CE 383P – Prestressed Concrete

Unique Number 15875

Instructor:

Oguzhan Bayrak
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ECJ 4.720 232-7826
Ferguson Lab 232-6409

Class Hours:

MWF 10:00AM – 11:00AM ECJ ~~7.208~~ 7.202

Office Hours:

WF 1:30 – 4:30
Additional times are available by request.

Prerequisites:

CE 331 (or equivalent) and graduate standing.

Texts:

1. Lin, T.Y., and Burns, N.H., *Design of Prestressed Concrete Structures*, John Wiley and Sons, Inc., Third Edition, 1993.
2. Collins, M.P., and Mitchell, D., *Prestressed Concrete Structures*, Response Publications, 1998.
   OR
   Collins, M.P., Reinforced and Prestressed Concrete Structures, Brunner-Routledge; 2nd edition, 2005.
3. ACI 318-05, *Building Code Requirements for Structural Concrete and Commentary*

Grading:

Homework	20%
Design Project	15%
Midterm Exam 1	30%
Final Exam	35%

Students must have an average grade of at least 60% on the exams in order to receive a passing grade in the course. All students must take the final exam.

Final grades will be assigned according to the following scale:

> 90	A
80 – 89	B
70 – 79	C
60 – 69	D
< 60	F

Attendance:

Students are expected to attend all lectures.

Homework:

Homework will be assigned regularly. Homework submitted after the due date and time will be penalized 25% if submitted within the first 24 hours, 50% if submitted by the next lecture and will not be accepted after that point in time.

Examinations:

Students are expected to take the exams during the scheduled class periods. Students must notify the instructor in advance if they are unable to take an exam. Make-up exams will be given only when adequate supporting evidence is furnished, e.g. a doctor's note in case of an illness preventing you from taking an exam.

Project:

One project will be assigned during the semester.

Use of Computer:

Students will be expected to use spreadsheet programs, commercial and/or academic structural analysis programs to complete the homework assignments and the project. Use of drafting programs may be helpful.

Course Outline:

1. Introduction
2. Prestressing Technology
3. Material Properties
4. Response to Axial Load
5. Response to Flexure
6. Design for Flexure
7. Design for Shear
8. Design for Torsion
9. Prestressed Slab Systems
10. Analysis of Indeterminate Structures

Scholastic Dishonesty:

Giving aid to a student during an exam or taking information from another student's exam constitutes academic dishonesty. Submitting another student's homework assignment and representing it as your own work also constitutes scholastic dishonesty.

Students who violate University rules on scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and/or dismissal from the University. Since such dishonesty harms the individual, all students, and the integrity of the University, policies on scholastic dishonesty will be strictly enforced. For further information, visit the Student Judicial Services web site http://www.utexas.edu/depts/dos/sjs/.

Important Dates:

Tuesday, 30 ~~Monday, 29~~ October	Midterm Exam 5:30pm – 8:30ish
Friday, 7 December	Last Day of Class
Monday, 17 December	Final Exam (2:00 PM– 5:00 PM)

Drop Dates:

From the 1st through the 4th class day, graduate students can drop a course on Rose or TEX and receive a refund. From the 5th through the 12th class day, graduate students must initiate drops in their department; refunds are given on a dropped course through the 12th class day. After the 12th class day, no refund is given. Graduate students can drop a class until the last class day with permission from the departmental graduate advisor and the Dean. Students with 20 hr/week GRA/TA appointment or a fellowship may not drop below 9 hours.

Course evaluation:

Students will be given the opportunity to evaluate the instructor during the last week of class using the standard form provided by the Measurement and Evaluation Center.

The University of Texas at Austin provides, upon request, appropriate academic adjustments for qualified students with disabilities. Any student with a documented disability (physical or cognitive) who requires academic accommodations should contact the Services for Students with Disabilities area of the Office of the Dean of Students at 471-6259 as soon as possible to request an official letter outlining authorized accommodations. For more information, contact that Office, or TDD at 471-4641, or the College of Engineering Director of Students with Disabilities at 471-4321.

Web-based, password-protected class sites will be associated with all academic courses taught at the University. Syllabi, handouts, assignments and other resources are types of information that may be available within these sites. Site activities could include exchanging email, engaging in class discussions and chats, and exchanging files. In addition, electronic class rosters will be a component of the sites. Students who do not want their names included in these electronic class rosters must restrict their directory information in the Office of the Registrar, Main Building, Room 1. For information on restricting directory information, see Course Schedule or http://www.utexas.edu/student/registrar/catalogs/gi02-03/app/appc09.html.

CE 383P – Prestressed Concrete
Project

Date assigned : October 17, 2007
Due date : December 5, 2007

Option 1

Develop a spreadsheet type program to evaluate the moment curvature response of prestressed concrete sections using layered section analysis approach. Use the following constitutive relationships in your spreadsheet:

- Parabolic and/or HSC stress-strain curve for concrete
- Trilinear stress-strain curve for reinforcing bars
- Modified Ramberg-Osgood function for stress-strain response of prestressing strands
- Vecchio-Collins model for tension stiffening

Focus on double-tee sections that may have up to 12 layers of reinforcing steel and 12 layers of prestressing strands. Note that you do not have to develop a spreadsheet type programming approach, if you feel more comfortable in using another programming language/means (e.g. C++, Visual Basic, Fortran etc.)

The final submission should include the following:

- A disk containing the program
- A report that summarizes your work and simplifying assumptions you may have made.
- Program verification: Comparison of the analysis results from your computer program/spreadsheet with those from a well-established and verified software (e.g. Response) and hand calculations.
- Concluding remarks

Option 2

Alternatively, you may choose a particular topic in prestressed concrete and critically examine several recently published papers on this topic. If you choose this alternative, you should consult your instructor and discuss the particular topic and papers you would like to work on before you fully embark on your report.

CE 383P – Prestressed Concrete

Midterm
October 30, 2007
Examiner: O. Bayrak

Last Name:
HOVELL

Name:
CATHERINE

Instructions:

1. This is an open book / notes exam.
2. Read questions carefully.
3. Make reasonable assumptions for missing parameters (if any).
4. Answer all questions.

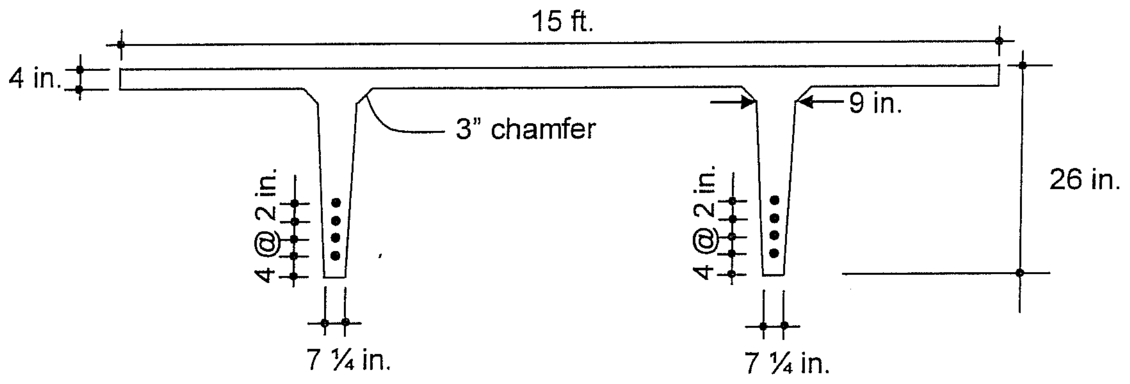
Question	Grade
1	25/25
2	23 /25
3	15 /25
4	25 /25
very good!	88 /100

(except for problem3)

Problem 1 (25 points):

The precast, pretensioned double tee beam shown below spans 26 feet as part of a floor system. Calculate the nominal moment capacity of the section.

Pretopped 15DT26



$f_c' = 5{,}000$ psi

$\varepsilon_c' = 2.5\times10^{-3}$

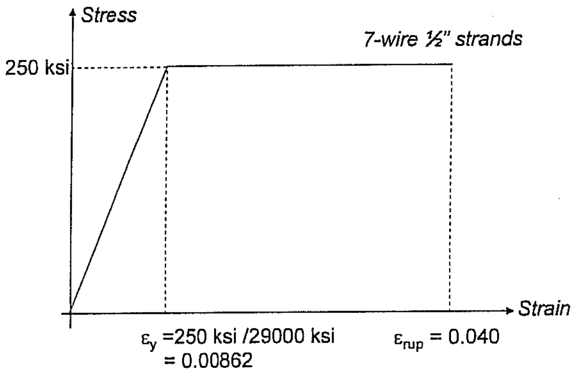
A_p = 8- ½ in. low relaxation strands (0.153 in^2 / strand)

E_p = 29000 ksi

$f_{py} = f_{pu}$ =250 ksi

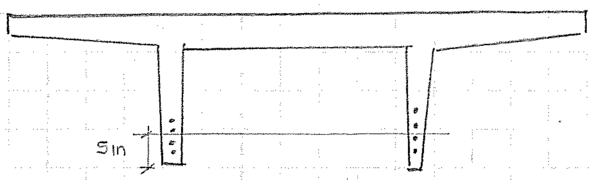
$\Delta\varepsilon_p = 6.00\times10^{-3}$

Strand ruptures at $\varepsilon_{rup} = 0.040$



MIDTERM

1.



$f'_c = 5\text{ ksi}$

$\varepsilon'_c = 2.5\times10^{-3}$

$A_p = 8(0.153\text{ in}^2) = 1.224\text{ in}^2$

$E_p = 29000\text{ ksi}$

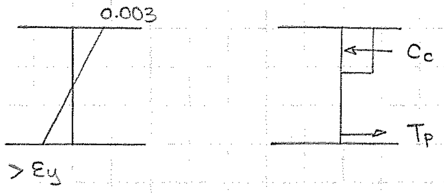
$f_{py} = f_{pu} = 250\text{ ksi}$

$\Delta\varepsilon_p = 6.0\times10^{-3}$

$\varepsilon_{rup} = 0.040$

$L = 26\text{ ft}$

Assume concrete crushing occurs before strand rupture



use rectangular stress block

$C_c = \alpha_1\beta_1 f'_c b c$

$$\alpha_1\beta_1 = \frac{\varepsilon_{ct}}{\varepsilon'_c} - \frac{1}{3}\left(\frac{\varepsilon_{ct}}{\varepsilon'_c}\right)^2 = \frac{0.003}{0.0025} - \frac{1}{3}\left(\frac{0.003}{0.0025}\right)^2 \checkmark$$

$$\alpha_1\beta_1 = 0.72$$

$T_p = 1.224\text{ in}^2\ (250\text{ ksi}) = 306\text{ k} = C_c$

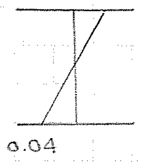
$$c = \frac{306\text{ k}}{0.72(5\text{ ksi})(180\text{ in})}, \quad c = 0.453\text{ in} \checkmark$$

$\uparrow$ if $c < 4\text{ in}$ ✓

check:

$$\varepsilon_{p1} = \frac{0.003}{0.453\text{ in}}(24\text{ in} - 0.453\text{ in}) = 0.156 \gg \varepsilon_{rup} \checkmark$$

Now, strand rupture controls



$$306\text{ k} = \left[\frac{\varepsilon_{ct}}{0.0025} - \frac{1}{3}\left(\frac{\varepsilon_{ct}}{0.0025}\right)^2\right] f'_c(180\text{ in})\cdot c$$

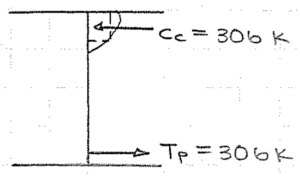
also, $\dfrac{\varepsilon_{ct}}{c} = \dfrac{0.04}{24\text{ in} - c}$, $\varepsilon_{ct} = \dfrac{0.04c}{24\text{ in} - c}$

solve for $c = 0.77\text{ in} \checkmark$, $\varepsilon_{ct} = 0.00133 \checkmark$

check $\alpha_1\beta_1 = 0.44$, $C_c = 306\text{ k}$ ✓ ✓

MIDTERM

1. (cont'd)



$$M_n = (306\text{k})\left[21\text{ in} - \tfrac{1}{2}\beta_1 c\right]$$

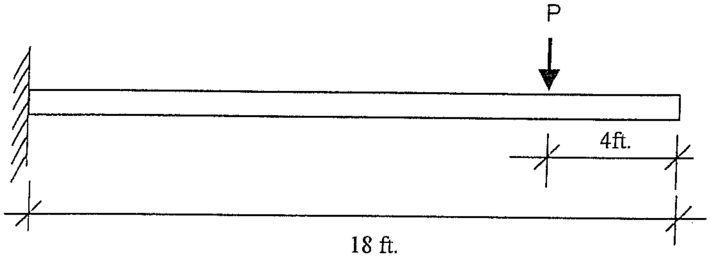
$$\beta_1 = \frac{4 - \varepsilon_{ct}/\varepsilon_c'}{6 - 2\varepsilon_{ct}/\varepsilon_c'} = \frac{4 - \frac{0.0013}{0.0025}}{6 - \frac{2(0.0013)}{0.0025}} = 0.703 \quad ✓$$

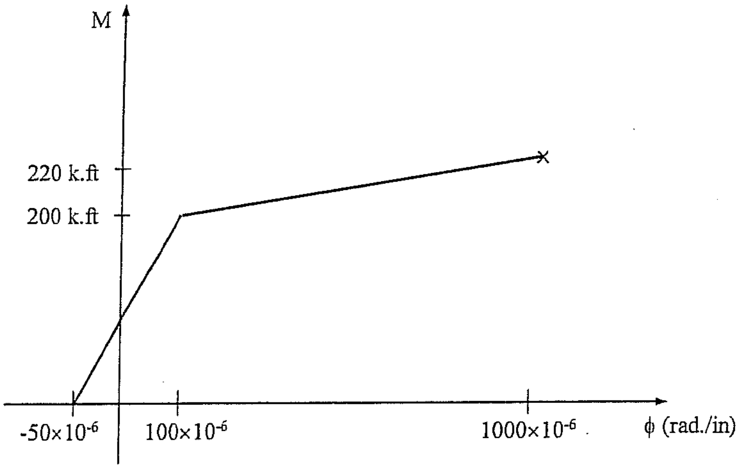
$$M_n = 306\text{ k}\left[21\text{ in} - \tfrac{1}{2}(0.703)(0.64\text{ in})\right] = 6343\text{ k}\cdot\text{in}$$

$$\underline{\underline{M_n = 529\text{ k}\cdot\text{ft}}} \quad ✓$$

Problem 2 (25 points)

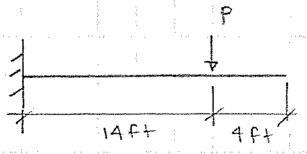
The moment-curvature relationship of the precast prestressed beam shown below is obtained using a sectional analysis program and further simplified as a bilinear relationship for your convenience. Calculate the tip deflection at failure.

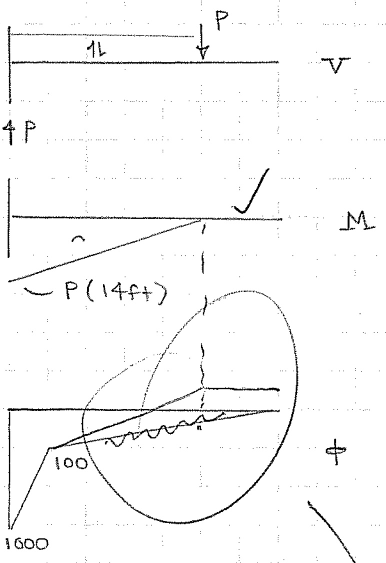


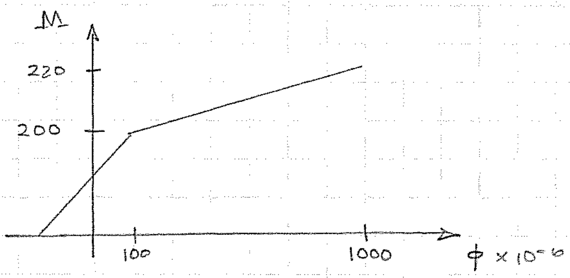


MIDTERM

2.







Load at failure, P:

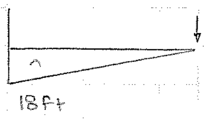
$$M_u = 220\,k\cdot ft = P(14ft)$$

$$P = 15.71\,k$$

M_u at $x = 0$, $\phi = 1000 \times 10^{-6}$

M_y at $x = 1.27\,ft$, $\phi = 100 \times 10^{-6}$

using virtual work,



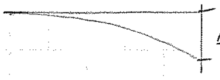
watch units!

$$\Delta = 900/in \cdot \tfrac{1}{2}(1.27ft)\left(1 - \tfrac{1}{3}\frac{1.27ft}{18ft}\right)(18ft)$$
$$+ 100/in\,(1.27ft)\left(1 - \tfrac{1}{2}\frac{1.27ft}{18ft}\right)(18ft)$$
$$+ \tfrac{1}{2}(100/in)(16.73ft)\left(1 - \frac{1.27 + \tfrac{1}{3}(16.73ft)}{18ft}\right)(18ft)$$

approx. due to error in φ diagram

$$\underline{\Delta = 0.25\,in}$$

check using moment-area

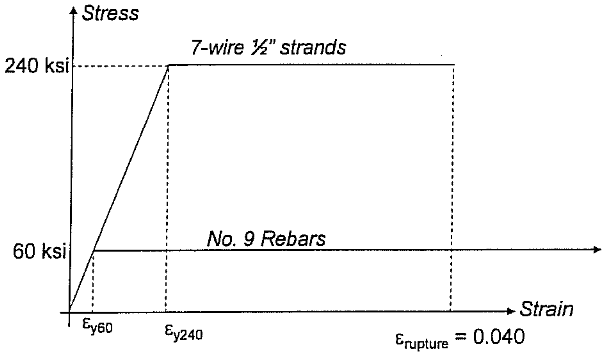


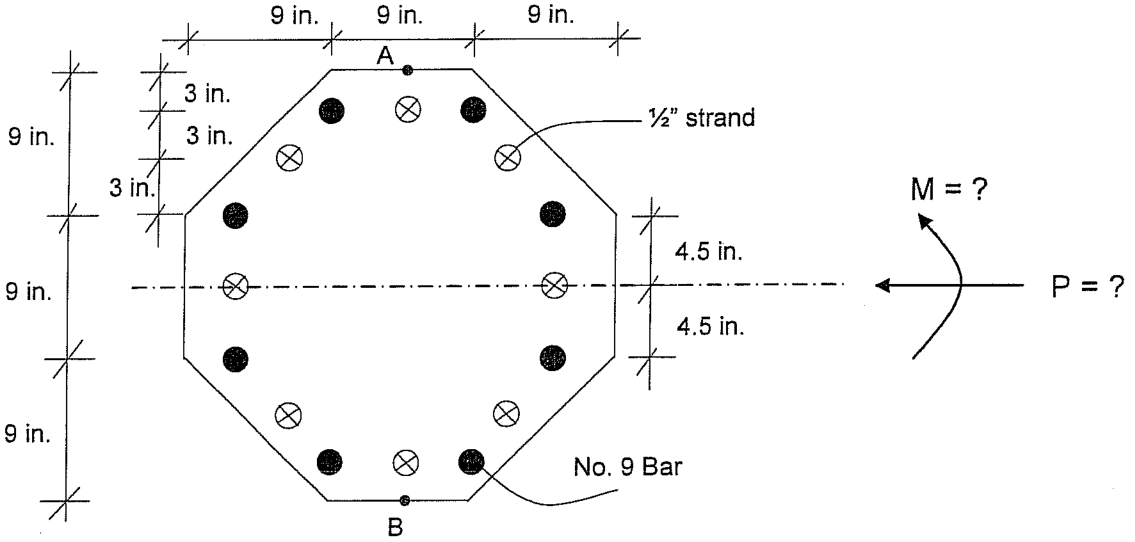
$$\Delta = \tfrac{1}{2}(100/in)(16.73ft)\tfrac{2}{3}(16.73ft) +$$
$$100/in\,(1.27ft)(16.73ft + \tfrac{1}{2}(1.27ft)) +$$
$$\tfrac{1}{2}\cdot 900/in\,(1.27ft)(16.73ft + \tfrac{2}{3}(1.27ft))$$
$$= 0.259\,in \checkmark$$

Problem 3 (25 points)

The precast prestressed octagonal pile shown below is subjected to an axial load and bending moment combination. The octagonal pile is reinforced with 8 No.9 bars and 8 7-wire ½" strands. Determine the axial load and bending moment combination that will cause a compressive strain of 0.0024 at point A and a tensile strain of 0.0010 at point B.

$f_c' = 6{,}500$ psi
$\varepsilon_c' = 2.2\times10^{-3}$
$E_p = 29000$ ksi
$f_{py} = f_{pu} = 240$ ksi
$\Delta\varepsilon_p = 7.00\times10^{-3}$
Strands rupture at $\varepsilon_{rupture} = 0.040$

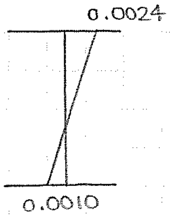




4/6

MIDTERM

3.



$$0.0024(27\,\text{in}) = 0.0010c + 0.0024c$$

$$c = 19.1\,\text{in}$$

$$\varepsilon_{cen} = \frac{0.0024}{19.1\,\text{in}}(13.5\,\text{in} - 19.1\,\text{in}) = -7.04\times10^{-4}$$

$$\phi = \frac{0.0024}{19.1\,\text{in}} = 1.26\times10^{-4}$$

$$A_{trans} = 567\,\text{in}^2 + \left(\frac{E_s}{E_c} - 1\right)\left[8(0.153\,\text{in}^2) + 8(1.0\,\text{in}^2)\right]$$

$E_s = E_p = 29000\,\text{ksi}$

$$E_c = \frac{2f'_c}{\varepsilon'_c} = \frac{2(6.5\,\text{ksi})}{2.2\times10^{-3}} = 5910\,\text{ksi}$$

$$A_{trans} = 603\,\text{in}^2$$

I_Δ about base

$$I_{trans} = \tfrac{1}{12}(27\,\text{in})(27\,\text{in})^3 - 4\left[\tfrac{1}{12}(9\,\text{in})(9\,\text{in})^3 + \tfrac{1}{2}(9\,\text{in})^2(4.5\,\text{in})^2\right]$$
$$+ (2)\left(\frac{E_s}{E_c} - 1\right)\left[(2.153\,\text{in}^2)(10.5\,\text{in})^2 + 2(0.153\,\text{in}^2)(7\,\text{in})^2 + (2.0\,\text{in}^2)(4.5\,\text{in})^2\right]$$
$$= 41108\,\text{in}^4$$

$$\varepsilon_{cen} = \frac{N - N_o}{E_c A_{trans}}, \quad N_o = E_p \Delta\varepsilon_p A_p = (29000\,\text{ksi})(7\times10^{-3})(8)(0.153\,\text{in}^2) = 248.5\,\text{K}$$

$$N = \varepsilon_{cen} E_c A_{trans} + N_o$$
$$= (-7.04\times10^{-4})(5910\,\text{ksi})(603\,\text{in}^2) + 248.5\,\text{K} = -2260\,\text{K}$$

$$\phi = \frac{M - M_o}{E_c I_{trans}}, \quad M_o = 0$$ as P/S is centered around C.A.

$$M = \phi E_c I_{trans} = (1.26\times10^{-4})(5910\,\text{ksi})(41108\,\text{in}^4) = 30611\,\text{K}\cdot\text{in}$$
$$= 2551\,\text{K}\cdot\text{ft}$$

$$N = -2260\,\text{K}$$
$$M = 2551\,\text{K}\cdot\text{ft}$$

Section cracked.

Problem 4 (25 points)

A three span post-tensioned bridge girder is shown in the figure below. The tendons are stressed from one end and the stressing procedure is to jack to 0.75 f_{pu} and then anchor. The post tensioned tendon consists of 32- 1/2" 7-wire strands. Assuming that $\mu = 0.3$, $K = 0.0015/\text{ft}$ and $\Delta_{set} = ¼$ in, calculate and plot the tendon force variation prior to and after anchorage. Also calculate the tendon elongation due to stressing operation (prior to anchorage). Note $f_{pu} = 270$ ksi for the low relaxation strands.

MIDTERM

4. $f_p = 0.75 f_{pu} = 0.75(270\,\text{ksi}) = 202.5\,\text{ksi}$

$A_s = 32(0.153\,\text{in}^2)$

$\mu = 0.3$

$K = 0.0015/\text{ft}$

$\Delta_{set} = 1/4\,\text{in}$

$F_p = (202.5\,\text{ksi})(32)(0.153\,\text{in}^2) = 991.4\,\text{K}$

Average force:

$$\frac{1}{360\,\text{ft}}\cdot\frac{1}{2}\Big[(991\text{k}+872\text{k})(50\text{ft}) + (872\text{k}+763\text{k})(50\text{ft}) + (763\text{k}+703\text{k})(20\text{ft}) + (703\text{k}+639\text{k})(20\text{ft}) + (639\text{k}+555\text{k})(60\text{ft}) + (555\text{k}+476\text{k})(60\text{ft}) + (476\text{k}+436\text{k})(20\text{ft}) + (436\text{k}+386\text{k})(10\text{ft}) + (386\text{k}+337\text{k})(30\text{ft}) + (337\text{k}+307\text{k})(40\text{ft})\Big] = 608.2\,\text{K}$$

Calculate elongation

$$\Delta = \frac{P_{av}\cdot L}{A_{ps}\,E_{ps}} = \frac{(608.2\text{k})(360\,\text{ft})}{32(0.153\text{in}^2)(29000\text{ksi})} = 18.5\,\text{in}$$

$$\underline{\underline{\Delta = 18.5\,\text{in}}}$$

Anchorage:

$$L_{set} = \left[\frac{\Delta_{set}\cdot A_s\cdot E}{\underline{p}}\right]^{\frac{1}{2}}$$

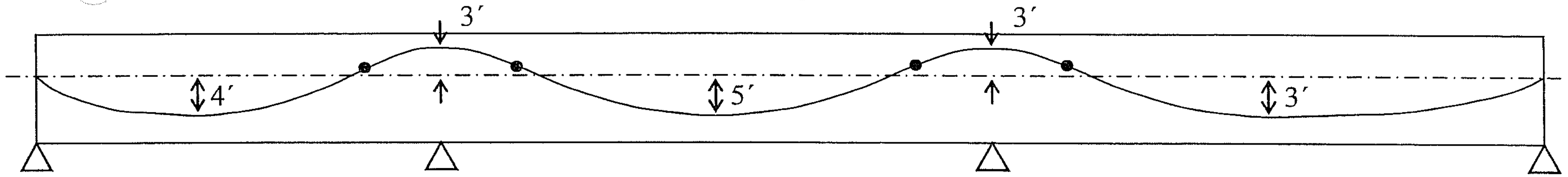
If all loss occurs in first section ($L_{set} < 50\,\text{ft}$), $\underline{p} = 2.38\,\text{k/ft}$

$$L_{set} = \left[\frac{0.25\,\text{in}\ (32)(0.153\,\text{in}^2)(29000\,\text{ksi})}{2.38\,\text{k/ft}}\right]^{\frac{1}{2}} = 35.3\,\text{ft}$$

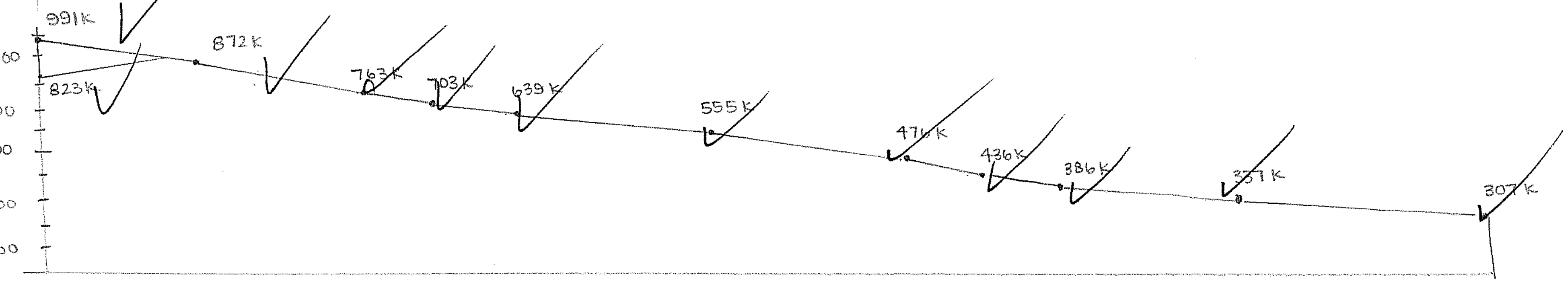
$< 50\,\text{ft}$ ✓

$$\Delta P = 2\underline{p}\,L_{set} = 2(2.38\,\text{k/ft})(35.3\,\text{ft}) = 168.0\,\text{k}$$

$$991\text{k} - 168.0\text{k} = 823\,\text{k}$$



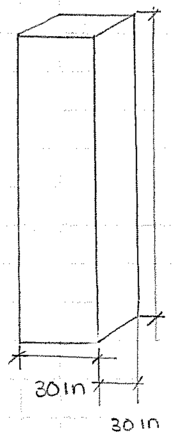
		50′		50′		20′		20′		60′		60′		20′		10′		30′		40′	
α		$\frac{2(4ft)}{50ft}=0.16$			0.20				0.20				0.20				0.30			0.15	
μα+Kx		0.3(0.16)+ 0.0015(50ft) = 0.123		0.135		0.09		0.09		0.15		0.15		0.09		0.105		0.135		0.105	
(μα+Kx)	0		0.123		0.258		0.348		0.438		0.588		0.738		0.828		0.933		1.068		1.173
$e^{-Σ}$			0.88		0.77		0.71		0.645		0.56		0.48		0.44		0.39		0.34		0.31
$e^{+Σ}$			1.13		1.29		1.42														



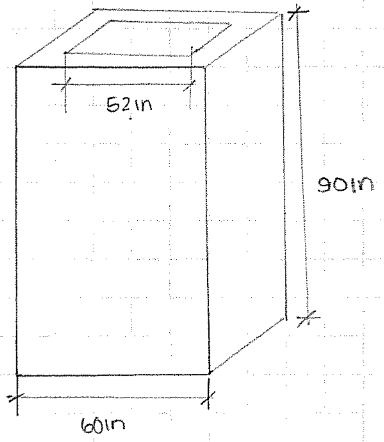
tendon force variation plot

HOMEWORK #1

1.



(1) $A = 900 \text{ in}^2$



(2) $A = 896 \text{ in}^2$

$f = 3000$ psi
7 days post-pour, $f_c' = 7000$ psi
$H = 50\%$

Estimate: initial strain
10, 1000, 10000 days

$$\varepsilon_{tot} = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth} + \varepsilon_{cr}$$

creep:
$t_i = 7$ days
$t = 10, 1000, 10000$ days

$$E_{ct} = \frac{-2 f_c'}{\varepsilon_c'} = \frac{-2(7000 \text{ psi})}{2 \times 10^{-3} \text{ in/in}} = 7000 \text{ ksi}$$

$$f_c = -f_c' \left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2 \right]$$

$$-3000 \text{ psi} = -7000 \text{ psi} \left[2\left(\frac{\varepsilon_{cf}}{-2 \times 10^{-3}}\right) - \left(\frac{\varepsilon_{cf}}{-2.0 \times 10^{-3}}\right)^2 \right]$$

solve for ε_{cf}, $\varepsilon_{cf} = -0.000488$ in/in

approximation: $\frac{-3000 \text{ psi}}{7000 \text{ ksi}} = 0.42 \times 10^{-3}$

column ratios:

$V_1 = 81{,}000 \text{ in}^3$
$SA_1 = 10{,}800 \text{ in}^2$
$(V/SA)_1 = 7.5$

$V_2 = 80{,}640 \text{ in}^3$
$SA_2 = 40{,}320 \text{ in}^2$
$(V/SA)_2 = 2$

HOMEWORK #1

1. (cont'd)

Creep:

	10	1000	10,000
k_{c1}	0.1	0.5	0.65
k_{c2}	0.82	0.92	0.95

$$k_g = \left[{}^{2}/_{3} + {}^{f_c'}/_{9000} \right]^{-1}$$

$$= \left[{}^{2}/_{3} + {}^{7}/_{9} \right]^{-1} = 0.692$$

$$\phi = 3.5 k_c (0.692)(1.58 - {}^{50}/_{120})(7)^{-0.118} \frac{(\Delta t)^{0.6}}{10 + \Delta t^{0.6}}$$ ✓

$\Delta t = 10, 1000, 10000$

φ	10	1000	10000
C1	0.064	0.967	1.40
C2	0.52	1.78	2.04

ε_{cr} calculated in Excel; spreadsheet pg.

