^6 \text{ psi})\left(\frac{c - 2.5 \text{ in}}{c}\right)$$

$$304800 = 34680 c + 137460\left(\frac{c-2.5}{c}\right)$$

$$304800 c = 34680 c^2 + 137460(c - 2.5)$$

$$0 = 34680 c^2 + 137460 c - 343650 - 304800 c$$

$$0 = 34680 c^2 - 167340 c - 343650$$

$$c = 6.38 \text{ in} \qquad a = 5.42 \text{ in}$$

$$f'_s = \varepsilon_u E_s \left(\frac{c-d'}{c}\right) = .003(29 \times 10^6 \text{ psi})\left(\frac{6.38 - 2.5}{6.38}\right) = 52900 \text{ psi} = 53 \text{ ksi}$$

$$M_n = 0.85(4 \text{ ksi})(5.42 \text{ in})(12 \text{ in})\left(18 \text{ in} - \frac{5.42 \text{ in}}{2}\right) + 1.58 \text{ in}^2 (53 \text{ ksi})(18 \text{ in} - 2.5 \text{ in})$$

$$= 3381 + 1298 = 4679 \text{ in}\cdot\text{kips} = \underline{390 \text{ ft}\cdot\text{kips}}$$

$\underline{\underline{\text{OR}}}$ $$M_n = \left(A_s - \frac{f'_s}{f_y} A'_s\right) f_y \left(d - \frac{a}{2}\right) + A'_s (f'_s)(d - d')$$

$$= \left(5.08 \text{ in}^2 - \frac{53}{60}(1.58 \text{ in}^2)\right)(60 \text{ ksi})\left(18 \text{ in} - \frac{5.42}{2}\right) + 1.58 \text{ in}^2 (53 \text{ ksi})(18 - 2.5 \text{ in})$$

$$= 3380 + 1298 = 4678 \text{ in}\cdot\text{kips} = \underline{390 \text{ ft}\cdot\text{kips}}$$

$$\varepsilon_t = \varepsilon_u \left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{19.27 - 6.38}{6.38}\right) = 0.006 > 0.005$$

$\therefore \phi = 0.90$

where $d_t = d + \frac{1}{2}\text{bar} + \frac{1}{2}\text{space}$

$= 18 + 1.27$

$= 19.27 \text{ in}$

$$\boxed{\phi M_n = 351 \text{ ft}\cdot\text{kips}}$$

part a) iterative approach used, a, c are estimated and f'_s (stress in comp. steel) is calculated … Find converging values, use to calc M_n

part b) balanced forces equation is solved to determine c exactly. f_s, M_n are calculated directly.

c) $A_s = 6.00 in^2 \quad \rho = 0.0278 \qquad \rho_{max} = 0.0279$ (very close)

$$a = \frac{(6.00in - 1.58in)(60ksi)}{0.85(4ksi)(12in)} = 6.50 in \quad c = 7.65 in$$

Again $c = 7.65 < 8.06 in$ $\boxed{OR}$ $\rho = 0.0278 < 0.0289 = \bar{\rho}_{cy}$ ∴ A_s' does not yield

$$f_s' = \varepsilon_u E_s \left(\frac{c - d'}{c}\right) = 0.003(29 \times 10^6)\left(\frac{7.65 - 2.5}{7.65}\right) = 58570 psi = 58.6 ksi$$

Iterating; $a = 6.55 in \quad c = 7.71 in \quad f_s' = 58.8 ksi$ ⇒ close enough

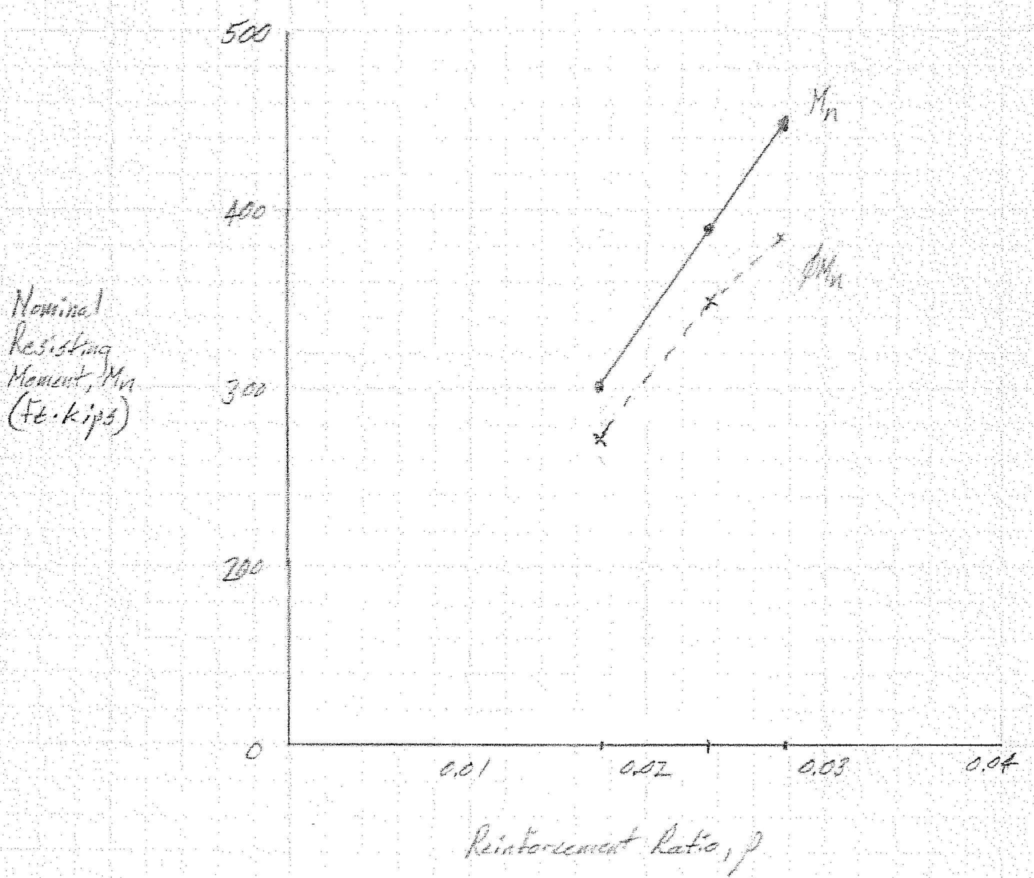
$$M_n = 0.85(4ksi)(6.55in)(12in)\left(18in - \frac{6.55in}{2}\right) + 1.58in^2(58.8ksi)(18in - 2.5in)$$

$$= 3935 + 1440 = 5375 \text{ in·kips} = \underline{448 \text{ ft·kips}}$$

$$\varepsilon_t = \varepsilon_u\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{19.13 - 7.65}{7.65}\right) = 0.0045 \quad ∴ \phi = 0.86$$

where $d_t = d + \frac{1}{2}bar + \frac{1}{2}space = 18 + 1.13 = 19.13$

$$\boxed{\phi M_n = 385 \text{ ft·kips}}$$



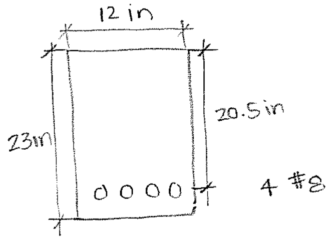
The relationship is nearly linear, showing significant increased capacity corresponding to increasing reinforcement. The peak point approaches roughly the minimum ductility limit and, thus, the maximum moment design for a beam of this size.

CE 326: HOMEWORK #6 – 6.3

CATHERINE HOVELL
MARCH 24, 2004

$$\frac{28}{30}$$

6.3



L = 18.5 ft, simply supported

$DL = 1.27\ \text{kip/ft}$

$LL = 2.69\ \text{kip/ft}$

$> M = \frac{w\ell^2}{8}$

$M_{DL} = \frac{(1.27\ \text{kip/ft})(18.5\ \text{ft})^2}{8} = 54.3\ \text{kip}\cdot\text{f}$ ✓

$M_{LL} = \frac{(2.69\ \text{kip/ft})(18.5\ \text{ft})^2}{8} = 115.1\ \text{kip}\cdot\text{ft}$

$f_y = 60\ \text{ksi}$

$f_c' = 4\ \text{ksi}$

$M_{cr} = \frac{f_r I_g}{y_t}$ $\quad I_g = \frac{1}{12}bh^3 = \frac{1}{12}(12\text{in})(23\text{in})^3 = 12167\text{in}^4$

$y_t = \frac{1}{2}(23\text{in}) = 11.5\text{in}$

$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000\text{psi}} = 474.3\text{psi}$

$M_{cr} = \frac{(474.3\text{psi})(12167\text{in}^4)}{11.5\text{in}} = 41.8\ \text{kip}\cdot\text{ft}$ ✓

$\frac{bx^2}{2} + A_s n x - A_s n d = 0$ solve for x

$(6\text{in})x^2 + (25.28\text{in}^2)x - (518.24\text{in}^3) = 0$

$x = 7.42\text{in}$

$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)\right]^3 I_{cr} \le I_g$

CASE 1:
$M_a = M_{DL} = 54.3\ \text{kip}\cdot\text{ft}$ ✓

CASE 2:
$M_a = M_{DL} + M_{LL} = 169.4\ \text{kip}\cdot\text{ft}$ ✓

$I_{cr} = \frac{1}{3}bx^3 + A_s n(d-x)^2$

$I_{cr} = \frac{1}{3}(12\text{in})(7.42\text{in})^3 + 4(0.79\text{in}^2)(8)(20.5\text{in} - 7.42\text{in}$

$I_{cr} = 5959.1\ \text{in}^4$

$I_{eff_1} = \left(\frac{41.8\ \text{kip}\cdot\text{ft}}{54.3\ \text{kip}\cdot\text{ft}}\right)^3(12167\text{in}^4) + \left[1 - \left(\frac{41.8\ \text{kip}\cdot\text{ft}}{54.3\ \text{kip}\cdot\text{ft}}\right)^{3}\right]^3(5959.1\text{in}^4)$

$I_{eff_1} = 5623.0\text{in}^4$ X −1

$\delta_{DL} = \frac{5w_{DL}\ell^4}{384 I_{eff_1}\cdot E} = \frac{5(1.27\ \text{kip/ft})(18.5\ \text{ft})^4(12\text{in/ft})^3}{384(3.6\times10^3\ \text{ksi})(5623.0\text{in}^4)} = 0.165\text{in}$

$I_{eff_2} = \left(\frac{41.8\ \text{kip}\cdot\text{ft}}{169.4\ \text{kip}\cdot\text{ft}}\right)^3(12167\text{in}^4) + \left[1 - \left(\frac{41.8\ \text{kip}\cdot\text{ft}}{169.4\ \text{kip}\cdot\text{ft}}\right)^{3}\right]^3(5959.1\text{in}^4) = 2729.6\text{in}^4$ X −1

$\delta_{DL+LL} = \frac{5(3.96\ \text{kip/ft})(18.5\text{ft})^4(12\text{in/ft})^3}{384(3.6\times10^3\text{ksi})(2729.6\text{in}^4)} = 1.06\text{in}$

$\delta_{LL} = \delta_{DL+LL} - \delta_{DL} = 1.06\text{in} - 0.165\text{in} = 0.897\text{in}$

$\boxed{a.\ \delta_{LL} = 0.897\text{in}}$

6.3 (cont'd)

$$\delta_{creep} = \lambda(\delta_{DL})\boxed{+\delta_{LL}}$$

$$\lambda = \frac{\xi}{1+50\rho'} \qquad \rho' = 0 \text{ for this beam}$$

$$\lambda = \xi$$

$\delta_{creep} = \xi(\delta_{DL}) + \delta_{LL}$ Assume creep in question is long-term (5yr+)

$\xi = 2.0$ ✓

$$\delta_{creep} = (2.0)(0.165\,in) + 0.897\,in = 1.23\,in$$ ✓

$$\boxed{\text{b. } \delta_{creep+load} = 1.23\,in}$$

deflection limitation: $\delta \le \ell/480$

$$\delta_{max} = \frac{18.5\,ft\,(12\,in/ft)}{480} = 0.463\,in < \delta_{creep+load}$$ ✓

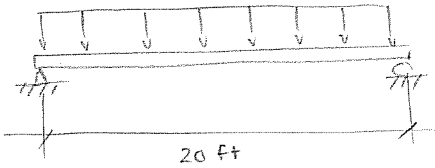
c. Does **NOT** meet limitations.

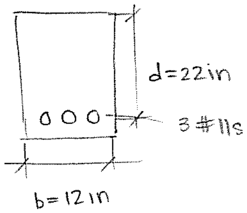
CE 326: HOMEWORK #7 – 4.2

CATHERINE HOVELL
APRIL 12, 2004

$\frac{28}{30}$

4.2





$w_{DL} = 1.63$ kip/ft
$w_{LL} = 3.26$ kip/ft

didn't factor! −1

$P_{tot} = (1.63 \text{ kip/ft} + \cancel{3.26 \text{ kip/ft}})(20 \text{ ft})$

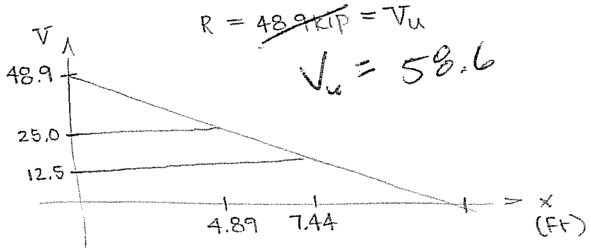
$P_{tot} = 97.8$ kip

$R = \cancel{48.9 \text{ kip}} = V_u$

$V_u = 58.6$

shear: using #3 U-stirrups

$f'_c = 4$ ksi $\quad f_y = 60$ ksi



$V_c = 2\sqrt{f'_c}\,bd = 2\sqrt{4000 \text{ psi}}\,(12 \text{ in})(22 \text{ in}) = 33.4$ kip

$\frac{V_u}{\phi} - V_c < 8\sqrt{f'_c}\,bd$?

$\frac{48.9 \text{ kip}}{0.75} - 33.4 \text{ kip} < 4(33.4 \text{ kip})$ section OK ✓

$\phi V_c = (0.75)(33.4 \text{ kip}) = 25.0 \text{ kip} < V_u$

$A_v = \frac{(V_u/\phi - V_c)s}{f_y d}$ $\quad s \le d/2$ (not $d/4$, as $V_u/\phi - V_c \le 4\sqrt{f'_c}\,bd$)

$s \le \frac{22 \text{ in}}{2} = 11$ in

$A_v = \frac{\left[\frac{48.9 \text{ kip}}{0.75} - 33.4 \text{ kip}\right](11 \text{ in})}{(60 \text{ ksi})(22 \text{ in})} = 0.265 \text{ in}^2$ since s is a max value, so is A_v

so, using $A_v = 0.22 \text{ in}^2$,

$s = \frac{A_v f_y d}{V_u/\phi - V_c} = \frac{(0.22 \text{ in}^2)(60 \text{ ksi})(22 \text{ in})}{48.9 \text{ kip}/0.75 - 33.4 \text{ kip}} = 9.13 \text{ in} \le d/2 = s_0$

$s_x = \frac{s_0(4.89 \text{ ft})}{4.89 \text{ ft} - x}$

$s_1 = 11.4$ in

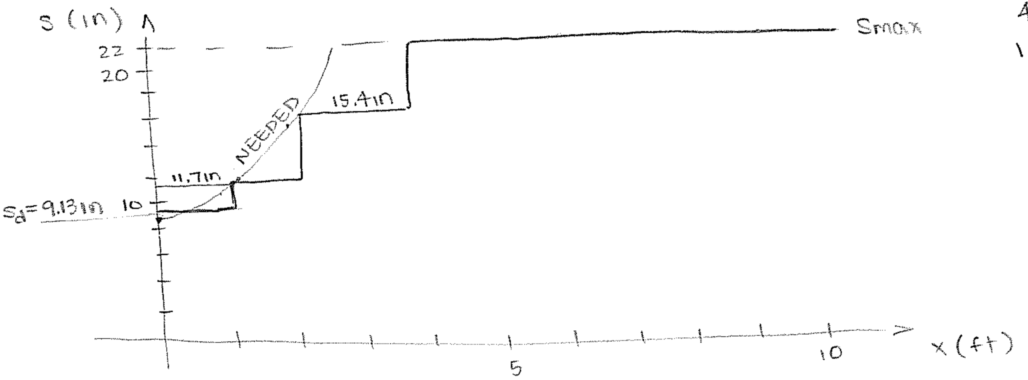
$s_2 =$

when is $V_u < \phi V_c$: $V_u < (0.75)(33.4 \text{ kip}) = 25.0$ kip at 4.89 ft

$V_u = \frac{1}{2}\phi V_c = 12.5$ kip at 7.44 ft

$s_{max} = \frac{A_v f_y}{\phi\sqrt{f'_c}\,b} = \frac{(0.22 \text{ in}^2)(60 \text{ ksi})}{0.75\sqrt{4000 \text{ psi}}\,(12 \text{ in})} = 23.2 \text{ in} \le \frac{A_v f_y}{50b} = \frac{(0.22 \text{ in}^2)(60 \text{ ksi})}{50(12 \text{ in})} = \underline{\underline{22 \text{ in}}}$

Wrong Spacings + locations −1

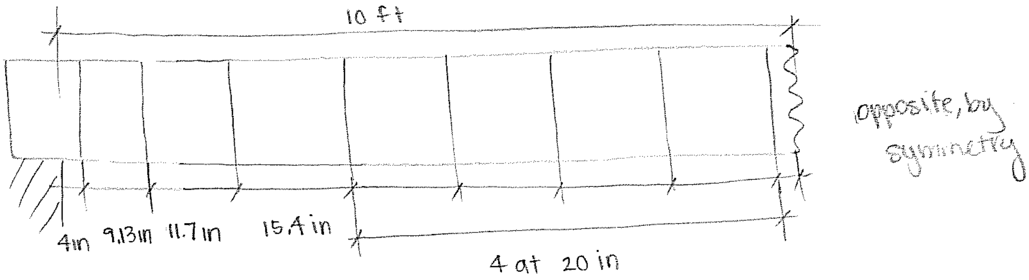


4'' space
1 at 9.13 in
1 at 11.7 in
1 at 15.4 in

drawing

C.HOVELL

CE 326: HOMEWORK #7 – 4.2



HW 7 Solution

4.2

$w_u = 1.2 \times 1.63 + 1.6 \times 3.26 = 7.17$ Kips/ft

$V_u = 7.17 \times 20/2 = 71.7$ Kips

$V_{u|d} = 71.7 \times 7.17 \times \frac{22}{12} = 58.6$ Kips

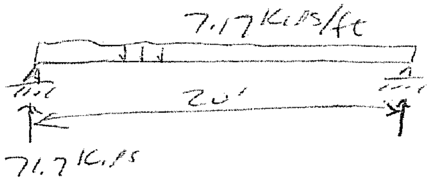
$\phi V_c = 0.75 \times 2\sqrt{4000} \times \frac{12 \times 22}{1000} = 25.0$ Kips

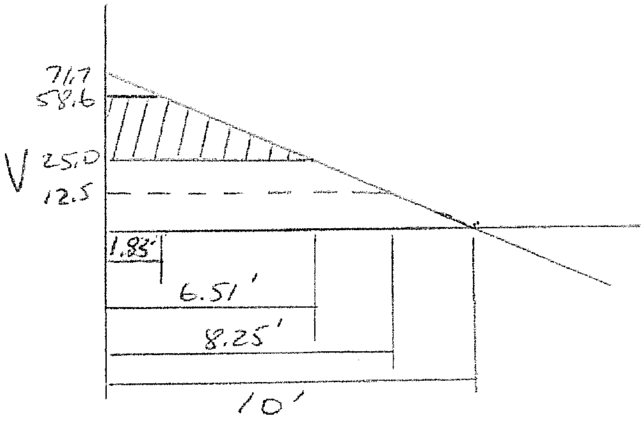
$$S_d = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60 \times 22}{58.6 - 25.0} = 6.48''$$

$s \le d/2 = \frac{22}{2} = 11''$

$s \le 24''$

$$s \le \frac{A_v f_y}{50 b_w} = \frac{0.22 \times 60{,}000}{50 \times 12} = 22''$$





CRITICAL SECTION @d FROM FACE OF SUPPORT

X = 1.83'   V = 58.6 Kips

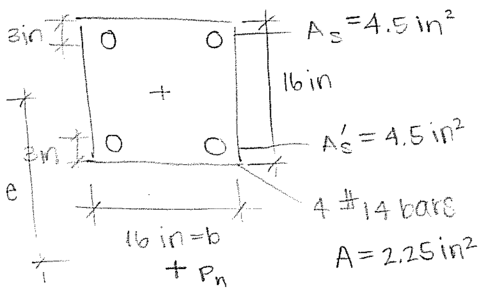
MIN. STIRRUPS REQUIRED TO $\frac{1}{2}\phi V_c$

CE 326: HOMEWORK #8 – Problem 8.1

CATHERINE HOVELL
APRIL 19, 2004

$\frac{26}{30}$

8.1.