	10	1000	10000
C1	0.27	0.91	1.23
C2	0.06	0.15	0.17

$-\varepsilon_{cr} \times 10^{-3}$

HOMEWORK #1

1. (cont'd)

Shrinkage:

$$\varepsilon_{sh} = -k_s k_n \left(\frac{t}{35+t}\right)(0.51\times10^{-3})$$

$t = 17, 1007, 10007$

k_s = approximately the same values as k_c, earlier

$k_n = 1.29$

ε_{sh}:

	17	1007	10007
C1	0.02	0.32	0.43
C2	0.18	0.59	0.62

$-\varepsilon_{sh} \times 10^{-3}$

Thermal $(\varepsilon_{th}) = 0$,
no loads applied

Loading strain:

$f_c = 3000\text{psi}$

$$\varepsilon_{cf} = \frac{\sigma}{E_c} = \frac{3000\text{ psi}}{4770\text{ ksi}} = 0.629\times10^{-3}\text{ in/in}$$

$57\sqrt{f_c'}$

Total strain is the sum of load, creep, and shrinkage strains

	10	1000	10000
C1	0.93	1.87	2.29
C2	1.41	2.72	2.95

$-\varepsilon_{tot} \times 10^{-3}$

comments:

The effects of shrinkage and creep depend highly on the volume / surface area ratio. Column 1 had more creep (high V/SA) while column 2 had more shrinkage. The total losses were comparable, however. I would advise using column 2 if only because a majority of the strain change accured in the first two weeks, meaning connections and long-term changes are less important or influential.

CREEP

load	3000	psi		V1	81000	1_10	0.1
f_c'	7000	psi		SA1	10800	1_1000	0.5
Ec	4768962	psi		V/SA1	7.5	1_10000	0.65
Ect	7000000	psi	4768962				
t_i	7	days		V2	80640	2_10	0.82
				SA2	40320	2_1000	0.92
k_g	0.692308			V/SA2	2	2_10000	0.95
Humid	50						

ε_c'	ε_cf	goalseek	approx
-0.002	-0.000488	-3.85E-07	~~-0.000629~~

		phi	Ec_eff	ε_c'_eff	ε_cf	goalseek	ε_creep
1_10	10	0.0638	4483.0	-0.003123	-0.000762	4.45E-06	-0.000274
1_1000	1000	0.9670	2424.5	-0.005774	-0.001409	2.51E-05	-0.000921
1_10000	10000	1.4006	1986.6	-0.007047	-0.00172	-1.19E-07	-0.001232
2_10	10	0.5231	3131.0	-0.004471	-0.001091	4.78E-06	-0.000603
2_1000	1000	1.7793	1715.9	-0.008159	-0.001991	0.000486	-0.001503
2_10000	10000	2.0470	1565.1	-0.008945	-0.002183	1.58E-07	-0.001695

ε_c'	ε_cf	goalseek
-0.002	-0.000488	-3.85E-07

SHRINKAGE

k_h	1.29	
		ε_sh
1_10	17	-2.15E-05
1_1000	1007	-0.000318
1_10000	10007	-0.000426
2_10	17	-0.000176
2_1000	1007	-0.000585
2_10000	10007	-0.000623

LOAD

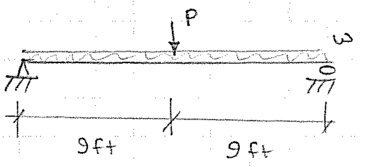
-0.000629

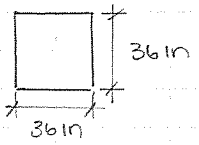
TOTALS

1_10	-0.000925	2_10	-0.001409
1_1000	-0.001868	2_1000	-0.002717
1_10000	-0.002287	2_10000	-0.002947

HOMEWORK #1

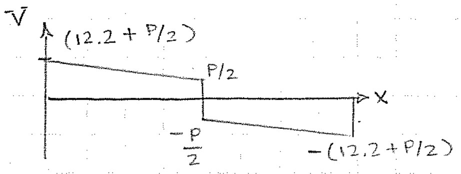
2.



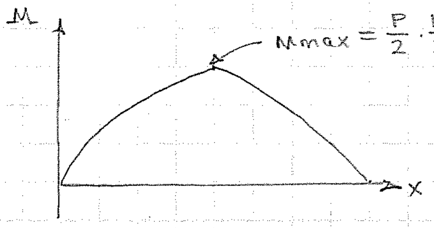


No reinforcement

$f_c' = 4200$ psi
$\omega = 150$ lb/ft



150 lb/ft³ → 24.3 kip/beam



$M_{max} = \frac{P}{2}\cdot\frac{L}{2} + \frac{1}{2}\cdot\frac{L}{2}(12.15) = 54P + 656.1$, in k·in

$$f_{max} = \frac{Mc}{I}$$

Beam tests indicate the tensile limit to be:

$$f_t = 7.5\sqrt{f_c'} = 7.5\sqrt{4200\text{ psi}} = 486\text{ psi}$$

When $f_{max} > f_t$, cracking will occur

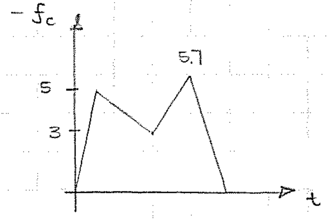
$I = \frac{1}{12} bh^3 = \frac{1}{12}(36\text{ in})(36\text{ in})^3 = 139{,}968\text{ in}^4$
$c = 18$ in

$$486\text{ psi} < \frac{18\text{ in}}{139{,}968\text{ in}^4}\left[54P + 656.1\right]$$

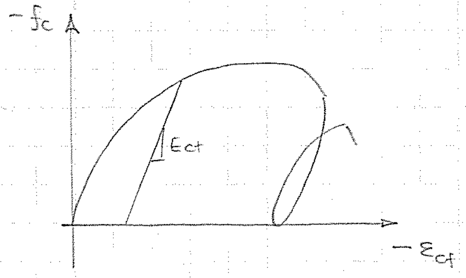
$$\underline{\underline{P > 57.8\text{ kip}}}$$

HOMEWORK #1

3. $f_c' = 6000$ psi



Loading/unloading



$$E_{ct} = \frac{2 f_c'}{\varepsilon_o}$$

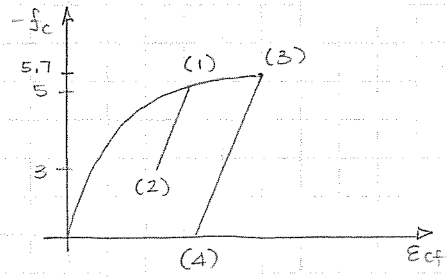
$\varepsilon_o = 2\times10^{-3}$ in/in

$f_c' = 6000$ ksi

$$E_{ct} = \frac{2(6000 \text{ ksi})}{2\times10^{-3}} = 6000 \text{ ksi}$$

$$f_c = -f_c'\left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2\right]$$

let $\varepsilon_c' = \varepsilon_o$



(1): $$-5000\text{psi} = -6000\text{psi}\left[2\left(\frac{\varepsilon_{ct}}{2\times10^{-3}}\right) - \left(\frac{\varepsilon_{cf}}{2\times10^{-3}}\right)^2\right]$$

using Excel,

$\varepsilon_{cf} = -0.00118$

(2): $\Delta f = E_{ct}\,\Delta\varepsilon$

$-(5000\text{psi} - 3000\text{psi}) = E_{ct}(\varepsilon_{cf_1} - \varepsilon_{cf_2})$

$\boxed{\text{a. } \varepsilon_{cf_2} = -0.00085}$

(3)
$$-5700\text{ psi} = -6000\text{psi}\left[2\left(\frac{\varepsilon_{cf}}{2\times10^{-3}}\right) - \left(\frac{\varepsilon_{ct}}{2\times10^{-3}}\right)^2\right]$$

$\boxed{\text{b. } \varepsilon_{cf_3} = -0.00155 \text{ in/in}}$

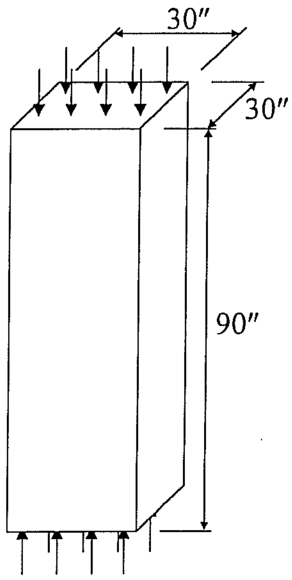
(4)
$$\varepsilon_{cf_4} = \varepsilon_{cf_3} - \frac{-5700\text{ psi}}{E_{ct}} = -0.00155 - \frac{-5700\text{ psi}}{6000\text{ ksi}}$$

$\boxed{\text{c. } \varepsilon_{cf_4} = -0.0006 \text{ in/in}}$

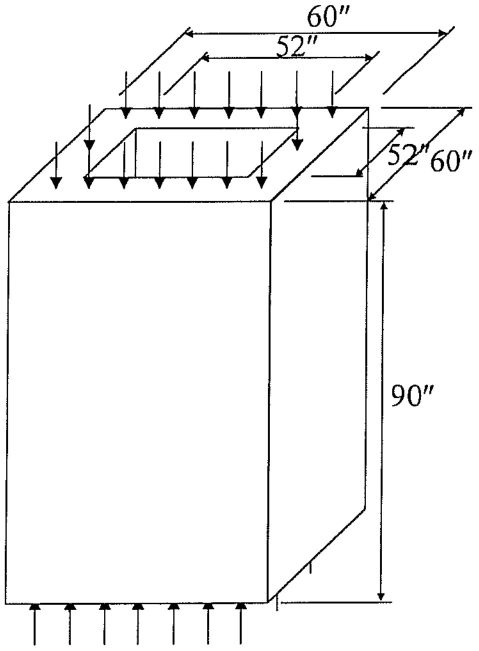
CE 383P – Prestressed Concrete
Homework Assignment # 1

Date assigned : September 10, 2007
Due date : September 14, 2007

Problem 1: Normal density concrete columns shown in the figure below were subjected to a compressive stress of 3000 psi 7 days after casting the concrete. The concrete strength at the time of loading was 7000 psi. Estimate the initial strain under the given loading conditions. Also, estimate the strain at 10 days, 1000 days, and 10,000 days after the loading if the average humidity was 50 %. Comment on your results.



Column 1
(A = 900 in^2)



Column 2
(A = 896 in^2)

Problem 2: A large plain concrete beam has a cross section of 36 x 36 in. The simply supported beam spans 18 ft and is subjected to a point load at midspan. The compressive strength of a standard cylinder cast from the same concrete as the beam was 4200 psi. Estimate the magnitude of the point load required to crack the beam. Assume that the normal density concrete weighs 150 lb/ft^3.

Problem 3: A normal density concrete has a compressive strength of 6000 psi. A compressive stress is applied to the concrete. This stress is increased in value from zero to 5000 psi over a short period of time and is then reduced to 3000 psi.
(i) Calculate the compressive strain in concrete at this point in time.
(ii) The stress is then increased to 5700 psi. Calculate the compressive strain at this point in time. Finally the stress is reduced to zero.
(iii) Calculate the residual compressive strain that remains in the concrete after the compressive stress has been removed.

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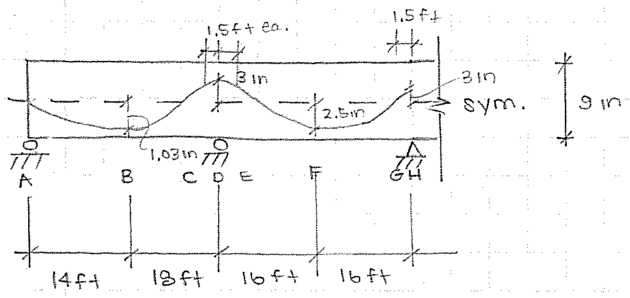
HOMEWORK #2



75% f_{pu}

$\mu = 0.27$
$K = 0.0025 / ft$
$\Delta_{set} = 0.25$ in
$f_{pu} = 270$ ksi
$E_{ps} = 29000$ ksi
$A_{ps} = 1.53$ in^2

$\mu\alpha + Kx$
0.0383

$$\alpha_{AB} = \frac{2(1.03 in)}{(14)(12) in} = 0.0123$$

$$\alpha_{BCD} = \frac{2(1.03 in + 3 in)}{(18)(12) in} = 0.0373$$
BC : 0.05132
CD : 0.01382

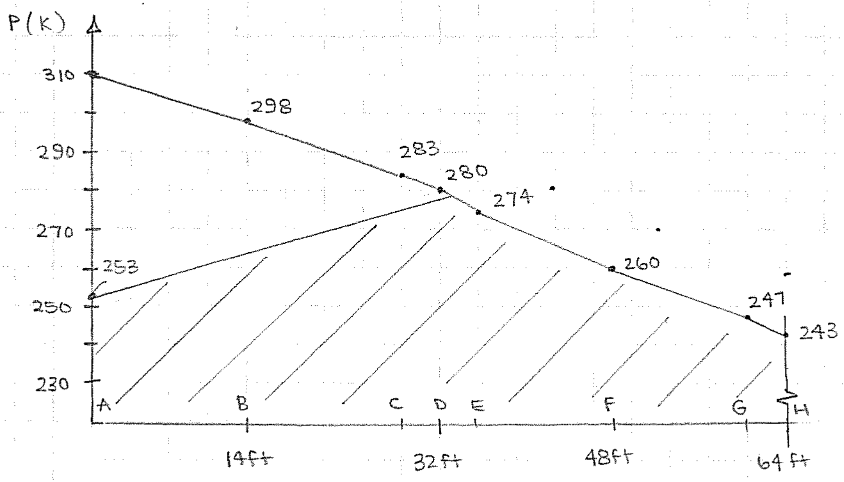
$$\alpha_{DEF} = \frac{2(3 in + 2.5 in)}{(16)(12) in} = 0.0573$$
DE : 0.01922
EF : 0.05172

$$\alpha_{FGH} = \frac{2(2.5 in + 3 in)}{(16)(12) in} = 0.0573$$
FG : 0.05172
GH : 0.01922

	$\Sigma(\mu\alpha + Kx)$	$e^{-\Sigma \dots}$
A =	0	1
B =	0.0383	0.962
C =	0.0896	0.914
D =	0.1034	0.9018
E =	0.12266	0.885
F =	0.17438	0.840
G =	0.2261	0.7976
H =	0.24532	0.7825

Initial force:

$P_{max} = 0.75\ (270 KSI)(1.53 in^2) = 309.8\ K \sim 310\ K$



HOMEWORK #2

Anchorage losses

$$\underline{p}_1 = \frac{310\text{k} - 298\text{k}}{14\text{ft}} = 0.857\text{ k/ft}$$

$$L_{set} = \left[\frac{(0.25\text{in})(1.53\text{in}^2)(29000\text{ KSI})}{0.857\text{k/ft} \cdot 1/12}\right]^{\frac{1}{2}} = 394\text{ in} = 32.8\text{ ft}$$

> 14 ft assumed

$$\underline{p}_2 = \frac{298\text{K} - 283\text{K}}{16.5\text{ ft}} = 0.909\text{ k/ft}$$

$$\underline{p}_{avg} = \frac{\underline{p}_1 + \underline{p}_2}{2} = \frac{1}{2}(0.857\text{K/ft} + 0.909\text{K/ft}) = 0.883\text{ K/ft}$$

$$L_{set} = \left[\frac{(0.25\text{in})(1.53\text{in}^2)(29000\text{KSI})}{1/12\,(0.883\text{ K/ft})}\right]^{\frac{1}{2}} = 388\text{ in} = 32.4\text{ ft}$$

close enough to limit of 30.5 ft

$$\Delta P = 2 L_{set}\,\underline{p} = 2(32.4\text{ft})(0.883\text{K/ft}) = 57.2\text{K}$$

310K − 57.2K = 253K − shown on graph

Average tendon forces:

$$P_{avg_1} = \frac{\frac{1}{2}(310\text{K}+298\text{K})(14\text{ft}) + \frac{1}{2}(298\text{K}+283\text{K})(16.5\text{ft}) + \frac{1}{2}(280\text{k}+283\text{k})(1.5\text{ft})}{32\text{ ft}}$$

$$= 296\text{K}$$

$$P_{avg_2} = \frac{1}{2}\,\frac{(280\text{k}+274\text{k})(1.5\text{ft}) + (274\text{k}+260\text{K})(14.5\text{ft}) + (247\text{k}+260\text{K})(14.5\text{ft}) + (243\text{K}+247\text{K})(1.5\text{ft})}{32\text{ ft}}$$

$$= 260\text{K}$$

Span 1 with losses:

$$P_{avg_{1L}} = \frac{1/2\,(280\text{K}+253\text{k})(32\text{ft})}{32\text{ ft}} = 267\text{K}$$

CE 383P: PRESTRESS

HOMEWORK #3

C. HOVELL
21 Sept 07

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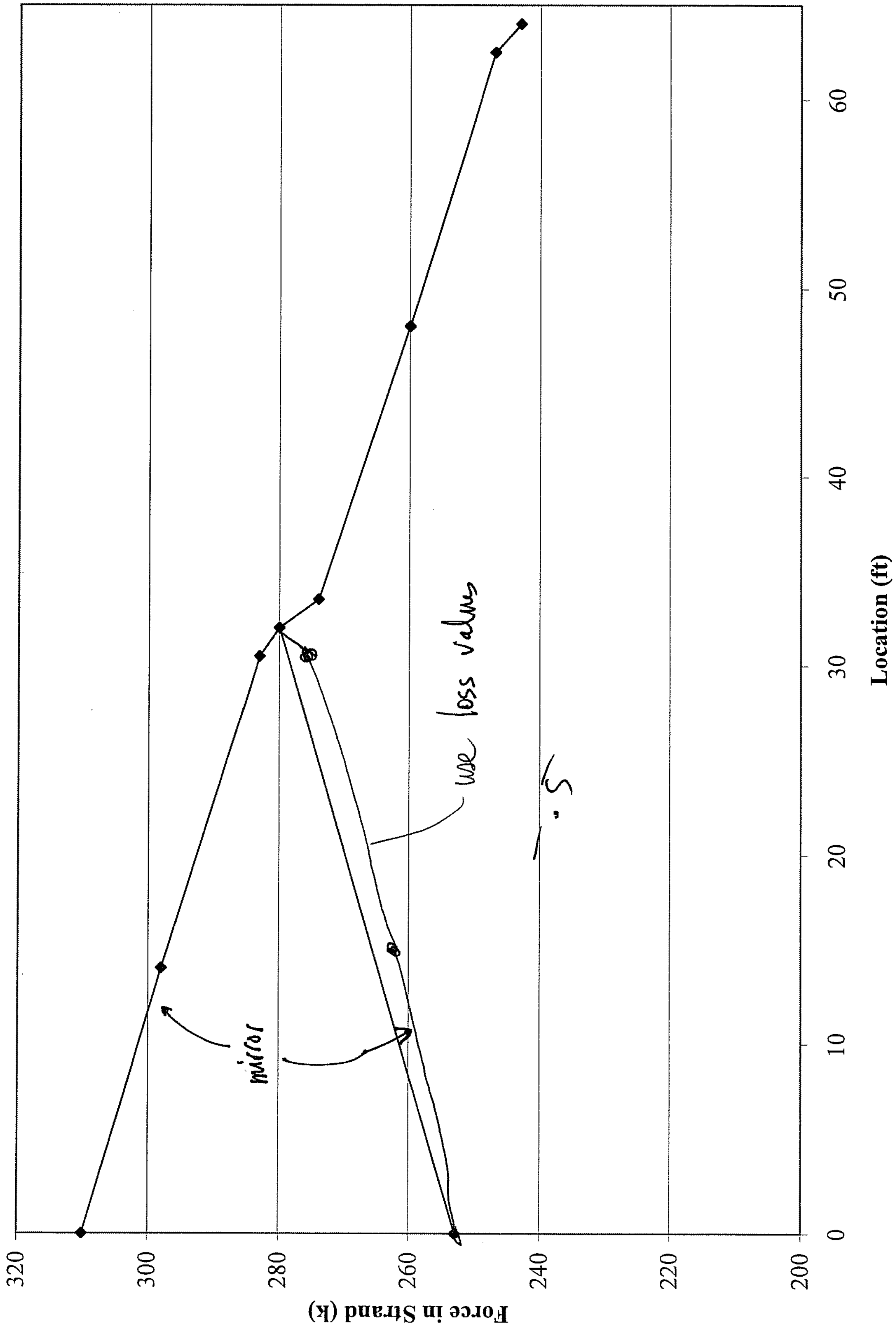
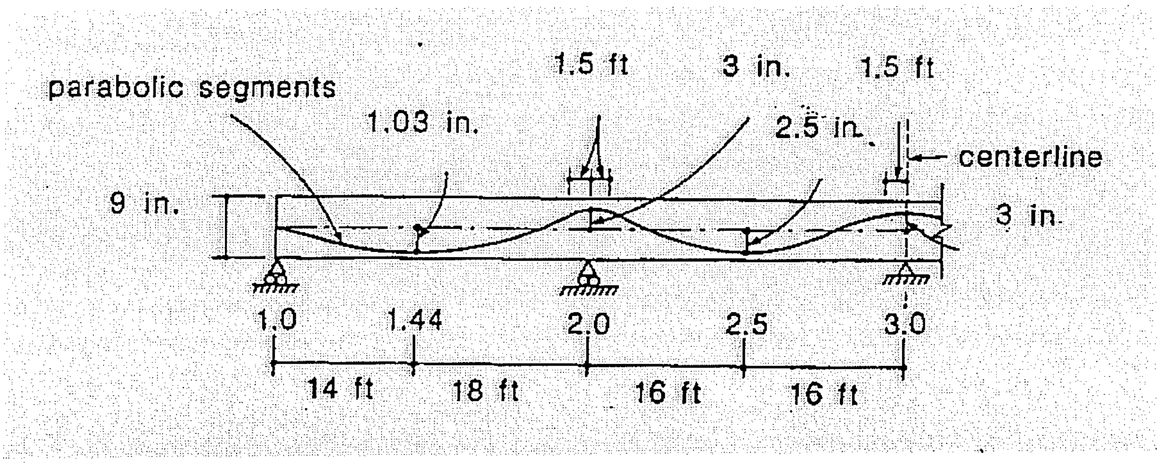


CE 383P – Prestressed Concrete
Homework Assignment # 2

Date assigned : September 17, 2007
Due date : September 21, 2007

Problem 1:

A four span post-tensioned one-way floor slab is shown in the figure below. The tendons are stressed from both ends and the stressing procedure is to jack to 0.75 f_{pu} and then anchor. Assuming that $\mu = 0.27$, K=0.0025 / ft and Δ_{set}=0.25 in, calculate and plot the tendon force variation along the length of the beam. Also calculate the average tendon force in each span. Assume f_{pu}=270 ksi, E_{ps}=29,000 ksi, and A_{ps}=1.53 in^2.



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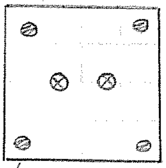
HOMEWORK #3

1.



12in x 12in

$f_c' = 8{,}000$ psi

$\varepsilon_0 = 2.2 \times 10^{-3}$

$A_s = 4 - \#8s = 3.14 \text{ in}^2$

$A_p = 4 - 1/2"$ 7 wire strands $= 0.612 \text{ in}^2$

$f_y = 60$ ksi

$f_{py} = 250$ ksi

$f_{pu} = 270$ ksi

$E_p = 29000$ ksi

$A_c = (12 \text{ in})^2 - 3.14 \text{ in}^2 - 0.612 \text{ in}^2 = 140.2 \text{ in}^2$

$L = 14$ ft

SHORT TERM!

Calculate $\Delta\varepsilon_p$

$$E_{cs} = 57\left[8000 \text{ psi}\right]^{1/2} = 5098 \text{ ksi}$$

$$E_{ct} = \frac{2\,(8000 \text{ psi})}{2.2 \times 10^{-3}} = 7273 \text{ ksi} \quad (90\% = 6545 \text{ ksi})$$

$$\varepsilon_{pi} = \frac{0.70\,(270 \text{ ksi})}{29000 \text{ ksi}} = 6.5 \times 10^{-3}$$

$$F_p = A_p f_p = (0.612 \text{ in}^2)(\underset{\uparrow\ 70\% f_{pu}}{189 \text{ ksi}}) = 115.7 \text{ KIP}$$

Using equilibrium,

$$A_c f_c + A_s f_s + A_p f_p = 0$$

$$(140.2 \text{ in}^2)(7273 \text{ ksi})\varepsilon_{ci} + (3.14 \text{ in}^2)(29000 \text{ ksi})\varepsilon_{ci} + 115.7 \text{ kip} = 0$$

$$\varepsilon_{ci} = -0.104 \times 10^{-3}$$

$$\Delta_i = \varepsilon_{ci} \cdot L = 0.017 \text{ in}$$

$f_{ci} = -0.758$ ksi , $F_{ci} = -106.4$ K

$f_{si} = -3.02$ ksi , $F_{si} = -9.49$ K

check: $\Sigma = 0$ ✓

$$\Delta\varepsilon_p = \varepsilon_{pi} - \varepsilon_{ci}$$

$$= (6.5 \times 10^{-3}) - (-0.104 \times 10^{-3})$$

$$\Delta\varepsilon_p = 6.62 \times 10^{-3}$$

Critical points:

$$\varepsilon_{cr} = \frac{4\,(8000 \text{ psi})^{1/2}}{7273 \text{ ksi}} = 0.05 \times 10^{-3}$$ ✓

$$\varepsilon_{py} = \frac{250 \text{ ksi}}{29000 \text{ ksi}} = 8.62 \times 10^{-3}$$

go to spreadsheet ...

HOMEWORK #3

2. LONG·TERM:

$\phi = 2.1$

$\varepsilon_{sh} = -0.6 \times 10^{-3}$

relax = 3%

$$E_{ceff} = \frac{E_{ct}}{1+\phi} = \frac{7273\text{ ksi}}{1+2.1} = 2346\text{ ksi}$$

$$\varepsilon'_{c_{eff}} = \frac{2f_c'}{E_{ceff}} = \frac{2(8\text{ ksi})}{2346\text{ ksi}} = 6.82 \times 10^{-3}$$

$$E_{Peff} = 0.97 E_P = (0.97)(29000\text{ ksi}) = 28,130\text{ ksi}$$

$$\varepsilon_{py}' = \frac{250\text{ ksi}}{28130\text{ ksi}} = 8.89 \times 10^{-3}$$

$$\varepsilon_{cr} = \frac{4(8000\text{ psi})^{1/2}}{2346\text{ ksi}} = 0.15 \times 10^{-3}$$

make calculations in Excel,
then plot ε·N curves

Attached Excel sheets:

Pg 3 — Excel calculations
data points for short & long·term curves

Pg 4 — ε·N curves

CE 383P: PRESTRESS

C. HOVELL
28 Sept 07

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HOMEWORK #3

Given values and simple calcs

A_c	140.25	in^2	f_c'	8.0	ksi	ε_o	0.0022	
A_s	3.14	in^2	f_y	60	ksi	ε_{sh}	-0.0006	
A_p	0.61	in^2	f_{py}	250	ksi	E_p	29000	ksi
			f_{pu}	270	ksi			

Initial Calculations

E_{ct}	7272.7	ksi	ΔL_i	-0.01749	in
ε_{pi}	0.00652		$\Delta\varepsilon_p$	0.00662	
F_p	115.67	kip			
ε_{ci}	-0.00010		ε_{py}	0.00862	
ε_{cr}	0.00005				

Long-term calculations

$E_{c,eff}$	2346.0	ksi
$\varepsilon_{c,eff}'$	0.00682	
$E_{p,eff}$	28130	ksi
ε_{py}'	0.00889	
ε_{cr}	0.00015	

-.5

SHORT-TERM RESPONSE

fy

	Concrete			Reinforcement			Pretressing				
ε_c	ε_{cf}	f_c ksi	F_c kip	ε_{sf}	f_s ksi	F_s kip	ε_{pf}	f_p ksi	F_p kip	N kip	
-0.003	-0.00300	-6.942	-973.6	-0.00300	-87.00	-273.2	0.00362	105.02	64.3	-1182.5	
-0.00275	-0.00275	-7.500	-1051.9	-0.00275	-79.75	-250.4	0.00387	112.27	68.7	-1233.6	
-0.00255	-0.00255	-7.798	-1093.6	-0.00255	-73.95	-232.2	0.00407	118.07	72.3	-1253.5	
-0.00207	-0.00207	-7.972	-1118.1	-0.00207	-60.03	-188.5	0.00455	131.99	80.8	-1225.8	
-0.00175	-0.00175	-7.665	-1075.0	-0.00175	-50.75	-159.4	0.00487	141.27	86.5	-1147.9	
-0.0015	-0.00150	-7.190	-1008.4	-0.00150	-43.50	-136.6	0.00512	148.52	90.9	-1054.1	
-0.001	-0.00100	-5.620	-788.2	-0.00100	-29.00	-91.1	0.00562	163.02	99.8	-779.5	
-0.0005	-0.00050	-3.223	-452.0	-0.00050	-14.50	-45.5	0.00612	177.52	108.6	-388.9	
0	0	0	0	0	0	0	0.00662	192.02	117.5	117.5	
0.00005	0.00005	0.362	50.7	0.00005	1.43	4.5	0.00667	193.45	118.4	173.6	$\varepsilon_c = \varepsilon_{cr-}$
0.00005	0.00005	0	0	0.00005	1.43	4.5	0.00667	193.45	118.4	122.9	$\varepsilon_c = \varepsilon_{cr+}$
0.00200	0.00200	0	0	0.00200	57.98	182.1	0.008621	250	153.0	335.1	$\varepsilon_p = \varepsilon_{py}$
0.00207	0.00207	0	0	0.00207	60	188.4	0.00869	250	153.0	341.4	$\varepsilon_s = \varepsilon_{sy}$
0.003	0.003	0	0	0.00300	60	188.4	0.00962	250	153.0	341.4	

LONG-TERM RESPONSE

	Concrete			Reinforcement			Pretressing				
ε_c	ε_{cf}	f_c ksi	F_c kip	ε_{sf}	f_s ksi	F_s kip	ε_{pf}	f_p ksi	F_p kip	N kip	
-0.01	-0.00940	-6.855	-961.4	-0.010	-60	-188.4	-0.00338	-95.04	-58.2	-1208.0	
-0.009	-0.00840	-7.571	-1061.8	-0.009	-60	-188.4	-0.00238	-66.91	-40.9	-1291.1	
-0.008	-0.00740	-7.942	-1113.9	-0.008	-60	-188.4	-0.00138	-38.78	-23.7	-1326.0	
-0.007	-0.00640	-7.970	-1117.7	-0.007	-60	-188.4	-0.00038	-10.65	-6.5	-1312.6	
-0.006	-0.00540	-7.653	-1073.3	-0.006	-60	-188.4	0.00062	17.48	10.7	-1251.0	
-0.005	-0.00440	-6.993	-980.7	-0.005	-60	-188.4	0.00162	45.61	27.9	-1141.2	
-0.004	-0.00340	-5.988	-839.8	-0.004	-60	-188.4	0.00262	73.74	45.1	-983.1	
-0.003	-0.00240	-4.640	-650.7	-0.003	-60	-188.4	0.00362	101.87	62.3	-776.8	
-0.002	-0.00140	-2.947	-413.4	-0.002	-58.00	-182.1	0.00462	130.00	79.6	-515.9	
-0.00045	0.00015	0.362	50.7	-0.00045	-12.98	-40.7	0.00617	173.67	106.3	116.3	$\varepsilon_c = \varepsilon_{cr-}$
-0.00045	0.00015	0	0	-0.00045	-12.98	-40.7	0.00617	173.67	106.3	65.5	$\varepsilon_c = \varepsilon_{cr+}$
0	0.00060	0	0	0	0	0	0.00662	186.26	114.0	114.0	
0.00207	0.00267	0	0	0.00207	60	188.4	0.00869	244.49	149.6	338.0	$\varepsilon_s = \varepsilon_{sy}$
0.00227	0.00287	0	0	0.00227	60	188.4	0.00889	250	153.0	341.4	$\varepsilon_p = \varepsilon_{py}$
0.003	0.00360	0	0	0.00300	60	188.4	0.00962	250	153.0	341.4	

CE 383P: PRESTRESS

HOMEWORK #3

C.HOVELL
28 Sept 07

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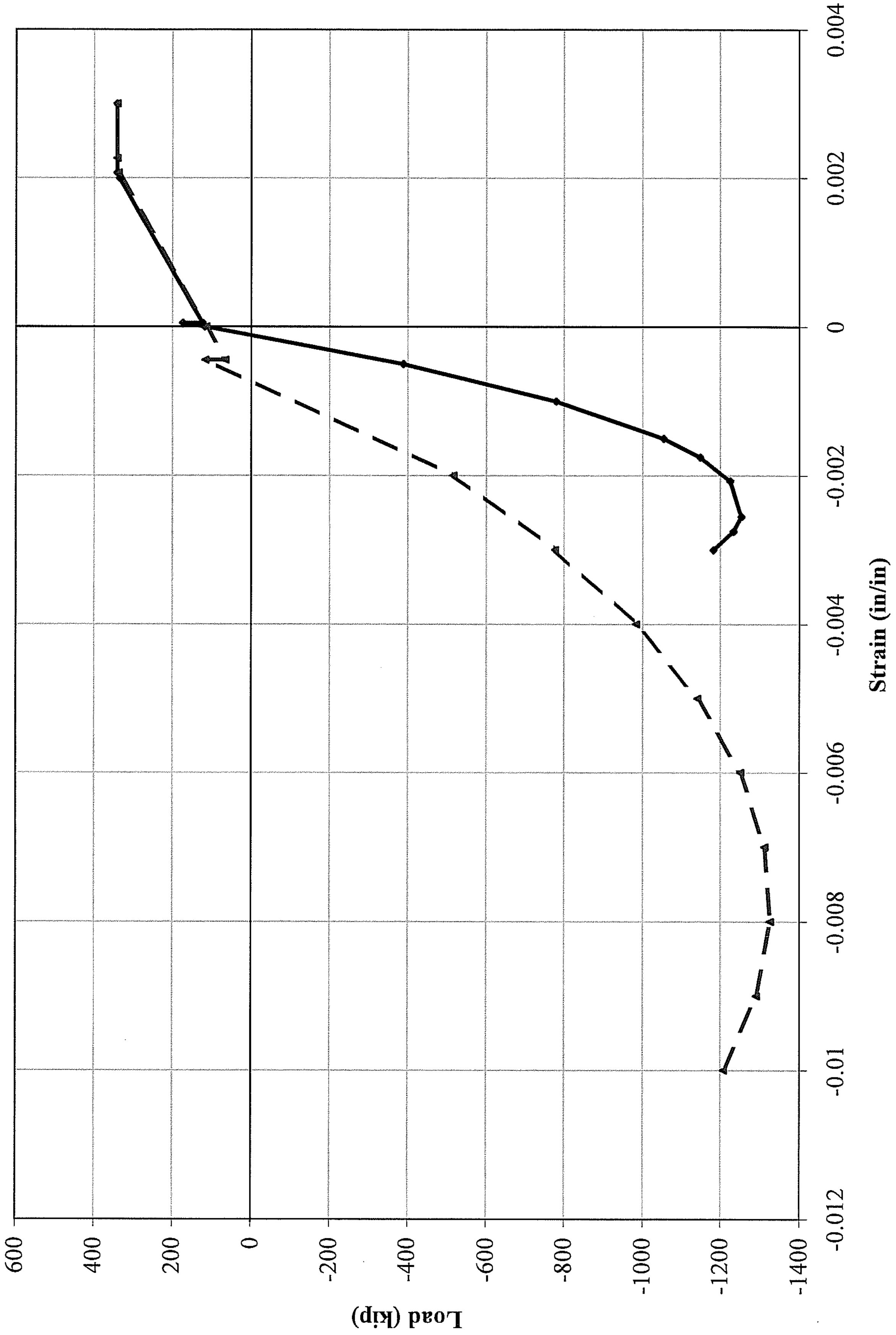


CE 383P – Prestressed Concrete

Homework Assignment #3

Date Assigned: September 24, 2007

Due Date: October 1, 2007

Problem 1:

A partially prestressed post-tensioned concrete member, 12 x 12 in. in cross section and 14 ft. in length, has the material properties shown. The low relaxation strands are stressed to $0.70f_{pu}$ and anchored. Assuming no friction or set losses, determine the short-term axial load-deformation response.

$f_c' = 8000$ psi

$\varepsilon_0 = 2.2 \times 10^{-3}$

A_s = 4-#8 bars

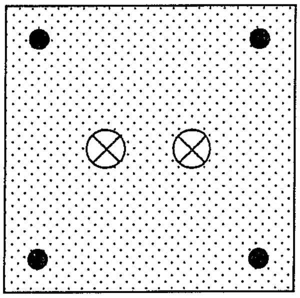
A_p = 4- ½" 7 wire strands

$f_y = 60$ ksi

$f_{py} = 250$ ksi

$f_{pu} = 270$ ksi

$E_p = 29000$ ksi



Problem 2:

Determine the long-term load-deformation response of the member above. Make allowances for creep and shrinkage of concrete and relaxation of the prestressing.

Note $\phi = 2.1$, $\varepsilon_{sh} = -0.6 \times 10^{-3}$ and relaxation in strands = 3%.

CE383P: PRESTRESS

CATHERINE HOVELL
08 october 2007

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HOMEWORK #4

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Given values and simple calcs

A_c	140.25	in^2	f_c'	8.0	ksi	ε_o	0.0022	
A_s	3.14	in^2	f_y	60	ksi	ε_{sh}	-0.0006	
A_p	0.61	in^2	f_{py}	250	ksi	E_p	29000	ksi
			f_{pu}	270	ksi			

Initial Calculations

E_{ct}	7272.7	ksi	ΔL_i	-0.01749	in
ε_{pi}	0.00652		$\Delta\varepsilon_p$	0.00662	
F_p	115.67	kip			
ε_{ci}	-0.00010		ε_{py}	0.00862	
ε_{cr}	0.00005				

Long-term calculations

$E_{c,eff}$	2346.0	ksi
$\varepsilon_{c,eff}'$	0.00682	
$E_{p,eff}$	28130	ksi
ε_{py}'	0.00889	
ε_{cr}	0.00015	

Tension Stiffening Constants

α_1	1.0	for deformed bars
α_1	0.7	for strands
$\alpha_{1weighted}$	0.95	
α_2	1.0	for short-term loading
α_2	0.7	for long-term loading
f_t'	357.8	psi

SHORT-TERM RESPONSE

	Concrete			Reinforcement			Pretressing				
ε_c	ε_{cf}	f_c ksi	F_c kip	ε_{sf}	f_s ksi	F_s kip	ε_{pf}	f_p ksi	F_p kip	N kip	
-0.003	-0.00300	-6.942	-973.6	-0.00300	-87.00	-273.2	0.00362	105.02	64.3	-1182.5	
-0.00275	-0.00275	-7.500	-1051.9	-0.00275	-79.75	-250.4	0.00387	112.27	68.7	-1233.6	
-0.00255	-0.00255	-7.798	-1093.6	-0.00255	-73.95	-232.2	0.00407	118.07	72.3	-1253.5	
-0.00207	-0.00207	-7.972	-1118.1	-0.00207	-60.03	-188.5	0.00455	131.99	80.8	-1225.8	
-0.00175	-0.00175	-7.665	-1075.0	-0.00175	-50.75	-159.4	0.00487	141.27	86.5	-1147.9	
-0.0015	-0.00150	-7.190	-1008.4	-0.00150	-43.50	-136.6	0.00512	148.52	90.9	-1054.1	
-0.001	-0.00100	-5.620	-788.2	-0.00100	-29.00	-91.1	0.00562	163.02	99.8	-779.5	
-0.0005	-0.00050	-3.223	-452.0	-0.00050	-14.50	-45.5	0.00612	177.52	108.6	-388.9	
0	0	0	0	0	0	0	0.00662	192.02	117.5	117.5	
0.00005	0.00005	0.362	50.7	0.00005	1.43	4.5	0.00667	193.45	118.4	173.6	$\varepsilon_c = \varepsilon_{cr-}$
0.00005	0.00005	0.294	41.3	0.00005	1.43	4.5	0.00667	193.45	118.4	164.1	$\varepsilon_c = \varepsilon_{cr+}$
0.00183	0.00183	0.174	24.4	0.00183	53.17	167.0	0.00845	245.19	150.1	341.4	
0.00200	0.00200	0.045	6.3	0.00200	57.98	182.1	0.008621	250	153.0	341.4	$\varepsilon_p = \varepsilon_{py}$
0.00207	0.00207	0	0.0	0.00207	60	188.4	0.00869	250	153.0	341.4	$\varepsilon_s = \varepsilon_{sy}$
0.003	0.003	0	0	0.00300	60	188.4	0.00962	250	153.0	341.4	

LONG-TERM RESPONSE

	Concrete			Reinforcement			Pretressing				
ε_c	ε_{cf}	f_c ksi	F_c kip	ε_{sf}	f_s ksi	F_s kip	ε_{pf}	f_p ksi	F_p kip	N kip	
-0.01	-0.00940	-6.855	-961.4	-0.010	-60	-188.4	-0.00338	-95.04	-58.2	-1208.0	
-0.009	-0.00840	-7.571	-1061.8	-0.009	-60	-188.4	-0.00238	-66.91	-40.9	-1291.1	
-0.008	-0.00740	-7.942	-1113.9	-0.008	-60	-188.4	-0.00138	-38.78	-23.7	-1326.0	
-0.007	-0.00640	-7.970	-1117.7	-0.007	-60	-188.4	-0.00038	-10.65	-6.5	-1312.6	
-0.006	-0.00540	-7.653	-1073.3	-0.006	-60	-188.4	0.00062	17.48	10.7	-1251.0	
-0.005	-0.00440	-6.993	-980.7	-0.005	-60	-188.4	0.00162	45.61	27.9	-1141.2	
-0.004	-0.00340	-5.988	-839.8	-0.004	-60	-188.4	0.00262	73.74	45.1	-983.1	
-0.003	-0.00240	-4.640	-650.7	-0.003	-60	-188.4	0.00362	101.87	62.3	-776.8	
-0.002	-0.00140	-2.947	-413.4	-0.002	-58.00	-182.1	0.00462	130.00	79.6	-515.9	
-0.00045	0.00015	0.362	50.7	-0.00045	-12.98	-40.7	0.00617	173.67	106.3	116.3	$\varepsilon_c = \varepsilon_{cr-}$
-0.00045	0.00015	0.187	26.2	-0.00045	-12.98	-40.7	0.00617	173.67	106.3	91.7	$\varepsilon_c = \varepsilon_{cr+}$
0.00000	0.00060	0.154	21.6	0	0	0	0.00662	186.26	114.0	135.6	
0.00196	0.00256	0.112	15.7	0.0020	56.71	178.1	0.00858	241.27	147.7	341.4	
0.00207	0.00267	0.024	3.4	0.00207	60	188.4	0.00869	244.49	149.6	341.4	$\varepsilon_s = \varepsilon_{sy}$
0.00227	0.00287	0	0.0	0.00227	60	188.4	0.00889	250	153.0	341.4	$\varepsilon_p = \varepsilon_{py}$
0.003	0.00360	0	0	0.00300	60	188.4	0.00962	250	153.0	341.4	

, CE383P: PRESTRESS

HOMEWORK #4

C. HOVELL
08 OCT. 07

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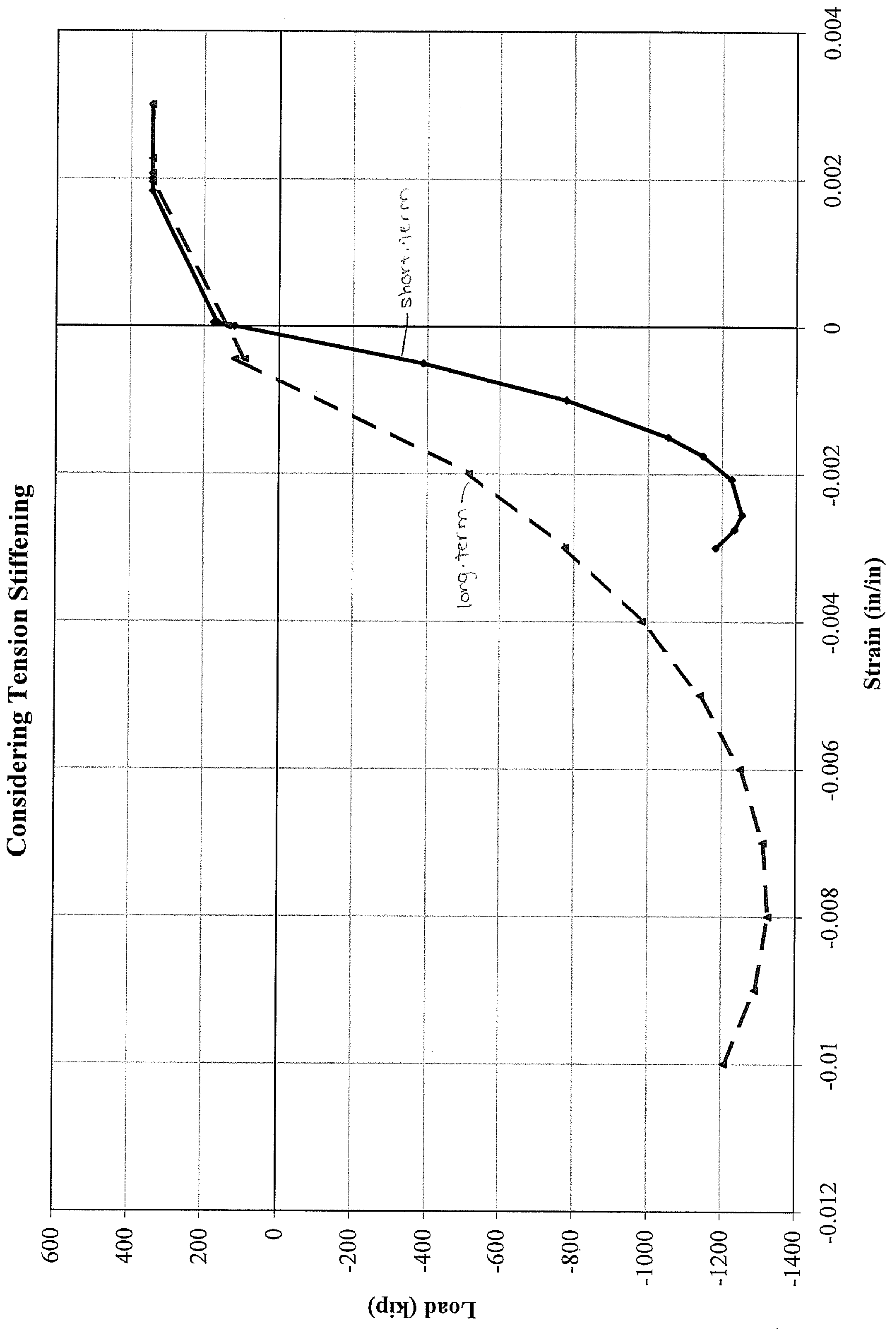

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HOMEWORK #4

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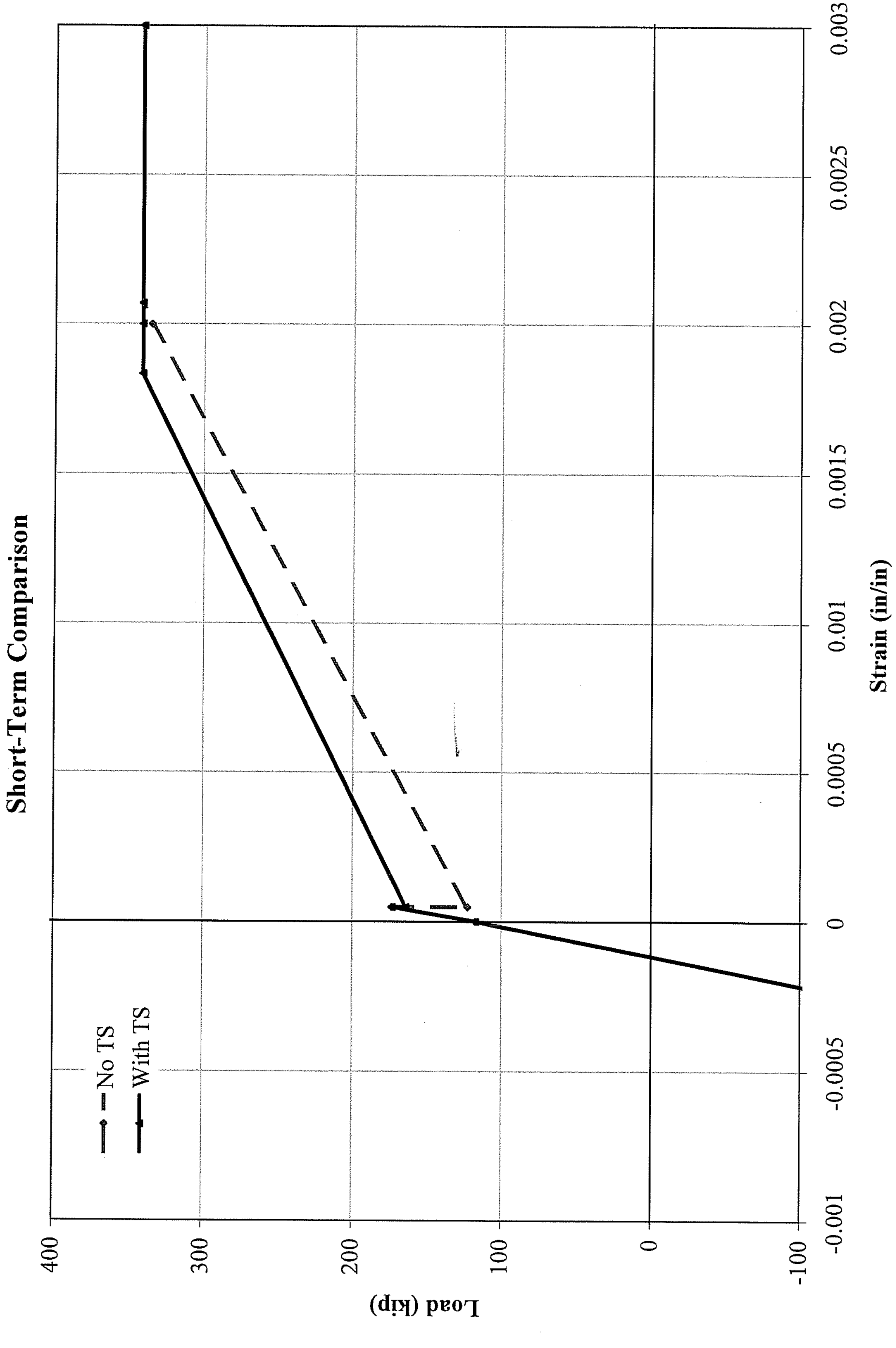

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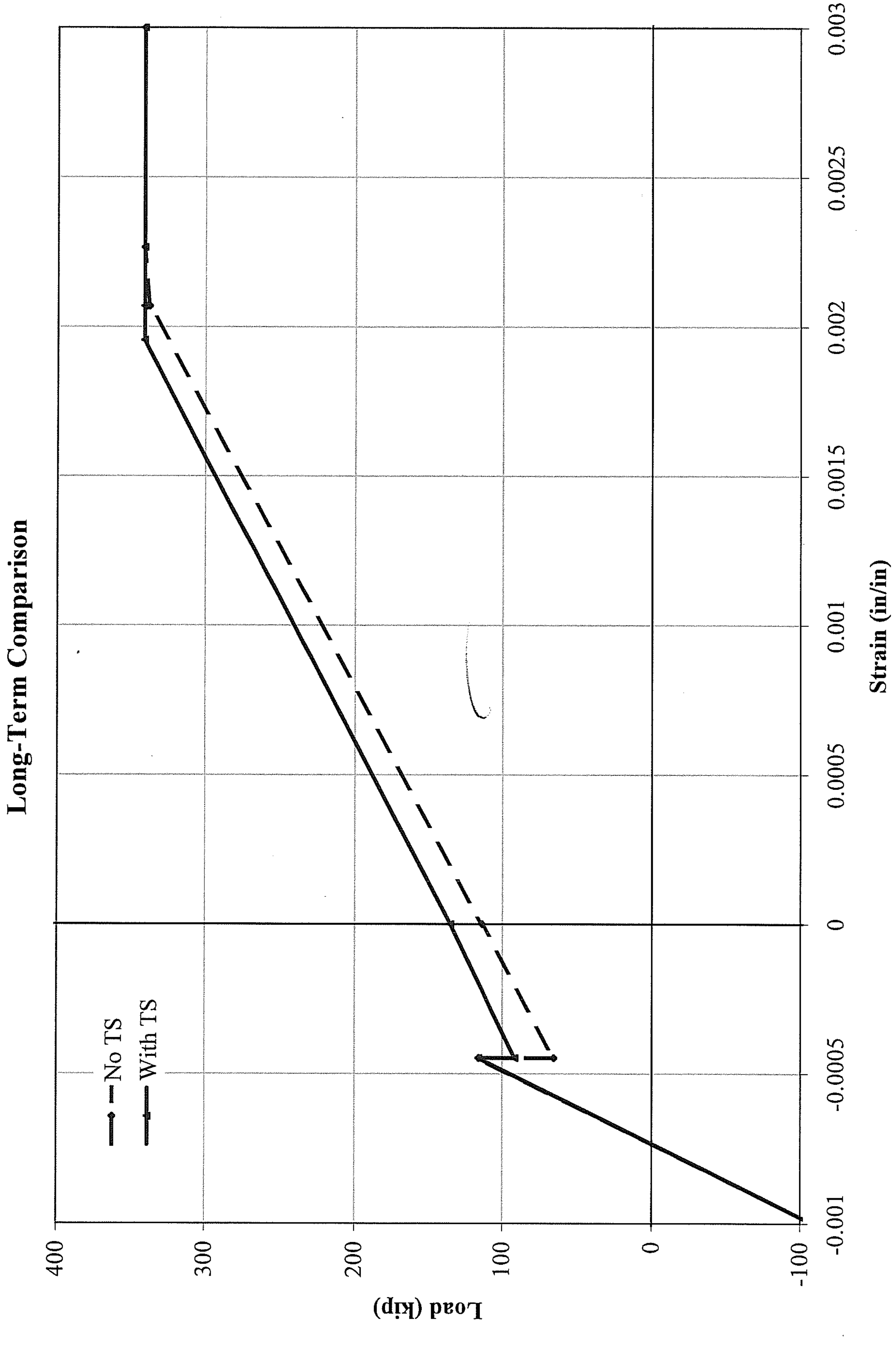

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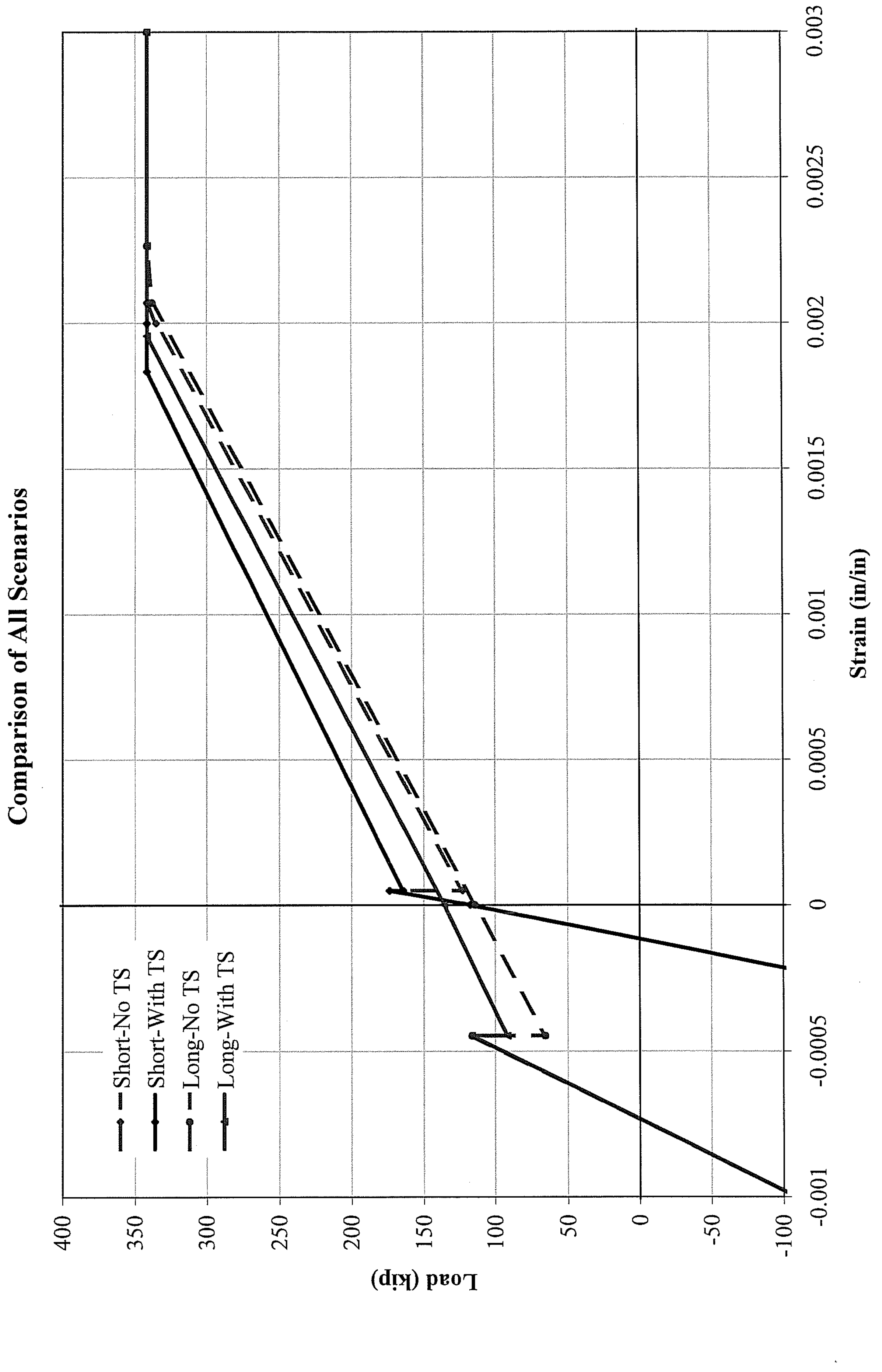

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HOMEWORK #4

Assignment comparison

Major comments:

- tension stiffening (TS) reduces drop in capacity experienced at point of concrete cracking
- effect of TS is greater in the short-term response
- TS increases difference between short and long-term post-cracking behavior
- as dictated by calculations, total capacity does not change

Response:

It is more important to consider tension stiffening for short-term serviceability calculations. If service loads are expected to be near Pcr, tension stiffening should be included for accurate predictive calculations.

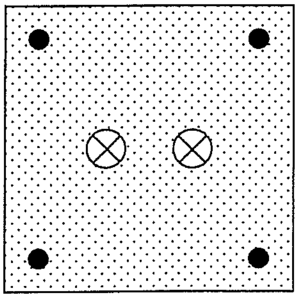
CE 383P – Prestressed Concrete
Homework Assignment No.4

Date Assigned: October 1, 2007
Due Date: October 8, 2007

Problem 1:

You have already determined the short-term and long-term responses for the partially prestressed post-tensioned concrete member shown below. (tension stiffening was ignored). As you know, the member is 12 x 12 in. in cross section and 14 ft. in length, has the material properties shown. The low relaxation strands are stressed to $0.70f_{pu}$ and anchored. Assuming no friction losses or set losses, determine the short-term and long-term axial load-deformation responses taking tension stiffening into account.

$f_c' = 8000$ psi
$\varepsilon_0 = 2.2 \times 10^{-3}$
A_s = 4-#8 bars
A_p = 4- ½" 7 wire strands
$f_y = 60$ ksi
$f_{py} = 250$ ksi
$f_{pu} = 270$ ksi
$E_p = 29000$ ksi

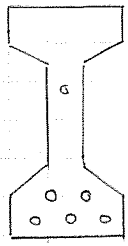


Problem 2:

Compare the results from Assignments No.3 and No. 4. You can plot short-term curves in one graph and long-term curves in another one and use these to come up with your comparative comments.

HOMEWORK #5

$\frac{9.2}{10}$



$f_c' = 7000\ psi$

$\varepsilon_c' = -2.5\times10^{-3}$

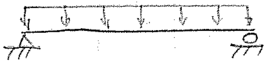
$A_p = 6\text{-}3/8"$, $A_{2p} = 0.17\ in^2$, $A_{1p} = 0.085\ in^2$

$A_{3p} = 0.255\ in^2$

$f_{pu} = 270\ ksi$

$\Delta\varepsilon_p = 6.1\times10^{-3}$

$A = 0.025,\ B = 118,\ C = 10$



$L = 18\ ft$

$w_D = 1.0\ k/ft + SW$

$\quad \llcorner\ 0.150\ k/ft^3\ (102\ in^2)$

ignore strands

$= 1.106\ k/ft$

$w_L = 2.0\ k/ft$

$$E_{ct} = \frac{2(7000\ psi)}{-2.5\times10^{-3}} = 5600\ ksi$$

$$E_{ceff} = \frac{E_{ct}}{1+\phi} = \frac{5600\ ksi}{1+2.5} = 1600\ ksi$$

$$\varepsilon_{c\,eff}' = \frac{2(7000\ psi)}{1600\ ksi} = -0.00875$$

$E_{peff} = 0.95(29000\ ksi) = 27550\ ksi$, $\varepsilon_{rupt} = 0.054$

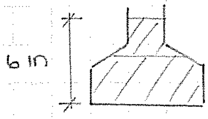
$M_{max} = \frac{wL}{4}$, $w = 3.106\ k/ft$ unfactored

$w_\phi = 1.2(1.106\ k/ft) + 1.6(2.0\ k/ft) = 4.53\ k/ft$ factored

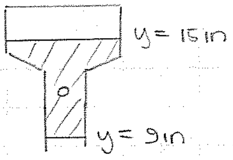
$M_{max} = \frac{18\ ft}{4}\cdot w = 14\ k\cdot ft$ unfactored $= 168\ k\cdot in$

$20.4\ k\cdot ft$ factored $= 245\ k\cdot in$

when T/S is included, $7.5d_b = 7.5(3/8\ in) = 2.81$ or $3\ in$ ✓



$\alpha_1\alpha_2 = (0.7)(1.0) = 0.7$



HOMEWORK #5

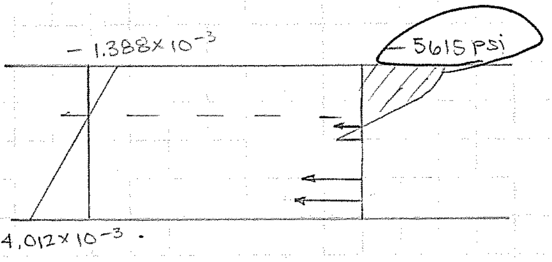
1. (cont'd)

$M_{max} = 141.7 \text{ k·ft}$

$90\% M_{max} = 127.5 \text{ k·ft}$ ✓

$\phi (M = 128.1 \text{ k·ft}) = 300 \times 10^{-6} / \text{in}$

At crack – no T/S:

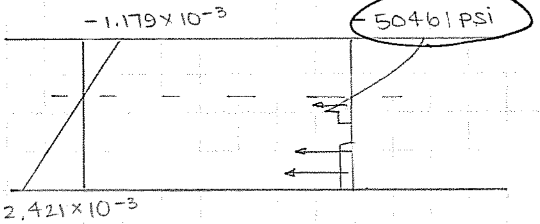


~~4700 psi~~
5400 psi

$\varepsilon_{cen} = 0.412 \times 10^{-3}$ tens.

-.5

Average – with T/S:



4700 psi

$\varepsilon_{cen} = 0.021 \times 10^{-3}$ tens.

$\phi (M = 128.1 \text{ k·ft}) = 200 \times 10^{-6} / \text{in}$

Your section appears to be a bit more flexible.

→ what did you use for tensile strength of concrete?