$f_c' = 5 \text{ ksi}$

$f_y = 60 \text{ ksi}$

BALANCED:

$\varepsilon_u = 0.003 \quad \varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29000 \text{ ksi}} = 0.00207$

$c_b = d \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = (16\text{in} - 3\text{in}) \frac{0.003}{0.003 + 0.00207} = 7.69 \text{ in}$

$a = \overset{0.80}{\cancel{0.85}} c = 6.54 \text{ in}$

$\rightarrow f_s = f_y \quad f_s' = \varepsilon_u E_s \frac{c - d'}{c} = (0.003)(29000 \text{ksi}) \frac{7.69\text{in} - 3\text{in}}{7.69\text{in}}$

$f_s' = 53.1 \text{ ksi} < f_y$

$C = 0.85 f_c' ab = (0.85)(5\text{ksi})(16\text{in})(6.54\text{ in}) = 445 \text{ kip}$

$P_b = 0.85 f_c' ab + A_s' f_s' - A_s f_s$

$P_b = 445 \text{kip} + (4.5 \text{in}^2)(53.1 \text{ksi} - 60 \text{ksi})$

$P_b = \cancel{413.8 \text{ kip}}$ – 1

$M_n = 0.85 f_c' ab (h/2 - a/2) + A_s' f_s' (h/2 - d') + A_s f_s (d - h/2)$

$M_n = (0.85)(5\text{ksi})(6.54\text{in})(16\text{in})(1/2)(16\text{in} - 6.54\text{in}) + (4.5\text{in}^2)\left[(53.1\text{ksi})\left(\frac{16\text{in}}{2} - 3\text{in}\right) + (60\text{ksi})\left(13\text{in} - \frac{16\text{ in}}{2}\right)\right]$

–1

$M_b = \cancel{387.4 \text{ft} \cdot \text{kip}} = P_b e \quad e = 11.2 \text{in}$

Pt. 4 $e = 11.2\text{in}, (387.4 \text{ ft} \cdot \text{kip}, 413.8 \text{ kip})$

PURE AXIAL LOAD:

$P_n = 0.85 f_c' bh + A_{s_T} f_y = 0.85(5\text{ksi})(16\text{in})(16\text{in}) + (9.0\text{in}^2)(60\text{ksi}) = 1628 \text{ kip}$

$P_o = \cancel{1628 \text{ kip}}$ –1 **Pt. 1** $e = 0, (0, 1628 \text{ kip})$

PURE MOMENT, NO AXIAL LOAD:

$f_s = f_y$ assume $f_s' < f_y \quad 0.85 f_c' ab + A_s' \left(\frac{c - d'}{c}\right) \varepsilon_u E_s = A_s f_y$

$(0.85)^2 (5\text{ksi})(16\text{in}) c + (4.5\text{in}^2)\left(\frac{c - 3\text{in}}{c}\right)(0.003)(29000\text{ksi}) = (4.5\text{in}^2)(60\text{ksi})$

SOLVE FOR C.

$c = 3.58 \text{ in}, \ a = 3.04 \text{ in}$

$\rightarrow$ check: $f_s' < f_y$? $f_s' = \frac{c - d'}{c} \cdot \varepsilon_u E_s = \frac{3.58\text{in} - 3\text{in}}{3.58\text{in}} (0.003)(29000 \text{ksi})$

$f_s' = 9.27 \text{ ksi} < f_y$ ✓

compressive steel needed:

$A_{s_1} = \frac{A_s' f_s'}{f_y} = \frac{(4.5\text{in}^2)(9.27\text{ksi})}{60\text{ksi}} = 0.695 \text{ in}^2 \quad A_{s_2} = 4.5\text{in}^2 - 0.695\text{in}^2$

$A_{s_2} = 3.80 \text{in}^2$ tensile stee balancing concrete

$M_n = A_{s_2} f_y (d - a/2) + A_s' f_s' (d - d')$

$M_n = (3.80\text{in}^2)(60\text{ksi})\left(13\text{in} - \frac{3.04\text{in}}{2}\right) + (4.5\text{in}^2)(9.27\text{ksi})(13\text{in} - 3\text{in}) = 263.6 \text{ kip} \cdot \text{ft}$

–1

CE 326: HOMEWORK #8 - Problem 8.1

C. HOVELL

8.1 (cont'd)

$c < c_b = 7.69\text{in} \quad c = 5\text{in},\ f_s = f_y$

$f_s' = \varepsilon_u E_s \dfrac{c-d'}{c} = (0.003)(29000\text{ksi}) \dfrac{5\text{in} - 3\text{in}}{5\text{in}} = 34.8\text{ ksi}$

$a = 0.85(5\text{in}) = 4.25\text{in}$

$C = 0.85 f_c' ab = (0.85)(5\text{ksi})(4.25\text{in})(16\text{in}) = 289\text{ kip}$

$P_n = C + A_s' f_s' - A_s f_s = 289\text{ kip} + (4.5\text{in}^2)(34.8\text{ ksi} - 60\text{ksi}) = 175.6\text{ kip}$

$M_n = 0.85 f_c' ab \frac{1}{2}(h-a) + A_s' f_s' (h/2 - d') + A_s f_s (d - h/2)$

$M_n = 289\text{ kip}(1/2)(16\text{in} - 4.25\text{in}) + (4.5\text{ in}^2)[(34.8\text{ksi})(\frac{16\text{in}}{2} - 3\text{in}) + (60\text{ksi})(13\text{in} - \frac{16\text{in}}{2}$

$M_n = 319.2\text{ ft}\cdot\text{kip}$ <u>Pt. 5</u> (319.2 ft·kip, 175.6 kip)

$e = 21.8\text{ in} > e_b$

$c > c_b,\ c = 12\text{in},\ a = 10.2\text{in}$

$C = (0.85)(5\text{ksi})(16\text{in})(10.2\text{in}) = 693.6\text{ kip}$

$f_s = \varepsilon_u E_s \dfrac{d-c}{c} = 0.003(29000\text{ksi}) \dfrac{13\text{in} - 12\text{in}}{12\text{in}} = 7.25\text{ ksi}$

$f_s' = \varepsilon_u E_s \dfrac{c-d'}{c} = 0.003(29000\text{ksi}) \dfrac{12\text{in} - 3\text{in}}{12\text{in}} = 65.25\text{ksi} > f_y$

$f_s' = f_y$

$P_n = 693.6\text{ kip} + (4.5\text{in}^2)(60\text{ksi}) - (4.5\text{in}^2)(7.25\text{ ksi}) = 931.\text{kip}$

$M_n = (693.6\text{ kip})(1/2)(16\text{in} - 10.2\text{in}) + (4.5\text{in}^2)[(60\text{ksi})(\frac{16\text{in}}{2} - 3\text{in}) + (7.25\text{ksi})(13\text{in} - \frac{16\text{in}}{2}$

$M_n = 293.7\text{ ft}\cdot\text{kip} \quad e = 3.79\text{ in}$

<u>Pt. 2,3</u> (293.7 ft·kip, 931 kip)

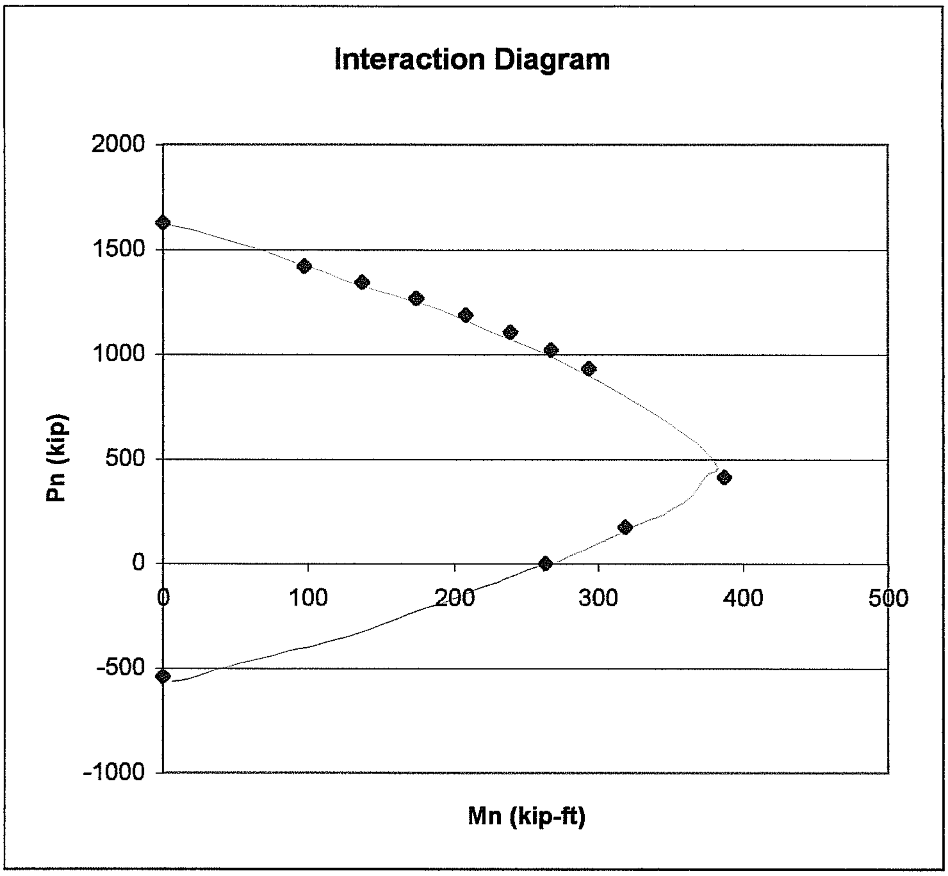
PURE TENSILE LOAD:

$P_n = -(A_s + A_s')(f_y) = (9.0\text{in}^2)(60\text{ ksi})$

$P_n = -540\text{ kip}$ <u>Pt. 7</u> (0, −540 kip)

plot, with more points between 1 and 4, on next page.

CE 326: HOMEWORK #8 - Problem 8.1 Catherine Hovell



As= 4.5
As'= 4.5

fy= 60
f'c= 5

Es= 29000
Epu= 0.003

h= 16
b= 16

d= 13
d'= 3

Point	Mn kip-ft	Pn kip	e in
1	0	1628	0
	97.5325	1419.15	0.824712
	137.5772	1344.718	1.227713
2	174.3941	1268.206	1.650149
	208.1563	1189.2	2.100467
	239.0865	1107.164	2.59134
	267.4763	1021.4	3.142466
3	293.7	931	3.785607
4	387.4	413.8	11.23441
5	319.2	175.6	21.81321
6	263.6	0	
7	0	-540	

c=	a=	C=	fs=	fs'=	Pn=	Mn=
18	15.3	1040.4	-24.1667	60	1419.15	1170.39
17	14.45	982.6	-20.4706	60	1344.718	1650.927
16	13.6	924.8	-16.3125	60	1268.206	2092.729
15	12.75	867	-11.6	60	1189.2	2497.875
14	11.9	809.2	-6.21429	60	1107.164	2869.039
13	11.05	751.4	0	60	1021.4	3209.715
12	10.2	693.6	7.25	60	930.975	3524.565

pts found by choosing values for c between points 1 and 4.

8.1

BALANCE POINT

$$C_b = \frac{0.003}{60/29\times10^3 + 0.003} \times 13 = 7.69''$$

$a_b = 0.80 \times 7.69 = 6.15''$

$\epsilon_{s}' = 0.003(7.69-3)/7.69 = 0.00183, f_s' = 53.06^{ksi}$

$P_b = 0.85\times5\times6.15\times16 + 4.5(53.06-0.85\times5)$
$\quad - 4.5\times60 \quad = 368^{kips}$

$M_b = 0.85\times5\times6.15\times16(8-6.15/2)$
$\quad + 4.5(53.06-0.85\times5)5$
$\quad + 4.5\times60\times5 = 4508''^{-k} = 376'^{-k}$

$P_o = 0.85\times5(16^2-9) + 9\times60 = 1590^{kips}$

M_o: From ~~iterations~~ $C = 3.73$
$\quad a = 2.98$
$\quad \epsilon_s' = 0.00059 \rightarrow f_s' = 17.11^{ksi}$
$\quad \epsilon_s = 0.00746 \rightarrow f_s = 60^{ksi}$
$\quad P = 0$
$\quad M_o = 0.85\times5\times2.98\times16\left(8-\frac{2.98}{2}\right)$
$\quad + 4.5(17.11-0.85\times5)\times5$
$\quad + 4.5\times60\times5 = 2959''^{-k}$
$\quad = 247'^{-k}$

SMALLER C → TRY $C = 5.6''$ ①
$\quad a = 4.48''$
$\quad \epsilon_s' = 0.00139 \quad f_s' = 40.3^{ksi}$
$\quad \epsilon_s = 0.00396 \quad f_s = 60^{ksi}$

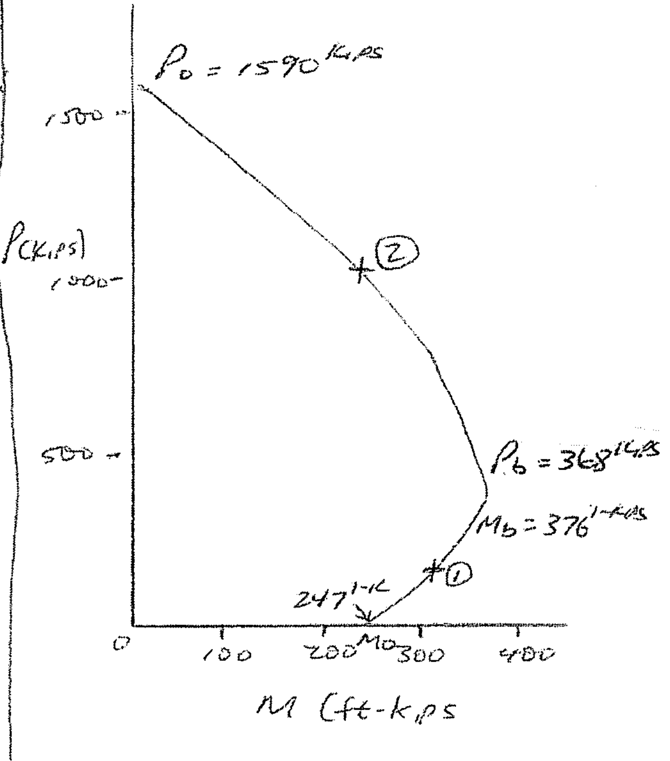
$P_n = 0.85\times5\times4.48\times16$
$\quad + 4.5(40.3-0.85\times5)$
$\quad - 4.5\times60 = 197^{kips}$

$M_n = 0.85\times5\times4.48\times16\left(8-\frac{4.48}{2}\right)$
$\quad + 4.5(40.3-0.85\times5)\times5$
$\quad + 4.5\times60\times5 = 3916''^{-kips}$
$\quad = 326'^{-kips}$

LARGER C → TRY $C = 14.4$ ②
$\quad a = 11.52''$
$\quad \epsilon_s' = 0.00238 \quad f_s' = 60^{ksi}$
$\quad \epsilon_s = 0.00029 \quad f_s' = 8.4^{ksi}$ COMPRESSION

$P_n = 0.85\times5\times11.52\times16$
$\quad + 4.5(60-0.85\times5)$
$\quad + 4.5(8.4-0.85\times5) = 1053^{kips}$

$M_n = 0.85\times5\times11.52\times16\left(8-\frac{11.52}{2}\right)$
$\quad + 4.5(60-0.85\times5)\times5$
$\quad - 4.5(8.4-0.85\times5)\times5 = 2916''^{-k}$
$\quad = 243'^{-k}$



C. Hovell

Hmm... Hoos Hungover? January 16, 2004

Reinforced concrete

Bridges - columns, piers, decks, girders, beams, foundations

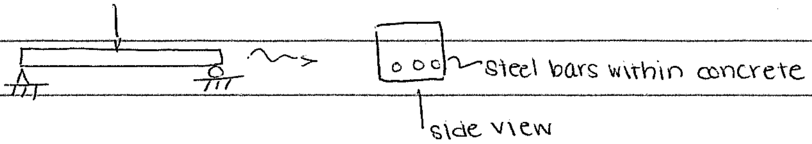
Parking garages	Sports arenas	Dams
office buildings	Music/Entertainment venues	Canals / Aquducts
Apartment buildings	sidewalks	underground pipes
Retaining walls	Roads / Highways	Tanks

* basically, it's used a bunch

Why is it good? Benefits...

- relative cost / abundance of materials
- formable
- relatively strong
- durable
- fire resistant
- low maintenance

strong in compression, but not intension... so steel is added



Portland cement concrete "ingredients"

> aggregate (course and fine)

> sand

> cement - pH > 12 (!!) steel is less likely to corrode

> water - hydration, starts reaction

> air - entrained (we put it there), ~~entraped~~ entrapped (naturally occuring)

spaces allow for the expansion of water during freeze / thaw

> admixtures

Cement types: I - ordinary, normal

SO4

II - lower generated heat, gives moderate resistance to sulfate attack

III - higher heat of hydration - good for cold weather; cures faster

IV - even cooler than II; slow strength gain

V - greatest sulfate exposure resistance

Aggregate of varying types

... crushed limestone, rounded river gravel...

almost always local (< 50 miles from site)

C.Hovell

Ugh... My head hurts... January 16, 2004

water/cement ratio

usual #s: 0.35 to 0.5+

need enough water to hydrate cement, and make it workable

too much weakens concrete in the end

Cementitious materials

portland cement

> Fly ash (10-30% replacement of cement)

> microsilica / silica fume

> blast furnace slag, ground and granulated

WOW. I AM REALLY IMPRESSED WITH ANYONE WHO CAN MAKE CONCRETE THIS INTERESTING.

Disadvantages of concrete

- low tensile strength
- quality control
- time-dependent material properties
- nonlinear material – not homogeneous
- heavy

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I Hate This Car Crap January 19,2004

Concrete: Material Properties

- > compressive strength / tensile strength (f'_c = design compressive strength) age = 28 days
- > unit weight (density)
- > slump (before curing); workability
- > durability
  - ex. freeze-thaw durability (ASTM C666)
- > modulus of elasticity (Young's modulus)
- > creep - deformation under a sustained load
- > shrinkage
  - loss of water: brine ; autogeneous - changes in chemical reaction
  - ↳ 2 gal H_2O + 2 gal conc ≠ 4 gal cement...
- > thermal effects

> important with precast / prestrained members

introduce a strain in steel before setting concrete

ASTM C39 and C31 - controlling manufacture and testing of concrete cylindars

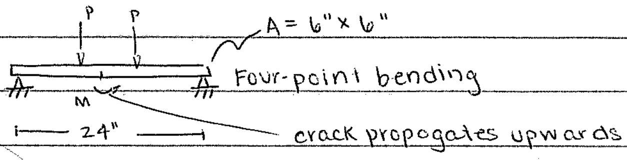
Tensile strength

10-12% of compressive strength

modulus of rupture, $f_r = 7.5 (f'_c)^{1/2}$ psi

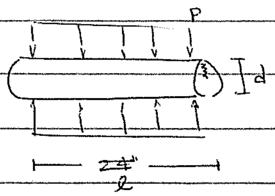
ACI = American Concrete Institute (International)

ASTM C78



$$\boxed{f_r = \frac{6M}{bh^2}}$$

ASTM C496



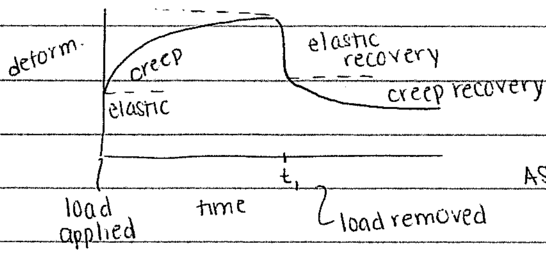
~ strength of concrete in tension

$$\boxed{f_{ct} = \frac{2P}{\pi l d}}$$

P = total load

$f_r \sim 1.5 f_{ct}$

Creep, etc



Shrinkage - ASTM C157

0.0004 to 0.001 in/in

ASTM C512

C.Hovell

Concrete on concrete

January 21, 2004

Reinforcing Steel

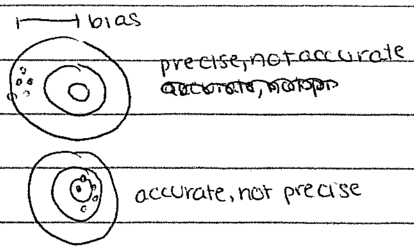
Standard sizes: #3, 4, 5 ... 11, 14, 18

$\llcorner$ 3/8" diameter for all, approximately (#/8)

grades: 40 (f_y = 40 ksi), 50, 60

modulus of elasticity = 29000 ksi = E

Variability and Reliability



Accuracy - how close a measured result is to the actual value

Precision - how closely one can make the measurement

Bias - how far off the actual value the data is (ew) (sic)

tendency to deviate from true value

Load and Resistance Factor Design (LRFD)

resistances ≥ load effects

- design strength = (nominal strength)(resistance factor) (< 1)
- factored load = (nominal load)(load factor) (> 1)

load effects: any external or internal force which acts on a structure or member

includes dead loads, live loads, soil, wind, snow, rain, flood, earthquakes

limit states: condition beyond which a structure or member becomes unfit for service

limit state design

- ultimate limit states: loss of equilibrium, ~~re~~rupture, progressive collapse
  formation of a plastic mechanism, instability, fatigue
- servicibility limit states: becomes unfit for service, reason for existance
  excessive deflections, excessive crack width
  undesirable vibrations, ~~special limit~~
- special limitstates
  damage or collapse due to earthquakes, structural effects from extreme incidents
  deterioration (corosion, chemical attack ...)

C.Hovell

FOUR MONTHS LEFT!

January 23, 2004

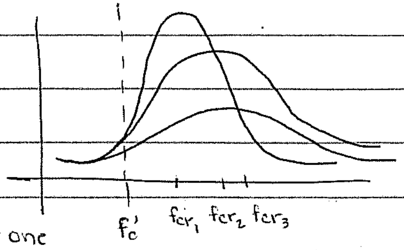
Specification of compressive strength

f'_c : specified compressive strength

ACI 5.32 (f_{cr} = required mean strength)

$$\boxed{\begin{aligned} f_{cr} &= f'_c + 1.34s \\ f_{cr} &= f'_c + 2.33s - 500\text{ psi} \end{aligned}}$$

where s = standard deviation of the test



ensures a probability of not ~~that not~~ more than one in 100 that the average of any 3 consecutive strength tests will be less than f'_c

ensures ~~t~~ a probability of one in 100 that any individual strength test is no more than 500 psi less than f'_c.

CHECK BOTH!

Load factors – account for variability in loadings

specifically snow, wind, earthquakes ← highly variable loads

~~IE~~

Resistance factors – account for variability in material strengths

– dimensional tolerances

– simplifying assumptions in design process

ACI ~~3~~318-02 Chapt. 9 and Appendix C : Strength reduction factors

resistance ≥ load effects

$\phi R_n \geq \alpha_1 S_1 + \alpha_2 S_2 \ldots$ α = load factors, S = type of load

ϕ = strength reduction factor

moment: $\phi_M M_n \geq \alpha_D M_D + \alpha_L M_L \ldots$ (or shear, V, or axial, P)

Dead Loads – self weight, walls, floors, ceilings, plumbing, electrical, HVAC

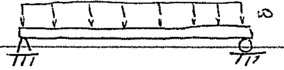
Live Loads – all non-fixed loads related to occupancy of a structure

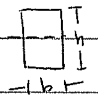
both uniform and concentrated load

for design purposes, the location of the loads must consider "worst case"

ASCE 702, Chapter 4, Table 4-1 load discussion

Oh boy! Concrete Design! Review of bending beam theory...





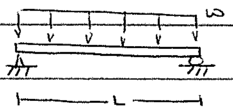
$M_{max} = \frac{w\ell^2}{8}$

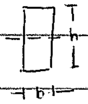
C.Hovell

ABC-123-##

January 26, 2004

Bending





a. max bending-internal couple

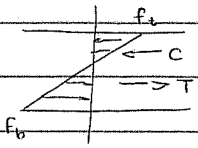
b. check with flexure formula

$M_{max} = \frac{w l^2}{8}$

Find f_t, f_B (top, bottom)

$M = Cz$ or $M = Tz$ z=moment arm!

$C = A_{triang\ stress} \cdot width_{beam}$



Flexure formula $f_t = \frac{Mc}{I}$

Service Load Analysis

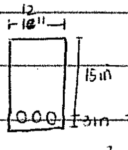
assumptions for reinforced concrete

- plane sections before bending remain plane after bending
- Hooke's Law ($\sigma = E\varepsilon$)
- tensile strength of concrete is neglected
- concrete and steel have a perfect bond - no slip with induced stress

three load stages

1. uncracked
2. service load (concrete is cracked, steel carries tensile load)
3. failure (ultimate) load

Example:

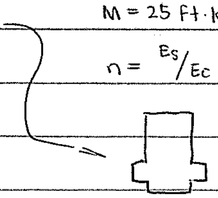


$A_s = 3\ in^2$

$f'_c = 4$ ksi — bending stress at top, bottom

M = 25 ft·kip — cracking moment

$n = E_s/E_c = 9$ using transformed section approach



$(n-1)A_s$ = transformed area

find central axis, at 9.6"

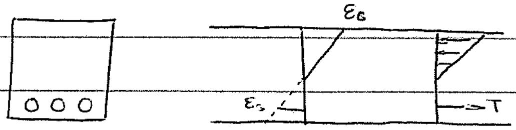
$F_r = 7.5\sqrt{f'_c}$

C. Hovell

Snow, Snow, Go Away... January 28, 2004

OK, SO, THIS GUY NEVER USES UNITS... AND I ACTUALLY KIND OF MISS THEM. I'VE COME A LONG WAY, BABY...

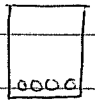
Cracked Section



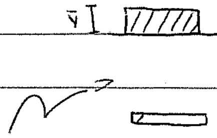
at the service load
(concrete has cracked)

all tensile load is then carried by the steel – concrete cannot carry any.

example:



stresses in steel, concrete?



NA, below which concrete carries no load

$\Sigma M_{NA} = 0$

ignore concrete in here, as it has failed and carries no load – calculate location of N.A. without it $A_c\bar{y}_n \cdot \frac{\bar{y}}{2} = A_s(h-\bar{y})$ solve for $\bar{y}$

Failure (oh boy!)

Ultimate Load State

$F_s \geq f_y$ stress in steel is greater than the yield strength of the steel

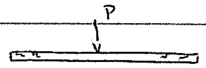
underreinforced $\overset{\text{is}}{\cancel{\neq}}$ better than over-reinforced

↳ yields first ↳ fails without warning

compressed concrete crushes (secondary crushing)

$f_c \geq f_c'$ concrete crushes

happens more in shear regions away from load



in a perfectly balanced situation, both occur at the same time

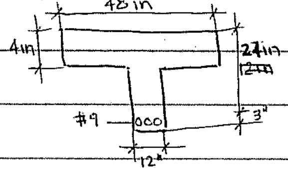
steel yields, concrete crushes simultaneously

ρ = reinforcement ratio = $\frac{A_s}{bd}$ bd = area of effective concrete

C. Hovell

FRIDAY! THAT'S KEY! January 30, 2004

A new shape!

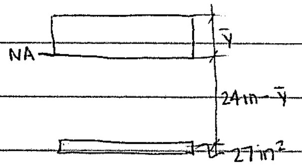


$n = 9 = {}^{E_s}/_{E_c}$ find stress in concrete and steel

$M = 130$ ft·kip

↳ assume this has caused cracking

so, below $\bar{y}$, concrete holds no load

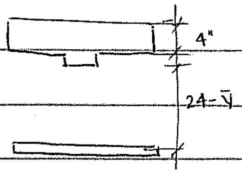


effective area of steel: $9\,(3) = 27\,in^2$

↑ n   ↑ #(in^2) #9 steel bars have area ~ 1 in^2

ΣM_{NA}: $\bar{y}\,(48\,in)\frac{1}{2}\bar{y} = (27\,in^2)(24\,in - \bar{y})$ solve for $\bar{y}$

$\bar{y} = 4.66$ in, which goes past the 4" flange



$(48\,in)(4\,in)(\bar{y} - 2\,in) + (\bar{y} - 4\,in)^2(12\,in)\frac{1}{2} = (27\,in^2)(24\,in - \bar{y})$

$\boxed{\bar{y} = 4.7\,in}$

$I = 1/3\,(48\,in)(4.7\,in)^3 - 1/3\,(36\,in)(0.7\,in)^3 + 27\,in^2(19.3\,in)^2$

$I = 11714\,in^4$

Now, check stresses – was moment assumption correct?

$f_{cb} = \frac{(130\,ft\cdot kip)(12\,in/ft)(22.3\,in)}{11714\,in^4} = \frac{Mc}{I}$ where $c = 27\,in - \bar{y}$

$f_{cb} = 2.97$ ksi f_r (modulus of rupture) $= 7.5\sqrt{f'_c} = 7.5\sqrt{4000}$ ← typical value for concre

compared to: $= 0.474$ ksi

thus assumption is correct!

$f_{cT} = \frac{Mc}{I} = \frac{\cancel{(130\,ft\cdot kip)(12\,in/ft)(4.7\,in)}}{11714\,in^4} = 0.626$ ksi comp.

linear behavior until ~ $\frac{1}{2} f'_c$, in psi

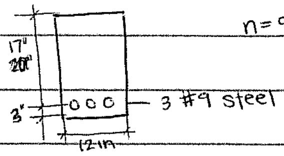
4000 psi $= f'_c$

$f_s = \frac{(130\,ft\cdot kip)(12\,in/ft)(19.3\,in)}{11714\,in^4} \cdot (9) = 23.1$ ksi tension

↖ modular ratio!

$f_{ys} = 60$ ksi for 60-grade steel

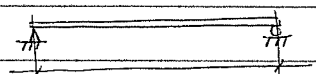
Allowable stress → allowable load



$n = 9$ allowable stresses: cracked properties:

$f_c = 1350\,psi < \frac{1}{2} f'_c$ $\bar{y} = 6.78\,in$, $I = 4065\,in^4$

$f_s = 20000\,psi < f_y$



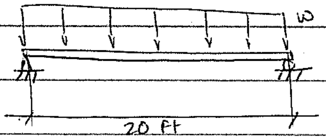
find the allowable superimposed distributed load $f = \frac{Mc}{I}$ $M_c = \frac{f_c I}{c_T}$

C. Hovell

Notes, continued...

January 30, 2004

Allowable stress, find allowable loads (superimposed)



$\bar{y} = 6.78$ in

$I = 4065$ in^4

$$M_c = \frac{f_c I}{c_T} = \frac{(1350 \text{ psi})(4.065 \text{ in}^4)}{6.78 \text{ in}} \boxed{= 67.5 \text{ ft}\cdot\text{kip}} \text{ (limits)}$$

$$M_s = \frac{(20000 \text{ psi})(4065 \text{ in}^4)}{\underset{\substack{\uparrow \\ \text{modular ratio}}}{9}(10.22 \text{ in})} = \underset{\substack{\uparrow \\ \text{allowable moments}}}{73.7 \text{ ft}\cdot\text{kip}}$$

allowable: $f_c = 1350$ psi

$f_s = 20{,}000$ psi

$$\text{max moment} = \frac{w l^2}{8} \qquad w = \frac{(67.5 \text{ ft}\cdot\text{kip})(8)}{(20 \text{ ft})^2} = 1.35 \text{ kip/ft}$$

but! superimposed allowable load

include weight of the beam!

beam weight: $\boxed{\sim 150 \text{ lb/ft}^3}$, which includes steel

$A \cdot \rho = 0.25$ kip/ft for this beam

so, additional load = 1.10 kip/ft

C.HOvell

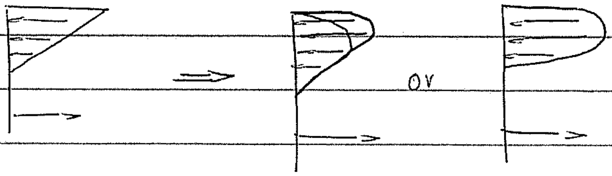
Oddly Not Hungover

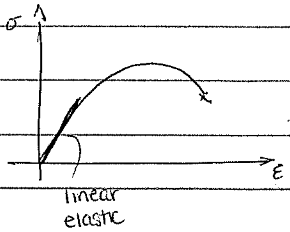
February 2, 2004

Strength Design

assume: linear elasticity

in actuality:





for concrete

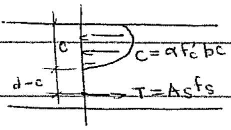
how do you predict this shape?

☆ effects on stress distribution:
- (strength of) forces in concrete (f'_c)
- (modulus (E))
- loading rate
- duration of the load (shrinkage, creep)

1. at or near ultimate load, stresses are no longer linearly elastic
2. near ultimate load, the stress distribution varies in both magnitude and geometry, depending on f'_c, loading rate, and duration of load
3. experimental observations have shown ultimate concrete strain of $0.003 = \varepsilon_u$ is a conservative indicator of failure

$0.003 <$ actual value < 0.004 (usually)

Note: what we really need to know for design is the total resultant compressive force, C, and its vertical location.



d = effective depth (or something)

c = depth to NA

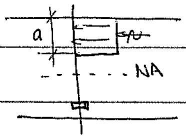
If $C = f_{av}\, b \cdot c$

f_{av} = average compressive concrete stress

let $\alpha = f_{av}/f'_c$, so $\boxed{C = \alpha f'_c bc}$ at a depth $\boxed{\beta c}$

Now find α, β – found experimentally – fig. ~~pg~~ 3.7 table 3.1 depend on f'_c.

Equivalent Rectangular Stress Block (whitney)



there is a rectangular approximation

$\beta/\alpha = 0.59$

force acts at $a/2 = \beta c$

$C = \alpha f'_c c \cdot b = \gamma f'_c ab$ so $\gamma = \alpha \frac{c}{a}$ $a = \beta_1 c$ $\beta_1 = \frac{\alpha}{\gamma} = 2\beta$

$C = 0.85 f'_c ab$

$\gamma \sim 0.85$

$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)$

$0.65 \leq \beta_1 \leq 0.85$

Reinforcement Ratio, ρ (if/when balanced, ρ_b)

$\rho = A_s/bd$ $c = \frac{\varepsilon_{uc}}{\varepsilon_{uc} + \varepsilon_{ys}} d$

I HAVE ZERO ABILITY TO SIT STILL TODAY. I'M SLEEPY YET NOT. HUNGOVER YET NOT. HUNGRY YET NOT. ALAS...

C.Hovell

Margaritas Tonight!

February 4, 2004

Homework questions / notes

$E_c \sim 57000\sqrt{f_c'}$ prob #1, $n = 9$

- graphs should be clearly labelled
- show work!

Back to last class...

$C_b = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d$ $\rho_b = \frac{\alpha f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$ failure balanced between the ultimate strain of concrete (~ 0.003) and the yield strain of the steel ($\varepsilon_y = 0.00207$)

theoretically, concrete and steel would fail simultaneously

Equivalent stress block

$C = T$, so $\rho_b f_y b d = 0.85 f_c' a b$ $\rightarrow a = \beta_1 c$

$$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

err on the side of too little - underreinforced, not over

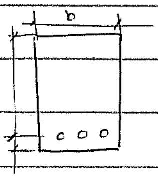
to ensure there is enough steel...

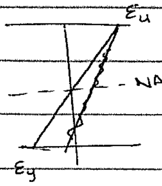
$$\boxed{As_{min} = \frac{3\sqrt{f_c'}}{f_y} bd}$$

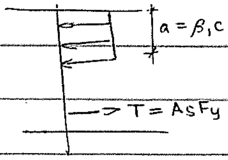
but also not less than $200bd/f_y$

YAY! MARGARITAS AT AMIGO'S TONIGHT. I FEEL LIKE THERE SHOULD BE AN H IN THERE.. MARGHARITAS? MARGAHRITAS?

I don't get this...







$c = \frac{\varepsilon_u}{\varepsilon_u + f_y/E_s} d = \frac{\rho_b f_y d}{\beta_1 (0.85)f_c'}$

$a = \frac{A_{sb} f_y}{0.85 f_c' b}$

ρ_b = reinforcement ratio = $\frac{A_s}{\text{total}A} \rightarrow bd$, where d = depth to steel

A_s = actual area of steel

amount of steel in comparison to concrete

Appendix 1A has the properties of steel beams (ex. #8 $A = 0.79 in^2$, $d = 1 in$)

Method: solve for $\rho = A_s/bd$

solve for $\rho_b = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$ etc

if $\rho < \rho_b$, beam is underreinforced

find M_n, knowing $f_s = f_y$; $C = T$ $a = \frac{A_s f_y}{0.85 f_c' b}$ ~~$f_c' = 4$ ksi (assume)~~ $f_y = 60$ ksi (assume)

$\frac{1}{\beta_1} a$ = depth to neutral axis $\beta_1 = 0.85$

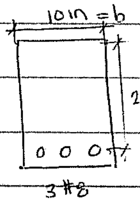
C. HOVELL

T.G.I.F.

February 6, 2004

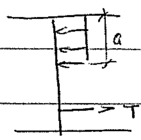
I AM SO HAPPY IT'S FINALLY WEEKEND. NOT LIKE TODAY WON'T BE DEATHLY LONG, BUT...

Another example



$A_s = 2.37\ in^2$

stress:



$C = 0.85 f_c' ab = a/\beta_1$

$\beta_1 = 0.85$ for 4000 psi (f_c')

find M_n

$f_c' = 4$ ksi

$f_y = 60$ ksi

Assume: $f_s = f_y$ $C = T$

$0.85 f_c' ab = A_s f_s$

$a = 4.18$ in depth of stress block

NOT depth to neutral axis

⇒ M_n = nominal moment capacity of the section

take them along the ~~a~~ line of action of C or T

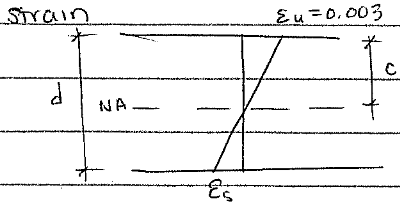
$M_n = A_s F_s (d - a/2)$ OR $M_n = \underbrace{(0.85 f_c' a b)}_{= C}(d - a/2)$

(around C) (around T)

in this ex. $M_n = \overset{\uparrow \text{ no decimal}}{29.70}$ in·kip

⇒ aim for underreinforcement, not balance

Strain



with more stress, NA moves ↑

solve using similar triangles $\frac{\varepsilon_u}{c} = \frac{\varepsilon_s}{d}$ $\varepsilon_{y\,steel} = 0.00207$

if $\varepsilon_s > \varepsilon_y$, $f_s = f_y$

thus, underreinforced - yay!

too little steel:

$A_s = \frac{3\sqrt{f_c'}}{f_y} bd$ or $200bd / f_y = A_s$

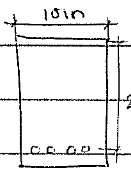
C. Hovell

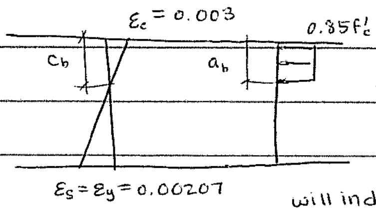
<u>Being Sick = So sucky</u> February 9, 2004

What As is required for balanced failure?

3 #8 → underreinforced

strain: stress:





← $C_b = 0.85 f'_c a_b b$

→ $T = A_{s_b} f_y$

will induce balanced failure

using similar triangles: $\frac{\varepsilon_s}{d-\cancel{a_b}\,c_b} = \frac{\varepsilon_c}{\cancel{a_b}\,c_b}$ $c_b = 13.61$ in $a_b = \beta_1 c$ $\beta_1 = 0.85$

$\underline{\underline{a_b = 11.57\text{ in}}}$ $\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 40}{1000}\right.$

so, $A_{s_b} = {}^{C_b}/f_y$

$A_{s_b} = 6.56\text{ in}^2$

M_n = nominal moment capacity = $C(d - a/2) = 6776$ in·kip

ways to check for $f_s = f_y$:

1. Geometry (strain distribution)

   is $\varepsilon_s < \varepsilon_y$?

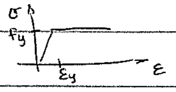
2. compare ρ_{actual} to ρ_b (Table A5)
3. compare ratio of depth of stress block, a, to effective depth, d, to balanced ratio

   a/d vs. a_b/d (copied and on toolkit)

Strength Design

Assumptions: > maximum usable strain in concrete = 0.003

> steel is elastic (linear) then perfectly plastic



> tensile strength of concrete is neglected

> concrete compression zone is rectangular, with a width = $0.85f'_c$ and height = $a = \beta_1 c$

> β_1 is a function of concrete strength

Advantages: ° non-linear behavior of concrete is included

° realistic load factors are used (1.2 DL + 1.6 LL) DL = dead load, LL = live load

(table 1.2 in book) ⇒ $\boxed{M_u \le \phi M_n}$ ϕ = resistance factor < 1 ~0.9

° steel is used more effectively – worked til steel yield

° smaller sections are usualy possible

WHY MUST HE ALWAYS START SOMETHING NEW WITH 5 MINUTES LEFT? I HATE IT AND DON'T LISTEN...

Margaritas Tonight?