↳ shouldn't affect the local stress/strain conditions

→ check geometry of input section

→ try finer analysis

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Short-Term Moment-Curvature Response

```
RESPONSE          OUTPUT      FILE
ANALYSIS:         Axial-Load
SECTION           NAME        :           HW5c
CONCRETE          MODEL:      Parabolic   Material    Factors:     C:0.600    S:0.850    P:0.900
TENSION-STIFF.:   No          FACTORED:   No          ACCURACY:    well_done
Axial-Load:       0.00kip     Moment:     0.00kft     Shear:       0.00kip
                  dN/dM:                0 dN/dV:                 0 dM/dV:               0
                  Axial-Load  Moment      Curvature   @-Axis       Bottom     Top        Iter.
                  kips        ft*kip      rad/10^6in  [--------Milli-Strain------]
                          0       37.81           0       -0.181     -0.181     -0.181        1
                      -0.01       68.67          20       -0.247     -0.007     -0.367        2
                       0.07       89.09          40       -0.284      0.196     -0.524        3
                          0       93.09          60       -0.253      0.467     -0.613        4
                          0       98.02          80       -0.218      0.742     -0.698        5
                        0.1      102.33         100       -0.175      1.025     -0.775        6
                       0.07      106.26         120       -0.128      1.312     -0.848        7
                       0.05      109.87         140       -0.078      1.602     -0.918        8
                       0.03      113.17         160       -0.025      1.895     -0.985        9
                       0.02      116.18         180        0.031      2.191     -1.049       10
                      -0.02      118.88         200        0.089      2.489     -1.111       11
                       0.02      121.28         220        0.149      2.789     -1.171       12
                          0      123.38         240        0.212      3.092     -1.228       13
                      -0.01       125.2         260        0.276      3.396     -1.284       14
                      -0.01      126.76         280        0.343      3.703     -1.337       15
90% Mmax              -0.02      128.08         300        0.412      4.012     -1.388       16
                      -0.02      129.21         320        0.482      4.322     -1.438       17
                      -0.02      130.18         340        0.554      4.634     -1.486       18
                      -0.02         131         360        0.627      4.947     -1.533       19
                      -0.02      131.72         380          0.7       5.26      -1.58       20
                      -0.02      132.35         400        0.775      5.575     -1.625       21
                      -0.02      132.91         420         0.85       5.89      -1.67       22
                      -0.02      133.41         440        0.926      6.206     -1.714       23
                      -0.03      133.86         460        1.002      6.522     -1.758       24
                      -0.02      134.28         480        1.079      6.839     -1.801       25
                      -0.02      134.66         500        1.156      7.156     -1.844       26
                      -0.02      135.02         520        1.233      7.473     -1.887       27
                      -0.02      135.36         540         1.31       7.79      -1.93       28
                      -0.02      135.67         560        1.388      8.108     -1.972       29
                      -0.02      135.97         580        1.465      8.425     -2.015       30
                      -0.02      136.26         600        1.543      8.743     -2.057       31
                      -0.02      136.53         620        1.621      9.061     -2.099       32
                      -0.02       136.8         640          1.7       9.38      -2.14       33
                      -0.02      137.05         660        1.778      9.698     -2.182       34
                      -0.02      137.29         680        1.856     10.016     -2.224       35
                      -0.02      137.53         700        1.934     10.334     -2.266       36
                      -0.02      137.75         720        2.013     10.653     -2.307       37
                      -0.02      137.97         740        2.091     10.971     -2.349       38
                      -0.02      138.18         760        2.169     11.289     -2.391       39
                      -0.02      138.39         780        2.248     11.608     -2.432       40
                      -0.02      138.58         800        2.326     11.926     -2.474       41
                      -0.02      138.78         820        2.404     12.244     -2.516       42
                      -0.02      138.96         840        2.482     12.562     -2.558       43
                      -0.02      139.14         860         2.56      12.88       -2.6       44
                      -0.02      139.31         880        2.638     13.198     -2.642       45
                      -0.02      139.48         900        2.715     13.515     -2.685       46
```

-0.02	139.64	920	2.793	13.833	-2.727	47
-0.02	139.8	940	2.87	14.15	-2.77	48
-0.02	139.95	960	2.947	14.467	-2.813	49
-0.02	140.09	980	3.024	14.784	-2.856	50
-0.03	140.24	1000	3.1	15.1	-2.9	51
-0.02	140.37	1020	3.176	15.416	-2.944	52
-0.02	140.5	1040	3.252	15.732	-2.988	53
-0.02	140.62	1060	3.327	16.047	-3.033	54
-0.02	140.74	1080	3.402	16.362	-3.078	55
-0.02	140.85	1100	3.477	16.677	-3.123	56
-0.02	140.96	1120	3.55	16.99	-3.17	57
-0.02	141.06	1140	3.624	17.304	-3.216	58
-0.02	141.16	1160	3.696	17.616	-3.264	59
-0.02	141.25	1180	3.768	17.928	-3.312	60
-0.02	141.33	1200	3.839	18.239	-3.361	61
-0.03	141.41	1220	3.91	18.55	-3.41	62
-0.03	141.48	1240	3.979	18.859	-3.461	63
-0.03	141.54	1260	4.047	19.167	-3.513	64
-0.03	141.6	1280	4.114	19.474	-3.566	65
-0.03	141.64	1300	4.179	19.779	-3.621	66
-0.03	141.68	1320	4.243	20.083	-3.677	67
-0.03	141.7	1340	4.305	20.385	-3.735	68
-0.03	141.71	1360	4.365	20.685	-3.795	69
-0.03	141.71	1380	4.422	20.982	-3.858	70
-0.03	141.69	1400	4.477	21.277	-3.923	71
-0.04	141.64	1420	4.527	21.567	-3.993	72
-0.04	141.57	1440	4.573	21.853	-4.067	73
-0.04	141.46	1460	4.613	22.133	-4.147	74
-0.03	141.31	1480	4.644	22.404	-4.236	75
-0.02	141.07	1500	4.662	22.662	-4.338	76
0.05	140.69	1520	4.662	22.902	-4.458	77
0.08	139.81	1540	4.573	23.053	-4.667	78

0	37.81	0	-0.181	-0.181	-0.181	1
-0.09	-8.6	-30	-0.094	-0.454	0.086	2
-0.02	-12.66	-60	0.14	-0.58	0.5	3
-0.06	-15.63	-90	0.402	-0.678	0.942	4
-0.03	-17.49	-120	0.679	-0.761	1.399	5
-0.01	-18.9	-150	0.964	-0.836	1.864	6
0	-20.06	-180	1.254	-0.906	2.334	7
0	-21.05	-210	1.548	-0.972	2.808	8
0.06	-21.87	-240	1.845	-1.035	3.285	9
0	-22.68	-270	2.144	-1.096	3.764	10
0	-23.35	-300	2.444	-1.156	4.244	11
-0.01	-23.94	-330	2.745	-1.215	4.725	12
0	-24.45	-360	3.048	-1.272	5.208	13
0	-24.89	-390	3.352	-1.328	5.692	14
0	-25.27	-420	3.657	-1.383	6.177	15
0	-25.6	-450	3.963	-1.437	6.663	16
0	-25.9	-480	4.27	-1.49	7.15	17
0	-26.17	-510	4.577	-1.543	7.637	18
0	-26.41	-540	4.885	-1.595	8.125	19
0	-26.63	-570	5.194	-1.646	8.614	20
0	-26.83	-600	5.503	-1.697	9.103	21
0	-27.02	-630	5.813	-1.747	9.593	22
0.07	-27.14	-660	6.124	-1.796	10.084	23
-0.03	-27.39	-690	6.433	-1.847	10.573	24
0	-27.52	-720	6.744	-1.896	11.064	25

0	-27.67	-750	7.055	-1.945	11.555	26
0	-27.81	-780	7.366	-1.994	12.046	27
0	-27.94	-810	7.677	-2.043	12.537	28
0	-28.07	-840	7.989	-2.091	13.029	29
0	-28.19	-870	8.3	-2.14	13.52	30
0	-28.3	-900	8.612	-2.188	14.012	31
0	-28.41	-930	8.923	-2.237	14.503	32
0	-28.52	-960	9.235	-2.285	14.995	33
0	-28.62	-990	9.546	-2.334	15.486	34
0	-28.72	-1020	9.857	-2.383	15.977	35
0	-28.82	-1050	10.169	-2.431	16.469	36
0	-28.91	-1080	10.48	-2.48	16.96	37
0	-28.99	-1110	10.791	-2.529	17.451	38
0	-29.08	-1140	11.102	-2.578	17.942	39
0	-29.15	-1170	11.413	-2.627	18.433	40
0	-29.23	-1200	11.723	-2.677	18.923	41
0	-29.3	-1230	12.034	-2.726	19.414	42
0	-29.37	-1260	12.344	-2.776	19.904	43
0	-29.43	-1290	12.654	-2.826	20.394	44
0	-29.49	-1320	12.963	-2.877	20.883	45
-0.01	-29.55	-1350	13.272	-2.928	21.372	46
-0.01	-29.6	-1380	13.581	-2.979	21.861	47
-0.01	-29.65	-1410	13.889	-3.031	22.349	48
-0.01	-29.7	-1440	14.197	-3.083	22.837	49
-0.01	-29.74	-1470	14.505	-3.135	23.325	50
-0.01	-29.78	-1500	14.812	-3.188	23.812	51
-0.01	-29.81	-1530	15.118	-3.242	24.298	52
-0.01	-29.84	-1560	15.424	-3.296	24.784	53
-0.01	-29.87	-1590	15.729	-3.351	25.269	54
-0.01	-29.89	-1620	16.033	-3.407	25.753	55
-0.01	-29.9	-1650	16.336	-3.464	26.236	56
-0.01	-29.91	-1680	16.639	-3.521	26.719	57
-0.02	-29.92	-1710	16.94	-3.58	27.2	58
-0.02	-29.92	-1740	17.24	-3.64	27.68	59
-0.02	-29.91	-1770	17.539	-3.701	28.159	60
-0.02	-29.89	-1800	17.837	-3.763	28.637	61
-0.02	-29.87	-1830	18.133	-3.827	29.113	62
-0.02	-29.84	-1860	18.428	-3.892	29.588	63
-0.03	-29.8	-1890	18.72	-3.96	30.06	64
-0.03	-29.75	-1920	19.011	-4.029	30.531	65
-0.03	-29.69	-1950	19.298	-4.102	30.998	66
-0.03	-29.62	-1980	19.583	-4.177	31.463	67
-0.04	-29.53	-2010	19.864	-4.256	31.924	68
-0.04	-29.42	-2040	20.14	-4.34	32.38	69
-0.05	-29.29	-2070	20.411	-4.429	32.831	70
-0.05	-29.12	-2100	20.675	-4.525	33.275	71
-0.05	-28.91	-2130	20.93	-4.63	33.71	72
-0.06	-28.63	-2160	21.169	-4.751	34.129	73
-0.02	-28.22	-2190	21.389	-4.891	34.529	74
-0.01	-27.61	-2220	21.562	-5.078	34.882	75
0	-26.98	-2250	21.723	-5.277	35.223	76
0	-26.39	-2280	21.886	-5.474	35.566	77
0	-25.83	-2310	22.052	-5.668	35.912	78
0.01	-25.32	-2340	22.221	-5.859	36.261	79
0	-24.84	-2370	22.39	-6.05	36.61	80
0.01	-24.38	-2400	22.563	-6.237	36.963	81
0.01	-23.96	-2430	22.736	-6.424	37.316	82
0.01	-23.56	-2460	22.912	-6.608	37.672	83

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```
                 0.01      -23.19     -2490      23.089    -6.791    38.029       84
                 0.01      -22.85     -2520      23.268    -6.972    38.388       85
                 0.01      -22.53     -2550      23.449    -7.151    38.749       86
                 0.01      -22.23     -2580      23.632    -7.328    39.112       87
                 0.01      -21.95     -2610      23.816    -7.504    39.476       88
                 0.01       -21.7     -2640      24.003    -7.677    39.843       89
                 0.03      -21.46     -2670      24.194    -7.846    40.214       90
                 0.01      -21.25     -2700      24.383    -8.017    40.583       91
                 0.02      -21.05     -2730      24.577    -8.183    40.957       92
                 0.01      -20.88     -2760      24.774    -8.346    41.334       93
                 0.02      -20.71     -2790      24.973    -8.507    41.713       94
                 0.01      -20.57     -2820      25.175    -8.665    42.095       95
                 0.01      -20.44     -2850       25.38     -8.82     42.48       96
                 0.01      -20.32     -2880      25.588    -8.972    42.868       97
                 0.01      -20.22     -2910      25.799    -9.121    43.259       98
                 0.06      -20.11     -2940      26.019    -9.261    43.659       99
                 0.06      -20.03     -2970      26.236    -9.404    44.056      100
END           OF       RESPONSE   OUTPUT   FILE

              max=         141.71      1380
                          127.539
                           128.08       300
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Long-Term Moment-Curvature Response

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RESPONSE         OUTPUT      FILE
ANALYSIS:        Axial-Load
SECTION          NAME        :           HW5c
CONCRETE         MODEL:      Parabolic   Material    Factors:    C:0.600     S:0.850     P:0.900
TENSION-STIFF.:  No          FACTORED:   No          ACCURACY:   well_done
Axial-Load:      0.00kip     Moment:     0.00kft     Shear:      0.00kip
                 dN/dM:                0 dN/dV:                0 dM/dV:                0
                 Axial-Load  Moment      Curvature   @-Axis      Bottom      Top         Iter.
                 kips        ft*kip      rad/10^6in  [--------Milli-Strain------]
                        0.08       33.68           0       -0.56       -0.56       -0.56           1
                           0       47.92          30      -0.666      -0.306      -0.846           2
                           0       62.09          60      -0.776      -0.056      -1.136           3
                        0.01       76.13          90      -0.889       0.191      -1.429           4
                        0.11        87.8         120      -0.981       0.459      -1.701           5
                       -0.09       88.56         150       -0.94        0.86       -1.84           6
                        0.01       92.78         180      -0.928       1.232      -2.008           7
                        0.01        97.1         210      -0.916       1.604      -2.176           8
                        0.01       101.1         240      -0.897       1.983      -2.337           9
                        0.01      104.74         270      -0.871       2.369      -2.491          10
                        0.01      107.99         300      -0.837       2.763      -2.637          11
                        0.01      110.81         330      -0.794       3.166      -2.774          12
                        0.01      113.23         360      -0.744       3.576      -2.904          13
                        0.01      115.24         390      -0.686       3.994      -3.026          14
                        0.01      116.91         420       -0.62        4.42       -3.14          15
                        0.01      118.29         450      -0.548       4.852      -3.248          16
                           0      119.43         480       -0.47        5.29       -3.35          17
                           0      120.39         510      -0.388       5.732      -3.448          18
                           0       121.2         540      -0.301       6.179      -3.541          19
                           0      121.91         570      -0.212       6.628      -3.632          20
                           0      122.54         600       -0.12        7.08       -3.72          21
                           0       123.1         630      -0.026       7.534      -3.806          22
                           0      123.62         660        0.07        7.99       -3.89          23
                        0.11      124.07         690        0.17        8.45       -3.97          24
                           0      124.56         720       0.266       8.906      -4.054          25
                        0.01      124.98         750       0.366       9.366      -4.134          26
                        0.03      125.41         780       0.466       9.826      -4.214          27
                       -0.03      125.82         810       0.566      10.286      -4.294          28
                       -0.11      126.23         840       0.666      10.746      -4.374          29
                           0      126.57         870       0.772      11.212      -4.448          30
                           0      126.94         900       0.875      11.675      -4.525          31
                           0      127.29         930        0.98       12.14        -4.6          32
                           0      127.64         960       1.084      12.604      -4.676          33
                           0      127.97         990        1.19       13.07       -4.75          34
                           0       128.3        1020       1.296      13.536      -4.824          35
                           0      128.63        1050       1.402      14.002      -4.898          36
                           0      128.95        1080       1.509      14.469      -4.971          37
                           0      129.26        1110       1.616      14.936      -5.044          38
                           0      129.56        1140       1.724      15.404      -5.116          39
                           0      129.86        1170       1.832      15.872      -5.188          40
                           0      130.15        1200        1.94       16.34       -5.26          41
                           0      130.44        1230       2.049      16.809      -5.331          42
                           0      130.72        1260       2.159      17.279      -5.401          43
                           0         131        1290       2.269      17.749      -5.471          44
                           0      131.27        1320       2.379      18.219      -5.541          45
                           0      131.54        1350       2.489      18.689      -5.611          46
                           0       131.8        1380         2.6       19.16       -5.68          47
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0	132.06	1410	2.712	19.632	-5.748	48
0	132.31	1440	2.823	20.103	-5.817	49
0	132.56	1470	2.935	20.575	-5.885	50
0	132.8	1500	3.047	21.047	-5.953	51
0	133.04	1530	3.16	21.52	-6.02	52
0	133.28	1560	3.273	21.993	-6.087	53
0	133.51	1590	3.386	22.466	-6.154	54
0	133.74	1620	3.499	22.939	-6.221	55
0	133.97	1650	3.612	23.412	-6.288	56
0	134.19	1680	3.726	23.886	-6.354	57
0	134.41	1710	3.839	24.359	-6.421	58
0	134.63	1740	3.953	24.833	-6.487	59
0	134.85	1770	4.067	25.307	-6.553	60
0	135.06	1800	4.181	25.781	-6.619	61
0	135.27	1830	4.295	26.255	-6.685	62
0	135.48	1860	4.41	26.73	-6.75	63
0	135.68	1890	4.524	27.204	-6.816	64
0	135.89	1920	4.639	27.679	-6.881	65
0	136.09	1950	4.753	28.153	-6.947	66
0	136.29	1980	4.868	28.628	-7.012	67
0	136.49	2010	4.983	29.103	-7.077	68
0	136.68	2040	5.098	29.578	-7.142	69
0	136.88	2070	5.213	30.053	-7.207	70
0	137.07	2100	5.328	30.528	-7.272	71
0	137.26	2130	5.443	31.003	-7.337	72
0	137.45	2160	5.558	31.478	-7.402	73
0	137.64	2190	5.673	31.953	-7.467	74
0	137.83	2220	5.788	32.428	-7.532	75
0	138.02	2250	5.903	32.903	-7.597	76
0	138.2	2280	6.018	33.378	-7.662	77
0	138.39	2310	6.133	33.853	-7.727	78
0	138.57	2340	6.248	34.328	-7.792	79
0	138.75	2370	6.363	34.803	-7.857	80
0	138.93	2400	6.478	35.278	-7.922	81
0	139.11	2430	6.593	35.753	-7.987	82
0	139.29	2460	6.708	36.228	-8.052	83
0	139.46	2490	6.822	36.702	-8.118	84
0	139.64	2520	6.937	37.177	-8.183	85
0	139.81	2550	7.052	37.652	-8.248	86
0	139.99	2580	7.166	38.126	-8.314	87
0	140.16	2610	7.28	38.6	-8.38	88
0	140.33	2640	7.395	39.075	-8.445	89
0	140.5	2670	7.509	39.549	-8.511	90
0	140.67	2700	7.623	40.023	-8.577	91
0	140.84	2730	7.737	40.497	-8.643	92
0	141.01	2760	7.85	40.97	-8.71	93
0	141.18	2790	7.964	41.444	-8.776	94
0	141.34	2820	8.077	41.917	-8.843	95
0	141.51	2850	8.191	42.391	-8.909	96
0	141.67	2880	8.304	42.864	-8.976	97
0	141.84	2910	8.417	43.337	-9.043	98
0	142	2940	8.53	43.81	-9.11	99
0	142.16	2970	8.642	44.282	-9.178	100

0.08	33.68	0	-0.56	-0.56	-0.56	1
0	19.32	-30	-0.459	-0.819	-0.279	2
0	4.93	-60	-0.361	-1.081	-0.001	3
0	-9.45	-90	-0.266	-1.346	0.274	4

-0.04	-12.46	-120	-0.06	-1.5	0.66	5
0.01	-11.91	-150	0.203	-1.597	1.103	6
-0.01	-13.34	-180	0.451	-1.709	1.531	7
0	-14.53	-210	0.708	-1.812	1.968	8
0	-15.56	-240	0.972	-1.908	2.412	9
0	-16.48	-270	1.241	-1.999	2.861	10
0	-17.3	-300	1.515	-2.085	3.315	11
0	-18.03	-330	1.792	-2.168	3.772	12
0	-18.67	-360	2.073	-2.247	4.233	13
0	-19.24	-390	2.357	-2.323	4.697	14
0	-19.74	-420	2.643	-2.397	5.163	15
-0.13	-20.29	-450	2.929	-2.471	5.629	16
-0.12	-20.66	-480	3.221	-2.539	6.101	17
-0.11	-20.98	-510	3.514	-2.606	6.574	18
-0.09	-21.26	-540	3.808	-2.672	7.048	19
-0.08	-21.51	-570	4.104	-2.736	7.524	20
-0.07	-21.74	-600	4.401	-2.799	8.001	21
-0.06	-21.94	-630	4.7	-2.86	8.48	22
-0.05	-22.13	-660	4.999	-2.921	8.959	23
-0.04	-22.3	-690	5.298	-2.982	9.438	24
-0.04	-22.47	-720	5.599	-3.041	9.919	25
-0.03	-22.62	-750	5.9	-3.1	10.4	26
-0.02	-22.77	-780	6.201	-3.159	10.881	27
-0.02	-22.91	-810	6.503	-3.217	11.363	28
-0.01	-23.05	-840	6.805	-3.275	11.845	29
-0.01	-23.19	-870	7.107	-3.333	12.327	30
0	-23.32	-900	7.41	-3.39	12.81	31
0	-23.45	-930	7.713	-3.447	13.293	32
0.01	-23.57	-960	8.016	-3.504	13.776	33
0.01	-23.7	-990	8.319	-3.561	14.259	34
0.01	-23.82	-1020	8.623	-3.617	14.743	35
0	-23.95	-1050	8.927	-3.673	15.227	36
-0.02	-24.08	-1080	9.23	-3.73	15.71	37
-0.04	-24.22	-1110	9.534	-3.786	16.194	38
-0.04	-24.33	-1140	9.839	-3.841	16.679	39
-0.04	-24.44	-1170	10.144	-3.896	17.164	40
-0.05	-24.55	-1200	10.449	-3.951	17.649	41
-0.05	-24.66	-1230	10.754	-4.006	18.134	42
-0.05	-24.76	-1260	11.06	-4.06	18.62	43
-0.05	-24.87	-1290	11.366	-4.114	19.106	44
-0.05	-24.97	-1320	11.672	-4.168	19.592	45
-0.05	-25.07	-1350	11.978	-4.222	20.078	46
-0.05	-25.16	-1380	12.285	-4.275	20.565	47
-0.05	-25.26	-1410	12.591	-4.329	21.051	48
-0.05	-25.35	-1440	12.898	-4.382	21.538	49
-0.05	-25.45	-1470	13.205	-4.435	22.025	50
-0.06	-25.54	-1500	13.512	-4.488	22.512	51
0.06	-25.55	-1530	13.82	-4.54	23	52
0.07	-25.63	-1560	14.127	-4.593	23.487	53
0.09	-25.71	-1590	14.435	-4.645	23.975	54
0.1	-25.79	-1620	14.743	-4.697	24.463	55
0	-25.96	-1650	15.048	-4.752	24.948	56
0	-26.04	-1680	15.356	-4.804	25.436	57
0	-26.13	-1710	15.664	-4.856	25.924	58
-0.06	-26.26	-1740	15.97	-4.91	26.41	59
-0.06	-26.34	-1770	16.278	-4.962	26.898	60
-0.06	-26.43	-1800	16.587	-5.013	27.387	61
-0.05	-26.5	-1830	16.895	-5.065	27.875	62

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                -0.05          -26.58       -1860        17.204      -5.116      28.364         63
                -0.05          -26.66       -1890        17.512      -5.168      28.852         64
                -0.05          -26.74       -1920        17.821      -5.219      29.341         65
                -0.05          -26.82       -1950         18.13       -5.27       29.83         66
                -0.07          -26.91       -1980        18.438      -5.322      30.318         67
                -0.08          -26.99       -2010        18.747      -5.373      30.807         68
                -0.08          -27.07       -2040        19.056      -5.424      31.296         69
                -0.06          -27.13       -2070        19.366      -5.474      31.786         70
                -0.06           -27.2       -2100        19.675      -5.525      32.275         71
                -0.06          -27.28       -2130        19.985      -5.575      32.765         72
                -0.06          -27.35       -2160        20.294      -5.626      33.254         73
                -0.06          -27.42       -2190        20.604      -5.676      33.744         74
                -0.06          -27.49       -2220        20.914      -5.726      34.234         75
                -0.06          -27.56       -2250        21.224      -5.776      34.724         76
                -0.07          -27.63       -2280        21.534      -5.826      35.214         77
                -0.06           -27.7       -2310        21.844      -5.876      35.704         78
                -0.06          -27.76       -2340        22.155      -5.925      36.195         79
                -0.06          -27.83       -2370        22.465      -5.975      36.685         80
                -0.06          -27.89       -2400        22.776      -6.024      37.176         81
                -0.06          -27.96       -2430        23.086      -6.074      37.666         82
                -0.06          -28.02       -2460        23.397      -6.123      38.157         83
                -0.06          -28.08       -2490        23.708      -6.172      38.648         84
                -0.06          -28.15       -2520        24.019      -6.221      39.139         85
                -0.06          -28.21       -2550         24.33       -6.27       39.63         86
                -0.06          -28.27       -2580        24.641      -6.319      40.121         87
                -0.07          -28.33       -2610        24.953      -6.367      40.613         88
                -0.07          -28.39       -2640        25.264      -6.416      41.104         89
                -0.07          -28.44       -2670        25.576      -6.464      41.596         90
                -0.07           -28.5       -2700        25.888      -6.512      42.088         91
                -0.06          -28.55       -2730          26.2       -6.56       42.58         92
                -0.06          -28.61       -2760        26.512      -6.608      43.072         93
                -0.06          -28.66       -2790        26.824      -6.656      43.564         94
                -0.06          -28.71       -2820        27.136      -6.704      44.056         95
                -0.06          -28.76       -2850        27.449      -6.751      44.549         96
                -0.06          -28.82       -2880        27.761      -6.799      45.041         97
                -0.06          -28.87       -2910        28.074      -6.846      45.534         98
                -0.06          -28.91       -2940        28.387      -6.893      46.027         99
                -0.05          -29.01       -3000        29.013      -6.987      47.013          1
END             OF       RESPONSE   OUTPUT   FILE
```

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Short-Term Moment-Curvature Response with Tension Stiffening

```
RESPONSE          OUTPUT      FILE
ANALYSIS:         Axial-Load
SECTION           NAME        :           HW5c
CONCRETE          MODEL:      Parabolic   Material      Factors:     C:0.600    S:0.850    P:0.900
TENSION-STIFF.:   Yes         FACTORED:   No            ACCURACY:    well_done
Axial-Load:       0.00kip     Moment:     0.00kft       Shear:       0.00kip
                  dN/dM:                0 dN/dV:                   0 dM/dV:             0
                  Axial-Load  Moment      Curvature     @-Axis       Bottom     Top        Iter.
                  kips        ft*kip      rad/10^6in    [-------Milli-Strain------]
                           0       37.81            0       -0.181     -0.181     -0.181       1
                           0       68.67           20       -0.247     -0.007     -0.367       2
                           0        95.1           40       -0.305      0.175     -0.545       3
                           0      103.07           60       -0.296      0.424     -0.656       4
                        0.11      108.11           80       -0.265      0.695     -0.745       5
                        0.01      112.01          100       -0.224      0.976     -0.824       6
                        0.07      115.78          120       -0.181      1.259     -0.901       7
                        0.05      119.34          140       -0.135      1.545     -0.975       8
                        0.03      122.55          160       -0.086      1.834     -1.046       9
                        0.02      125.46          180       -0.034      2.126     -1.114      10
    90% Mmax            0.01      128.05          200        0.021      2.421     -1.179      11
                       -0.01      130.35          220        0.078      2.718     -1.242      12
                           0      132.36          240        0.138      3.018     -1.302      13
                       -0.01      134.09          260          0.2       3.32      -1.36      14
                       -0.01      135.56          280        0.264      3.624     -1.416      15
                       -0.02       136.8          300        0.331      3.931     -1.469      16
                       -0.02      137.84          320        0.399      4.239     -1.521      17
                       -0.02      138.71          340        0.469      4.549     -1.571      18
                       -0.02      139.44          360         0.54       4.86      -1.62      19
                       -0.02      140.06          380        0.612      5.172     -1.668      20
                       -0.03       140.6          400        0.685      5.485     -1.715      21
                       -0.03      141.05          420        0.759      5.799     -1.761      22
                       -0.03      141.46          440        0.833      6.113     -1.807      23

                           0       37.81            0       -0.181     -0.181     -0.181       1
                       -0.09        -8.6          -30       -0.094     -0.454      0.086       2
                        -0.1      -15.75          -60        0.124     -0.596      0.484       3
                       -0.05      -20.12          -90        0.372     -0.708      0.912       4
                       -0.06      -21.85         -120        0.648     -0.792      1.368       5
                       -0.01      -23.03         -150        0.933     -0.867      1.833       6
                           0      -24.04         -180        1.223     -0.937      2.303       7
                           0      -25.04         -210        1.513     -1.007      2.773       8
                           0      -25.87         -240        1.807     -1.073      3.247       9
                           0      -26.58         -270        2.103     -1.137      3.723      10
                           0       -27.2         -300        2.402     -1.198      4.202      11
                           0      -27.72         -330        2.701     -1.259      4.681      12
                       -0.01      -28.18         -360        3.002     -1.318      5.162      13
                       -0.01      -28.57         -390        3.305     -1.375      5.645      14
                       -0.01      -28.91         -420        3.608     -1.432      6.128      15
                           0       -29.2         -450        3.912     -1.488      6.612      16
                           0      -29.45         -480        4.218     -1.542      7.098      17
                           0      -29.67         -510        4.524     -1.596      7.584      18
                           0      -29.86         -540        4.831     -1.649      8.071      19
                           0      -30.05         -570        5.137     -1.703      8.557      20
                           0      -30.21         -600        5.445     -1.755      9.045      21
                           0      -30.36         -630        5.753     -1.807      9.533      22
```

-0.01	-30.51	-660	6.061	-1.859	10.021	23
0	-30.63	-690	6.369	-1.911	10.509	24
0	-30.75	-720	6.678	-1.962	10.998	25
0	-30.86	-750	6.987	-2.013	11.487	26
0	-30.97	-780	7.296	-2.064	11.976	27
0	-31.07	-810	7.606	-2.114	12.466	28
0	-31.17	-840	7.915	-2.165	12.955	29
0	-31.26	-870	8.225	-2.215	13.445	30
0	-31.34	-900	8.535	-2.265	13.935	31
0	-31.42	-930	8.844	-2.316	14.424	32
0	-31.5	-960	9.154	-2.366	14.914	33
0	-31.57	-990	9.463	-2.417	15.403	34
0	-31.64	-1020	9.773	-2.467	15.893	35
0	-31.71	-1050	10.082	-2.518	16.382	36
0	-31.77	-1080	10.392	-2.568	16.872	37
-0.01	-31.83	-1110	10.701	-2.619	17.361	38
-0.01	-31.88	-1140	11.01	-2.67	17.85	39
-0.01	-31.94	-1170	11.318	-2.722	18.338	40
-0.01	-31.98	-1200	11.627	-2.773	18.827	41
-0.01	-32.03	-1230	11.935	-2.825	19.315	42
-0.01	-32.07	-1260	12.243	-2.877	19.803	43
-0.01	-32.11	-1290	12.55	-2.93	20.29	44
-0.01	-32.14	-1320	12.857	-2.983	20.777	45
-0.01	-32.17	-1350	13.164	-3.036	21.264	46
-0.01	-32.2	-1380	13.47	-3.09	21.75	47
-0.01	-32.22	-1410	13.776	-3.144	22.236	48
-0.01	-32.24	-1440	14.081	-3.199	22.721	49
-0.01	-32.25	-1470	14.386	-3.254	23.206	50
-0.01	-32.26	-1500	14.69	-3.31	23.69	51
-0.01	-32.27	-1530	14.993	-3.367	24.173	52
-0.02	-32.27	-1560	15.295	-3.425	24.655	53
-0.02	-32.26	-1590	15.597	-3.483	25.137	54
-0.02	-32.25	-1620	15.897	-3.543	25.617	55
-0.02	-32.24	-1650	16.196	-3.604	26.096	56
-0.02	-32.22	-1680	16.495	-3.665	26.575	57
-0.02	-32.19	-1710	16.792	-3.728	27.052	58
-0.02	-32.15	-1740	17.087	-3.793	27.527	59
-0.03	-32.1	-1770	17.381	-3.859	28.001	60
-0.03	-32.05	-1800	17.673	-3.927	28.473	61
-0.03	-31.99	-1830	17.962	-3.998	28.942	62
-0.03	-31.91	-1860	18.25	-4.07	29.41	63
-0.04	-31.82	-1890	18.534	-4.146	29.874	64
-0.04	-31.72	-1920	18.815	-4.225	30.335	65
-0.04	-31.59	-1950	19.092	-4.308	30.792	66
-0.05	-31.44	-1980	19.363	-4.397	31.243	67
-0.05	-31.26	-2010	19.628	-4.492	31.688	68
-0.06	-31.04	-2040	19.885	-4.595	32.125	69
-0.06	-30.76	-2070	20.129	-4.711	32.549	70
-0.04	-30.36	-2100	20.355	-4.845	32.955	71
0.01	-29.76	-2130	20.546	-5.014	33.326	72
0	-29.06	-2160	20.705	-5.215	33.665	73
0.01	-28.41	-2190	20.867	-5.413	34.007	74
0.01	-27.8	-2220	21.031	-5.609	34.351	75
0.01	-27.23	-2250	21.196	-5.804	34.696	76
0.01	-26.69	-2280	21.363	-5.997	35.043	77
0.01	-26.19	-2310	21.531	-6.189	35.391	78
0.01	-25.71	-2340	21.699	-6.381	35.739	79
0.01	-25.27	-2370	21.869	-6.571	36.089	80

0.01	-24.85	-2400	22.04	-6.76	36.44	81
0.01	-24.46	-2430	22.212	-6.948	36.792	82
-0.01	-24.11	-2460	22.382	-7.138	37.142	83
0.03	-23.76	-2490	22.563	-7.317	37.503	84
0.01	-23.46	-2520	22.738	-7.502	37.858	85
0.02	-23.17	-2550	22.917	-7.683	38.217	86
0.01	-22.91	-2580	23.098	-7.862	38.578	87
0.02	-22.67	-2610	23.282	-8.038	38.942	88
0.01	-22.45	-2640	23.467	-8.213	39.307	89
-0.02	-22.27	-2670	23.652	-8.388	39.672	90
0.06	-22.05	-2700	23.855	-8.545	40.055	91
0	-21.92	-2730	24.042	-8.718	40.422	92
0.01	-21.77	-2760	24.247	-8.873	40.807	93
0.08	-21.6	-2790	24.451	-9.029	41.191	94
0.07	-21.49	-2820	24.646	-9.194	41.566	95
0.01	-21.37	-2850	24.809	-9.391	41.909	96
0.01	-21.22	-2880	24.978	-9.582	42.258	97
0.01	-21.08	-2910	25.151	-9.769	42.611	98
0.01	-20.97	-2940	25.33	-9.95	42.97	99
0.01	-20.87	-2970	25.514	-10.126	43.334	100

-0.03	141.81	460	0.908	6.428	-1.852	24
-0.03	142.13	480	0.983	6.743	-1.897	25
-0.03	142.43	500	1.059	7.059	-1.941	26
-0.03	142.7	520	1.134	7.374	-1.986	27
-0.03	142.95	540	1.21	7.69	-2.03	28
-0.03	143.18	560	1.286	8.006	-2.074	29
-0.03	143.4	580	1.362	8.322	-2.118	30
-0.03	143.61	600	1.439	8.639	-2.161	31
-0.02	143.81	620	1.515	8.955	-2.205	32
-0.02	144	640	1.591	9.271	-2.249	33
-0.02	144.18	660	1.668	9.588	-2.292	34
-0.02	144.35	680	1.744	9.904	-2.336	35
-0.02	144.52	700	1.82	10.22	-2.38	36
-0.02	144.67	720	1.896	10.536	-2.424	37
-0.02	144.83	740	1.973	10.853	-2.467	38
-0.02	144.97	760	2.049	11.169	-2.511	39
-0.02	145.12	780	2.125	11.485	-2.555	40
-0.02	145.25	800	2.2	11.8	-2.6	41
-0.02	145.38	820	2.276	12.116	-2.644	42
-0.02	145.51	840	2.352	12.432	-2.688	43
-0.02	145.63	860	2.427	12.747	-2.733	44
-0.02	145.74	880	2.502	13.062	-2.778	45
-0.02	145.85	900	2.577	13.377	-2.823	46
-0.02	145.95	920	2.651	13.691	-2.869	47
-0.02	146.05	940	2.725	14.005	-2.915	48
-0.03	146.14	960	2.799	14.319	-2.961	49
-0.03	146.23	980	2.872	14.632	-3.008	50
-0.03	146.31	1000	2.945	14.945	-3.055	51
-0.03	146.39	1020	3.018	15.258	-3.102	52
-0.03	146.46	1040	3.089	15.569	-3.151	53
-0.03	146.53	1060	3.161	15.881	-3.199	54
-0.03	146.59	1080	3.231	16.191	-3.249	55
-0.03	146.64	1100	3.301	16.501	-3.299	56
-0.03	146.68	1120	3.37	16.81	-3.35	57
-0.03	146.72	1140	3.438	17.118	-3.402	58
-0.03	146.75	1160	3.505	17.425	-3.455	59
-0.03	146.77	1180	3.57	17.73	-3.51	60

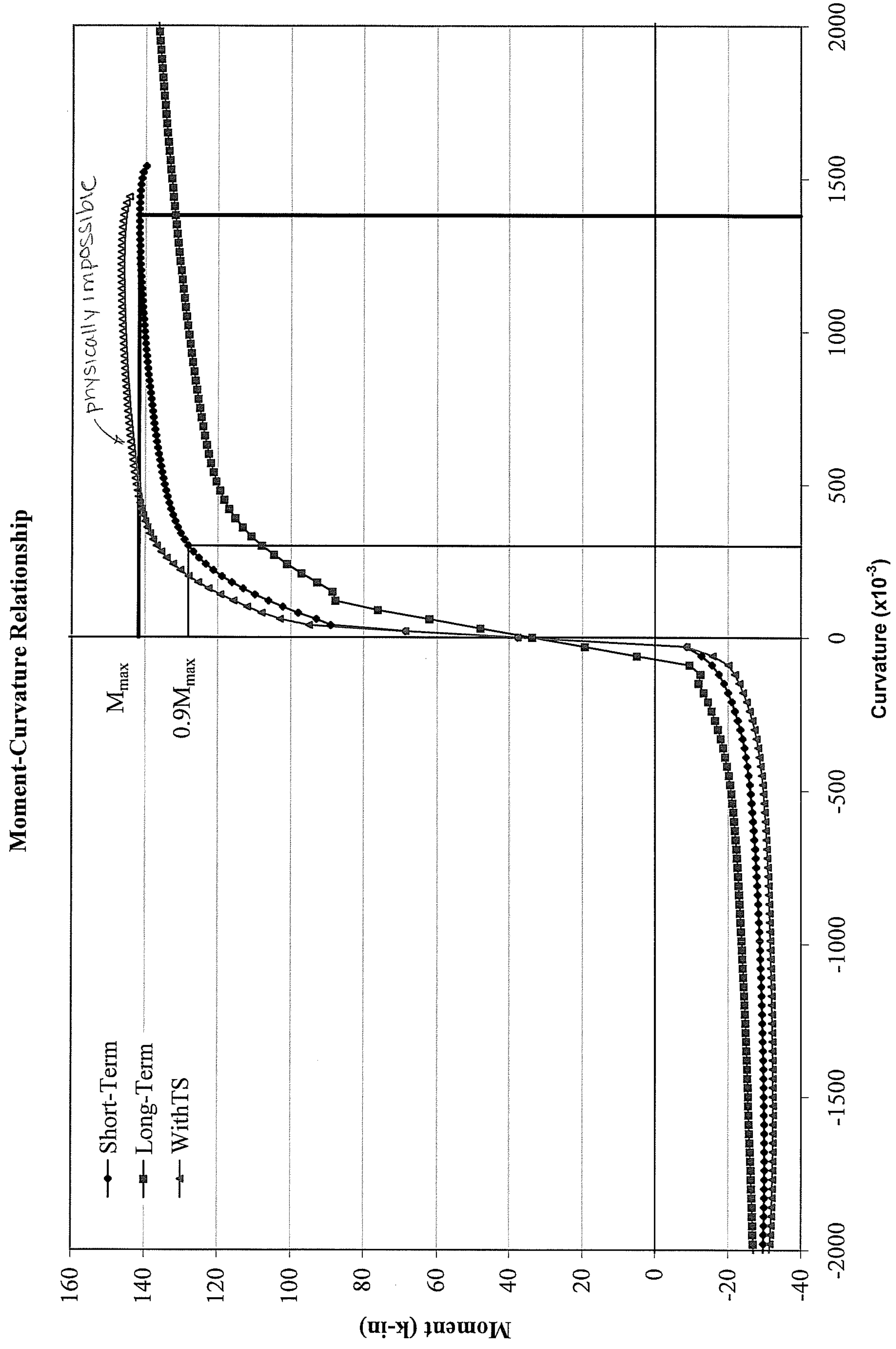
```
                    -0.03      146.79       1200       3.634     18.034     -3.566        61
                    -0.03      146.79       1220       3.697     18.337     -3.623        62
                    -0.03      146.78       1240       3.758     18.638     -3.682        63
                    -0.03      146.76       1260       3.817     18.937     -3.743        64
                    -0.04      146.72       1280       3.873     19.233     -3.807        65
                    -0.04      146.66       1300       3.927     19.527     -3.873        66
                    -0.04      146.58       1320       3.976     19.816     -3.944        67
                    -0.04      146.47       1340       4.021     20.101     -4.019        68
                    -0.04      146.32       1360        4.06      20.38       -4.1        69
                    -0.04      146.13       1380       4.091     20.651     -4.189        70
                    -0.03      145.85       1400       4.109     20.909     -4.291        71
                     0.06      145.42       1420       4.109     21.149     -4.411        72
                     0.07      144.51       1440       4.029     21.309     -4.611        73
END            OF         RESPONSE   OUTPUT    FILE
```

CE 383P: PRESTRESS

C.HOVELL
15 Oct 07

15/16

HOMEWORK #5



16/16

HOMEWORK #5

2. Part IV – no T/S

$$\varepsilon_{ct} = -1.388 \times 10^{-3}$$

$$\varepsilon_{pf} = -\varepsilon_{ct} \frac{d-c}{c} + \Delta\varepsilon_p$$

educated guess: c = 4.63 in

$$\varepsilon_{pf_1} = -(-1.388 \times 10^{-3}) \frac{18\text{ in} - 1.5\text{ in} - 4.63\text{ in}}{4.63\text{ in}} + 6.1 \times 10^{-3}$$

$$\varepsilon_{pf_1} = 9.64 \times 10^{-3}$$

$$\varepsilon_{pf_2} = 1.388 \times 10^{-3} \frac{10.12\text{ in}}{4.63\text{ in}} + 6.1 \times 10^{-3} = 9.11 \times 10^{-3}$$

$$\varepsilon_{pf_3} = 1.388 \times 10^{-3} \frac{1.37\text{ in}}{4.63\text{ in}} + 6.1 \times 10^{-3} = 6.51 \times 10^{-3}$$

$$T_p = 29000\text{ ksi} \left[(9.64)(0.255\text{ in}^2) + (9.11)(0.17\text{ in}^2) + (6.51)(0.085) \right] \times 10^{-3}$$

use Ramberg-Osgood

-.1

$$T_p = 132.2\text{ k}$$

$$C_c = \alpha_1 \beta_1 f_c' c b$$

$$\beta_1 = \frac{4 - \varepsilon_{ct}/\varepsilon_c'}{6 - 2\varepsilon_{ct}/\varepsilon_c'} = \frac{4 - \frac{-1.388}{-2.5}}{6 - \frac{2(1.388)}{2.5}} = 0.705$$

$$\alpha_1 = \frac{1}{\beta_1} \left[\frac{\varepsilon_{ct}}{\varepsilon_c'} - \frac{1}{3} \left(\frac{\varepsilon_{ct}}{\varepsilon_c'} \right)^2 \right] = \frac{1}{0.705} \left[\frac{1.388}{2.5} - \frac{1}{3} \left(\frac{1.388}{2.5} \right)^2 \right]$$

$$\alpha_1 = 0.642$$

section tapers

-.2

$$C_c = (0.642)(0.705)(7000\text{ psi})(4.63\text{ in})(9\text{ in})$$

$$C_c = 132.0\text{ K}$$ – very close

$$M = (-132.0\text{ k}) \left[\beta_1 c/2 - 9\text{ in} \right] + (132.2\text{ k}) \left[3.83\text{ in} - 9\text{ in} \right]$$

	Response	☐ stress block	
ε_{ct}	-1.388×10^{-3}		
ε_{cb}	4.01×10^{-3}	4.0×10^{-3}	
ϕ	300×10^{-6}	300×10^{-6}	
M	128.1 k·ft	138 k·ft	– Not great; other values are good

check: no yield ✓

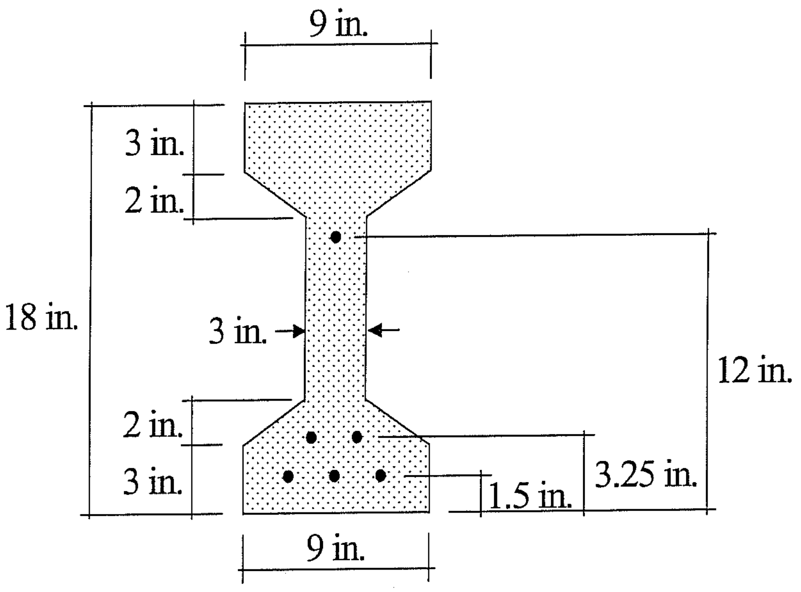
CE 383P – Prestressed Concrete
Homework Assignment #5

Date Assigned: October 8, 2007
Due Date: October 15, 2007

Problem 1:

A post-tensioned beam with grouted tendons has the cross-section shown. The beam is simply supported and spans 18 ft. In addition to its own weight, it carries a superimposed dead load of 1.0 kip/ft and a live load of 2.0 kip/ft. Assume that the concrete weighs 150 lb/ft^3. Using the software of your choice, determine:

i) the short-term moment-curvature response.
ii) the long-term moment-curvature response assuming $\phi = 2.5$ and 5 % relaxation in strands.
iii) Average stress/strain conditions when $M = 0.9\ M_{max}$ (short-term).
iv) Local stress/strain conditions at a crack when $M = 0.9\ M_{max}$ (short-term).



$f_c' = 7000$ psi
$\varepsilon_c' = -2.5 \times 10^{-3}$
A_p = 6- 3/8 in. strands (0.085 in^2/strand)
$f_{pu} = 270$ ksi
$\Delta\varepsilon_p = 6.1 \times 10^{-3}$
low-relaxation strands:
(A = 0.025, B = 118, C = 10)

Problem 2:

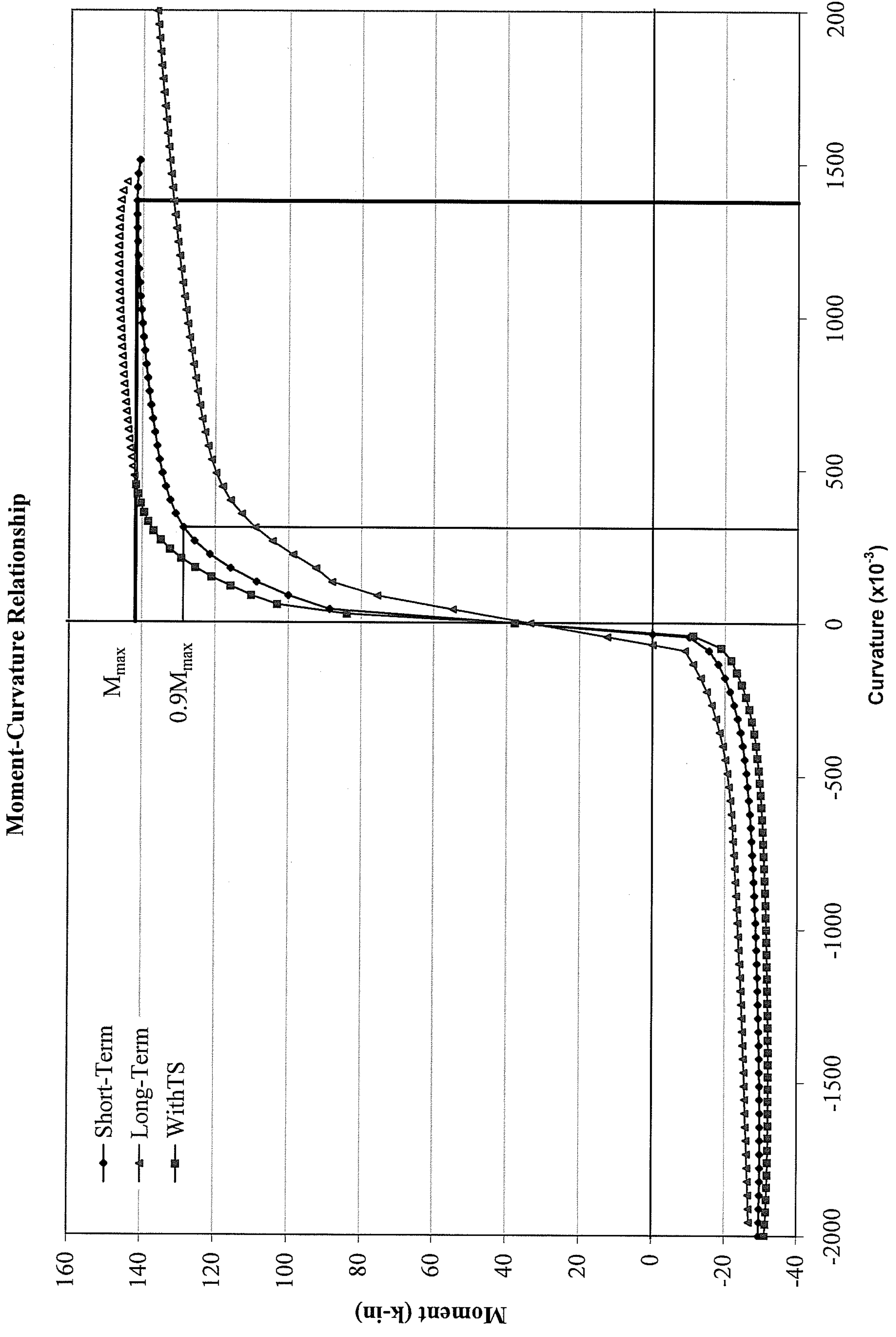
Repeat (iv) using the rectangular stress block.

CE 383P: PRESTRESS

C.HOVELL

HOMEWORK #5

modified curve



CE383P: PRESTRESS

HOMEWORK #6

9.5/10

CATHERINE HOVELL
26 October 2007

1/3

Short-Term Camber

i.

x	M	φ	i	Average φ	Δxi	dxi	φ · Δxi · dxi
0	0	-22.011					
1	0.901	-21.488	1	-21.750	12	6	-0.002
2	1.696	-21.027	2	-21.257	12	18	-0.005
3	2.385	-20.627	3	-20.827	12	30	-0.007
4	2.968	-20.288	4	-20.457	12	42	-0.010
5	3.445	-20.011	5	-20.150	12	54	-0.013
6	3.816	-19.796	6	-19.904	12	66	-0.016
7	4.081	-19.642	7	-19.719	12	78	-0.018
8	4.24	-19.550	8	-19.596	12	90	-0.021
9	4.293	-19.519	9	-19.535	12	102	-0.024
						TOTAL:	-0.116

M self-weight ↑ ↖ linear fit

Long-Term Camber

ii

x	M	φ	i	Average φ	Δxi	dxi	φ · Δxi · dxi
0	0	-64.170					
1	0.901	-62.470	1	-63.320	12	6	-0.005
2	1.696	-60.970	2	-61.720	12	18	-0.013
3	2.385	-59.670	3	-60.320	12	30	-0.022
4	2.968	-58.570	4	-59.120	12	42	-0.030
5	3.445	-57.670	5	-58.120	12	54	-0.038
6	3.816	-56.970	6	-57.320	12	66	-0.045
7	4.081	-56.470	7	-56.720	12	78	-0.053
8	4.24	-56.170	8	-56.320	12	90	-0.061
9	4.293	-56.070	9	-56.120	12	102	-0.069
						TOTAL:	-0.335

Short-Term with Load

iii.

x	M	φ	i	Average φ	Δxi	dxi	φ · Δxi · dxi
0	0.0	-22.01					
1	26.4	-6.69	1	-14.350	12	6	-0.001
2	49.7	6.83	2	0.071	12	18	0.000
3	69.9	18.55	3	12.689	12	30	0.005
4	87.0	28.46	4	23.504	12	42	0.012
5	100.9	42.50	5	35.481	12	54	0.023
6	111.8	85.02	6	63.760	12	66	0.050
7	119.6	110.02	7	97.520	12	78	0.091
8	124.2	169.95	8	139.985	12	90	0.151
9	125.8	170.30	9	170.125	12	102	0.208
						TOTAL:	0.540

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26 OCT 07

HOMEWORK #6

N	Positive Flexure				Negative Flexure			
	M	Φ	ε_b	ε_t	M	Φ	ε_b	ε_t
kip	kip-ft	$\times 10^{-6}$ rad/in	$\times 10^{-3}$	$\times 10^{-3}$	kip-ft	$\times 10^{-6}$ rad/in	$\times 10^{-3}$	$\times 10^{-3}$
0	141.76	1348.44	20.557	-3.715	-29.91	-1720	-3.6	27.36
100	187.42	658.67	8.519	-3.337	-84.41	-846.22	-3.314	11.918
150	204.05	468	5.13	-3.294	-107.74	-640	-3.313	8.207
200	210.12	336	2.872	-3.176	-126.36	-462.22	-3.298	5.022
250	207.41	256	1.537	-3.071	-136.9	-328.89	-3.215	2.705
300	198.25	205.33	0.687	-3.009	-138.28	-245.78	-3.143	1.281
350	180.55	178.67	0.25	-2.966	-128.51	-202.22	-3.129	0.511
450	131.02	146.67	-0.284	-2.924	-83.17	-165.33	-3.104	-0.129
600	54.05	73.33	-1.398	-2.718	-6.06	-90.67	-2.875	-1.243
658	23.14	0	-2.705	-2.705				

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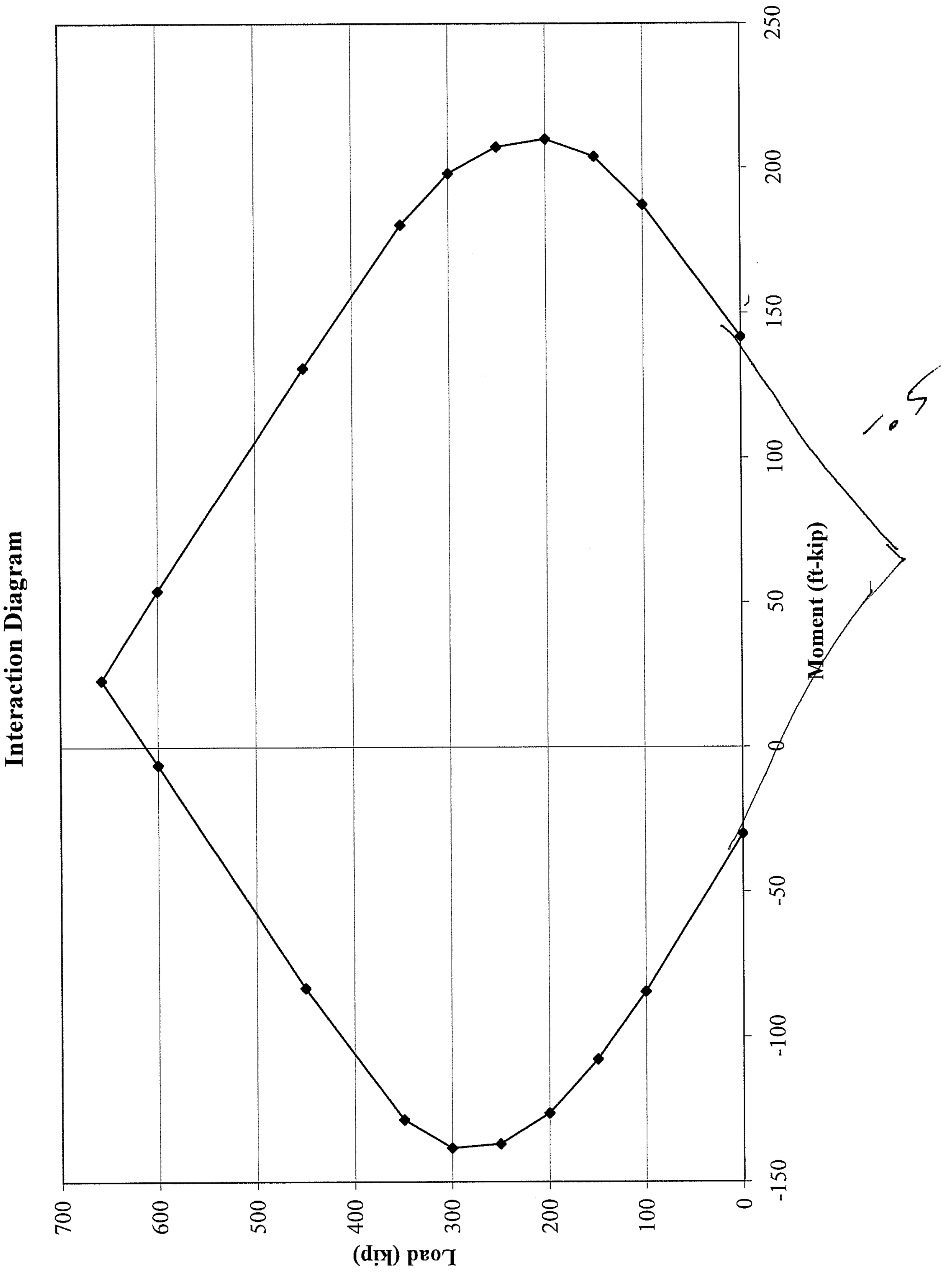
26 Oct. 07

HOMEWORK #6



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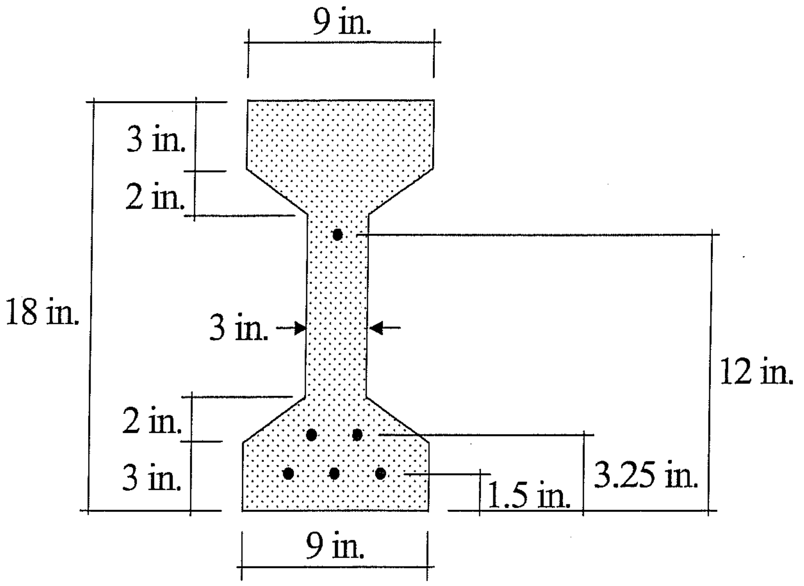
CE 383P – Prestressed Concrete
Homework Assignment #6

Date Assigned: October 15, 2007
Due Date: October 22, 2007

Problem 1:

A post-tensioned beam with grouted tendons has the cross-section shown. The beam is imply supported and spans 18 ft. In addition to its own weight, it carries a superimposed dead load of 1.0 kip/ft and a live load of 2.0 kip/ft. Assume that the concrete weighs 150 lb/ft^3. Using the software of your choice, determine:

- i) Short-term camber.
- ii) Long-term camber, assuming $\phi = 2.5$ and 5 % relaxation in strands.
- iii) Short-term deflection under the dead + superimposed dead + live loads



$f_c' = 7000$ psi
$\varepsilon_c' = -2.5 \times 10^{-3}$
$A_p = 6$- 3/8 in. strands (0.085 in^2/strand)
$f_{pu} = 270$ ksi
$\Delta\varepsilon_p = 6.1 \times 10^{-3}$
low-relaxation strands:
(A = 0.025, B = 118, C = 10)

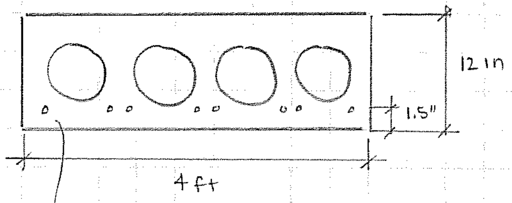
Problem 2:

For the cross-section above, determine the complete N-M interaction diagram.

$\frac{9.5}{10}$

HOMEWORK #7

1.



8 - ½"φ strands
low-lax, $f_{pu} = 270$ ksi
75% stressing
$f_{pi} = 187$ ksi
$f_{pf} = 157$ ksi

$A = 262 \text{ in}^2$
$I = 4949 \text{ in}^4$
$y_b = 6$ in
$y_t = 6$ in
$e = 4.5$ in
$L = 40$ ft
$f'_{ci} = 4$ ksi
$f'_c = 6$ ksi

$S_b = 825 \text{ in}^3$
$S_t = 825 \text{ in}^3$
$b_w = 8.0$ in
SW = 273 plf
= 68 psf
V/S = 2.18 in
DL = 10 psf
LL = 60 psf (30% sustained)

Assume uncracked
tensile at service: $7.5\sqrt{f'_c}$
comp. at transfer: $0.6 f'_{ci}$
tension at transfer: $3\sqrt{f'_{ci}}$

i. ends – $L_d = 25$ in
ii. midspan – $L = 0.4L = 192$ in

Loads / moments

$M = 0.12 wL^2$ at 0.4L, $= 0.125 wL^2$ at midspan

$M = \frac{1}{150} wL^2$ at transfer length

$w_{SW} = (0.068 \text{ ksf})(4 \text{ ft}) = 0.272$ k/ft
$w_{DW} = (0.01 \text{ ksf})(4 \text{ ft}) = 0.04$ k/ft
$w_{LL} = (0.06 \text{ ksf})(4 \text{ ft}) = 0.24$ k/ft
$w_{sus} = 0.30(0.06 \text{ ksf})(4 \text{ ft}) = 0.072$ k/ft

	transfer	0.4L	midspan
SW	34.8 k-in	626.7	652.8
DW	5.1	92.2	96.0
LL	30.7	553.0	576.0

Stress calculations performed in Excel, on following sheets
Gross properties used

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HOMEWORK #7

Gross Area	262.0	in^2
Gross Moment of Inertia	4949	in^4
Height of section	12.0	in
y-top	6.0	in
y-bot	6.0	in
Width of top slab	4.0	ft
Length of span	40	ft

CONCRETE PROPERTIES

f_c'	4.0	ksi