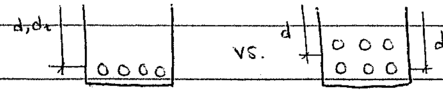
February 11, 2004

ACI Code on underreinforcement of beams

1. minimal tensile reinforcement strain when loaded al the nominal moment
2. Strength reduction factors, $\phi < 1$, based on ε_t (strain in furthest tensile steel)

at a depth d_t = at the reinforcement furthest from the compression face

$d = d_t$ if there is only one row ~~one~~ of steel



$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c}$$

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

For axial load less than 10% of $f_c' A_g$ (gross area of section) $A_g = b \cdot h$ for rectangle

$\varepsilon_t \geq 0.004$ ($\varepsilon_s = 0.00207$, so steel has yielded) (forget steel)

[$\varepsilon_s = 0.00259$ for grade 75 steel $- f_y = 75$ ksi]

thus, underreinforced

Thus... $\boxed{\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}}$ with low axial load

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

tension-controlled member – one that, at nominal load, $\varepsilon_t \geq 0.005$

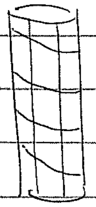
$\phi = 0.9$

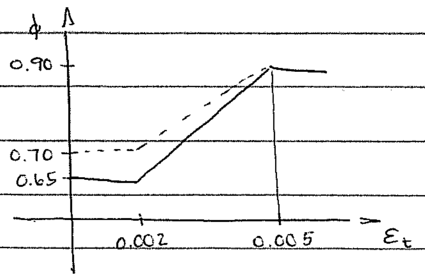
compression controlled member – any member where, at nominal load, $\varepsilon_t < 0.002$

steel has not reached yield. $\phi = 0.65$ (much more conservative!)

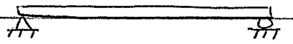
between $\varepsilon_t = 0.002$ and $\varepsilon_t = 0.005$, ϕ varies linearly from $\phi = 0.65$ to $\phi = 0.9$

If spirally reinforced, $\phi = 0.70$





So, example:

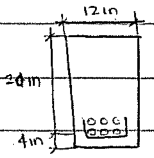


$w_{DL} = 2$ k/ft $f_y = 40$ ksi

$w_{LL} = 3$ k/ft $f_c' = 4$ ksi

$w_u = 1.2(w_{DL}) + 1.6(w_{LL}) = 7.2$ k/ft

$M_u = \frac{w_u l^2}{8} = 325$ ft·kip

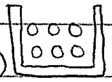


d = 20 in

Assume 2 rows

Need: 1.5 in cover

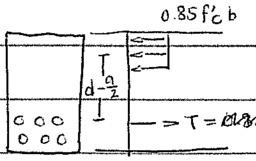
0.5 in stirrup (#4)



use #10 ($\overset{d}{\text{[illegible]}} = 1.27 \text{ in}^2$) $d \sim \#/8$

0.5 in spacing between bars

depth from bottom ~ 3.77 in = 4 in



$T = \cancel{0.85f_c'}\, A_s f_y$

$M_u \leq \phi M_n$

Now calc. from $T = A_s f_y = 0.85 f_c' \cdot b \cdot a$ $A_s = 6.44 \text{ in}^2$

$M_u = \phi \left[0.85 f_c' \cdot b \cdot a \left(d - a/2 \right) \right]$

USE TABLE 8.2

C. Hovell

FRIDAY THE THIRTEENTH! February 13, 2004

On homework #2, prob 3:

$\bar{y} = 5.71\text{ in}$ $I = 8000\text{ in}^4$ $f_s = 22.9\text{ ksi} < f_y$ $f_{cT} = 1.28\text{ ksi} < f_c'$

on homework #3, Prob 3.4:

find nominal flexural strength (M_n), use to find design strength (ϕM_n)

use ε_t (in steel) to find ϕ $\varepsilon_t = \varepsilon_u \left(\frac{d_t - c}{c} \right)$

Back to the problem from Wednesday...

avoid using bars larger than #10s

check limits — As must be greater than value calculated

$\rho = \frac{A_s}{bd}$ $A_{s\,min} = \frac{3\sqrt{f_c'}}{f_y} bd$ $A_{s\,min} = 200 \frac{bd}{f_y}$

must be in psi

$$\left[\rho_{max} = 0.75 \rho_b \quad \rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \right) \right]$$

— OLD CODE —

$\boxed{M_u \le \phi M_n}$

Now: $\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$ ← steel MUST have yielded by this point

try mixing bars... how about 3 #10s, 3 #9s?

check As directly, go back to beginning and change assumptions

find ϕ... $\varepsilon_t = \varepsilon_u \left(\frac{d_t - c}{c} \right)$ d_t = distance from ~~b~~ top to most tensile steel bar (center of it)

0.005 $c = a/0.85 = 7.42\text{ in}$

→ if $\varepsilon_t > 0.005$, beam is underreinforced, $\phi = 0.9$

a has changed... $a = \frac{A_s f_y}{0.85 f_c' b}$ Now, $a = 6.68\text{ in}$, $c = 7.85\text{ in}$ (calculate first, before ϕ)

$M_n = (0.85 f_c' a b)(d - a/2)$ or $A_s f_y (d - a/2)$ $M_n = 378\text{ ft}\cdot\text{kip}$ $M_u = 340\text{ ft}\cdot\text{kip}$

which is larger than given M_u, so we're good!

CODE

10.4 — Distance between lateral supports of flexural members

10.4.1 — Spacing of lateral supports for a beam shall not exceed 50 times the least width b of compression flange or face.

10.4.2 — Effects of lateral eccentricity of load shall be taken into account in determining spacing of lateral supports.

10.5 — Minimum reinforcement of flexural members

10.5.1 — At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4, the area A_s provided shall not be less than that given by

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \qquad (10\text{-}3)$$

and not less than $200 b_w d / f_y$

10.5.2 — For statically determinate members with a flange in tension, the area $A_{s,min}$ shall be equal to or greater than the value given by Eq. (10-3) with b_w replaced by either $2b_w$ or the width of the flange, whichever is smaller.

10.5.3 — The requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis.

10.5.4 — For structural slabs and footings of uniform thickness the minimum area of tensile reinforcement in the direction of the span shall be the same as that

COMMENTARY

R10.4 — Distance between lateral supports of flexural members

Tests[10.10,10.11] have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that causes torsion.

Laterally unbraced beams are frequently loaded off center (lateral eccentricity) or with slight inclination. Stresses and deformations set up by such loading become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports spaced closer than $50b$ may be required by loading conditions.

R10.5 — Minimum reinforcement of flexural members

The provision for a minimum amount of reinforcement applies to flexural members, which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tensile reinforcement, the computed moment strength as a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. Failure in such a case can be sudden.

To prevent such a failure, a minimum amount of tensile reinforcement is required by 10.5.1 in both positive and negative moment regions. When concrete strength higher than about 5000 psi is used, the $200/f_y$ value previously prescribed may not be sufficient. Equation (10-3) gives the same amount of reinforcement as $200 b_{wd}/f_y$ when f_c' equals 4440 psi. When the flange of a section is in tension, the amount of tensile reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression. A higher amount of minimum tensile reinforcement is particularly necessary in cantilevers and other statically determinate members where there is no possibility for redistribution of moments.

R10.5.3 — The minimum reinforcement required by Eq. (10-3) is to be provided wherever reinforcement is needed, except where such reinforcement is at least one-third greater than that required by analysis. This exception provides sufficient additional reinforcement in large members where the amount required by 10.5.1 or 10.5.2 would be excessive.

R10.5.4 — The minimum reinforcement required for slabs should be equal to the same amount as that required by 7.12 for shrinkage and temperature reinforcement.

C.Hovell

Qdoba For Lunch!

February 16, 2004

Singly-Reinforced Beam Design

but, $\boxed{\text{we don't know b or h!}}$ design for a given moment M_u

① $\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$ ← because if $\varepsilon_t > 0.005$, beam is underreinforced

$M_u = \phi M_n$

② $M_u = \phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$ solve for bd^2 → choose appropriate values for b and d $d \sim 1.5b$

③ use $A_s = \rho d b$ to find $A_{s\,required}$

④ determine h by considering d, cover, stirrups, and half the bar diameter

round h up to the next highest even number of inches → ex. $h = 16.8 \to h = 18$ in.

cover = 1.5" stirrup = 0.5"

$\frac{1}{2}$ bar = depends on A_s, bars, etc.

⑤ revise A_s to match the new depth d

check moment and ϕ

moment ϕM_n must be greater than M_u – then ok!

→ calculate A_s, find a combination of bars that makes that A_s

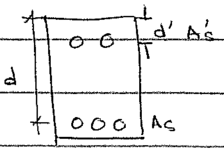
err on the side of too little, not too much.

Doubly Reinforced Beam Design

not enough concrete is present to cause steel to yield

beam cross-section is limited so concrete cannot develop adequate force to balance (yield) the ste

(among other reasons)



1. if ~~poρρb~~ $\rho < \rho_b$, the compression steel may be disregarded
2. if $\rho > \rho_b$, the compression steel must be considered

Case 1: both steels yield

Case 2: tensile steel yields, compression steel does not

C.Hovell

My Bologna Has A First Name... February 18, 2004

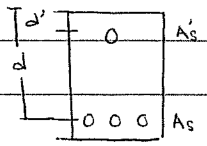
THREE MORE HOURS OF WORK (ER, CLASS) AND THEN I MUST GO HOME AND WRITE THIS PAPER.

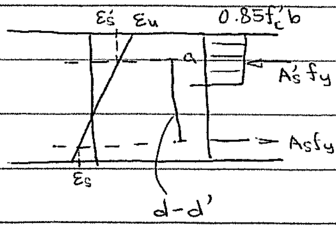
Double-reinforced beams

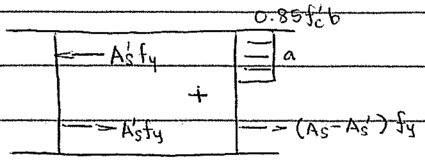
Case 1: both steels yield (tensile and compression)

simpler to analyze

theoretically, f_y for compression and tension are equivalent







$M_{n_1} = A_s' f_y (d - d')$ (compression steel) $M_{n_2} = (A_s - A_s') f_y (d - a/2)$ (concrete)

$$\boxed{a = \frac{(A_s - A_s') f_y}{0.85 f_c' \cdot b}} \quad A_s = \rho b d \quad A_s' = \rho' b d \quad \boxed{a = \frac{(\rho - \rho') f_y d}{0.85 f_c'}}$$

$$\boxed{M_n = A_s' f_y (d - d') + (A_s - A_s') f_y (d - a/2)}$$

accounts for both steel sections and the concrete

$$\boxed{\begin{array}{l} \bar{\rho}_b = \rho_b + \rho' \\ \bar{\rho}_{max} = \rho_{max} + \rho' \end{array}}$$

ρ_b = balanced ratio for single-reinforced beam

why use double-reinforced beams?

> space restrictions

> increases ductility of tensile steel

> reduces sustained load deflections (creep)

steel does not creep, whereas concrete does

C.Hovell

TEST MONDAY. February 20, 2004

1/2 closed book, theory; 1/2 open book, problems

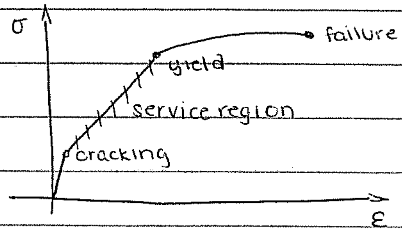
does not include double-reinforced beams

when using equivalent stress block, beam is at yield point

strain is linear, stress is not

up to cracking, use all of concrete (n-1)

after cracking, in service load region, neglect concrete (n)

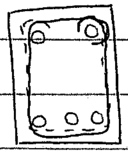


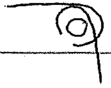
More details of double-reinforced beams: (ACI Code Req)

- 7.11.1 enclose compression steel with ties

(restrain lateral motion)

prevents against buckling

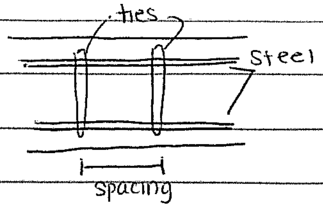




stirrup comes across top, too

spacing: ① ~~16 d_b (top)~~
② ~~48 tied d_b~~
③ ~~least of b or h~~

find least conservative (smallest)

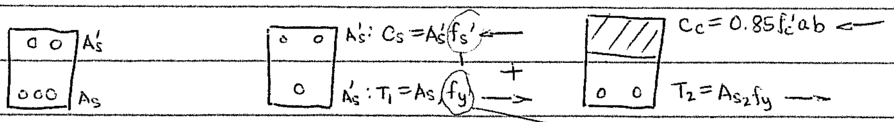


size: 1. steel ≤ #10, use #3 tie

2. steel > #10, use #4 tie (normal #3, 4 steel, just bent)

compression steel

Beam Details...



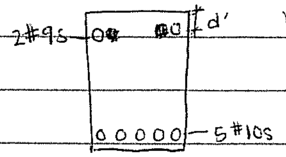
$A_{s1} = A_s'$; $A_{s2} = A_s - A_s'$

CASE II: tensile steel yields, compression steel does not necessarily

so, $f_s' \neq f_y$ is used

$$A_s' f_s' + 0.85 f_c' ab = A_s' f_y + (A_s - A_s') f_y$$

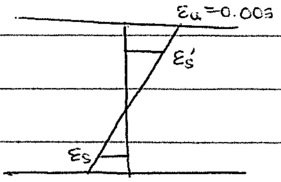
Example of case I:



b = 14 in, d = 20 in

d' = 2.5 in

$f_y = 50$ ksi, $f_c' = 3$ ksi



Assume A_s' yields ($f_s' = f_y$)

(assume same grade steel)

$A_{s1} f_y = A_s' f_y$ $A_{s1} = A_s'$

so, to balance A_s', $A_{s1} = 2$ in^2, $A_{s2} = 4.3$

$$a = \frac{A_{s2} f_y}{0.85 f_c' b} = 6.06 \text{ in}, \; c = 7.13 \text{ in}$$

solve for ε_s' using similar triangles

$\varepsilon_s' = 0.00195$ ... ε_y(50 grade) = 0.0017, so, assumption is correct. ✓

$$M_u = M_n \phi = \phi \left[A_{s2} f_y (d - a/2) + A_s' f_y (d - d') \right] = 4880 \text{ kipin}$$

A_{smin}? f_y in tensile?

C. Hovell

OK... I'm Still Alive...

February 25, 2004

Double-Reinforced Beams: Case II

Tensile steel yields, compression steel does not

$\bar{\rho}_b = \rho_b + \rho' \left(\frac{f_s'}{f_y}\right)$ fore case I, $f_s' = f_y$, equation simplifies

$$\boxed{f_s' = E_s \varepsilon_s' = E_s \left[\varepsilon_u - d'/d \left(\varepsilon_u + \varepsilon_y\right)\right] \leq f_y}$$

(uses similar triangles!)

$\bar{\rho}_{max} = \rho_{max} + \rho' \left(\frac{f_s'}{f_y}\right)$ $f_s' = E_s \left[\varepsilon_u - \frac{d'}{d} \left(\varepsilon_u + 0.004\right)\right] \leq f_y$

(for ρ_{max}, $\varepsilon_y = 0.004$)

reinforcement ratio necessary to cause compressive steel to yield (minimum value)

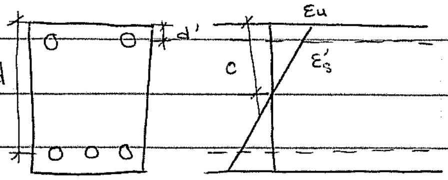
$$\boxed{\rho_{cy} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} + \rho'}$$

$[\rho > \rho_{cy}$, then yield occurs$]$

use to check Case I/II $\rho' = A_s'/bd$

at ultimate concrete strain (0.003), relate stress in compression reinforcement to section properties (using similar triangles)

$f_s' = \varepsilon_u E_s \left(\frac{c-d'}{c}\right)$



balancing internal forces, T=C at failure

$T = C_c + C_s$

$$\boxed{A_s f_y = 0.85 f_c' \beta_1 b c + A_s' \varepsilon_u E_s \left(\frac{\cancel{d}c-d'}{c}\right)}$$

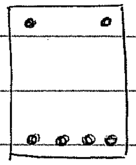
$\varepsilon_s' = \varepsilon_u \frac{c-d'}{c}$

solve for c; $a = c\beta_1$

Nominal moment:

$$\boxed{M_n = 0.85 f_c' a b \left(d - a/2\right) + A_s' f_s' \left(d - d'\right)}$$

Example



$d' = 2.5$ in $d = 24$ in

$b = 14$ in

$f_c' = 4$ ksi, $f_y = 50$ ksi

$A_s = 5.08 \text{ in}^2$ $A_s' = 1.20 \text{ in}^2$

$A_{s_1} = 1.20 \text{ in}^2$, $A_{s_2} = 3.88 \text{ in}^2$
(steel) (concrete)

Finding the balance between steel, concrete; moment, etc.

1. Assume A_s' yields
2. calculate $a = \frac{A_{s_2} f_y}{0.85 f_c' b}$ for concrete, $c = a/\beta_1$
3. find $\varepsilon_s' = \varepsilon_u \frac{c-d'}{c}$, compare to $\varepsilon_y = \frac{f_y}{E_s} = 0.0017$ (check original assumption)
4. If not yielded, use $T = C_c + C_s$ equation, solve for c.
5. calculate ε_s' again, with new c.
6. calculate f_s', which is less than f_y
7. $A_s' f_s' = A_{s_1} f_y$ what is A_{s_1}? calculate $A_{s_2} = A_s - A_{s_1}$

$\underline{M_u} = \phi \left[A_{s_2} f_y \left(d - a/2\right) + A_s' f_s' \left(d - d'\right)\right]$ capacity of section — is load below this?

I Hate Being Late   February 27, 2004

So, what was the point of that example?

analyze a double-reinforced beam

- given section, material properties

1. determine if compression reinforcement is needed

let $\rho = \rho_{max} = 0.85\beta_1 f'_c / f_y \frac{\varepsilon_u}{\varepsilon_u + 0.004}$ (table A.4)

calculate M_n as if single-reinforced, $A_s = \rho_{max} bd$

compare to M_u: if too small, more steel is needed

$\hookrightarrow \phi = 0.82$

2. decide if compression steel will yield

calculate ρ_{cy}, greater than ρ? steel does not yield

Case I: compression steel yields

$A_{s_1} = A'_s$, $A_{s_2} = A_s - A_{s_1}$ (notes 2/20)

3a. $M_n = A'_s f_y (d - d') + (A_s - A_s') f_y (d - a/2)$

Case II: steel does not yield (notes 2/25)

3b. $A_{s_2} f_y = 0.85\beta_1 f'_c bc + A'_s \varepsilon_u E_s \left(\frac{c-d'}{c}\right)$

solve for c, a

4. $M_n = 0.85 f'_c ab (d - a/2) + A'_s f'_s (d - d')$

$f'_s = \varepsilon_u E_s \left(\frac{c-d'}{c}\right)$

IS M_n (of M_u) LARGE ENOUGH TO SUPPORT GIVEN LOADS (MOMENTS)? (DING! POINT!)

4./5. check strain to find ϕ $\varepsilon_t = \varepsilon_u \left(\frac{d-c}{c}\right)$

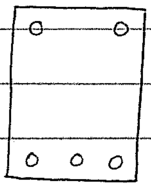
$\phi M_n \overset{\geq}{\underset{<}{=}} M_u$? is beam acceptable?

$\phi M_n \geq M_u$ for successful beam

Design of Double-Reinforced Beams

1. Is compression steel needed?
2. Has compression steel yielded?
3. How much compression steel is needed?

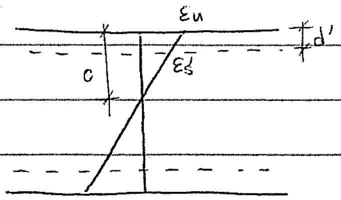
★ generally, double-reinforcement is used when size is constrained, single can't hold load



given h, b

$d' \sim 2.5\text{-}3in$

$d \sim h - (2.5\text{-}3in)$



assume d is smaller, not lar

given moments... $M_T = 1.2(M_{DL*}) + 1.6(M_{LL})$

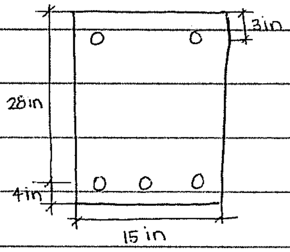
1. use ρ_{max} to find M_n with single-reinforcement. Is this enough? [$f'_c = 4ksi$, $f_y = 60ksi$, $\rho_{max} = 0.0206$

C.Hovell

I Want to Go Back to Bed...

March 1, 2004

Design of Double-Reinforced Beams (cont'd)



d' ~~given~~, d assumed (4in from bottom)

h, b given

$f_c' = 4$ ksi

$f_y = 60$ ksi

$> \rho_{max} = 0.0206$

1. use ρ_{max} to find M_n with single reinforcement. is compression steel needed?

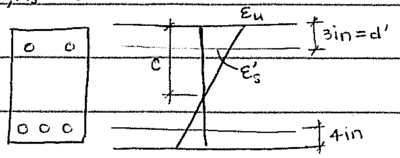
$\rho_{max} \longrightarrow A_{s\,max}$, calculate $a \longrightarrow M_n$. use $\phi = 0.82$ to compare to M_u

if $\phi M_n < M_u$, single-reinforcement is not enough.