f_{ci}'	6.0	ksi

STRAND PROPERTIES

f_{pu}	270	ksi
f_{py}	243	ksi
Number of strands	8	
Area of one strand	0.153	in^2
End eccentricity	4.5	in
0.4L eccentricity	4.5	in
Center eccentricity	4.5	in
f_{pi}	187	ksi
f_{pf}	157	ksi

LOADS

Self-weight	68	psf
Superimposed dead load	10	psf
Live load	60	psf
Sustained loads	18	psf

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HOMEWORK #7

	at TL	0.4L	midspan	
PRESTRESSING				
P_{pi}	228.9	228.9	228.9	
M_{pi}	1030.0	1030.0	1030.0	k-in
f_{ctop}	0.375	0.375	0.375	ksi
f_{cbot}	-2.122	-2.122	-2.122	ksi
P_{pf}	192.2	192.2	192.2	kip
M_{pf}	864.8	864.8	864.8	k-in
f_{ctop}	0.315	0.315	0.315	ksi
f_{cbot}	-1.782	-1.782	-1.782	ksi
APPLIED MOMENTS				
Self-weight	34.8	626.7	652.8	k-in
f_{ctop}	-0.042	-0.760	-0.791	ksi
f_{cbot}	0.042	0.760	0.791	ksi
Superimposed dead load	5.1	92.2	96.0	k-in
f_{ctop}	-0.006	-0.112	-0.116	ksi
f_{cbot}	0.006	0.112	0.116	ksi
Live load	30.7	553.0	576.0	k-in
f_{ctop}	-0.037	-0.670	-0.698	ksi
f_{cbot}	0.037	0.670	0.698	ksi
STRESSES AT TRANSFER				
f_{ctop}	0.333	-0.385	-0.416	ksi
f_{cbot}	-2.080	-1.363	-1.331	ksi
STRESSES FROM SUSTAINED LOADS				
f_{ctop}	0.255	-0.758	-0.802	ksi
f_{cbot}	-1.722	-0.709	-0.665	ksi
STRESSES FROM SERVICE LOADS				
f_{ctop}	0.229	-1.227	-1.291	ksi
f_{cbot}	-1.696	-0.240	-0.176	ksi

at 25" ?

These numbers are too small

$M_D \simeq 129$ k-in

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LIMITS							
Tension stress at transfer, middle	--	PASS	PASS	psi	$3\sqrt{f_{ci}'}$	232.4	psi
Tension stress at transfer, ends	PASS	--	--	psi	$6\sqrt{f_{ci}'}$	464.8	psi
Compressive stress at transfer	PASS	PASS	PASS	ksi	$0.60f_{ci}'$	-3.60	ksi
Tension stress from sustained loads	PASS	PASS	PASS	psi	$7.5\sqrt{f_{ci}'}$	580.9	psi
Compressive stress from sustained loads	PASS	PASS	PASS	ksi	$0.45f_c'$	-1.80	ksi
Tension stress at service loads	PASS	PASS	PASS	psi	$7.5\sqrt{f_{ci}'}$	580.9	psi
Compressive stress at service loads	PASS	PASS	PASS	ksi	$0.60f_c'$	-2.40	ksi

379 k-in

section is adequate for anticipated loads

$\frac{4}{5}$

HOMEWORK #7

2. Deflection calculations

$$\Delta = \delta_p + \delta_{sus} + \delta_L$$

Using handout, $\delta_p = \frac{1}{8}\frac{PeL^2}{EI_g}$ – use long-term E

$$\delta_{sus} = \frac{5}{384}\frac{(W_{SW} + W_{SD} + 0.3W_{LL})L^4}{E_{eff}\,I_g}$$

Initial camber:

$$\Delta_i = \frac{1}{8}\frac{P_i e L^2}{E_{ci}\,I_g} + \frac{5}{384}\frac{W_{SW} L^4}{E_{ci}\,I_g}$$

$$= -\frac{1}{8}\frac{(228.9\text{K})(4.5\text{in})(40\text{ft})^2}{\boxed{(3605\text{KSI})}(4949\text{ in}^4)} + \frac{5}{384}\frac{(0.068\text{ ksf})(4.0\text{ft})(40\text{ft})^4}{\boxed{(3605\text{ KSI})}(4949\text{ in}^4)}$$

$$= -0.78\text{ in (up)}$$

use $57 \cdot \sqrt{f'_c}$ rather than f'_{ci} b/c modulus gain is very quick

Final camber:

check w/ solution.

$$\Delta_f = \frac{1}{8}\frac{P_f e L^2}{E_{eff}\,I_g} + \frac{5}{384}\frac{(W_{SW} + W_{SD} + 0.3W_{LL})L^4}{E_{ceff}\,I_g}$$

$$= \frac{-1/8(192.2\text{K})(4.5\text{in})(40\text{ft})^2 + 5/384\left[(4\text{ft})(0.068\text{KSF} + 0.01\text{ KSF} + 0.018\text{KSF})\right]^4}{\underline{(1202\text{ KSI})}\underline{(4949\text{ in}^4)}}$$

$$= -0.47\text{ in (up)}.$$

Immediate deflection:

$$\Delta_L = \frac{5}{384}\frac{W_L L^4}{E_{ci}\,I_g} = \frac{5}{384}\frac{(0.7)(0.06\text{ ksf})(4\text{ft})(40\text{ft})^4}{\boxed{(3605\text{KSI})}(4949\text{in}^4)}$$

$$= 0.54\text{ in (down)}$$

Total service deflection:

$$\Delta_t = \Delta_f + \Delta_L = 0.07\text{ in (down)}$$

ACI limits:

$$\Delta_L < L/360 = \frac{40\text{ft}}{360} = 1.33\text{ in} \checkmark$$

$$\Delta_t < L/480 = 1.0\text{ in} \checkmark$$

Δ_i = 0.78 in UP
Δ_f = 0.47 in UP
Δ_L = 0.54 in down
Δ_t = 0.07 in down
ALL OKAY

HOMEWORK #7

2. (cont'd)

Final camber approximation:

$$\Delta = (-1.66\text{ in})(2.45) + (0.88\text{ in})(2.70) = -1.69\text{ in}$$

total camber from P/S and SW using rigorous calcs:

$$\Delta = \frac{-(192.2\text{K})(4.5\text{in})(40\text{ft})^2}{8(1202\text{ksi})(4949\text{in}^4)} + \frac{5}{384}\frac{(4\text{ft})(0.068\text{ksf})(40\text{ft})^4}{(1202\text{ksi})(4949\text{in}^4)}$$

$$= -1.55\text{ in}$$

Not terribly accurate, but rigorous calcs still use gross moment of inertia.

CE 383P – Prestressed Concrete
Homework Assignment # 7

Date assigned : November 5, 2007
Due date : November 12, 2007

Problem 1:

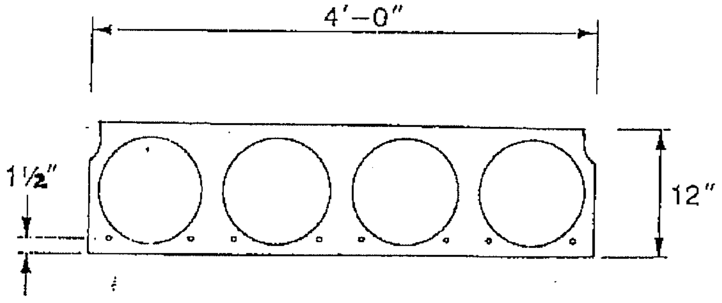
A PCI 4HC12 – 88S hollow core one way slab element contains 8 ½”-7-wire strands low-relaxation strands (f_{pu} = 270 ksi) and spans 40’. The straight strands have an eccentricity of 4.5”. The floor system is designed to carry a superimposed dead load of 10 psf and a service live load of 60 psf (30% sustained). The concrete strength of the normal density concrete at transfer, f_{ci}’, is 4000 psi and the minimum specified 28-day strength, f_c’, is 6000 psi. The prestressing steel is pretensioned to $0.75f_{pu}$ in the tensioning bed. The initial and final stresses in the strands, f_{pi} and f_{pf}, are 187 ksi and 157 ksi respectively. Check if the stress limit requirements of the ACI 318-05 are satisfied at transfer and under service load conditions

i. At the simply supported ends of the beam
ii. At the mid span

Note that the hollow core slab system is to be used in an industrial building in Houston and the creep coefficient for this application can be taken as 2 (ϕ=2).

HOLLOW-CORE

4'-0" x 12"
Normal Weight Concrete



A	=	262	in²
I	=	4,949	in⁴
y_b	=	6.00	in.
y_t	=	6.00	in.
S_b	=	825	in³
S_t	=	825	in³
b_w	=	8.00	in.
wt	=	273	plf
		68	psf
V/S	=	2.18	in.

Problem 2:

Check if the floor element in Problem 1 satisfies the deflection limits of ACI 318-05. Note that the floor system is not supporting or attached to nonstructural elements likely to be damaged by large deflections. Calculate deflections using the material and sectional properties given above. For long-term deflection calculations use ϕ=2. Use the PCI multipliers to check the rigorous long-term deflection calculations. Comment on your results.

HOMEWORK #8

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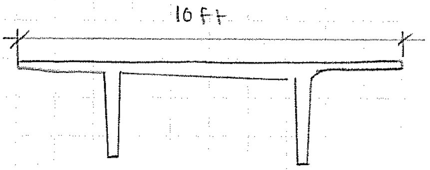
1.



$A = 855\ in^2$
$I = 80780\ in^4$
$L = 66\ ft$

$LL = 95\ psf = 950\ plf$
$SW = 89\ psf = 891\ plf$
$SDL = 0$

$f'_{ci} = 4\ ksi$
$f'_c = 6\ ksi$
$\phi = 0.5\ in$ low·lax
$f_{pu} = 270\ ksi$
$f_{pi} = 187\ ksi$
$f_{pf} = 157\ ksi$

Choose No. of strands

$$M_{max} = \frac{wL^2}{8} = \frac{(66ft)^2}{8}\left[891plf + 950plf\right] = 1002\ k\cdot ft = 12029\ k\cdot in$$

$$M_u = \frac{(66ft)^2}{8}\left[1.2(891plf) + 1.6(950plf)\right] = 1410\ k\cdot ft = 16918\ k\cdot in$$

$$(e+k_t)_{double\cdot T} = 0.7h = 0.7(34in) = 23.8in$$

$$f'_t = 7.5\sqrt{f'_c} = 581\ psi$$

$$P_f \geq \frac{M_{max} - S_b f_b}{e+k_t} = \frac{12029\ k\cdot in - (3222 in^3)(0.581 ksi)}{23.8in}$$

$$P_f \geq 427K, \quad A_{ps} \geq \frac{P_f}{f_{pf}} = \frac{427K}{157 ksi} = 2.72 in^2$$

No. strands = 18

or,

$$\phi M_n \geq M_u \sim 0.77 h A_{ps} f_{pu}$$

$$A_{ps} = \frac{0.9(16918\ k\cdot in)}{0.77(34in)(270ksi)} = 2.15 in^2,\ 14/16\ \text{strands}$$

Use 18 strands (for now)

HOMEWORK #8

1. (cont'd)

Choose tendon profile

max stresses: at end $= 6\sqrt{f'_{ci}} = 379$ psi

$$e \leq k_b + \frac{M_{min} + S_t f_t}{P_i} \quad , \quad k_b = \frac{S_t}{A}$$

$$M_{min} = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{891 \text{ plf}}{2}\left[(66\text{ft})(25\text{in}) - (25\text{in})^2\right]$$

$$= 712 \text{ k}\cdot\text{in}$$

$$P_i = 18\,(0.153\text{in}^2)(187\text{ksi}) = 515\text{k}$$

$$e \leq \frac{9046\text{in}^3}{855\text{in}^2} + \frac{712\text{k}\cdot\text{in} + (9046\text{in}^3)(379\text{psi})}{515\text{k}}$$

$$e \leq 18.6\text{in}$$

From PCI, use 188D1 : $e_e = 25.07\text{in} - 14.39\text{in} = 10.68\text{in}$

$e_c = 25.07\text{in} - 4\text{in} = 21.07\text{in}$

Stress checks do not work. Vary parameters in Excel until they do.

they should. double check.

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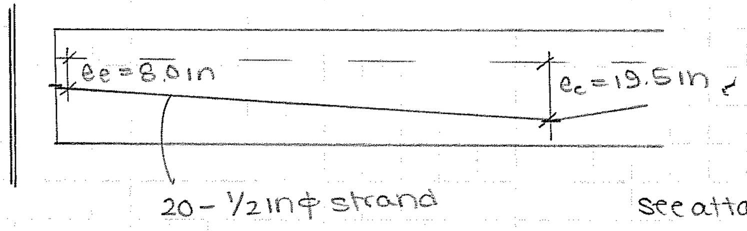
\# strands = 20

$e_e = 8.0\text{in}$

$e_c = 19.5\text{in}$

FINAL DESIGN



See attached spreadsheets for stresschecks.

More checks follow

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HOMEWORK #8

SECTION PROPERTIES

Gross Area	855.0	in^2
Gross Moment of Inertia	80,780	in^4
Height of section	34.0	in
y-top	8.9	in
y-bot	25.1	in
Width of top slab	10.0	ft
Length of span	66	ft
Creep coefficient	2.5	--

assumed

CONCRETE PROPERTIES

f_c'	6.0	ksi
f_{ci}'	4.0	ksi
f_r	580.9	psi
E_{ci}	4415	ksi
E_{ceff}	1261	ksi

STRAND PROPERTIES

f_{pu}	270	ksi
f_{py}	243	ksi
E_{steel}	29000	ksi
$E_{steeleff}$	28420	ksi
Number of strands	20	--
Area of one strand	0.153	in^2
End eccentricity	8.0	in
0.4L eccentricity	17.2	in
Center eccentricity	19.5	in
f_{pi}	187.0	ksi
f_{pf}	157.0	ksi

LOADS

Self-weight	89	psf
Superimposed dead load	0	psf
Live load	95	psf
Sustained loads		psf

SECTION CALCULATIONS

	at TL	0.4L	midspan	
A_{net}	851.9	851.9	851.9	in^2
I_{net}	80,583	79,874	79,616	in^4
y_{net}	25.10	25.13	25.14	in

ST SECTION CALCULATIONS

n	6.6			
A_{trans}	872.0	872.0	872.0	in^2
I_{trans}	81,849	85,725	87,136	in^4
y_{trans}	24.91	24.73	24.69	in

LT SECTION CALCULATIONS

n	22.5			
A_{trans}	920.9	920.9	920.9	in^2
I_{trans}	84,695	98,878	104,042	in^4
y_{trans}	24.50	23.84	23.67	in

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SHORT·TERM

	at TL	0.4L	midspan	
PRESTRESSING				
P_{pi}	572.2	572.2	572.2	
M_{pi}	4577.8	9842.2	11158.3	k-in
f_{ctop}	-0.163	0.419	0.564	ksi
f_{cbot}	-2.090	-3.724	-4.132	ksi
P_{pf}	480.4	480.4	480.4	kip
M_{pf}	3843.4	8263.2	9368.2	k-in
f_{ctop}	-0.137	0.352	0.474	ksi
f_{cbot}	-1.755	-3.126	-3.469	ksi
APPLIED MOMENTS				
Self-weight	310.1	5582.6	5815.3	k-in
f_{ctop}	-0.034	-0.617	-0.643	ksi
f_{cbot}	0.096	1.733	1.805	ksi
Superimposed dead load	0.0	0.0	0.0	k-in
f_{ctop}	0.000	0.000	0.000	ksi
f_{cbot}	0.000	0.000	0.000	ksi
Live load	331.1	5959.0	6207.3	k-in
f_{ctop}	-0.037	-0.659	-0.686	ksi
f_{cbot}	0.103	1.849	1.926	ksi
STRESSES AT TRANSFER				
f_{ctop}	-0.197	-0.198	-0.079	ksi
f_{cbot}	-1.994	-1.991	-2.327	ksi
STRESSES FROM SUSTAINED LOADS				
f_{ctop}	-0.171	-0.266	-0.169	ksi
f_{cbot}	-1.658	-1.394	-1.665	ksi
STRESSES FROM SERVICE LOADS				
f_{ctop}	-0.208	-0.924	-0.855	ksi
f_{cbot}	-1.556	0.456	0.262	ksi

LIMITS							
Tension stress at transfer, middle	--	OK	OK	psi	$3\sqrt{f_{ci}'}$	189.7	psi
Tension stress at transfer, ends	OK	--	--	psi	$6\sqrt{f_{ci}'}$	379.5	psi
Compressive stress at transfer	OK	OK	OK	ksi	$0.60f_{ci}'$	-2.40	ksi
Tension stress from sustained loads	OK	OK	OK	psi	$7.5\sqrt{f_{ci}'}$	474.3	psi
Compressive stress from sustained loads	OK	OK	OK	ksi	$0.45f_c'$	-2.70	ksi
Tension stress at service loads	OK	OK	OK	psi	$7.5\sqrt{f_{ci}'}$	474.3	psi
Compressive stress at service loads	OK	OK	OK	ksi	$0.60f_c'$	-3.60	ksi

$7.5\sqrt{f_c'}$ for service load

– are you checking at 0.5L, 0.4L, or both?

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HOMEWORK #8

LONG TERM

	at TL	0.4L	midspan				
PRESTRESSING							
P_{pi}	572.2	572.2	572.2				
M_{pi}	4577.8	9842.2	11158.3	k-in			
f_{ctop}	-0.166	0.421	0.570	ksi			
f_{cbot}	-2.097	-3.768	-4.195	ksi	Net section properties		
P_{pf}	480.4	480.4	480.4	kip			
M_{pf}	3843.4	8263.2	9368.2	k-in			
f_{ctop}	-0.139	0.354	0.479	ksi			
f_{cbot}	-1.761	-3.164	-3.522	ksi	Net section properties		
APPLIED MOMENTS							
Self-weight	310.1	5582.6	5815.3	k-in			
f_{ctop}	-0.034	-0.620	-0.647	ksi			
f_{cbot}	0.097	1.757	1.836	ksi	Net section properties		
Superimposed dead load	0.0	0.0	0.0	k-in			
f_{ctop}	0.000	0.000	0.000	ksi			
f_{cbot}	0.000	0.000	0.000	ksi	LT section properties		
Live load	331.1	5959.0	6207.3	k-in			
f_{ctop}	-0.037	-0.644	-0.663	ksi			
f_{cbot}	0.101	1.719	1.759	ksi	ST section properties		
STRESSES AT TRANSFER							
f_{ctop}	-0.200	-0.199	-0.077	ksi			
f_{cbot}	-2.001	-2.012	-2.359	ksi			
STRESSES FROM SUSTAINED LOADS							
f_{ctop}	-0.174	-0.266	-0.169	ksi			
f_{cbot}	-1.664	-1.407	-1.686	ksi			
STRESSES FROM SERVICE LOADS							
f_{ctop}	-0.210	-0.910	-0.832	ksi			
f_{cbot}	-1.564	0.312	0.073	ksi			
LIMITS							
Tension stress at transfer, middle	--	OK	OK	psi	$3\sqrt{f_{ci}'}$	189.7	psi
Tension stress at transfer, ends	OK	--	--	psi	$6\sqrt{f_{ci}'}$	379.5	psi
Compressive stress at transfer	OK	OK	OK	ksi	$0.60f_{ci}'$	-2.40	ksi
Tension stress from sustained loads	OK	OK	OK	psi	$7.5\sqrt{f_{ci}'}$	474.3	psi
Compressive stress from sustained loads	OK	OK	OK	ksi	$0.45f_c'$	-2.70	ksi
Tension stress at service loads	OK	OK	OK	psi	~~$7.5\sqrt{f_{ci}'}$~~	~~474.3~~	~~psi~~
Compressive stress at service loads	OK	OK	OK	ksi	$0.60f_c'$	-3.60	ksi

6/7

HOMEWORK #8

1. (cont'd)

check flexural strength

$M_u = 1410 \text{ k·ft}$ at midspan

$= 1353 \text{ k·ft}$ at 0.4L , $d_p = 17.2\text{in} + 8.931\text{n} = 26.1\text{ in}$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f'_c}\right)$$

$$= (270\text{ksi})\left[1 - \frac{0.28}{0.75}\cdot\frac{20(0.153\text{in}^2)}{(120\text{in})(26.1\text{in})}\cdot\frac{270\text{ ksi}}{6\text{ksi}}\right]$$

$$= 266\text{ ksi}$$

$$a = \frac{A_{ps}f_{ps}}{0.85\, b f'_c} = \frac{(266\text{ksi})(20)(0.153\text{in}^2)}{0.85(6\text{ksi})(120\text{in})} = 1.33\text{in}$$

$$c = 1.77\text{in}$$

$$\omega_p = \frac{A_{ps}f_{ps}}{f'_c b d_p} = \frac{20(0.153\text{in}^2)(266\text{ ksi})}{(6\text{ksi})(120\text{in})(26.1\text{in})} = 0.043 < 0.32\beta_1 \checkmark$$

$$\phi M_n = 0.9(20)(0.153\text{in}^2)(266\text{ksi})\left[26.1\text{in} - \tfrac{1}{2}(1.33\text{in})\right]$$

$$= 1553\text{ k·ft}$$

$$\frac{\phi M_n}{M_u} = \frac{1553\text{k·ft}}{1353\text{k·ft}} = 1.15$$

$\phi M_n = 1695\text{k·ft}$ at midspan

ratio $= 1.20$

Reserve strength after cracking

$f_r = 7.5\sqrt{f'_c} = 581\text{ psi}$

$f_{b_{midspan}} = 73\text{psi}$ (long term)

additional $f = 508\text{psi}$

$\Delta f \cdot S_b = (508\text{psi})(3222\text{in}^3) = 136\text{k·ft}$

$=$ additional load to crack

$M_{cr} = 1002\text{k·ft} + 136\text{ k·ft} = 1138\text{k·ft}$

$\frac{\phi M_n}{M_{cr}} = 1.36$ – extra capacity ✓

Deflections are okay.

check! –2

HOMEWORK #8

2. Using Response, calculate stress in strands

$M_{max} = 1002\ k\cdot ft$, $\varepsilon \sim 0.00602$

$M_n = 1726\ k\cdot ft$, $\varepsilon \sim 0.012$

$$f_{ps} = E\varepsilon_p\left[A + \frac{1-A}{(1+(\beta\varepsilon_p)^c)^{1/c}}\right], \quad A = 0.025,\ \beta = 118,\ C = 10$$

~~f_{ps} at M_{max} = 174 ksi~~

f_{ps} at M_n = 248 ksi — check Response model.

$f_{ps_{EQ}}$ = 266 ksi

The stress calculated by the simplified equation is used to calculate the capacity of the section. Thus, comparing that stress (266ksi) to the stress given by RESPONSE and the Ramberg osgood equation (248ksi) shows good correlation. The stress at Mmax (174ksi), however, is much less than that expected at ultimate.

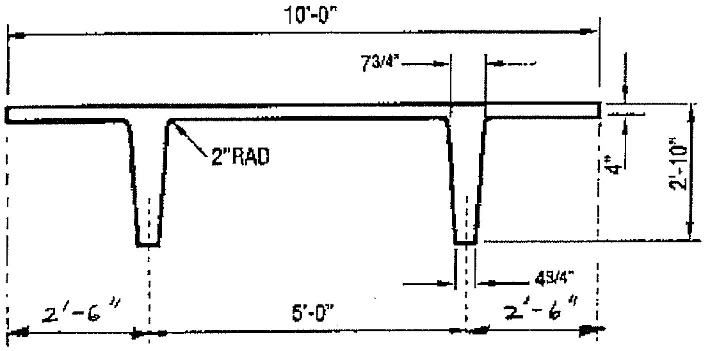
CE 383P – Prestressed Concrete　　Date Assigned: November 12, 2007

Homework Assignment # 8　　Due Date: November 19, 2007

Problem 1:

The pretopped double-tee floor member shown in the figure below spans 66 ft. in a parking structure. No partition that is likely to be damaged due to large deflections is attached to the floor members. The service live load is 95 psf and the superimposed dead load is 0 psf. For the normal weight concrete used, the specified strength at transfer is f_{ci}' = 4,000 psi and the minimum specified 28-day strength is f_c' = 6,000 psi. Use ½" diameter low relaxation stands with f_{pu} = 270 ksi, which is to be tensioned to 0.75f_{pu} in the stressing bed. Assume f_{pi} = 187 ksi and f_{pf} = 157ksi. Design the double-tee.

Pretopped Double-Tee 10′-0″ × 34″



Section Properties:

Normal weight

A = 855 in^2
I = 80780 in^4
y_b = 25.07 in
y_t = 8.93 in
S_b = 3222 in^3
S_t = 9046 in^3
Weight= 891 plf
89 psf
V / S = 2.32 in

Problem 2:

For the cross-section above, determine the moment capacity at mid-span using a sectional analysis program of your choice. Compare the stress in prestressing strands when M = M_{max} with that obtained from the Ch.18 simplified formula. Note that for analysis purposes you can assume $\Delta\varepsilon_p = 6.0 \times 10^{-3}$, low-relaxation strands A = 0.025, B = 118, C = 10, $\varepsilon_c' = -2.3 \times 10^{-3}$.

$\Delta\varepsilon_p$: P/S strain differential

ε_{cf} : concrete strain due to stress

ε_{cr} : strain due to creep

ε_{sh} : strain due to shrinkage

ε_{ci} : initial concrete strain

$= 0$ for pretensioned members

$$\boxed{\varepsilon_{tot} = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth}}$$

long. term (creep) effects calculated using modified E, etc.

$\varepsilon_t' = \dfrac{f_t'}{E_{ct}}$: cracking strain

$f_t' = 4, 6, 7.5\sqrt{f_c'}$ (direct, split, rupture)

f_c: applied stress on member $= -f_c'\left[2\left(\dfrac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\dfrac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2\right]$, solve for ε_{cf}

(−) for compression

ε_c' : max strain on stress parabola

(or strain at max stress)

$\sim 2.0\times10^{-3}$

$\ell_d = \frac{1}{3} f_{sc} d_b + (f_{ps} - f_{se}) d_b$

$E_{cs} = 57\sqrt{f_c'}$

E_{ct} : initial tangent modulus $= \dfrac{2f_c'}{\varepsilon_o = \varepsilon_c'}$

$$\varepsilon_c' = \frac{-2f_c'}{E_{ct}}$$

$$\varepsilon_{c_{eff}}' = \frac{-2f_c'}{E_{c_{eff}}},\quad E_{c_{eff}} = \frac{E_{ci}}{1+\phi(t,t_i)}$$

t : time since casting

t_i : time of stress application

$E_{ci} = E_{ct}$ at time t_i

$$\phi(t,t_i) = 3.5K_cK_g(1.58 - H/120)t_i^{-0.118}\frac{(t-t_i)^{0.6}}{10+(t-t_i)^{0.6}}$$

K_c from graph, pg 5ish

$K_g = [2/3 + f_c'/9000]^{-1}$

H = relative humidity

C R E E P

$$E_{c_{eff}} = \frac{E_{ct}}{1+\phi}$$

$$\varepsilon_{sh} = -K_sK_h\left(\frac{t}{35+t}\right)(0.51\times10^{-3})$$

t : time in days since casting

K_s : volume /surface area ratio factor

K_h : relative humidity factor

shrinkage
pg 8/10

$\varepsilon_{th} = \alpha_c \Delta T$

$\sim 5.5\times10^{-6}/°F$

temp. effects

$\alpha_s = 6.0\times10^{-6}/°F$

$f_{py} = 0.9f_{pu}$ for lowlax strands

$E_{p_{eff}} = \dfrac{f_p}{f_{pi}}E_p$ to consider strand relaxation

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{10}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

relaxation
pg 12

$P_B = P_A e^{-\mu\alpha - kx}$ pg 17

k: wobble coefficient

μ: friction coefficient

α: total angle change

x: tendon length from A to B

$$\Delta = \frac{P_{av}\cdot L}{A_{ps}\cdot E_p}$$

$$L_{set} = \sqrt{\frac{\Delta_{set}\cdot A_{ps}\cdot E_p}{\underline{p}}}$$

$$\Delta P = 2\underline{p}\cdot L_{set}$$

Axially Loaded Members

$N = A_c f_c + A_s f_s + A_p f_p$

$\Delta\varepsilon_p = \varepsilon_{pi} - \varepsilon_{ci}$

tensile capacity determined by f_y, f_{py}; not concrete

$N = A_{trans}\cdot E_c\varepsilon_c + N_o$

in linear elastic range

$-0.5f'_c < f_c < 0.5f'_c$

no yield of strands, rebar

$$A_{trans} = A_c + \frac{E_s}{E_c}A_s + \frac{E_p}{E_c}A_p$$

long-term, use E_{ceff}, E_{peff}

Tension stiffening – included in δ calcs

$\alpha_1 = 1.0$ deformed bars

$\alpha_1 = 0.7$ strand

$\alpha_2 = 1.0$ short-term loads

$\alpha_2 = 0.7$ sustained loads

$$f_c = \frac{\alpha_1\alpha_2 f'_t}{1+\sqrt{500\varepsilon_{cf}}}$$

$\varepsilon_{cf} = \varepsilon_c - \varepsilon_{cth} - \varepsilon_{sh}$

Deflections – pg 32

$w_m = \varepsilon_{cf}\cdot S_m$, $S_m = 3c_{max}$

$\varepsilon_c = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth}$

$\varepsilon_s = \varepsilon_{sf} + \varepsilon_{sth}$

$\varepsilon_p = \varepsilon_{pf} + \varepsilon_{pth}$

$\varepsilon_c = \varepsilon_s$

$$\beta_1 = \frac{4 - \varepsilon_{ct}/\varepsilon'_c}{6 - 2\,\varepsilon_{ct}/\varepsilon'_c}, \quad \alpha_1 = \frac{1}{\beta_1}\left[\frac{\varepsilon_{ct}}{\varepsilon'_c} - \frac{1}{3}\left(\frac{\varepsilon_{ct}}{\varepsilon'_c}\right)^2\right]$$

$C_c = \alpha_1\beta_1 f'_c\, c\, b$

$$\varepsilon_{cen} = \frac{N - N_o}{E_c\cdot A_{trans}}, \quad \phi = \frac{M - M_o}{E_c\cdot I_{trans}}$$

$N_o = E_p\Delta\varepsilon_p A_p$ short-term pg 41

$M_o = -E_p\Delta\varepsilon_p y A_p$

Flexure

$$A_{s\,req'd} = \frac{N_c}{0.6 f_y} = \frac{1/2\, f_{ct} \cdot c \cdot b}{0.6 f_y}$$

f_{ct} = top fiber stress

$$f = \frac{-P}{A} \pm \frac{Mc}{I}$$

[P/S force is negative; remember this in M = Pe calculation]

ACI Stress Limits

initial:
$0.6 f'_{ci}$ comp.
3 or $6\sqrt{f'_{ci}}$ tension (6 at ends)

final:
$0.6 f'_c$ comp. (P/S + total load)
$0.45 f'_c$ comp. (P/S + sust. load)
$7.5\sqrt{f'_c}$ tension (service loads)

$$\boxed{f_c = \frac{P_f}{A_{net}} \pm \frac{(P_f e)y}{I_{net}} \pm \frac{M_D\, y}{I_{net}} \pm \frac{M_S\, y}{I_{trans.\ \text{Long term}}} + \frac{M_L\, Y}{I_{trans.\ \text{short term}}}}$$

If section is uncracked

Deflections

$$\Delta = \frac{2e_c + e_e}{24} \cdot \frac{PL^2}{EI}$$ single haping point

$$\Delta = \frac{1}{8} Pe \frac{L^2}{EI}$$ constant eccentricity

Strand Stress

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(w - w')\right]\right]$$

$\gamma_p = 0.28$ low-lax

$\rho_p = \frac{A_{ps}}{b d_p}$

d = distance comp. to non-P/S reinf.

d_p = d for P/S

No non-P/S ∴

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f'_c}\right]$$

Now:
$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b}, \quad M_n = A_{ps} f_{ps}(d_p - a/2)$$

Design

Keys of Design

$$P_i = \frac{M_{min}}{e - k_b} , \quad P_f = \frac{M_{max}}{e + k_t} , \quad M_{min} = \text{moment due to SW}$$

$$M_{max} = \frac{wL^2}{8} , \quad M_u = 1.2DL + 1.6LL$$

- choose largest e possible at midspan
- select smallest P_f from equation
- detail

If cracking is allowed,

$$P_f \geq \frac{M_{max} - S_b f_b}{e + k_t} , \quad f_b = \text{tensile stress limit}$$

1. Depth of section (fromspa
2. choose P/S

   P_f, $f_{pf} = 157$ KSI

   $f_{pi} = 200$ KSI

   $\phi M_n = M_u = 0.77h A_{ps}f_p$

3. choose tendon profile
4. check concrete stresses
5. ~~deflection calcs~~

   flexural strength

6. check reserve strength after cracking
7. check deflections

Shear

$$V_c = \left[0.6\sqrt{f'_c} + 700 \frac{\$ V_u d}{M_u}\right] b_w d , \quad 2\sqrt{f'_c} \leq V_c \leq 5\sqrt{f'_c}$$

Flexure·Shear cracking

$$V_{ci} = 0.6\sqrt{f'_c} b_w d + \frac{\overset{V_u - V_d}{V_i} M_{cr}}{\underset{M_u - M_d}{M_{max}}} + V_d \geq 1.7\sqrt{f'_c} b_w d$$

V_d : shear force from unfactored SW & superimposed DL

$V_i = V_u - V_d$

Shear Notes
- $\sqrt{f'_c} \leq 100$ psi
- V_u at $h/2$ fromsupport

Web·shear cracking

$$V_{cw} = \left[3.5\sqrt{f'_c} + 0.3 f_{pc}\right] b_w d + \underset{\text{vertical component of P/s force}}{V_p}$$

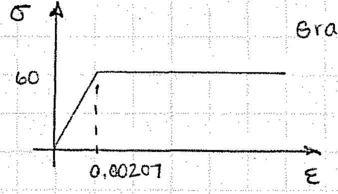
composite construction

Non·uniform strain profile!

INTRODUCTION

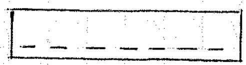
Prestressed concrete

Strain differential exists between concrete and reinforcement



Grade 60 rebar

- how much do you strain rebar before casting?



$\rho = 0.5\%$

using a factor of safety of 2, stress rebar to 30 ksi ($\varepsilon = 0.001$)

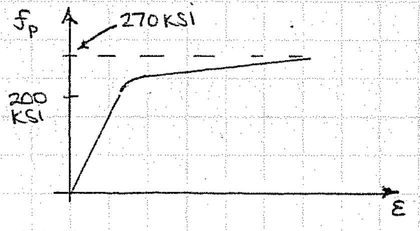
- how much do shrinkage, creep strain concrete?

$$\left.\begin{array}{l}\varepsilon_{sh} \sim 0.5\times10^{-3}\\ \varepsilon_{cr} \sim 0.5\times10^{-3}\end{array}\right\rangle \text{total } 0.001$$

loses all differential from prestressing

CONCLUSION: you can't prestress concrete using Grade 60 reinforcing bars

Prestressing steel



Now we can afford to lose 1 millistrain to concrete losses.

total strain in beam: ε

concrete strain: $\varepsilon = \varepsilon_{cf} + \varepsilon_{crp} + \varepsilon_{sh}$

reinforcing strain: $\varepsilon = \varepsilon_{sf} - \Delta\varepsilon_p$

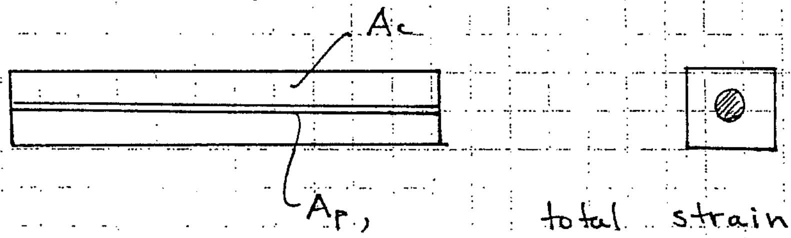
$\uparrow$ prestressing strain

PRESTRESSED CONCRETE

1. INTRODUCTION

1.1 PRESTRESSED CONCRETE

- Prestressed concrete is a type of reinforced concrete in which the steel reinforcement has been tensioned against the concrete.
- Tensioning operation results in self-equilibrating system of internal forces.
- Strain differential exists between concrete ~~and~~ reinforcement.



$\Delta\varepsilon_p$ = p/s strain

total strain = ε

concrete strain = $\varepsilon_c = \overset{\text{concrete stress}}{\varepsilon_{cf}} + \overset{\text{creep}}{\varepsilon_{crp}} + \overset{\text{Shrinkag}}{\varepsilon_{sh}}$

reinf strain = $\varepsilon = \varepsilon_{sf} - \Delta\varepsilon_p$

(more later)

$$N = f_c \cdot A_c + f_s \cdot A_s + f_p \cdot A_p$$

Why:

- Concrete weak in tensile strength ($f'_t \ll f'_c$)
- After cracking, considerable loss in stiffness
- Precompression substantially increases the external load required to crack the concrete resulting in a member that is strong, tough and stiff.

Origins:

- First developed by Eugene Freyssinet of France ca. 1930

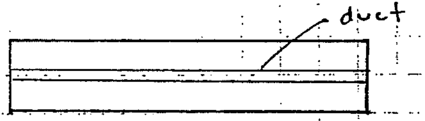
- use of high strength steel wire was 'breakthrough' step
  - concrete plagued by creep, shrinkage
  - if normal reinf. used, only small amount of prestrain can be applied before rebar yields; soon creep/shrinkage erode prestressing
  - with HS wire, much more prestrain possible → feasible ($f_y \simeq 1450$ MPa) $f_u = 260 - 270$ MPa
- By 1950, widespread use in bridges and buildings

1.2 PRESTRESSING SYSTEMS:

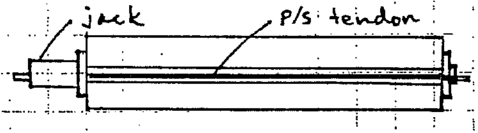
Two types :
- post-tensioning (Freysinnet)
- pretensioning (Hoyer, 1938)

POST-TENSIONING:

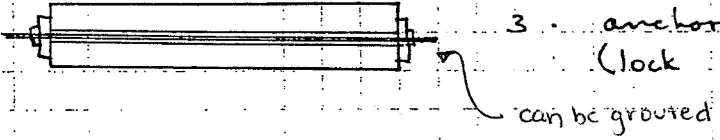
- p/s steel tensioned after member is cast; tension is free in duct



1. cast member with duct.

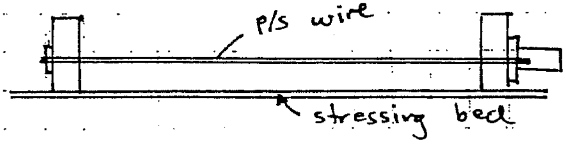


2. tension p/s tendon using jack after concrete has hardened

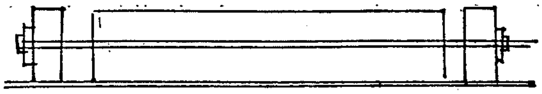


3. anchor p/s tendon (lock in strain differential)

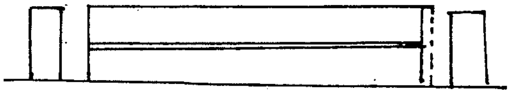
PRETENSIONING:



1. tension wire in p/s bed



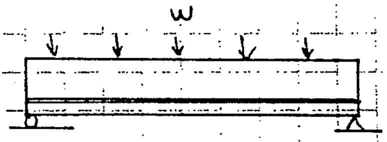
2. cast concrete member (Note: no duct; concrete must bond)



3. release strands from bed member shortens

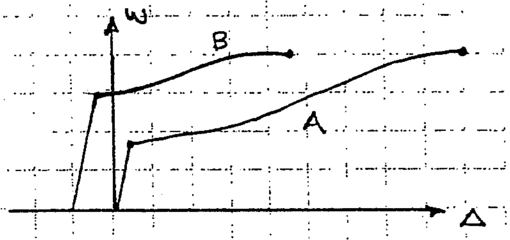
1.3 DESIGN CONCEPT

- P/s reinforcement is placed in those locations of a structure where tensile stresses will be caused by external loads. (same as conventional r/c)
- initial tensioning of reinf precompresses surrounding concrete giving it ability to resist higher loads prior to cracking



A: R/c   As = Ap

B: P/s



discuss :
- initial camber
- deformation
- ultimate capacity (same)

1.4 TYPICAL PRESTRESSED CONCRETE STRUCTURES

- Approx. 50 % of bridges constructed of prestressed concrete
- includes simple girders, box-section girders, cable-stayed bridges etc.
- parking garages (to minimize cracking), water towers, nuclear containment structures, tanks, towers, offshore structures
- buildings made from precast components

READ CHAPTER 1 : INTRODUCTION ←

PRESTRESSING

Pretensioning

order of construction

1. Place strands, stress
2. Place forms
3. Cast concrete
4. cut strands

concrete strength

4000 ~~6700 psi~~ at strand release

takes approximately 10 to 24 hours

precast yards look for 1 day turnovers (necessary to be successful)

Strain differential

$\Delta \varepsilon_p = \varepsilon_{pi} - \varepsilon_{ci}$

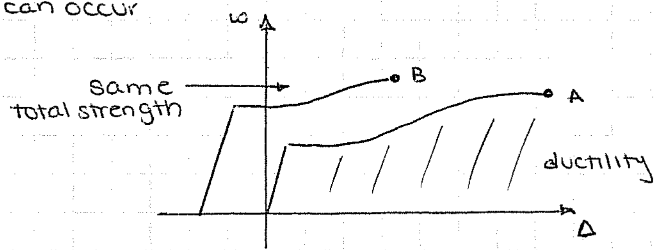
$\rightarrow$ = 0 for pretensioned

not zero for post-tensioned

Benefits of prestressing

under service loads, cracks do not form

in fact, negative deflections (camber) can occur



A: reinforced beam — can deform 4x more than prestressed wires (more ductility)

B: prestressed

Post-tensioning

Order of construction

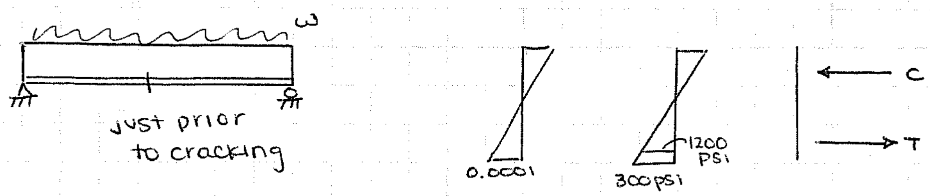
1. cast concrete with hollow duct
2. Place strands
3. Stress strands
4. Anchor strands

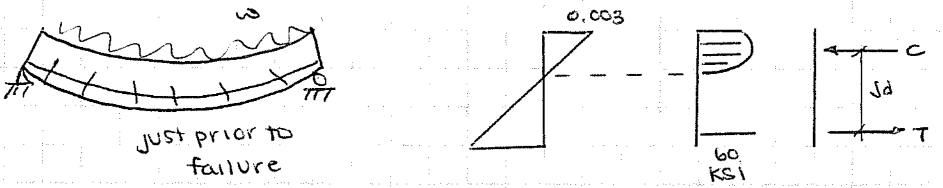
$\rightarrow$ Next step can be to grout within duct (vs. greased strand)

- Thought to help in durability; instead, left voids ⟶ bad
- Structurally, helps make several small cracks, not one big one
- grouting removes ability to replace, restress strands

BEAM COMPARISONS

Reinforced concrete beam

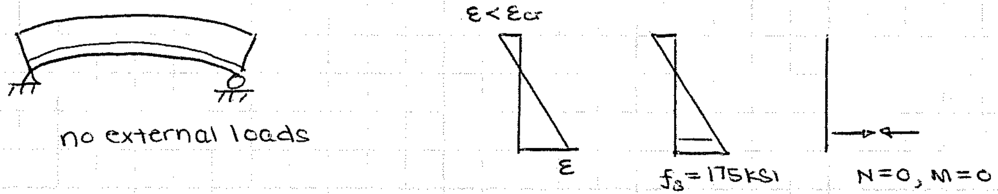


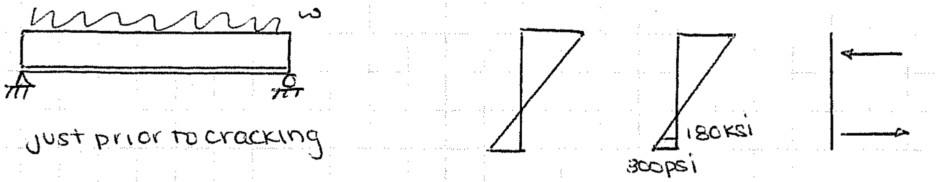


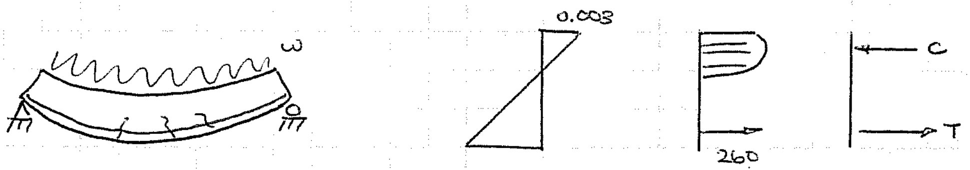
$M = C(jd) = T(jD)$

$= wL^2/8$

Prestressed concrete beam



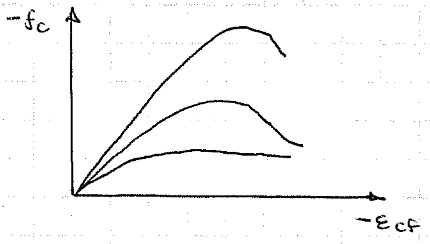




$M = wL^2/8$

MATERIAL PROPERTIES

concrete in uniaxial compression



represented using a parabola

$$f_c = -f_c' \left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2 \right]$$

initial tangent modulus

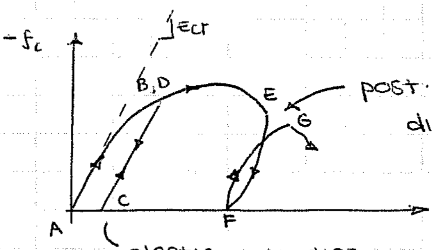
$$E_{ct} = 2\, f_c'/\varepsilon_o$$

(handout accompanies this)

ACI: $E_c = 57000\sqrt{f_c'}$ (psi)

all approximations of modulus

cyclic loading



post-failure unloading dissipates energy

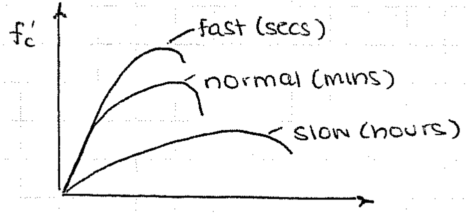
no real use for this part of curve outside seismic

elastic unloading, reloading

fatigue cycles on prestressed concrete works on strands, not concrete.

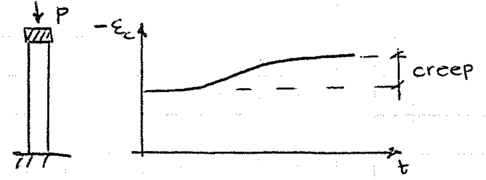
goal: don't let the beam crack

load rates



long term loading (years) doesn't feel slow-load effects because cement hydrates further, overcoming difference

Time-dependent behavior (creep)



modified stiffness

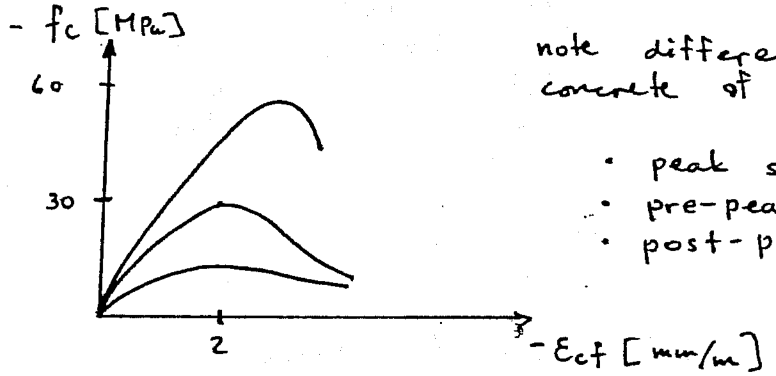
$$E_{ceff} = \frac{E_{ci}}{1+\underset{\uparrow \text{ tricky part}}{\phi}(t,t_i)}$$

2. MATERIAL PROPERTIES

2.1 CONCRETE IN UNIAXIAL COMPRESSION

- typically determined from concrete cylinder 150 mm ϕ x 300 m
- loading rate ≃ -1.00×10^{-3} mm/mm per minute (2-3 mins to failure

- typical response :

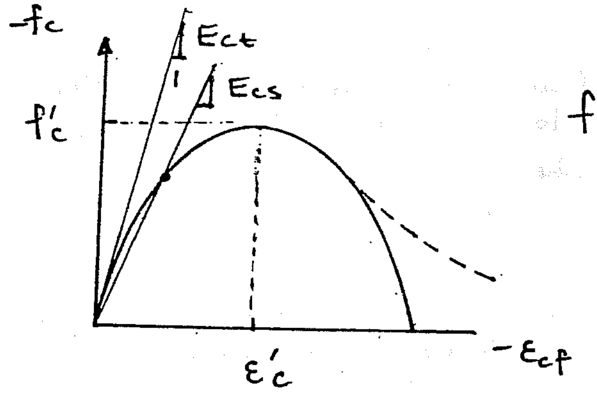


note differences in behaviour of concrete of different strengths

- peak strain, ε_o (ε_c')
- pre-peak nonlinearity
- post-peak brittleness

$-\frac{2f_c'}{\varepsilon_{ceff}}$

- for typical concrete in p/s applications, okay to represent by a parabola: for long term, use ε'_{ceff}, not ε'_c



$$f_c = -f_c'\left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2\right]$$

note: f_c, ε_{cf}, ε_c' are negative qu.

f_c' is positive quantities

let $\varepsilon_o = -\varepsilon_c'$ ~ -0.0002

f_c' = peak stress obtained from cylinder test
ε_{cf} = strain due to stress (NOT TOTAL STRAIN)
f_c = longitudinal stress

Initial Tangent Modulus ; E_{ct} :

$$E_{ct} = 2\,\frac{f_c'}{\varepsilon_o} \quad \text{(analytically, from parabola)}$$

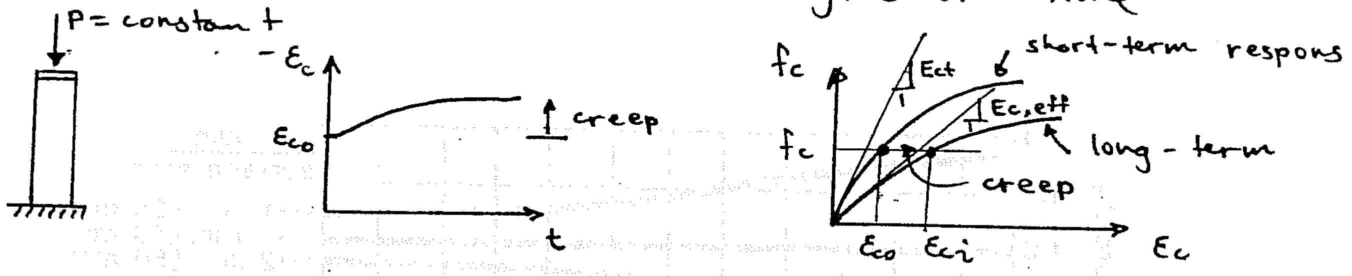
[(SI) $E_{ct} = 5500\sqrt{f_c'}$ (MPa) (from cylinder test)]

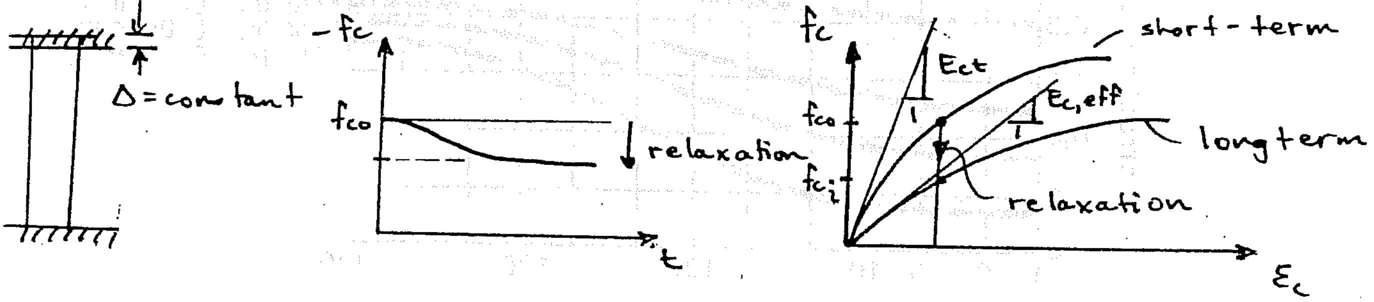
For low compressive stresses, can use approx. linear relation:

$$f_c = E_{cs}\cdot\varepsilon_{cf} \quad ; \quad E_{cs} = \text{secant modulus}$$

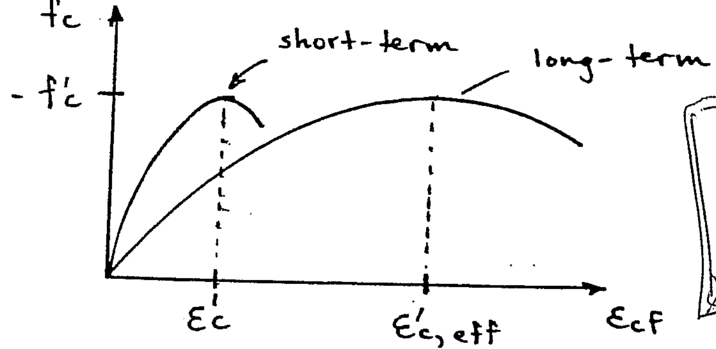
2.3 CREEP OF CONCRETE

- $f_c - \varepsilon_c$ response of concrete depends on:
  - rate of loading
  - time history
  - structural details (eg. vol./surf. ratio)
  - ambient conditions (eg. humidity, tem
- creep: strain increases that occur when stress held constant for some length of time
- relaxation: stress decrease that occurs when strain is held constant for some length of time





- creep and relaxation are inter-related, off course
- we will account for creep using a reduced initial stiffness, $E_{c,eff}$



long-term:

$$f_c = -f_c'\left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_{c,eff}'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_{c,eff}'}\right)^2\right]$$

$$\boxed{\varepsilon_c' = \frac{-2f_c'}{E_{ct}}} \qquad \boxed{\varepsilon_{c,eff}' = \frac{-2f_c'}{E_{c,eff}}}$$

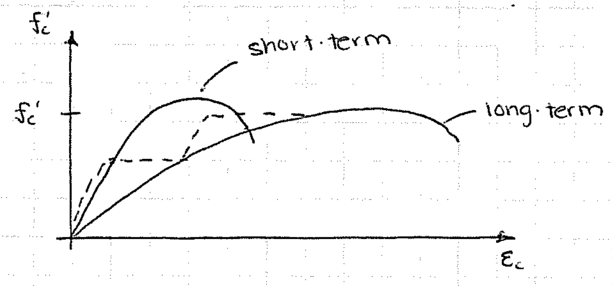
Long-term modulus can be estimated as follows:

$$\boxed{E_{c,eff} = \frac{E_{ci}}{1+\phi(t,t_i)}}$$

where

CONCRETE RESPONSE

Creep / Time Dependent Effects



load to a certain stress, then concrete creeps to long-term curve

long-term curve doesn't actually exist adding stress will still follow shape of short-term curve

$$E_{ceff} = \frac{E_{ci}}{1 + \phi(t, t_i)}$$

t = time measured from casting of concrete

t_i = time at which stress applied (loaded), in days

E_{ci} = E_{ct} at time t_i

$\phi(t, t_i)$ = creep coefficient

$$\phi(t, t_i) = 3.5 k_c k_g \left(1.58 - H/120\right) t_i^{-0.118} \frac{(t-t_i)^{0.6}}{10 + (t-t_i)^{0.6}}$$