2. use ρ_{cy} to calculate if compression steel has yielded

$\rho > \rho_{cy}$ for yield

OR, since A_s, A_s' are unknown...



$$\varepsilon_s' = \varepsilon_u \left(\frac{c - d'}{c} \right)$$

$$M_n = M_u/\phi = A_{s1} f_y (d - d') + A_{s2} f_y (d - a/2)$$

~~assume $A_{s2} = A_{smax} - A_s'$, solve for A_{s1}~~

make $A_{s2} = A_{max}$, use $a = \#$ determined using $A_{s\,max}$

solve for A_{s1}, $A_s = A_{s1} + A_{s2}$

3. using $A_{s1} = A_s'$, A_s, find steel bars which match

$A_s' = 1.24\text{ in}^2$, $A_s = 9.89\text{ in}^2$

2 #8s   8 #10s $M_u = 940$ ft·kip

Double Reinforced Beam Design (cont'd)

c) How much A_s' is needed?

$$M_{n_1} = {}^{M_u}\!/\!_{\phi} - M_{n_2} = {}^{940}\!/\!_{0.9} - 813 = 231 \text{ ft}\cdot\text{kips}$$

Since $M_{n_2} = A_s' f_y (d-d')$

$$A_s' = \frac{M_{n2}}{f_y(d-d')} = \frac{231 \text{ ft}\cdot\text{kips} \, (12 \text{ in/ft})}{(60 \text{ ksi})(28\text{in} - 3\text{in})} = 1.85 \text{ in}^2 \Rightarrow \begin{matrix} \text{Try } 3\#7 = 1.80 \text{ in}^2 \\ 2\#9 = 2.00 \text{ in}^2 \checkmark \end{matrix}$$

$$A_s' f_s' = A_{s_1} f_y \Rightarrow A_{s_1} = 2.00 \text{ in}^2$$

$$M_u = \phi A_{s_1} f_y (d-d') + \phi A_{s_2} f_y (d - {}^{a}\!/\!_{2})$$

$$940 \text{ ft}\cdot\text{kips} (12 \text{ in/ft}) = 0.9(2.00\text{in}^2)(60\text{ksi})(28\text{in} - 3\text{in}) + 0.9(A_{s_2})(60 \text{ksi})(28\text{in} - {}^{10.18\text{in}}\!/\!_{2})$$

$$11280 \text{ in}\cdot\text{kips} = 2700 + 1237 A_{s_2}$$

$$A_{s_2} = 6.94 \text{ in}^2 \qquad \therefore A_s = A_{s_1} + A_{s_2} = 8.94 \text{ in}^2$$

Choose: $\boxed{A_s' \Rightarrow 2 \#9\text{'s} = 2.00 \text{in}^2}$

$\boxed{A_s \Rightarrow 4\#10 + 4\#9 \text{ in 2 rows} = 9.08 \text{ in}^2}$ $\quad A_{s_2} = 7.08 \text{ in}^2$

$$d = 32\text{in} - \overset{\text{cover}}{1.5\text{in}} - \overset{\text{stirrup}}{0.375\text{in}} - \overset{\#10}{1.27\text{in}} - \overset{\frac{1}{2}\text{ space}}{1.27/2} = 28.22 \text{ in} \Rightarrow \text{stick w/ } d = 28 \text{ in.}$$

$$\varepsilon_t = \varepsilon_u\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{29.49\text{in} - 10.68}{10.68}\right) \quad ; \text{where} \quad a = \frac{A_s f_y}{0.85 f_c' b} = \frac{7.08\text{in}^2(60\text{ksi})}{0.85(4\text{ksi})(15\text{in})}$$

$$= .00603 > 0.005 \quad \therefore \phi = 0.90 \qquad a = 8.32 \text{in} \quad c = {}^{8.32}\!/\!_{.85} = 9.80 \text{in}$$

$$M_n = 2.00\text{in}(60\text{ksi})(28\text{in} - 3\text{in}) + (7.08\text{in}^2)(60\text{ksi})(28\text{in} - {}^{8.32\text{in}}\!/\!_{2})$$

$$= 13127 \text{ in}\cdot\text{kips} = 1093 \text{ ft}\cdot\text{kips}$$

$$\phi M_n = 0.9(1093 \text{ ft}\cdot\text{kips}) = \boxed{985 \text{ ft}\cdot\text{kips}} > M_u = 940 \text{ ft}\cdot\text{kips} \checkmark \text{OK}$$

d) check $A_{s,min}$

1) $\frac{3\sqrt{4000}}{60000}(15)(28) = 1.3 \text{ in}^2 \checkmark$

2) $\frac{200}{60000}(15)(28) = 1.4 \text{ in}^2 \checkmark$

e) Tie Design: Size #3 for #10 bars

Spacing:
1) $16(1.27\text{in}) = 20 \text{ in}$
2) $48(0.375\text{in}) = 18 \text{ in}$
3) $b = \underline{15 \text{ in}}$

Choose $\boxed{15 \text{ in o.c.}}$ most conservative

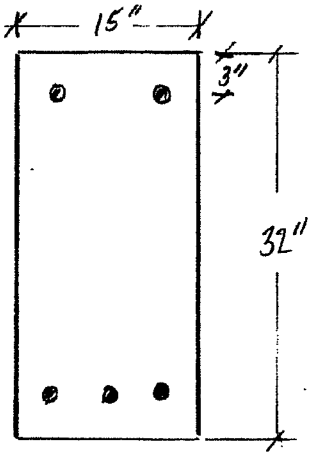
Example

Design of Double Reinforced Beam

$f'_c = 4000$ psi $f_y = 60{,}000$ psi

$M_D = 250^{k}$ (includes self-weight)

$M_L = 400^{k}$



a) Is compression steel needed?

b) Has A'_s yielded?

c) How much A'_s?

Since 2 layers will likely be needed, use d=28" to start

a) $M_u = 1.2(250) + 1.6(400) = 940$ ft·kips

$\rho_{max} = 0.85\beta_1\left(\frac{f'_c}{f_y}\right)\left(\frac{\varepsilon_u}{\varepsilon_u + 0.004}\right) = 0.0206$ (or see Table A.4)

$A_{s_1} = (0.0206)(15)(28) = 8.65\ in^2$

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{8.65(60)}{0.85(4)(15)} = \underline{10.18\ in}$ $c = 10.18/.85 = \underline{12.0\ in}$

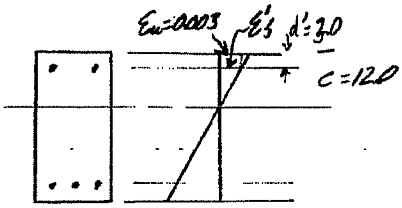
$M_n = A_s f_y (d - a/2) = 8.65(60)(28 - \frac{10.18}{2}) = 11890$ in·kips $= 991$ ft·kips

for $\varepsilon_t = 0.004$ @ $\rho_{max} \Rightarrow \phi = 0.82$

$\phi M_n = 0.82(991\ ft\cdot kips) = 813$ ft·kips $\ngtr M_u = 940$ ft·kips

∴ Compression reinforcement is needed

b) $a = 10.18\ in$ $c = 12.0\ in$



$\varepsilon'_s = \varepsilon_u\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{12-3}{12}\right) = 0.00225$

$\varepsilon'_s = 0.00225 > \varepsilon_y = \frac{60000}{29000000} = 0.00207$

∴ A'_s yields

C.Hovell

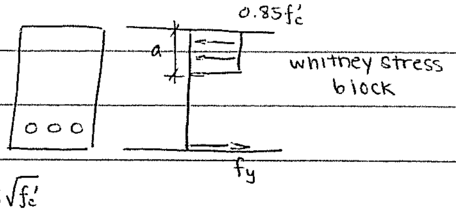
Crazy - A Sub!

March 15, 2004

Jose Gomez - take reinforced concrete, steel I, concrete II

Serviceability

at M_n, state of stress in concrete



f_r = modulus of rupture $= 7.5\sqrt{f'_c}$

= tensile strength of concrete

> behavior of the structure under service loads; behavior is quantified by deflection analysi

SERVICEABILITY

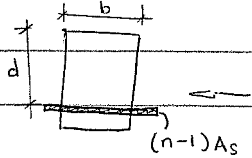
(pre)(non)-load factored loads

deflections

$\boxed{E_{conc} = 57000\sqrt{f'_c}} \sim 3.6 \times 10^6$ psi for $f'_c = 4$ ksi

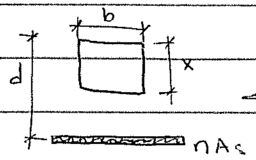
Steel → concrete to calculate I

n = modular ratio $= E_s/E_c$ (stress in steel = 8x that in concrete)



← uncracked transformed section

$f_t < f_r$



← cracked section

x = plastic neutral axis

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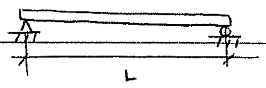
A Finite Project Idea! March 17, 2004

Deflection control

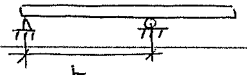
table of minimum thicknesses for non-prestressed beams

1.



$d = L/16$

2.

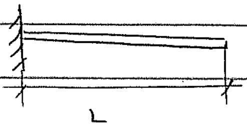


$d = L/18.5$

3.

$d = L/21$

4.



$d = L/8$

dead load sometimes causes the largest deflections on beam

I changes as concrete cracks

$$I_{effective} = \left(\frac{M_{crack}}{M_{apply}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

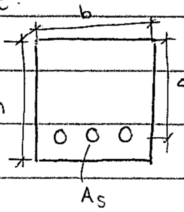
$$M_{cr} \simeq f_r I_g / y_t$$

I_g = gross moment of inertia = $1/12\, bh^3$

M_a = applied moment (from service loads)

I_{cr} = cracked moment of inertia

Example:



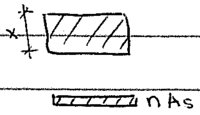
$M_{DL} = 80$ ft·kip
$M_{LL} = 95$ ft·kip

SERVICE loads – no safety factors

short term / immediate deflection

$$\delta = \frac{5wL^4}{384EI}$$

I_{cr}:



Find X:

$$b \cdot x \cdot x/2 = A_s \cdot n \cdot (d - x)$$

$$I_{cr} = 1/3 \cdot b \cdot x^3 + A_s \cdot n \cdot (d - x)^2$$

try two cases for I_e:

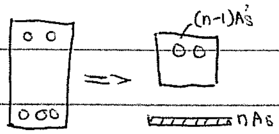
1. $M_a = M_{DL}$ $\qquad \delta_{DL} = \dfrac{5 w_{DL} L^4}{384 E (I_e)_{DL}}$

2. $M_a = M_{DL} + M_{LL}$ $\qquad \delta_{DL} + \delta_{LL} \cdots$ $\qquad \delta_{LL} = \delta_{DL+LL} - \delta_{DL}$

ACI Table 9.5(b): max permissible computed deflections

still to go: creep and shrinkage

Moment of inertia calculation, for a cracked, double-reinforced beam



$$I_g = 1/12\, bh^3$$

$$(b)(x)(x/2) + (n-1)A_s'(x - d') = (n)(A_s)(d - x)$$

SOLVE FOR X

$$\boxed{I_{cr} = 1/3\, bx^3 + (n-1)A_s'(x - d')^2 + (n)A_s(d - x)^2}$$

BIG TIME INTERVIEW TODAY! I'M A LITTLE BIT ON THE WORRIED SIDE OF LIFE. WISH ME LUCK!

C.Hovell

A Sub! Again!

March 19, 2004

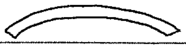
Long Term concrete Deflection

Material behavior

- shrinkage: 90% occurs within the first year

  happens as concrete loses water

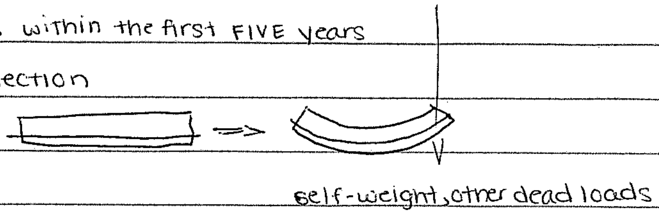
  rebar helps prevent against shrinking - can even cause opposite flexure of concrete (cracks on top)



- creep: continuous deflection (strain) due to sustained loads

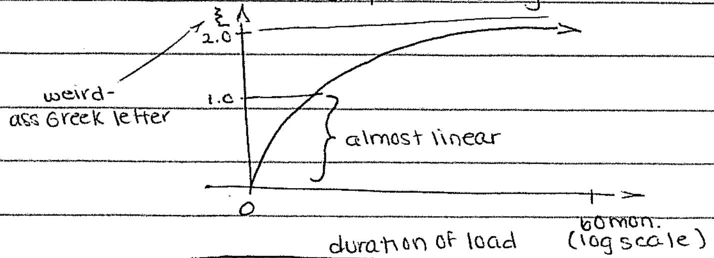
  90% occurs within the first FIVE years

  added deflection



highly non-linear problem

need a method for simple estimating



$$\boxed{\lambda = \frac{\xi}{1+50\rho'}}$$

compression steel ratio -

ξ = 5 yrs or more = 2.0

at 12 mon = 1.4

at 6 mon. = 1.2

at 3 mon = 1.0

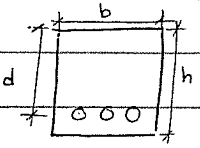
Steel used to counter deflections, not hold load

$\delta_{creep/shrinkage} = \boxed{= \lambda(\delta_{DL}) + \delta_{LL}}$

long term behavior:

function of water/cement (ε w/c) ratio, loads (dead), span length, curing reinforcing steel (tensile and compressive)

Going back to Wednesday's example



$\delta_{DL} = 0.57$ in, $\delta_{LL} = 0.75$ in

$\delta_{C/SH} = 0.57$ in (2.0) → given

$+ \delta_{LL}$ < limited to deflections (given. ex = L/480

If it fails: - compressive steel, larger h

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A second Test soon

March 22, 2004

Shear Design - chapt. 4.1-4.5

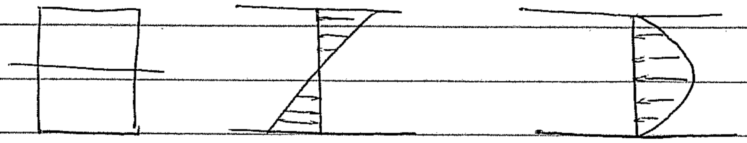
(more properly termed diagonal tension)

NOT FULLY UNDERSTOOD - equations are not theoretical~~ly~~, but empirical

→ or "shear collapse" - sudden failure with little or no warning signs - BAD.

design shear reinforcement to ensure flexural failure occurs first

shear design / analysis actually addresses diagonal tension stresses that result from a combination of shear stress and longitudinal flexure stress



bending stress

shear stress

$v = \frac{VQ}{Ib}$

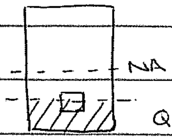
V = total shear at given xsec

Q = statical moment about N of portion of xsection betw line through a point in quest parallel to NA and the nearest horizontal face

I = moment of inertia about NA

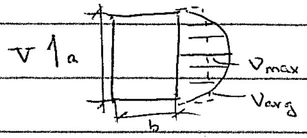
b = beam width

ex of Q:



Q = I ▨ about NA

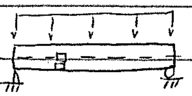
For pure shear:



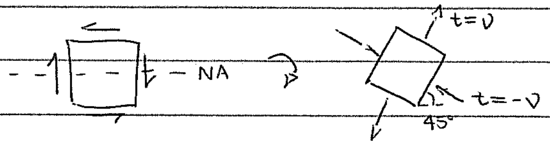
$v_{avg} = \frac{V}{ab}$

$v_{max} = 3/2\, v_{avg}$

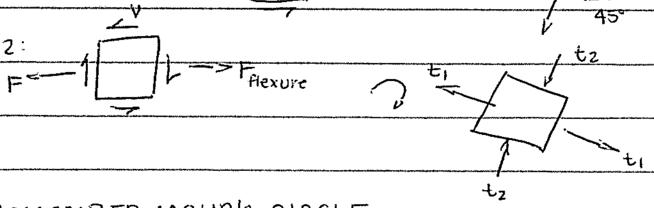
Example



Case 1:



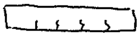
Case 2:



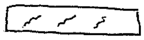
$$\boxed{\begin{aligned} t_{12} &= \frac{F}{2} \pm \sqrt{\frac{F^2}{4} + v^2} \\ \alpha &= \tan^{-1}\left(\frac{2v}{F}\right)\frac{1}{2} \end{aligned}}$$

REMEMBER MOHR'S CIRCLE.

Flexural cracking: near-vertical



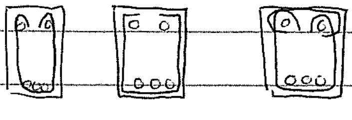
Shear cracking: start it center, grow out



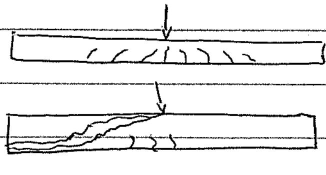
combination:

without shear reinforcement, this can result in large scale failure

Stirrups - many many orientations to resist shear



Failure patterns:



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I HATE SNIFFLERS!

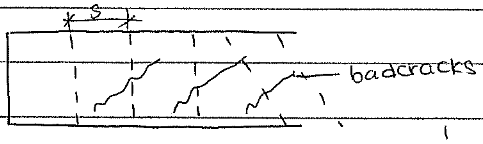
March 22, 2004

Failure Patterns

shear failure more likely the smaller the span-depth (L/d) ratio

failure often occurs when steel and concrete separate

stirrups:



stirrups can be vertical ¦ or on an angle \

Designing for shear failure

For design purposes, $V_n = V_c + V_s$ V_c = concrete shear capacity

↑ V_s = steel shear capacity

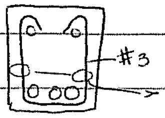
nominal shear capacity

$V_u \le \phi V_n$

For vertical stirrups $\boxed{V_u \le \phi V_c + \frac{\phi A_v f_y d}{s}}$ s = web reinforcement spacing

A_v = cross-sectional area of web reinforcement

f_y = yield stress; may or may not be the same as f_y for horizontal steel bars!



$A_v = 2(A_{\#3})$

stirrup is there twice

n = # of stirrups traversing a crack = $\frac{d}{s}$

Shear Code Reqs

11.1.3.2 — For prestressed members, sections located less than a distance $h/2$ from face of support shall be permitted to be designed for the same shear V_u as that computed at a distance $h/2$.

11.1.4 — For deep beams, brackets and corbels, walls, and slabs and footings, the special provisions of 11.8 through 11.12 shall apply.

11.2 — Lightweight concrete

11.2.1 — Provisions for shear and torsion strength apply to normalweight concrete. When lightweight aggregate concrete is used, one of the following modifications shall apply to $\sqrt{f_c'}$ throughout Chapter 11, except 11.5.4.3, 11.5.6.9, 11.6.3.1, 11.12.3.2, and 11.12.4.8

11.2.1.1 — When f_{ct} is specified and concrete is proportioned in accordance with 5.2, $f_{ct}/6.7$ shall be substituted for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f_c'}$.

11.2.1.2 — When f_{ct} is not specified, all values of $\sqrt{f_c'}$ shall be multiplied by 0.75 for all-lightweight concrete and 0.85 for sand-lightweight concrete. Linear interpolation shall be permitted when partial sand replacement is used.

11.3 — Shear strength provided by concrete for nonprestressed members

11.3.1 — Shear strength V_c shall be computed by provisions of 11.3.1.1 through 11.3.1.3, unless a more detailed calculation is made in accordance with 11.3.2.

11.3.1.1 — For members subject to shear and flexure only,

$$V_c = 2\sqrt{f_c'}\,b_w d \qquad (11\text{-}3)$$

11.3.1.2 — For members subject to axial compression,

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f_c'}\,b_w d \qquad (11\text{-}4)$$

Quantity N_u/A_g shall be expressed in psi.

11.3.1.3 — For members subject to significant axial tension, shear reinforcement shall be designed to carry total shear unless a more detailed analysis is made using 11.3.2.3.

11.3.2 — Shear strength V_c shall be permitted to be computed by the more detailed calculation of 11.3.2.1 through 11.3.2.3.

11.3.2.1 — For members subject to shear and flexure only,

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right)b_w d \qquad (11\text{-}5)$$

but not greater than $3.5\sqrt{f_c'}\,b_w d$. Quantity $V_u d/M_u$ shall not be taken greater than 1.0 in computing V_c by Eq. (11-5), where M_u is factored moment occurring simultaneously with V_u at section considered.

11.3.2.2 — For members subject to axial compression, it shall be permitted to compute V_c using Eq. (11-5) with M_m substituted for M_u and $V_u d/M_u$ not then limited to 1.0, where

$$M_m = M_u - N_u\frac{(4h-d)}{8} \qquad (11\text{-}6)$$

However, V_c shall not be taken greater than

$$V_c = 3.5\sqrt{f_c'}\,b_w d\sqrt{1 + \frac{N_u}{500A_g}} \qquad (11\text{-}7)$$

Quantity N_u/A_g shall be expressed in psi. When M_m as computed by Eq. (11-6) is negative, V_c shall be computed by Eq. (11-7).

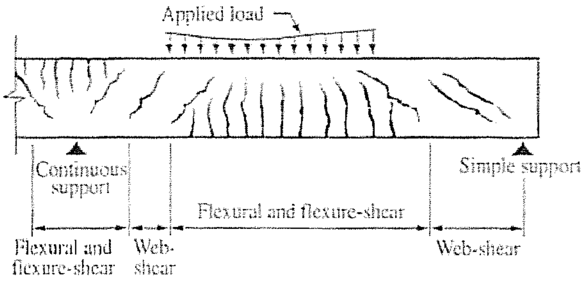


Fig. R11.4.2—Types of cracking in concrete beams

Skipping Design… March 24,2004

Shear design

$V_n = V_c + V_s$ $V_u \leq \phi V_n$ $\boxed{\phi = 0.75}$ ALWAYS.

for vertical stirrups:

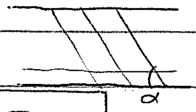
$$\boxed{V_u \leq \phi V_c + \frac{\phi A_v f_y d}{s}}$$

for inclined stirrups:

$$\boxed{V_u \leq \phi V_c + \frac{\phi A_v f_y d}{s}\left[\sin(\alpha) + \cos(\alpha)\right]}$$

α = angle of stirrup from the horizontal plane

$$\boxed{V_c = 2\sqrt{f'_c}\, b_w d}$$



very conservative, but accepted

OR: $\boxed{V_c = 1.9\sqrt{f'_c} + 2500\frac{\rho_w V_u d}{M_u} b_w d \leq 3.5\sqrt{f'_c}\, b_w d}$

$\sqrt{f'_c} \leq 100\text{psi}$ for plain concrete, unless minimum web reinforcement is used

$\rho_w = A_s / b_w d$

$V_u d / M_u \leq 1.0$, else use 1.0

slightly more precise than simpler equation

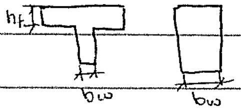
Minimum web reinforcement

even if $V_u \leq V_c$, web reinforcement is still needed by the code, but not to hold load

$$\boxed{A_{v_{min}} = 0.75\sqrt{f'_c}\, b_w s \frac{1}{f_y} \geq 50 b_w s / f_y}$$

A_v = total cross-sectional area within a distance s

(area of one stirrup)



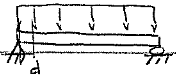
exceptions:

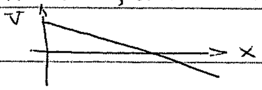
- $V_u \leq \frac{1}{2} V_c \cdot \phi$ minimum need not be met
- footings and slabs (width >> depth)
- concrete floor joist construction
- beams with a total depth, h < 10 in or h < $2.5h_f$ (flange depth)

  or $h < \frac{1}{2} b_w$ —> slab

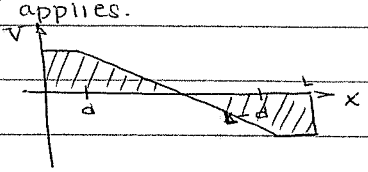
Critical section

For non-prestressed members (11.1.3.1): critical section is taken at a distance d from the face of the support. At x < d, assume the shear at d applies.





consider it as this:



d = effective depth

if load is hanging, not resting on top, critical length = 0!

Maximum spacing: • vertical (non-prestressed)

C. Hovell

Mom Comes Today – Car Tomorrow! March 31, 2004

I SHOULD PAY ATTENTION, SINCE WE HAVE A TEST MONDAY. I DON'T THINK I WILL, THOUGH.

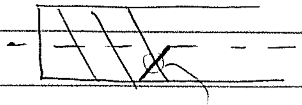
Maximum spacing:

$V_s \le 4\sqrt{f'_c}\, b_w d$, $s \le \frac{d}{2}$ or 24 in (smaller value)

$V_s > 4\sqrt{f'_c}\, b_w d$, $s \le \frac{d}{4}$ or 12 in

$V_s > 8\sqrt{f'_c}\, b_w d$, section must be modified / ~~eng~~ enlarged

For incline d stirrups : every 45° line that can be drawn from middepth toward reaction, a distance $d/2$ shall be crossed by at least 1 line of shear reinforcement



~~[illegible]~~ $V_n = V_c + V_s$ n=nominal, c=concrete, s=steel

V_u is max value on shear diagram ~~[illegible]~~

Web reinforcement failure modes

1. Stirrups yielding – desired failure

$$\boxed{V_s = A_v f_y d / s}$$

2. Stirrup pulls out of anchorage

to avoid: Stirrups must extend as close to the tension / compression faces as the cover requirements allow (~1.5 in)

Types of hooks and anchorages are specified in ACI 318 section 12.3 and 7.1

Shear Design

given: f'_c, b_w, d, f_y; V_u determined from loading (use critical section)

1. determine the concrete shear capacity, V_c

$V_c = 2\sqrt{f'_c}\, b_w d$

2. check maximum allowable shear steel $V_s = V_u/\phi - V_c \underline{<} 8\sqrt{f'_c}\, b_w d$, else redesign section

[is $V_s \overset{?}{} 4\sqrt{f'_c}\, b_w d$? s limited by d, given values (above)]

3. is shear steel even needed? $V_u > \phi V_c$? ~~[illegible]~~

a. NO.

calculate $A_{v\,min} = 0.75\sqrt{f'_c}\, b_w s \frac{1}{f_y}$ or $A_{v\,min} = 50 b_w \frac{s}{f_y}$

choose larger minimum

$s \le d/2 \le 24$ in

guess for A_v, calculate s

b. YES. $A_v = \frac{(V_u/\phi - V_c)s}{f_y d}$ $s \le d/2$

If $V_u/\phi - V_c > 4\sqrt{f'_c}\, b_w d$, use $s \le d/4$

[SHEAR / MOMENT CALCS : USE FACTORS "service load" does not.]

OKAY. MUST BAIL FROM ASCE CONFERENCE. TOO MUCH TO DO. SIGH.

Shear Design Example

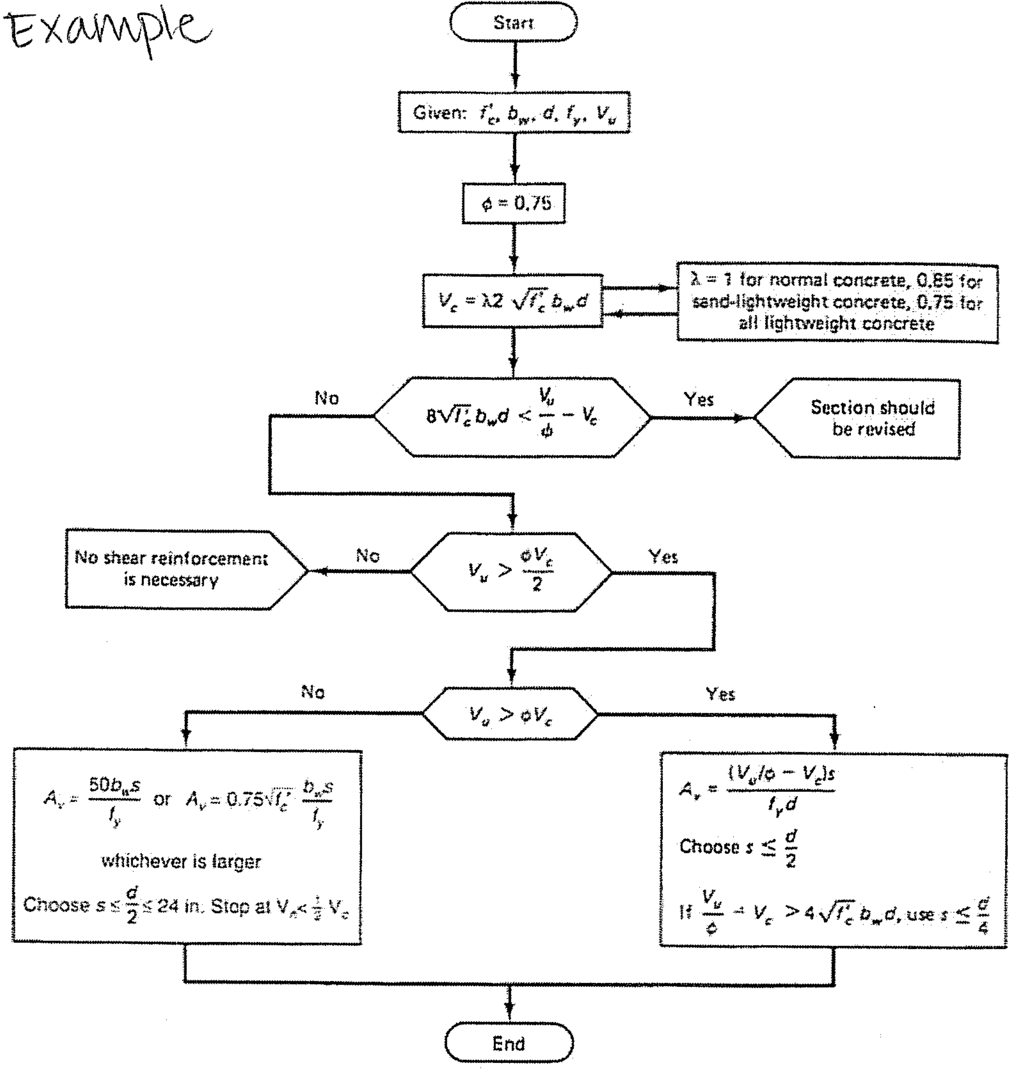


Figure 6.9 Flowchart for web reinforcement design procedure.

EXAMPLES OF THE DESIGN OF WEB STEEL FOR SHEAR

6.8.1 Example 6.1: Design of Web Stirrups

A rectangular isolated beam has an effective span of 25 ft (7.62 m) and carries a working live load of 7500 lb per linear foot (110 kN/m) and no external dead load except its self-weight. Design the necessary shear reinforcement. Use the simplified term of Eq. 6.9 for calculating the capacity V_c of the plain concrete web. Given:

$f'_c = 4000$ psi (27.6 MPa), normal-weight concrete
$f_y = 60{,}000$ psi (414 MPa)
$b_w = 14$ in. (356 mm)
$d = 28$ in. (712 mm)
$h = 30$ in. (762 mm)
longitudinal tension steel: six No. 9 bars (diameter 28.6 mm)
no axial force acts on the beam

Solution:

Factored shear force (Step 1)

$$\text{beam self-weight} = \frac{14 \times 30}{144} \times 150 = 438\ \text{lb/ft}$$

$$\text{total factored load} = 1.2 \times 438 + 1.6 \times 7500 = 12{,}526\ \text{lb/ft}$$

The factored shear force at the face of the support is

$$V_u = \frac{25}{2} \times 12{,}526 = 156{,}575\ \text{lb}$$

The first critical section is at a distance $d = 28$ in. from the face of the support of this beam (half-span = 150 in.).

$$V_u \text{ at } d = \frac{150 - 28}{150} \times 156{,}575 = 127{,}348\ \text{lb}$$

Shear capacity (Step 2)

The shear capacity of the plain concrete in the web from the simplified equation for normal-weight concrete ($\lambda = 1.0$) is

$\sqrt{f_c'}\, b_w d = 24792$ lb

$$V_c = 2.0\lambda\sqrt{f_c'}b_w d = 2 \times 1.0\sqrt{4000} \times 14 \times 28 = 49{,}585\ \text{lb}$$

Check for adequacy of section for shear:

$$(8 + 2.0)\sqrt{f_c'}b_w d = 10\sqrt{f_c'}b_w d = 247{,}923\ \text{lb}$$

$$\text{required } V_n = \frac{V_u}{\phi} = \frac{127{,}348}{\cancel{0.85}\ 0.75} = 169{,}797\ \text{lb} \quad \text{cross-section O.K.}$$

$$V_u > \frac{1}{2}V_c \quad \text{hence stirrups are necessary}$$

Shear reinforcement (Steps 3 to 5)

Try No. 4 two-legged stirrups (area per leg = 0.20 in.^{2}).

$$A_v = 2 \times 0.2 = 0.40\ \text{in.}^2$$

From Eq. 6.15b,

$$s = \frac{A_v f_y d}{(V_u/\phi) - V_c} = \frac{0.4 \times 60{,}000 \times 28}{169{,}797 - 49{,}584}$$

$$= 5.6\ \text{in. (142 mm)}$$

Since $V_n - V_c > 4\sqrt{f_c'}b_w d$, the maximum allowable spacing $s = d/4 = 28/4 = 7$ in. At the critical section, $d = 28$ in. from the face of the support, the maximum allowable spacing would in this case be 5.6 in.

The shear force for distributed load decreases linearly from the support to midspan of the beam. Hence the web reinforcement can be reduced accordingly after determining the zone where minimum reinforcement is necessary and the zone where no web reinforcement is needed. The same size and spacing of stirrups needed at the critical section d from face of support should be continued to the support. Figure 6.10 illustrates the various values being calculated:

Critical plane x_d (consider the midspan as the origin): $V_n = 169{,}\cancel{993}\,797$ lb and from before, $s = 5.6$ in. x_d from the midspan point $= 150 - 28 = 122$ in.

Plane x_1 *at* $s = d/4$ *maximum spacing:*

$$V_{s1} = 4\sqrt{f_c'}b_w d = 4\sqrt{4000} \times 14 \times 28 = 99{,}169\ \text{lb}$$

$$V_{n1} = 99{,}169 + 49{,}585 = 148{,}754\ \text{lb}$$

$$x_1 \text{ from midspan point} = (150 - 28) \times \frac{148{,}754}{169{,}797} = 106.9\ \text{in.}$$

Plane x_2 at $s = d/2$ maximum spacing:

$$s = \frac{A_v f_y d}{V_u - V_c} \quad \text{or} \quad \frac{28}{2} = \frac{0.4 \times 60{,}000 \times 28}{V_s}$$

or

$$V_{s2} = 48{,}000 \text{ lb}$$

$$V_{u2} = 48{,}000 + 49{,}585 = 97{,}585 \text{ lb}$$

$$x_2 \text{ from midspan point} = 122 \times \frac{97{,}585}{169{,}797} = 70.1 \text{ in.}$$

From Figure 6.10a, the distance 36.73 in. is the transition zone from $s = 7$ in. to $s = 14$ in.; hence a stirrup spacing of 8 in. center to center is shown in Figure 6.10b.

Plane x_3 at shear force V_c:

$$V_c = 2\sqrt{f'_c}\, b_w d = 49{,}585 \text{ lb}$$

$$x_3 \text{ from midspan point} = 122 \times \frac{49{,}585}{169{,}797} = 35.6 \text{ in.}$$

Discontinue the stirrups at plane where $V_u \leq \frac{1}{2} V_c$.

Minimum web steel: Test when $V_u > \frac{1}{2}\phi V_c$ or $V_u > \frac{1}{2} V_c$

$$V_u = 127.248$$

$$\frac{1}{2} V_c = \frac{1}{2} \times 49{,}585 = 24{,}793 \text{ lb}$$

$$\text{minimum } A_v = \frac{50 b_w s}{f_y} = \frac{50 \times 14 \times 14}{60{,}000} = 0.16 \text{ in.}^2$$

$$< \text{actual } A_v = 0.40 \text{ in.}^2 \qquad \text{O.K.}$$

or

$$\text{maximum allowed } s = \frac{A_v f_y}{50 b_w} = \frac{0.40 \times 60{,}000}{50 \times 14} = 34.3 \text{ in.}$$

$$\text{versus maximum used } s = \frac{d}{2} = 14 \text{ in.} \qquad \text{O.K.}$$

$$x_4 = 122.0 \times \frac{24{,}793}{169{,}797} = 17.8 \text{ in. from midspan}$$

Proportion the spacing of the vertical stirrups accordingly.

The shaded area in Figure 6.10a is the shear force area for which stirrups must be provided. The spacing of the stirrups in Figure 6.10b is based on the practical consideration of the desirability of using whole spacing dimensions and varying the spacing as little as possible.

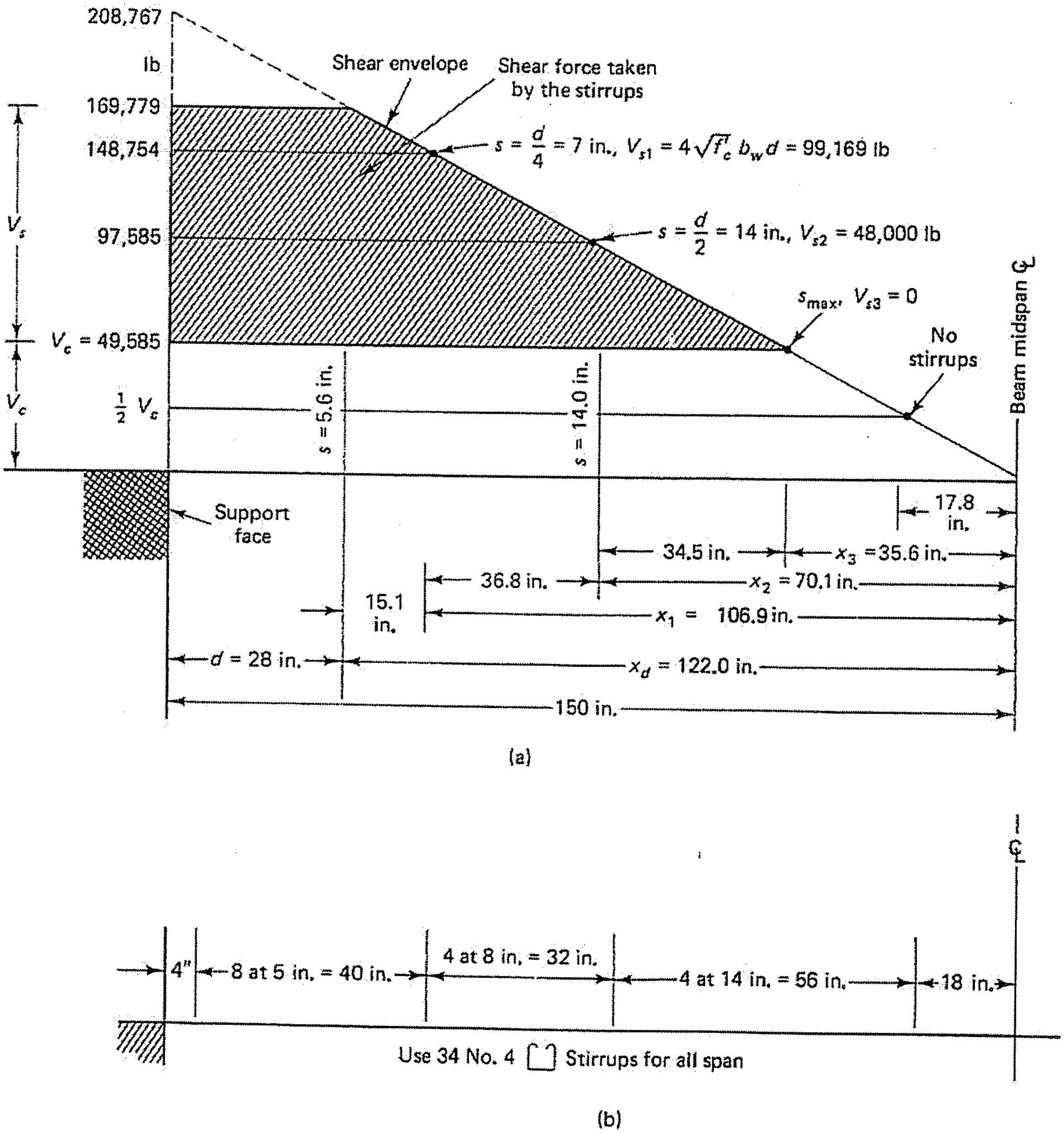


Figure 6.10 Stirrups arrangements for Ex. 6.1: (a) shear envelope and stirrup design segments; (b) vertical stirrups spacing.

C.Hovell

So So Tired

April 5, 2004

Columns

1. pedestals (ACI 2.1 and 10.15)

   $\ell / h \leq 3$, where h is the smallest lateral dimension

☆ 2. short columns (reinforced)

   failure at ultimate loading is caused by actual failure of materials (crushing, yielding)

3. long columns (reinforced)

   failure occurs from weak or strong-axis buckling

secondary moment $= P\Delta$

Types of columns:

A. tied columns – ties hold the longitudinal reinforcement together during construction, and serve to prevent bars from buckling under load

- can be square, rectangular, round, even octagonal

B. Spiral ~~tied~~ columns – spiral reinforcement holds longitudinal bars in place during construction and increase the resistance of concrete inside the spirals to axial compression by resisting expansion of concrete laterally

- spiral steel must yield prior to failure of the column

DUCTILE FAILURE!

Tie and Spiral Design

Axial load only

—> in practice, this does not exist, but we'll pretend

—> tests have shown that

- no matter which material (concrete or steel) approaches failure first, the other quickly "catches up" – stain must be compatible
- ultimate strength of an axially loaded column is:

$$\boxed{P_n = 0.85 f'_c (A_g - A_s) + f_y A_s}$$

$A_g = bd$ (gross area of concrete)

Failure Modes

large tie spacings: near failure, concrete flakes off near ties, longitudinal bars buckle, brittle (bad) failure occurs

small tie spacings: fails more like a concrete cylinder (crumbles)

spiral columns: near failure, concrete spalls around spiral (if pitch is small enough); but, column will continue to resist load through the confined concrete

C.Hovell

Damn – Stop Already!

April 5,2004

Tie and Spiral Design

Ties – ACI 318-7.10.5

for ≤#10 ~~longit~~ longitudinal bar, use #3 ties

>#10 (or bundled bars), use #4s

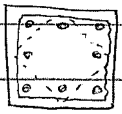
Spacing ≤ 16 (diameter of longitudinal bar)

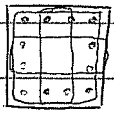
48 (tie bar diameter)

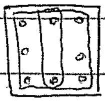
smallest lateral dimension

} Choose most conservative – smallest

- every corner bar must be tied
- every other longitudinal bar must be tied
- no longitudinal bars can be farther than 6" from a tied bar ~~for a circular bar pattern~~
- for a circular bar pattern, one circular tie is allowed







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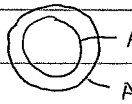
32-HOURS TO LEAVING!

April 7, 2004

Spiral design



A_c = area of inner core, $d = d_c$ (location of steel spiral)

A_g = gross area, including cover

shell strength = $0.85 f_c' (A_g - A_c)$

spiral strength = $2 \rho_s A_c f_y$ ρ_s = % spiral steel

confining effect of spiral (pg 256-257)

we want them to be equal

$$\rho_s = 0.425 \frac{(A_g - A_c) f_c'}{A_c f_y} \quad \text{exactly}$$

code says:

$$\boxed{\rho_s = 0.45 \left(\frac{A_s}{A_c} - 1 \right) \frac{f_c'}{f_y}}$$

$$\rho_s = \frac{\text{volume of spiral steel in one loop}}{\text{volume of concrete thru one loop}}$$

$$\boxed{\rho_s = \frac{A_{sp} \pi (d_c - d_b)}{\left(\frac{\pi d_c^2}{4} \right) s}}$$

OR:

$$\rho_s = \frac{4 A_{sp} (d_c - d_b)}{s d_c^2}$$

A_{sp} = area of spiral

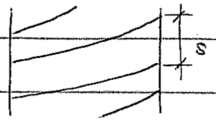
d_c = diameter of concrete core

d_b = diameter of spiral

s = pitch

limitations:

- clear spacing > 1 in, < 3 in



- 10.9.1

  minimum and maximum longitudinal reinforcement (A_{st})

  $\rho > 1\%$ to ensure ductile performance

  $\rho < 8\%$ to prevent overcrowding

  $\rho < 4\%$ if lap splices are used

- 10.9.2 – minimum # of longitudinal bars

  rectangular / square or circular ties = 4

  triangular ties = 3

  spirals = 6  MINIMUM.

- 9.3.2.2 (2002)

  $\phi = 0.65$ for a tied column, $\phi = 0.70$ for a spiral column

- 10.3.5 – design formula

spiral: $\phi P_{n_{max}} = \phi (0.85) \left[(0.85) f_c' (A_g - A_{st}) + f_y A_{st} \right]$

C.Hovell

CROSSWORD BREAKTHROUGH!

April 7, 2004

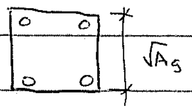
Design example

given $P_{LL}, P_{DL}, f_y, f'_c, \rho_s$

1. calculate P_u
2. use P_u to find equation for A_g, A_{st}

$$P_u = \phi P_n = \phi(0.80)\left[(0.85)f'_c(A_g - A_{st}) + f_y A_{st}\right]$$

3. $\rho_s = {}^{A_{st}}/A_g$
4. solve for A_g
5.



6. go back to equation in 2, solve for A_{st} using modified A_g (rounded)
7. find steel that matches that area

A_{st} = minimum, so find A above that!

8. ties - bars < #10, use #3

spacing is smallest of: minimum lateral dimension

$16(d_{long.bar})$

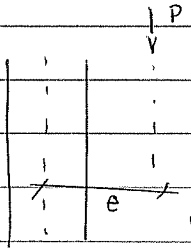
$48(d_{stirrup})$

C.Hovell

RAIN = SUCKY

April 12, 2004

Axial load and bending of columns



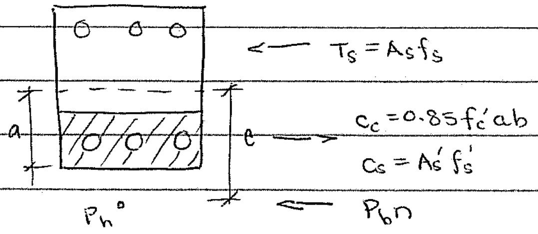
bending on column

previous formulas for axial load apply for small moments

$e \le 0.1\,h$ tied columns

$e \le 0.05\,h$ spiral columns

moments cause compression on one side, tension on the other



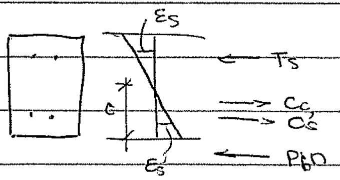
load cases:

1. pure axial load – crushing failure of concrete while bars yield in compression
2. large axial load, small moment – similar failure
3. large axial load, large moment – bars on tension face in tension ($< f_y$) and column fails when concrete crushes in compression
4. balanced loading – simultaneous crushing of concrete and yielding of both tension and compression bars
5. small axial load, large moment – reinforcement yields but failure is in crushing
6. no axial load, pure moment – beam behavior
7. negative axial load (tension), small moment – fails by steel rupture due to tension

Problem Solving

Case 1: $e = 0,\ M_n = 0$ $\quad P_n = 0.85 f_c' (A_g - A_{ST})$

Case 4:



calc $\varepsilon_s = \varepsilon_y = {f_y}/{E}$

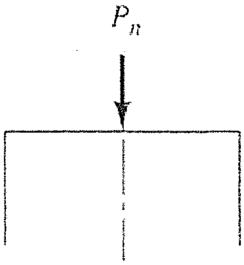
use similar triangles to calculate c, a

calculate ε_s', is it yielded? f_s' ? f_y ?

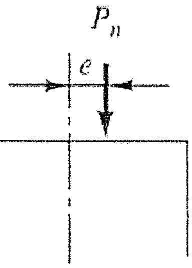
balancing forces: $P_{bn} = C_s' - T_s + C_c$

moments about centroid: $P_{bn} e_b + P_{bn}\left(\frac{h}{2} - \frac{a}{2}\right) + A_s f_s \left(\frac{h}{2}\cancel{a} - d\right)$

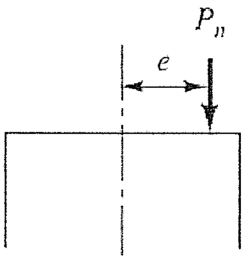
$+ A_s' f_s' \left(\frac{h}{2} - d'\right)$



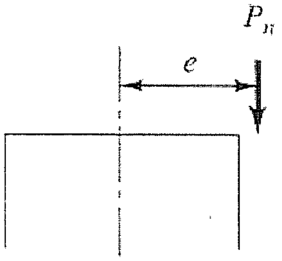
(a) Large axial load causes a crushing failure of the concrete with all bars reaching their yield points in compression.



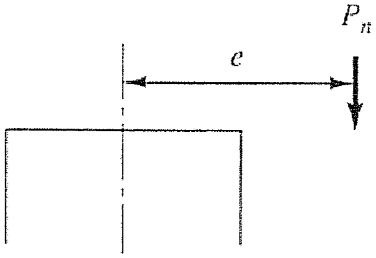
(b) Large axial load and small moment but entire cross section in compression. Failure occurs by crushing of the concrete, all bars in compression.



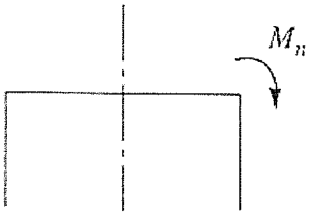
(c) Large axial load, moment larger than in (b). Bars on far side in tension but have not yielded. Failure occurs by crushing of the concrete



(d) Balanced loading condition—bars on tensile side yield at same time concrete on compression side crushes at $0.85 f'_c$.



(e) Large moment, relatively small axial load—failure initiated by yielding of tensile bars.



(f) Large bending moment—failure occurs as in a beam.

Figure 9.1 Column subject to load with larger and larger eccentricities.

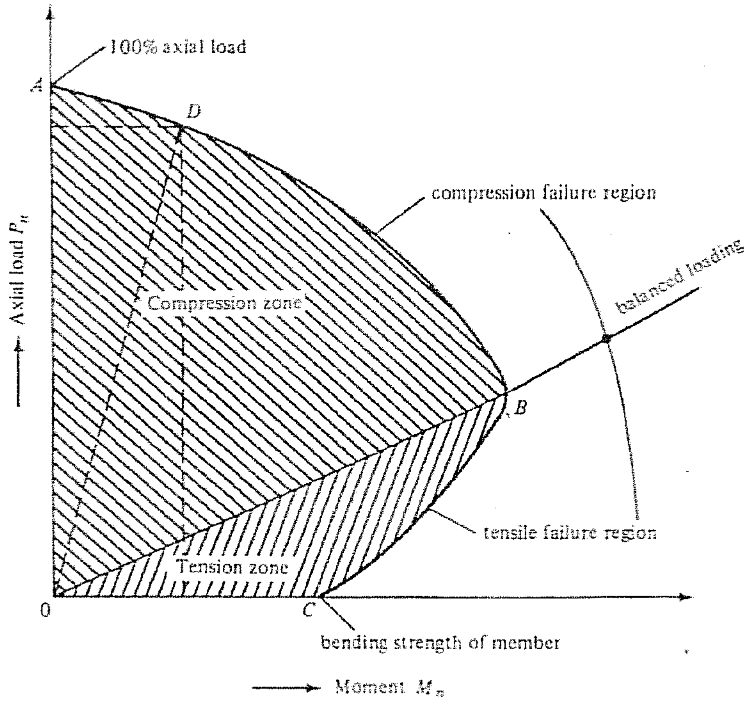


Figure 9.10 Column interaction diagram.

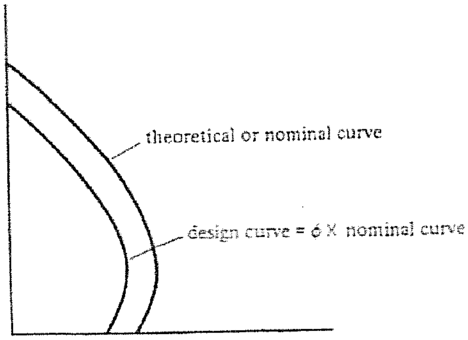


Figure 9.12

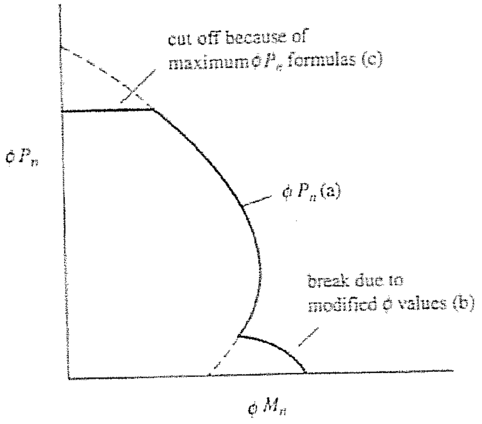
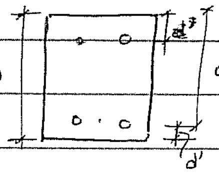


Figure 9.14 Shape of column design interaction curve.

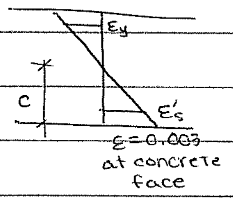
C. Hovell

ONE MORE CLASS TODAY (AFTER THIS) April 14, 2004

More Problem Solving



Case 6: no axial load, pure moment - acts like a beam

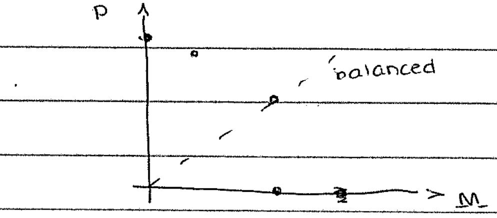


tensile steel at yield

assume $f_s' < f_y$

$C_c + C_s = T$

interaction diagram:



$0.85 f_c' a b + A_s' \left(\frac{c-d'}{c} \right) \varepsilon_c E_s = A_s f_y$ solve for C

check assumption, calc. $\varepsilon_s' < \varepsilon_y$? calc. $f_s' = \varepsilon_s' E_s$

calc. compressive steel $A_s' f_s' = A_{s_1} f_y$ $A_{s_1} = ?$ balances stee

$A_{s_2} = A_s - A_{s_1}$ = tensile steel balancing concrete

calc. $M_n = A_{s_2} f_y \left(d - a/2 \right) + A_s' f_s' \left(d - d' \right)$

stress block changes depth,

by using a = h to $a = a_b$, a

curve can be formed between pts.

Case 2: large axial, small moment

assume a = h - small number, calc. c

ε_y(50 ksi steel) = 0.00172

calc. ε_s', strain in compression steel, $= \varepsilon_c \left(\frac{c-d'}{c} \right)$

$\varepsilon_s = \varepsilon_c \left(\frac{d'}{c} \right)$, $f_s = \varepsilon_s E$

ΣM at tensile steel = 0 = $- P_n \left(h/2 - d' + e \right) + C_c \left(d - a/2 \right) + C_s \left(d - d' \right)$

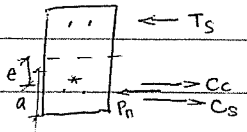
assume value for e, calculate P_n → $0.1h = e$

$\Sigma F = 0$: $C_s + C_c - P_n - T_s = 0$, check a

$A_s' f_y + 0.85 f_c' a b - P_n + A_s f_s$

Case 3: large axial, large moment

assume: e = value between case 2, e_b



assume a, calc. c

calc. ε_s', ε_s, f_s, f_s' (if necessary)

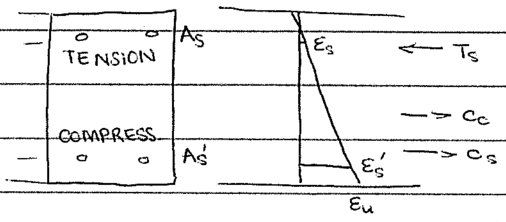
sum moments, forces (check a assumption)

C. Hovell

BUSY FRIDAY!

April 16, 2004

case 2, revisited: large axial, small moment



choose $c = h$, calculate a

calculate $\varepsilon_s', \varepsilon_s =$ $f_s E$

$\Sigma F = 0$, $C_c + C_s - T_s = P_n$

$A_s' f_s' + 0.85 f_c' ab - A_s f_s = P_n$

KEY EQS: $C_c = 0.85 f_c' ab$

$C_s = A_s' f_s'$

$T_s = A_s f_s$

$\Sigma M = 0$ $P_n e = M_n = C_c \frac{1}{2}(h-a) + C_s (d - \frac{h}{2}) - T_s(\frac{h}{2} - d')$

calculate M_n and then e

OR, $P_n(d - h/2 + e) = C_c(d - a/2) + C_s(d - d')$

case 5: small axial load, large moment

same procedure as for case 2, but choose $c < c_b$

case 7: pure tension

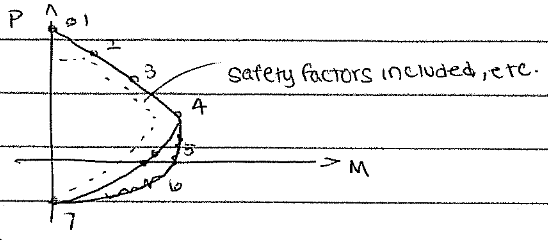
what's the total tensile ~~steel~~ strength of the steel? (concrete cracks)

$-P_{n_t} = (A_s' + A_s) f_y$, and that's it. $M_n = 0$

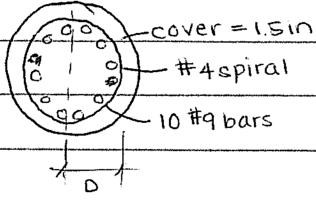
unless there is very little steel and concrete does not rupture before steel yields

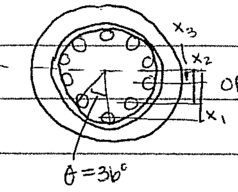
$f_r = 474$ psi

Total curve



Round columns

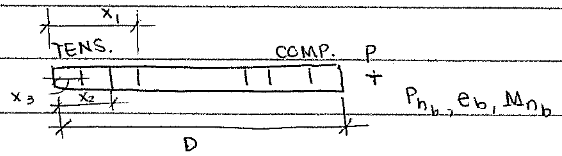




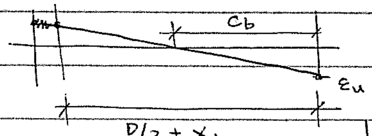
offsets: $x_1 = \frac{1}{2}(D - 2(cov) - 2(tie) - d_{st})$

$x_2 = x_1 \cos\theta$

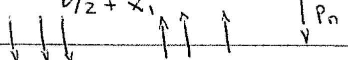
$x_3 = x_1 \sin\frac{\theta}{2}$



P_{n_b}, e_b, M_{n_b}



$$\frac{c_b}{\varepsilon_u} = \frac{x_1 + D/2}{\varepsilon_u + \varepsilon_{sy}}$$



T_1 is at yield, none of the others are

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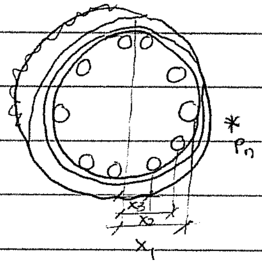
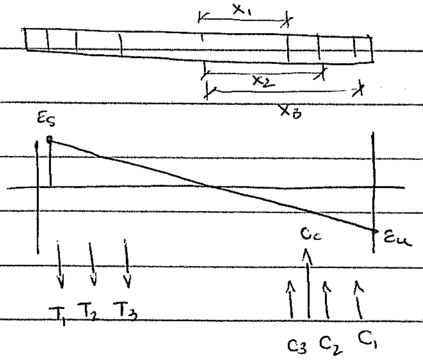
Seven Days of Class Left...

April 19, 2004

Round columns (cont'd)





$$\frac{c_b}{\varepsilon_u} = \frac{x_1 + D/2}{\varepsilon_u + \varepsilon_y}$$

T_1: $\varepsilon_{T_1} = \varepsilon_y$; $f_{s_{T_1}} = f_y$ $\quad T_1 = A_{s_{T_1}} f_y$

T_2: ~~$\frac{x_2}{x_3}\varepsilon_y = \varepsilon_{T_2}$~~ $\frac{(D/2 + x_1) - c - (x_1 - x_2)}{D/2 + x_1 - c}\varepsilon_y = \varepsilon_{T_2}$ $\quad T_2 = A_{s_{T_2}} f_{s_{T_2}}$

T_3: $\frac{(D/2 + x_1) - c - (x_1 - x_3)}{D/2 + x_1 - c}\varepsilon_y = \varepsilon_{T_3}$

C_1: $\varepsilon_u \frac{c - (D/2 - x_1)}{c} = \varepsilon_{c_1}$ $\quad$ and then calculate stresses

C_2: $\frac{c - (D/2 - x_2)}{c}\varepsilon_u = \varepsilon_{c_2}$

C_3: $\frac{c - (D/2 - x_3)}{c}\varepsilon_u = \varepsilon_{c_3}$

area of concrete:

$$A_c = h^2\left(\frac{\theta - \sin\theta\cos\theta}{4}\right)$$ θ in radians!

$$A\bar{y} = h^3\left(\frac{\sin^3\theta}{12}\right)$$

$$\theta = \cos^{-1}\left(\frac{h/2 - a}{h/2}\right)$$ $h = D$

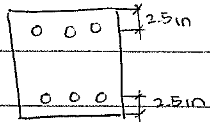
$c_c = 0.85 f'_c A_c$

$\boxed{P_b = \Sigma C_s + C_c - \Sigma T_s}$ $= C_1 + C_2 + C_3 + C_c - T_1 - T_2 - T_3$ for this situation

$$M_b = c_1 x_1 + c_2 x_2 + c_3 x_3 + C_c\bar{y} + T_1 x_1 + T_2 x_2 + T_3 x_3$$

Back to interaction diagram

given P, M, find geometry



> assume square column, 2.5 in cover

> calc. max load (1.2DL + 1.6LL), P_u, M_u

$\phi = 0.65$ for a tied column, for P_b, M_b

> calculate eccentricity, $\frac{M_b}{P_b} = e$

> $h >$ ~~3~~ $1/3 L$ ... but just guess a geometry, h, $\gamma h = (h - 2\text{cover})$

e/h also useful.

> use interaction diagram now! $\phi M_n / bh^2$ vs. $\phi P_n / bh$, and e/h

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I DON'T WANNA BE A COWBOY! April 21, 2004

Column Design

calculate γ, e/h, $\frac{\phi P_n}{A_g}$, $\frac{\phi M_n}{A_g h}$ and find point on graph – find ρ_g on $\gamma = 0.60$ graph

then find ρ_g on $\gamma = 0.75$ graph

use linear interpolation to find ρ_g to use

$$\longrightarrow \frac{h - 2(\text{cover})}{h} = \gamma$$

now determine As from ρ_g

$A_s = \rho_g b h = 6.86 \text{ in}^2$ (for in class example) – minimum value

ties, spacing:

16 (bar diameter) vs. 48 (tie diameter) vs. smallest dimension

Balanced Point on I.D.

calculate $\varepsilon_s = \varepsilon_y = \frac{f_y}{E_s}$

$$C_b = \frac{\varepsilon_u}{\varepsilon_s + \varepsilon_u} \cdot d \longrightarrow a = 0.85C$$

$\hookrightarrow$ $\beta = 0.80$, as $f_c' > 4$ ksi page 737 in text

$f_s' = 0.003 \frac{c - d'}{c} \cdot E_s$, how compared to f_y?

$P_b = 0.85 f_c' a b + A_s' f_s' - A_s f_y$

$\hookrightarrow$ more accurately, $(ab - A_s')$

ALWAYS SUBTRACT OUT AREA OF STEEL FROM COMPRESSION CONCRETE.

C.Hovell

TWENTY-ONE IN THIRTY DAYS!

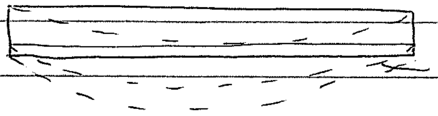
April 23,2004

Bond and development length

assumptions have been made about bond between concrete, steel

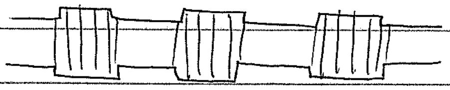
bond stress develops along length of bar

when bars were smooth:



steel would bend, but wouldn't elongate — it would slip at the ends

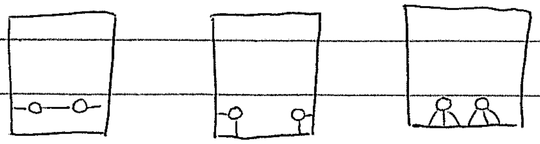
new bars:



idealized bar, with points that grip

Modes of failure

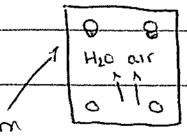
1. direct pullout — concrete around the bar crushes; there is nothing left to hold onto bar
2. splitting — cracks form as concrete by bar is put into tension



development length — length of embedment of a bar necessary to develop the full tensile strength of the bar

influenced by:
1. bar diameter
2. bar shape (deformations, shown above)
3. bar surface (mill scale, cleaned, coated)
4. compressive and tensile strength of concrete
5. fracture energy
6. cover and spacing (thus, # of bars)
7. transverse confining steel — think stirrups, ties
8. vertical location of bar — bleedwater pockets form

chapt 5.1,2



general equation:

$$\boxed{\ell_d = \left(\frac{3}{40}\,\frac{f_s}{\sqrt{f'_c}\left(\frac{c+k_{TR}}{d_b}\right)}\right)}$$

d_b = bar diameter

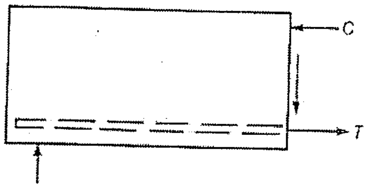
c = minimum of either the cover ~~of~~ or 1/2 spaci

A_{tr} = area transverse steel

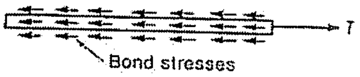
k_{TR} = factor relating to confining steel (transv

s = spacing of transverse bars

$$\boxed{= A_{tr} f_{yt} / 1500 \cdot s \cdot n}$$



(a) Internal forces in beam.



(b) Forces on reinforcing bar.

Fig. 8–1
Need for bond stresses.

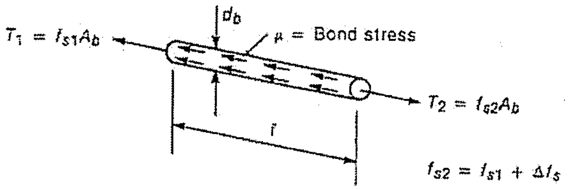
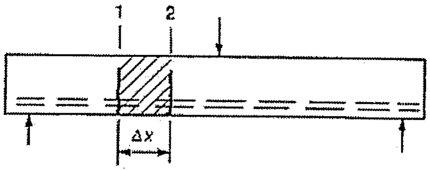
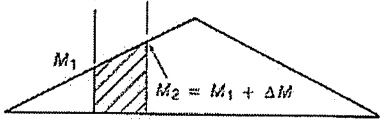


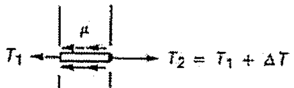
Fig. 8–2
Relationship between change in bar stress and bond stress.



(a) Beam.



(b) Moment diagram.



(c) Bar forces.

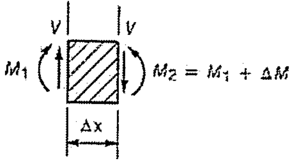


Fig. 8–3
Average flexural bond stress.

(d) Part of beam between sections 1 and 2.

If U is the magnitude of the local bond force per unit length of bar, then, by summing horizontal forces

$$U\,dx = dT \tag{b}$$

Thus

$$U = \frac{dT}{dx} \tag{5.1}$$

indicating that the local unit bond force is proportional to the rate of change of bar force along the span. Alternatively, substituting Eq. (*a*) in Eq. (5.1), the unit bond force can be written as

$$U = \frac{1}{jd}\frac{dM}{dx} \tag{c}$$

from which

$$U = \frac{V}{jd} \tag{5.2}$$

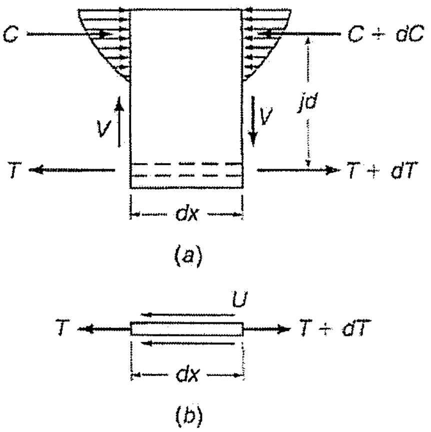


FIGURE 5.3
Forces acting on elemental length of beam: (*a*) free-body sketch of reinforced concrete element; (*b*) free-body sketch of steel element.

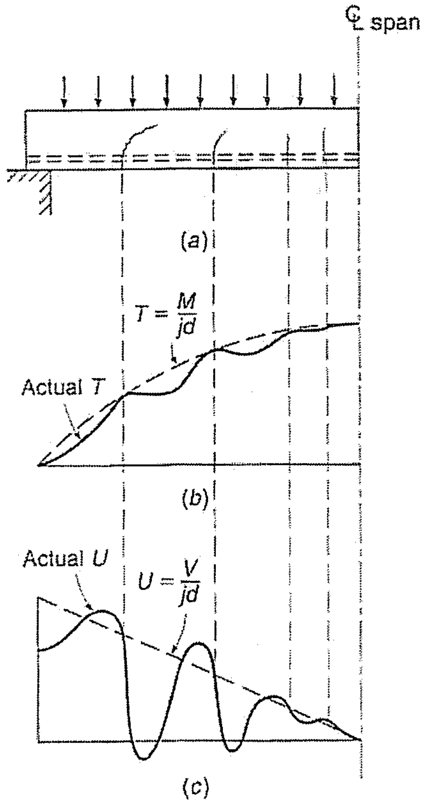
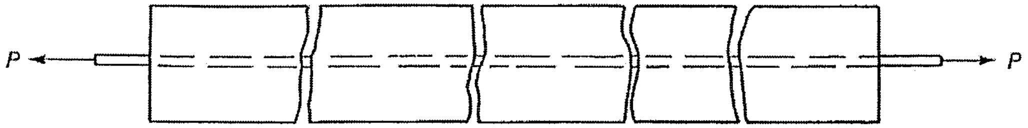
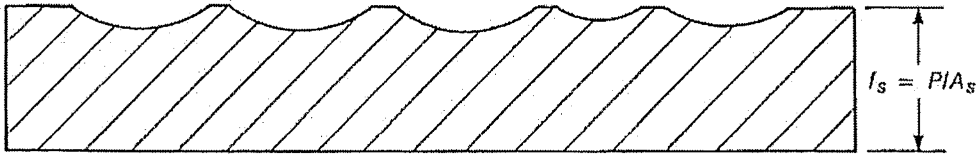


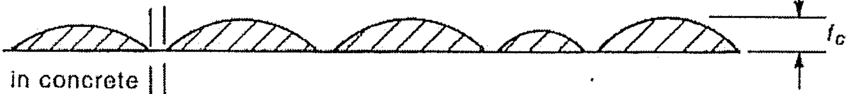
FIGURE 5.5
Effect of flexural cracks on bond forces in beam: (*a*) beam with flexural cracks; (*b*) variation of tensile force T in steel along span; (*c*) variation of bond force per unit length U along span.



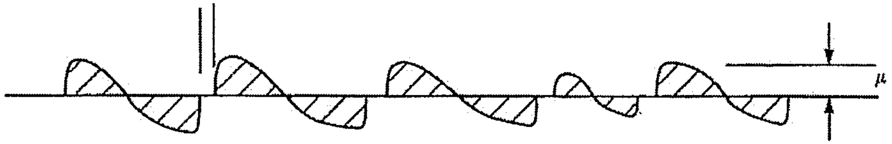
(a) Axially loaded prism.



(b) Variation in steel stress.

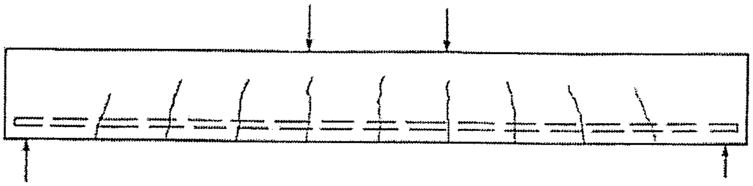


(c) Variation in concrete stress.

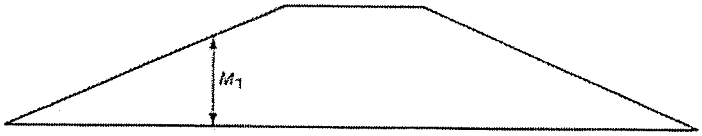


(d) Variation in bond stress.

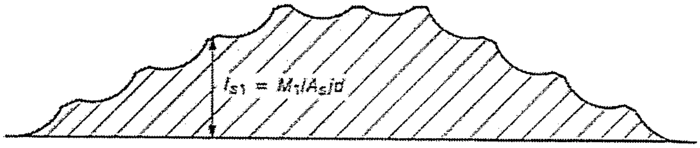
Fig. 8–4
Steel, concrete, and bond stress in a cracked prism.



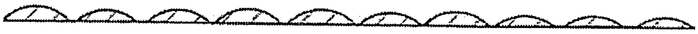
(a) Cracked beam.



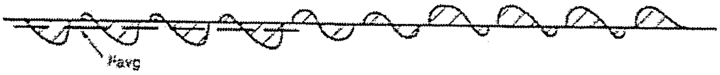
(b) Moment diagram.



(c) Variation in steel stress.



(d) Tensile stress in concrete.



(e) Bond stresses.

Fig. 8–5
Steel, concrete, and bond stresses in a cracked beam.

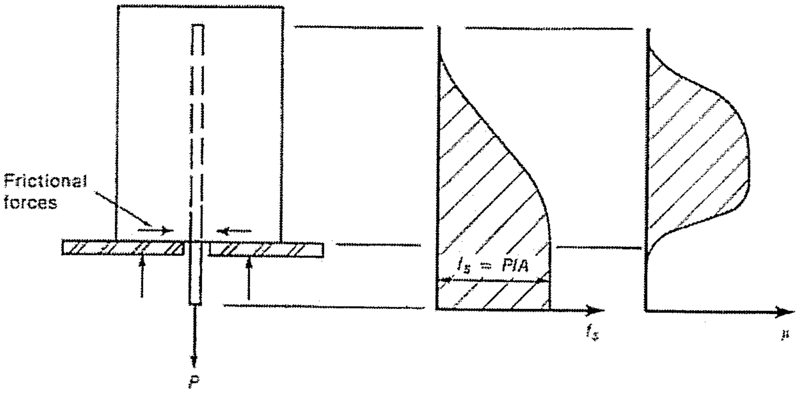
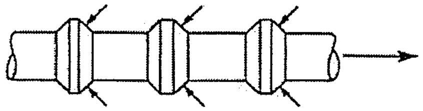
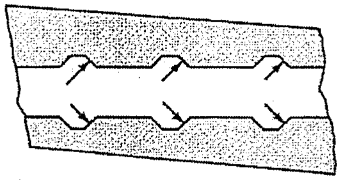


Fig. 8–6
Stress distribution in a pull-out test.

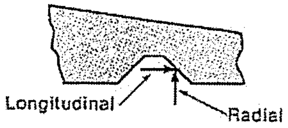
(a) Test method. (b) Bar stress. (c) Bond stress.



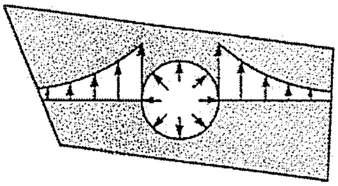
(a) Forces on bar.



(b) Forces on concrete.

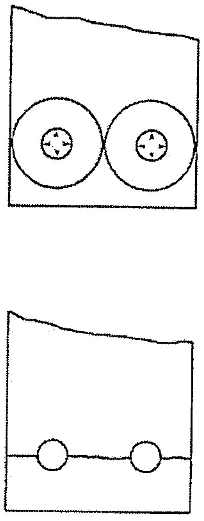


(c) Components of force on concrete.

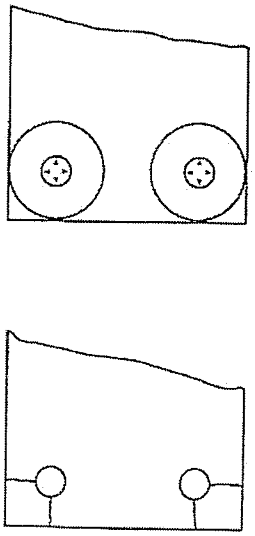


(d) Radial forces on concrete and splitting stresses shown on a section through the bar.

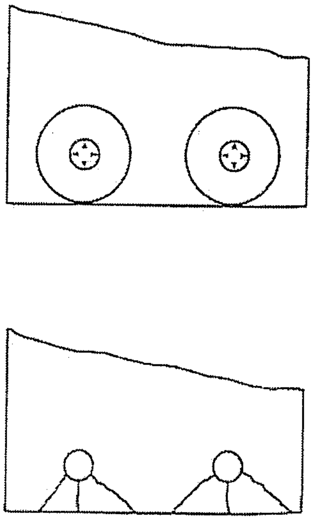
Fig. 8-7
Bond transfer mechanism.



(a) Side cover and half the bar spacing both less than bottom cover.



(b) Side cover = bottom cover, both less than half the bar spacing.



(c) Bottom cover less than side cover and half the bar spacing.

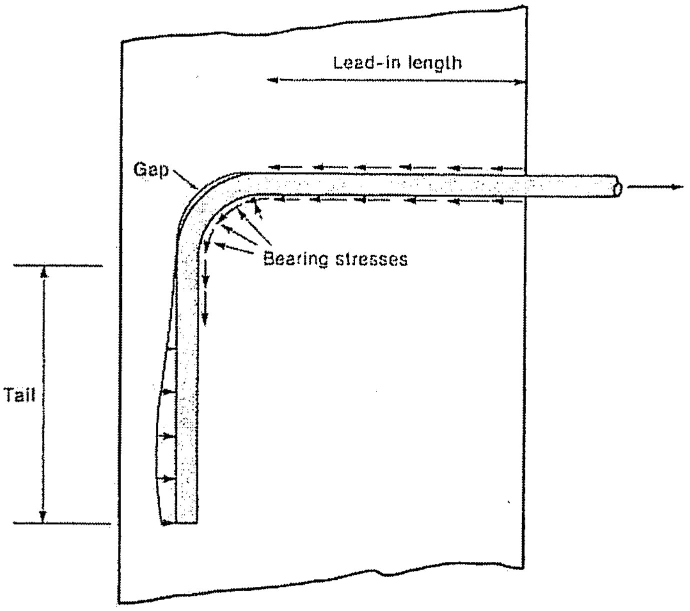
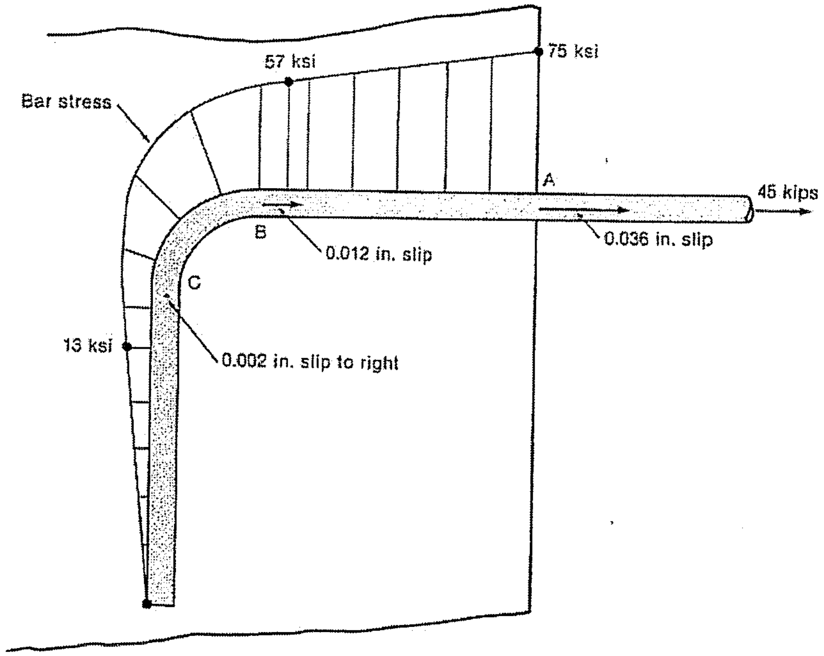


Fig. 8-13
Behavior of hooks.

(a) Forces acting on bar.



(b) Stresses and slip—90° standard hook.

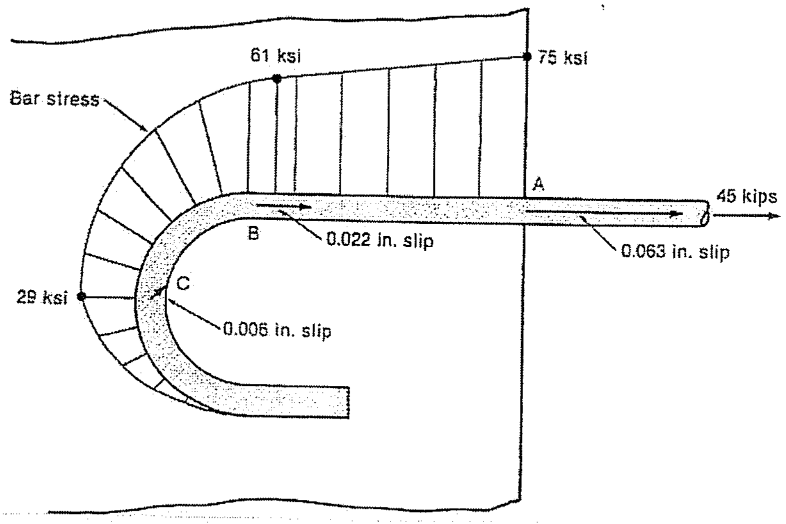


Fig. 8-13
Continued.

(c) Stresses and slip—180° standard hook

C.Hovell

SECOND-TO-LAST CLASS! April 26, 2004

What's the exam about?

cumulative ... short answer part } entirely open-book, open-notes
problems

major TOPICS

- beams in flexure (single, double reinforcement)
- deflections
- shear
- columns (one-axis bending)