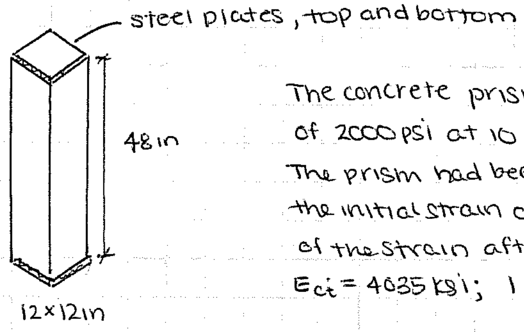
relative humidity (in %)

a factor accounting for the influence of concrete strength

$$k_g = \left[0.67 + f_c'/9000\right]^{-1} \quad \text{(in psi)}$$

a factor that accounts for the influence of the volume-to-surface area ratio for the member (figure 3.12 in handout 2)

Example on creep



The concrete prism was subjected to compression stress of 2000 psi at 10 days after casting. $f_c' = 5000$ psi. The prism had been steam cured for 1 day. Estimate the initial strain caused by the stress and the magnitude of the strain after 100 days of load at $H = 70\%$.

$E_{ci} = 4035$ ksi; 1 day steam cure = 7 days moist cure

CREEP OF CONCRETE

Example, cont'd

Summary of info:

$t_i = 9 + (1 \times 7) = 16$ days

$t = 116$ days

$t - t_i = 100$ days

Now:

$$\varepsilon_c' = \frac{-2f_c'}{E_{ct}} = \frac{-2\,(5000\text{ psi})}{4035\text{ ksi}} = -2.48 \times 10^{-3}\text{ in/in}$$

negative sign indicates compression

$$f_c = -f_c' \left[2\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^2 \right]$$

$$-2000\text{ psi} = 5000\text{ psi}\left[2\left(\varepsilon_{cf}/-2.48\times10^{-3}\right) - \left(\varepsilon_{cf}/-2.48\times10^{-3}\right)^2 \right]$$

solve for $\varepsilon_{cf} = -0.559 \times 10^{-3}$ in/in

Note: for a compressive stress $\leq 0.5 - 0.6 f_c'$, linear elastic approximation is acceptable.

$$\varepsilon_{cf} = \frac{-2000\text{ psi}}{4035 \times 10^3\text{ psi}} = -0.50 \times 10^{-3}\text{ in/in}$$

Not too far off other calculation

For K_c:

$$\frac{\text{volume}}{\text{surface area}} = \frac{(48\text{ in})(12\text{ in})(12\text{ in})}{4(48\text{ in})(12\text{ in})} = 3\text{ in}$$

exposed

No top and bottom because the steel plates "seal" those surfaces; no drying.

$K_c = 0.68$

For K_g:

$$K_g = \frac{1}{2/3 + 5000/9000} = 0.821$$

Then, ϕ:

$$\phi = 3.5\,(0.68)(0.821)\left[1.58 - 70/120\right](16)^{-0.118}\frac{(100)^{0.6}}{10 + (100)^{0.6}} = \underline{\underline{0.86}}$$

$$E_{c_{eff}} = \frac{4035\text{ ksi}}{1 + 0.86} = 2169\text{ ksi}$$ — nearly half what it once was!

$$\varepsilon_{c_{eff}}' = \frac{-2f_c'}{E_{c_{eff}}} = \frac{-2\,(5000\text{ psi})}{2169\text{ ksi}} = -4.61 \times 10^{-3}\text{ in/in}$$

CREEP OF CONCRETE

Example (cont'd more)

$E_{ceff} = 2169 \text{ ksi}$
$\varepsilon'_{ceff} = -4.61 \times 10^{-3}$ in/in

$$f_c = -5000 \text{ psi} \left[2 \left(\varepsilon_{ct} / -4.61 \times 10^{-3} \right) - \left(\varepsilon_{cf} / -4.61 \times 10^{-3} \right)^2 \right] = 2000 \text{ psi}$$

$\varepsilon_{cf} = -1.04 \times 10^{-3}$ in/in
(about double)

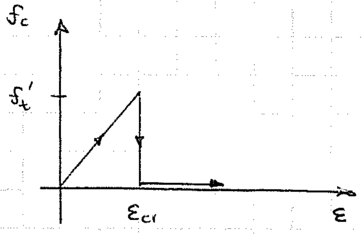
using linear approximation, $\varepsilon_{cf} = \frac{-2000}{2169 \times 10^3} = -0.92 \times 10^{-3}$

creep strain = total (calculated) minus initial
$-1.04 \times 10^{-3} + 0.559 \times 10^{-3}$
$= 0.0005$ in/in

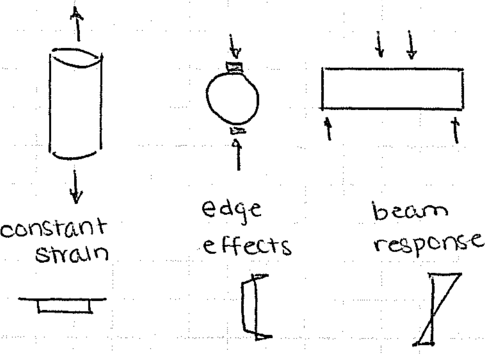
Is that a big value? depends what the structure is
can cause considerable stresses in
a floor slab, for instance

CONCRETE IN TENSION

Material response



measure tensile strength using tests:
- direct tension , $4\sqrt{f_c'}$
- split cylinder , $6\sqrt{f_c'}$
- modulus of rupture, $7.5\sqrt{f_c'}$



Shrinkage of concrete (from drying)

$$\varepsilon_{sh} = -k_s k_h \left(\frac{t}{35+t}\right)(0.51\times10^{-3})$$

t — Time in days since casting

k_h — relative humidity factor

k_s — volume / surface ratio factor

all modify base value 0.5 millistrain

Thermal properties

$$\varepsilon_{cth} = \alpha_c \, \Delta T$$

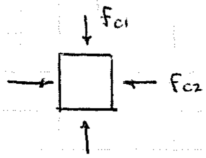
ΔT — temperature change

α_c — coefficient of thermal expansion

ranges from $3.3\times10^{-6}/°F$ to $7.2\times10^{-6}/°F$

generally use $5.5\times10^{-6}/°F$

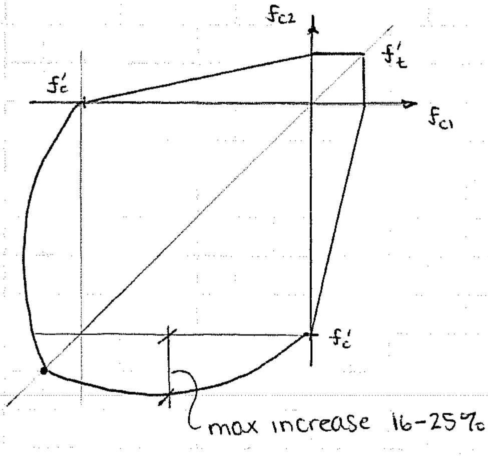
Response to confinement



tension · tension — no effect

tension on compression — decrease capacity

Compression · compression — increases capacity

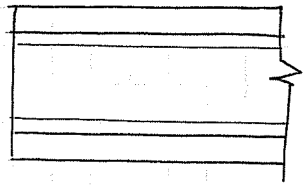


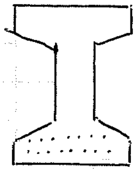
Kupfer's envelope

max increase 16-25% over f_c'

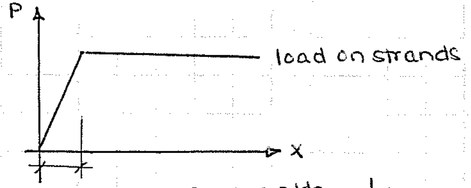
STRESSES IN CONCRETE

confinement and transfer





50 strands, 31 k/strand

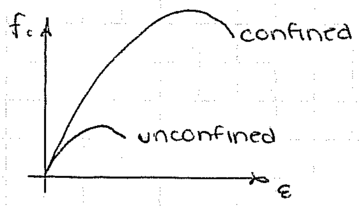


Lt – transfer length = $\frac{1}{2}$As

D-region

Triaxial confinement – Illinois

used geotech triaxial test setup



$f'_{c\,com} = f'_c + 4.1 f_e$ ← confinement stress

huge difference because even in 2D confined setting, out-of-plane expansion and failure occurs

increases:

stiffness, strength

AND ductility

All additional strains:

$$\varepsilon_{tot} = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth} \left[+ \varepsilon_{cr} \right]$$

considered with modified modulus

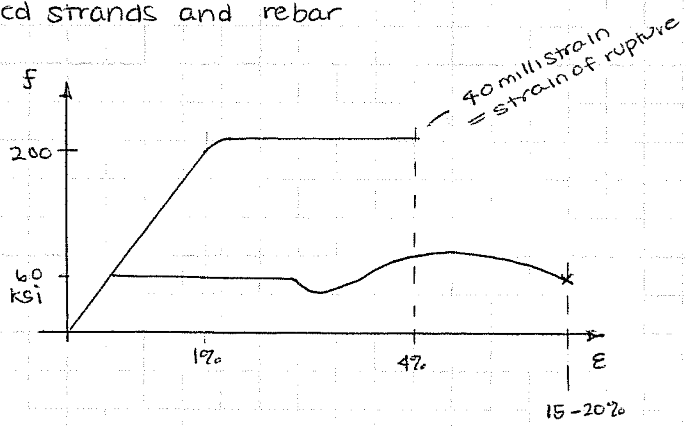
thermal strain

shrinkage strain

strain due to stress

REINFORCEMENT

Prestressed strands and rebar



f_{py} = 220-240 ksi — yield point is not well defined

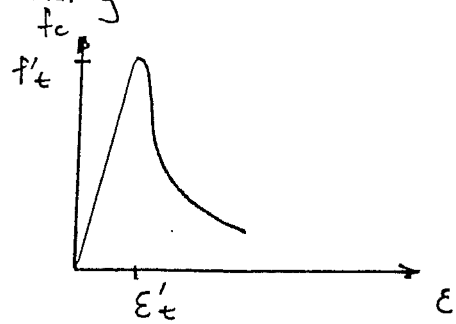
7-wire strands:

- stress-relieved → heat, no tension
- low relaxation (lolax) → strain tempering
  - heat to 660°F while strand is under tension
  - 4.5 to 5x less relaxation losses
- untreated

f_{pu} = 270 ksi
A = 0.153 in^2
weight = 0.53 lb/ft

2.4 CONCRETE IN TENSION

- Stress-strain response is essentially linear up to cracking



Some tension softening after cracking ; not significant

Also tension stiffening effect, in R/C, will discuss later

We will approximate relation as :

$$f_c = E_{ct} \cdot \varepsilon_{cf} \qquad 0 < \varepsilon_{cf} < \varepsilon'_t$$

$$= 0 . \qquad \varepsilon_{cf} > \varepsilon'_t$$

where E_{ct} = initial tangent modulus as for concrete in compression

$\simeq$ ~~$5500\sqrt{f'_c}$ (MPa)~~ $57000\sqrt{f'_c}$ (psi)

$\varepsilon'_t = \varepsilon_{cr}$ = cracking strain

$= \dfrac{f'_t}{E_{ct}}$

f'_t = cracking stress

Can be determined from tests, for example, from direct tension tests or from split cylinder tests among others. ; each give different results

Estimate direct cracking stress as:

$$f'_t = \left[0.33\sqrt{f'_c} \ (\text{MPa})\right] = \boxed{4\sqrt{f'_c} \ (\text{psi})} = 4\lambda\sqrt{f'_c} \ (\text{psi})$$

Estimate modulus of rupture as :

$$f_r = 0.6\sqrt{f'_c} \ (\text{MPa}) = \boxed{7.5\sqrt{f'_c} \ (\text{psi})} = 7.5\lambda\sqrt{f'_c} \ (\text{psi})$$

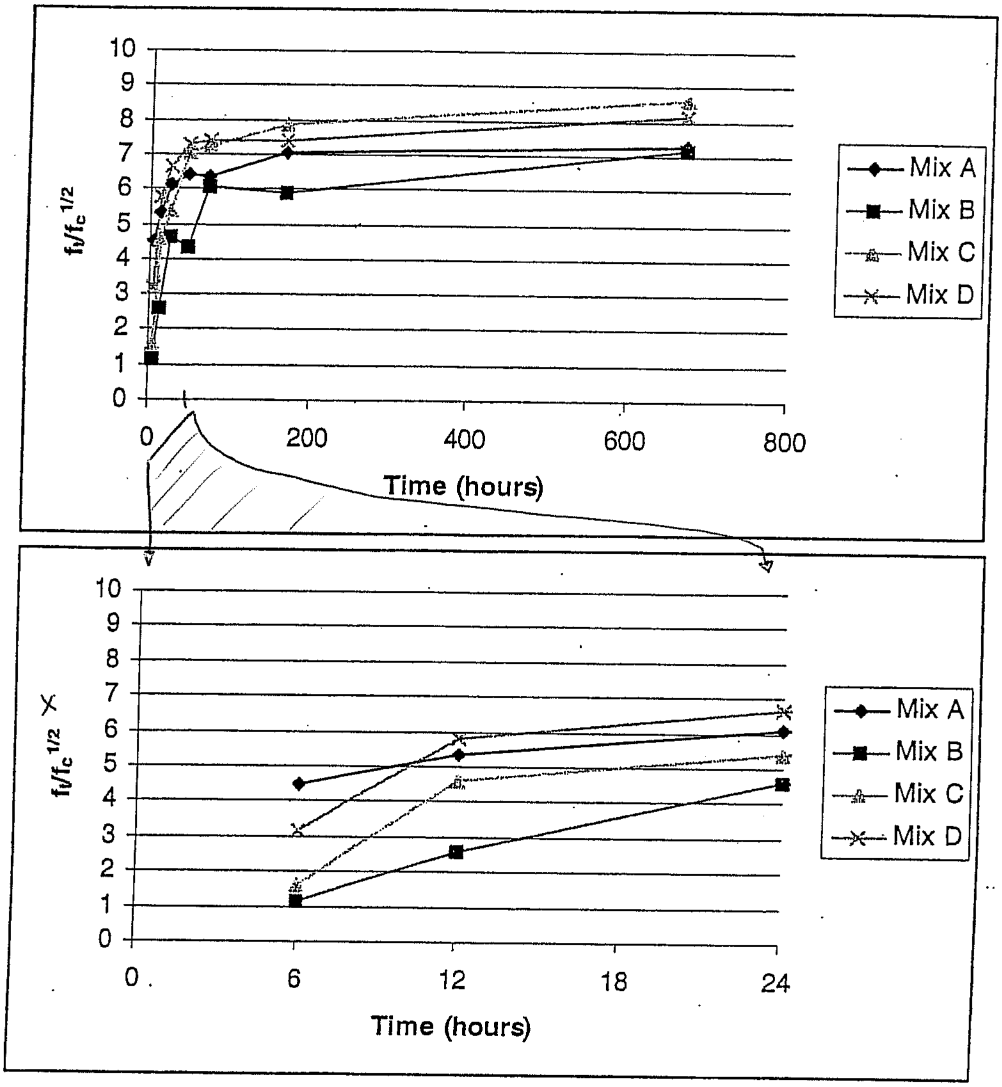
$\lambda = 1.0$ for normal weight concrete

$= 0.85$ for sand-light weight concrete

$= 0.75$ for all-light weight concrete

} often used to reduce shipping weights

Split cylinder tests



$\times\sqrt{f_c'}$

release of prestress strands → cracking of top of beam

2.5 SHRINKAGE OF CONCRETE

Concrete loses moisture with time and decreases in volume → process known as 'shrinkage'

Amount of shrinkage depends on:
- composition of concrete
- elapsed time
- ambient conditions
- volume/surface ratio

For moist-cured concrete:

$$\varepsilon_{sh} = -k_s k_h \left(\frac{t}{35+t}\right) 0.51 \times 10^{-3}$$

where t = time (days) concrete exposed to drying

k_s = correction factor for volume/surface ratio

k_h = correction factor for relative humidity

see Fig 3-18 of Collins/Mitchell for k_s, k_h factors

6 THERMAL PROPERTIES OF CONCRETE

- As with all materials, experiences thermal expansion/contractio

$$\varepsilon_{c\,th} = \alpha_c \cdot \Delta T$$

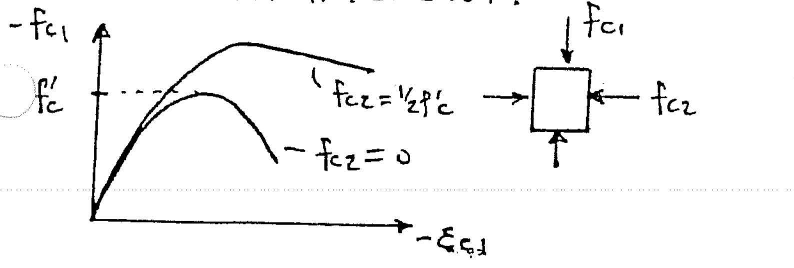
where
α_c = coeff of thermal expansion
ΔT = temperature change

α_c
- depends on type of aggregate used
- ranges from ~~$6 \times 10^{-6}/°C$~~ $3.3 \times 10^{-6}/°F$ to $13 \times 10^{-6}/°C$ $7.2 \times 10^{-6}/°F$
- customary to use $\alpha_c = (10 \times 10^{-6}/°C)$ $= 5.5 \times 10^{-6}/°F$

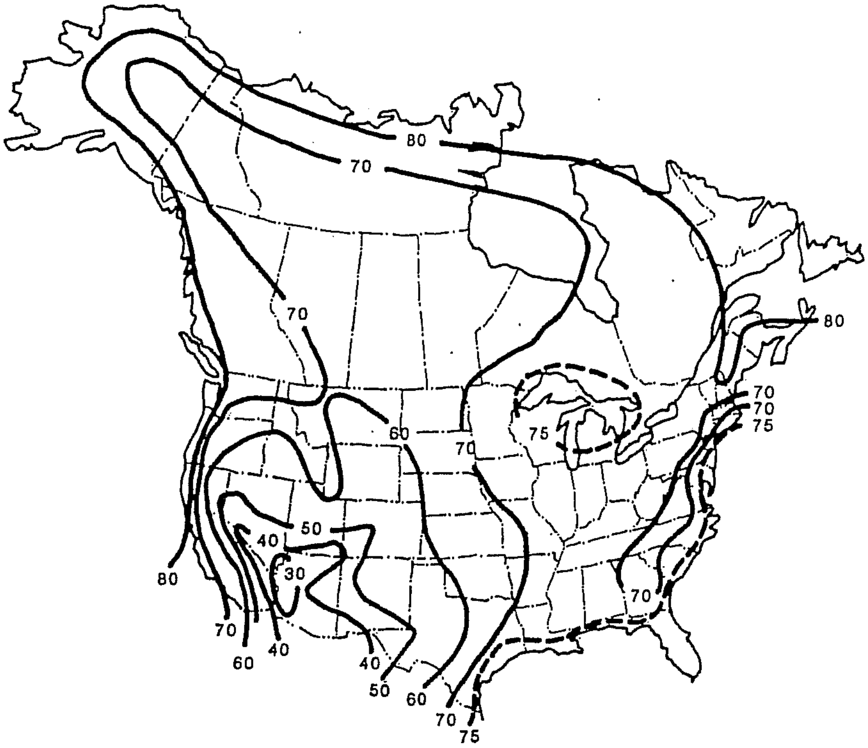
USE $\alpha_c = 5.5 \times 10^{-6}/°F$

2.7 RESPONSE OF CONFINED CONCRETE

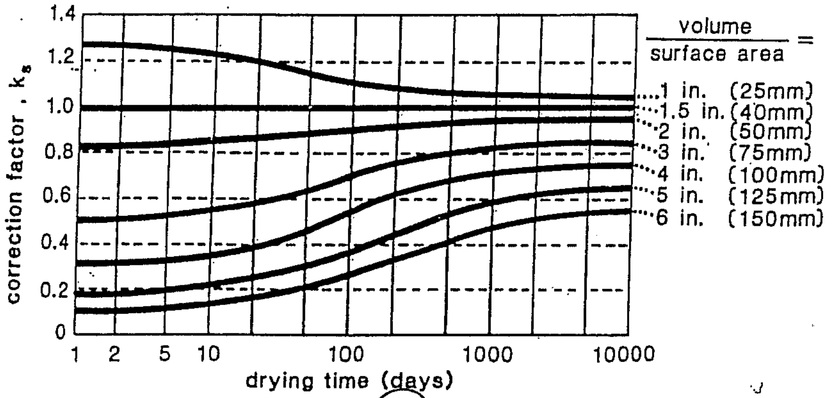
Biaxial Confinement:



- biaxial confinement results in increases in:
  - stiffness
  - strength
  - post-peak ductility



(a) Annual average ambient relative humidity



(b) Factor k_s

avg. ambient rel. hum.	k_h
40 %	1.43
50 %	1.29
60 %	1.14
70 %	1.00
80 %	0.86
90 %	0.43
100 %	0.00

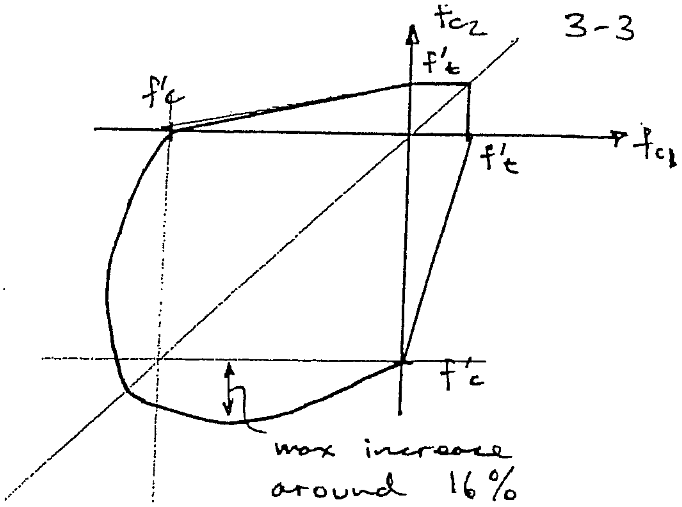
← can't evaporate

(c) Factor k_h

Figure 3-18 Correction factors for relative humidity, based on data given in Ref. 3-20.

Biaxial strength enhancement best represented by Kupfer envelope.

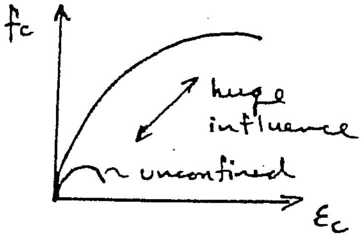
* note max increase occurs when normal stress ≈ 1/2 f'_c
* note linear transition between tension and compression
* note no interaction in biaxial tension



max increase around 16%

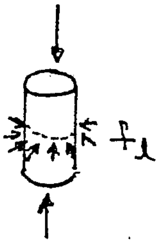
Triaxial Confinement:

Much more pronounced increases in stiffness, strength and ductility when triaxially confined



confined strength

$$f'_{c,con} = f'_c + 4.1 f_l$$



(Not too much of a factor in what we will cover in this course.)

2.8 CONCRETE COMPATIBILITY RELATION

$$\varepsilon_{c_{tot}} = \underset{\substack{\uparrow \\ \text{total} \\ \text{strain}}}{\varepsilon_{tot}} = \underset{\substack{\uparrow \\ \text{strain} \\ \text{due to} \\ \text{stress}}}{\varepsilon_{cf}} + \underset{\substack{\uparrow \\ \text{shrinkage} \\ \text{strain}}}{\varepsilon_{sh}} + \underset{\substack{\uparrow \\ \text{thermal} \\ \text{strain}}}{\varepsilon_{c_{th}}} \; (+ \underset{\substack{\nwarrow \\ \text{creep} \\ \text{strain}}}{\varepsilon_{cr}})$$

Note: in some models, creep strain treated as strain component; in our case, treated by modifying stress-strain relationship

In using concrete stress-strain relationships, must determine ε_{cf} and use it in relationship (not $\varepsilon_{c_{tot}}$)

More later.

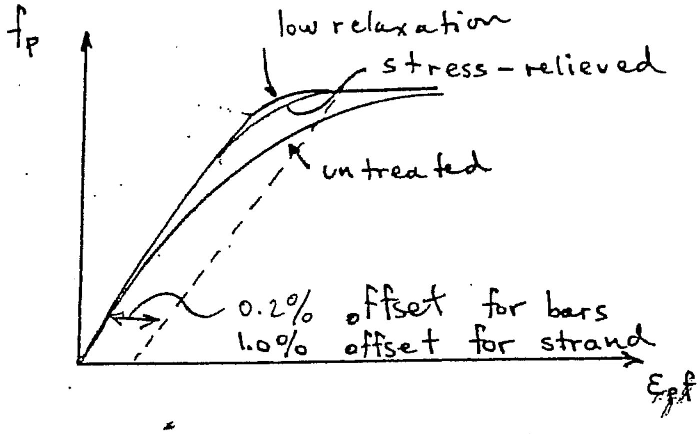
2.9 TYPES OF REINFORCEMENT

Prestressed concrete structures can be reinforced with:

- deformed rebars – passive
  - – prestressing (Dywidag)
- prestressing tendons – prestressing wire
  - – seven wire strand
- welded wire mesh

- See Table ~~3-5~~ 3.6 / Table ~~3-6~~ 3.7 for typical sizes and yield stresses

- most common type used is seven wire strand
  - two types of strands : – stress-relieved } see Fig 3-21
    low – relaxation } 3, 23



low relaxation slightly better in terms of having distinct proportional limit; determined by stress relieving process

- note 'yield' $f_{py} \simeq$ 1500 – 1700 MPa (220 – 240 ksi)

Deformed Bars (Prestressed): Dywidag Bars

- not commonly used
- note 'yield' $f_{py} \simeq$ 1000 – 1100 MPa $\cong$ 150 ksi

Welded Wire Fabric:

- normally used for temperature / shrinkage reinforcement ; not of much concern to us

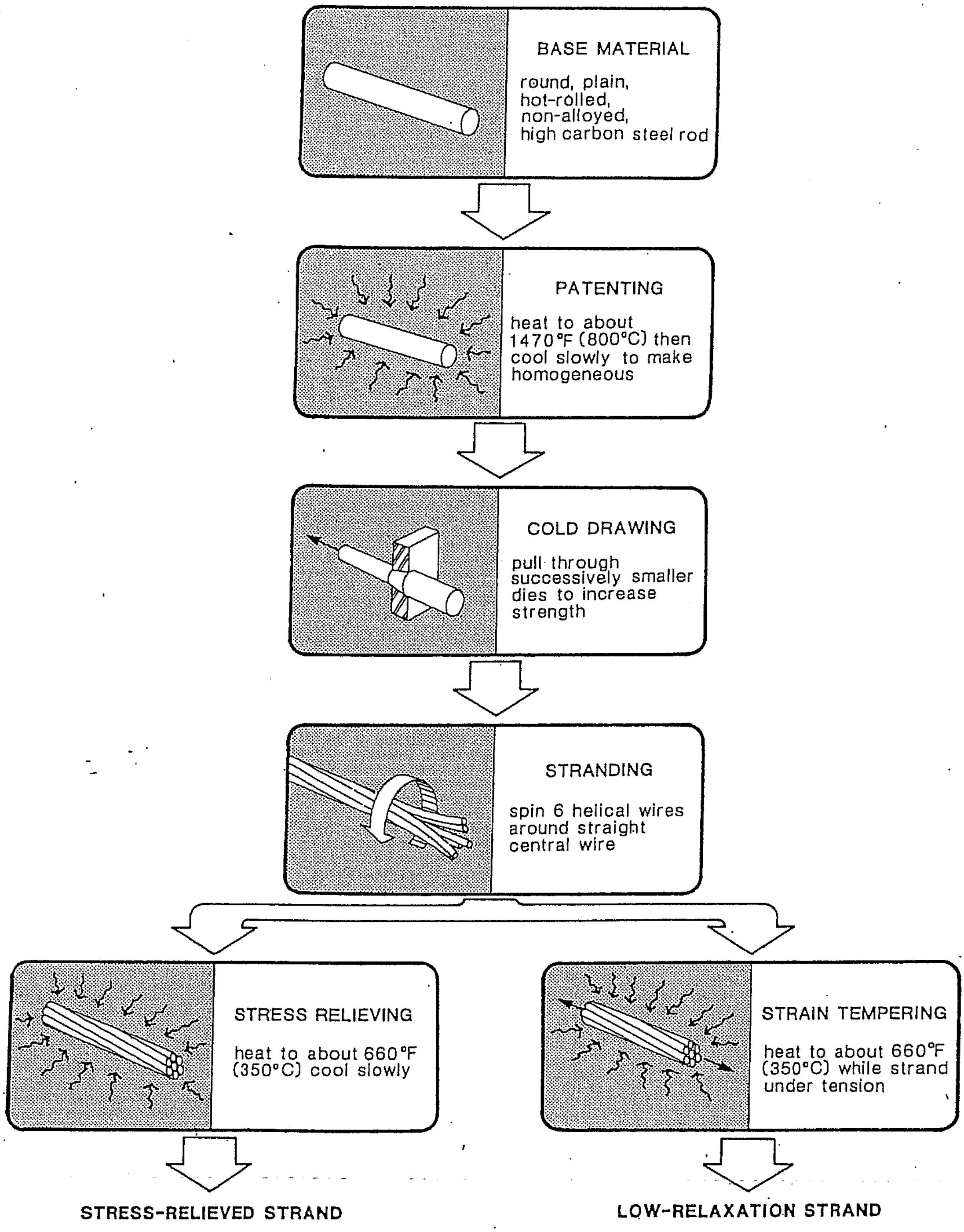


Figure 3-22 Production of seven-wire strand.

Table 3-8 Geometric properties of deformed reinforcing bars.

(a) ASTM standard reinforcing bars

Bar Size	Nominal Diameter in.	Nominal Area in^2	Weight plf
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.600

Table 3-9 ASTM requirements for reinforcing bars.*

	A615		
	Grade 40	Grade 60	A706
Minimum yield, ksi	40	60	60
Maximum yield, ksi	—	—	78
Minimum ultimate, ksi	70	90	$80 \not< 1.25f_y$
Minimum elongation in 8 in. gage length, %			
# 3	11	9	14
# 4, 5, 6	12	9	14
# 7, 8	—	8	12
# 9, 10, 11	—	7	12
# 14, 18	—	7	10
Pin diameter for 180° bend test			
# 3, 4, 5	$3.5d_b$	$3.5d_b$	$3d_b$
# 6	$5d_b$	$5d_b$	$4d_b$
# 7, 8	—	$5d_b$	$4d_b$
# 9, 10, 11	—	$7d_b$	$6d_b$
# 14, 18	—	$9d_b$	$8d_b$

weldable reinforcing bars

*f_y, yield stress; d_b, nominal diameter of reinforcing bar.

Table 3-10 Typical types of welded wire fabric.

Designation*	Wire Diameter in.	Wire Area in^2	Cross-Sectional Steel Area per Foot Width		Approximate Weight lb/ft^2
			Long. in^2	Transv. in^2	
6×6–W1.4×W1.4	0.135	0.014	0.029	0.029	0.21
6×6–W2.1×W2.1	0.162	0.021	0.041	0.041	0.30
6×6–W2.9×W2.9	0.192	0.029	0.058	0.058	0.42
6×6–W4.0×W4.0	0.225	0.040	0.080	0.080	0.58
6×6–W5.5×W5.5	0.264	0.055	0.110	0.110	0.80
4×4–W1.4×W1.4	0.135	0.014	0.043	0.043	0.31
4×4–W2.1×W2.1	0.162	0.021	0.062	0.062	0.44
4×4–W2.9×W2.9	0.192	0.029	0.087	0.087	0.62
4×4–W4.0×W4.0	0.225	0.040	0.120	0.120	0.85
4×4–W4.7×W4.7	0.244	0.047	0.141	0.141	1.02
4×4–W5.5×W5.5	0.264	0.055	0.165	0.165	1.19

*The first number is the longitudinal wire spacing and the second number is the transverse wire spacing. The third and the fourth numbers are the areas of the longitudinal and transverse wires in hundredths of a square inch, respectively.

Table 3-6 Standard prestressing strands, wires, and bars.

(a) Common types from *PCI Design Handbook* (Ref. 3-20)

Tendon Type	Grade f_{pu} ksi	Nominal Dimension Diameter in.	Area in^2	Weight plf
Seven-wire strand	250	1/4	0.036	0.12
	270	3/8	0.085	0.29
	250	3/8	0.080	0.27
	270	1/2	0.153	0.53
	250	1/2	0.144	0.49
	270	0.6	0.215	0.74
	250	0.6	0.216	0.74
Prestressing wire	250	0.196	0.0302	0.10
	240	0.250	0.0491	0.17
	235	0.276	0.0598	0.20
Deformed prestressing bars	157	5/8	0.28	0.98
	150	1	0.85	3.01
	150	1 1/4	1.25	4.39
	150	1 3/8	1.58	5.56

Table 3-7 Requirements for prestressing tendons specified by ASTM (Refs. 3-31 to 3-33).

Tendon Type	Minimum Tensile Strength ksi	Minimum "Yield"* Strength ksi	Minimum Elongation at Rupture %	Gage Length
0.5 and 0.6 in. stress-relieved strand	270	230	3.5	24 in.
0.5 and 0.6 in. low-relaxation strand	270	245	3.5	24 in.
0.276 in. wire	235	200	4.0	10 in.
1, 1 1/4, and 1 3/8 in. deformed prestressing bar	150	120	4.0	$20d_b$†

*Yield strength taken as the stress at an elongation of 1.0% for strand and wire and as the 0.2% offset stress for bars.

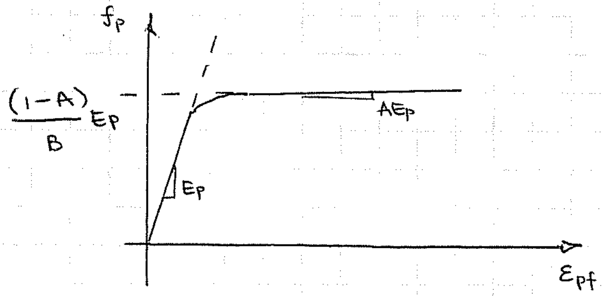
†d_b, nominal diameter of reinforcing bar.

REINFORCEMENT DETAILS

Stress-strain response

$$f_p = E_p \varepsilon_{pf} \left[A + \frac{1-A}{\left[1+(\beta \varepsilon_{pf})^C\right]^{\frac{1}{C}}} \right] \le f_{pis}$$

modified Ramberg-Osgood function

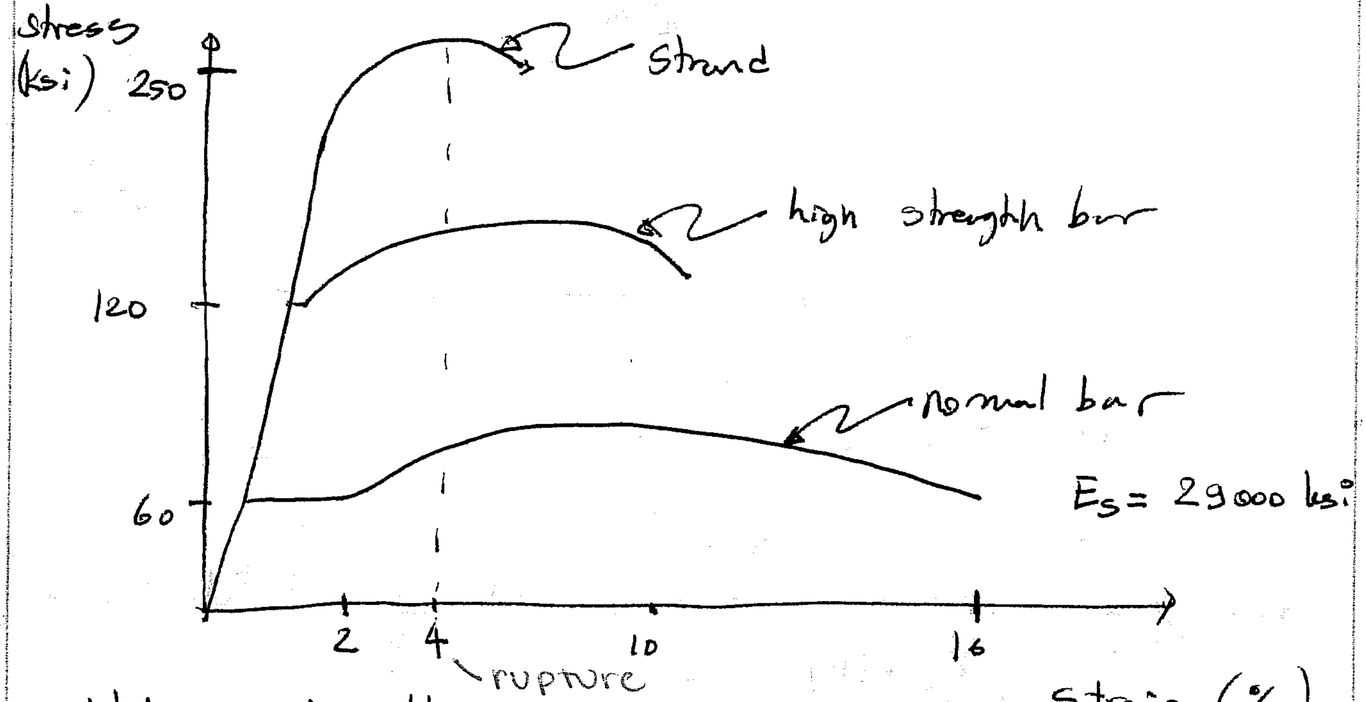


$$f_p = 29\times10^3 \varepsilon_{pf} \left\{ 0.025 + \frac{0.975}{\left[1+(118\varepsilon_{pf})^{10}\right]^{0.1}} \right\} \le 270\text{ ksi}$$

↑ for low-relaxation strands.
stress-relieved strands have slightly different constants (A, β, C ...)

Fatigue: if prestressed beam does not crack,
fatigue will not be a problem

2.10 Stress-Strain Response of Reinforcement.



Note: initial stiffness approx equal and constant for all types

$$f_p = E_p \, \varepsilon_{pf} \leq f_{py}$$ ← Bilinear relationship

A more accurate representation of the stress-strain response of prestressing strand can be obtained by using the modified Ramberg-Osgood function reccomended by Mattock.

$$f_p = E_p \, \varepsilon_{pf} \left[A + \frac{1-A}{[1+(B\varepsilon_{pf})^C]^{1/c}} \right] \leq f_{pu}$$

To estimate the "yield stress", use following.

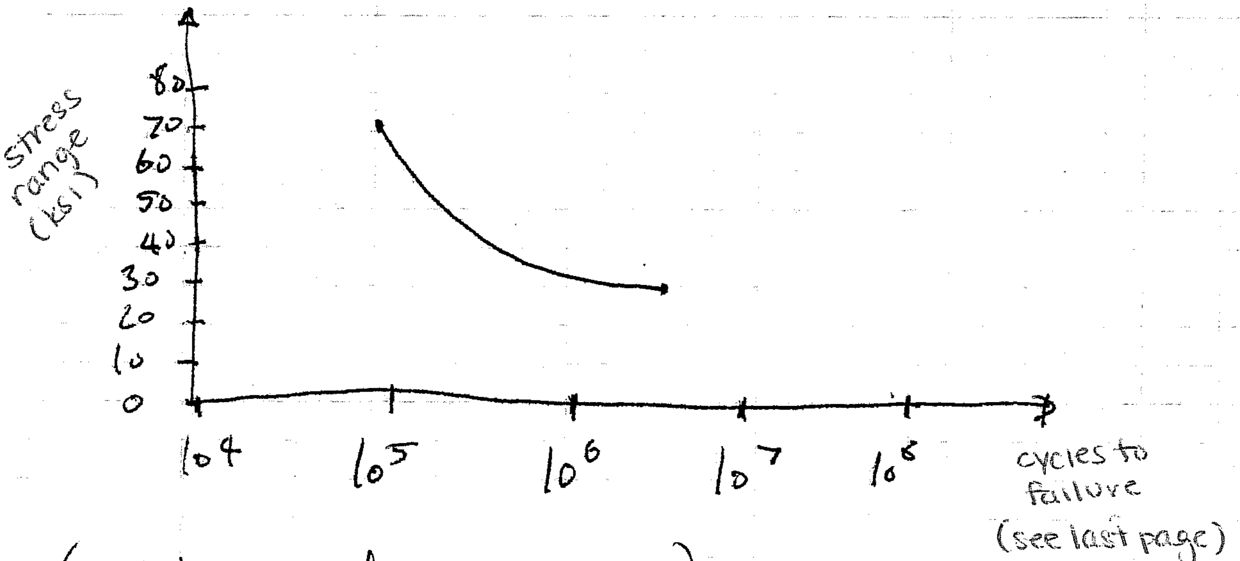
Type	f_{py}/f_{pu}
Low Relaxation Strand	0.90
Stress Relieved Strand	0.85
Plain P/s bars	0.85
Deformed P/s bars	0.80

2.10 FATIGUE CHARACTERISTICS OF REINFORCEMENT

function of stress range $(f_{s,max} - f_{s,min})$

number of cycles of such loading required to cause failure.

i.e S-N curves or Wöhler diagrams



(* Photocopy figure 3.32 *)

- After about 1-2 million cycles the S-N curves become nearly horizontal

The stress range at which the S-N curve becomes horizontal is called the fatigue limit or endurance limit. It's assumed that cycles of stress which have a stress range smaller than the fatigue limit can be endured indefinitely.

* CEP-FIP "Euro Code" defines the characteristic fatigue strength of reinforcing bars as the stress range which can be resisted $\boxed{2\times 10^6 \text{ times}}$ (with the maximum stress going to $0.7 f_y$) by nine bars out of ten. (~ 36 ksi) smooth bars
(~ 22 ksi) deformed bars
in the absence of tests data

* Major study on "Fatigue properties of Prestressing Strand" at Lehigh University in 1966.
Based on these tests "Tide and Van Horn" suggested that

$$\log N = 10 - 3.6 \log \left(\frac{100 \, \Delta f_p}{f_{pu}} \right)$$

∴ for a stress range of 36 ksi and $f_{pu} = 270$ ksi

a mean fatigue life of 892000 cycles can be expecte
~900 000
compared to 2×10^6 in Eurocode

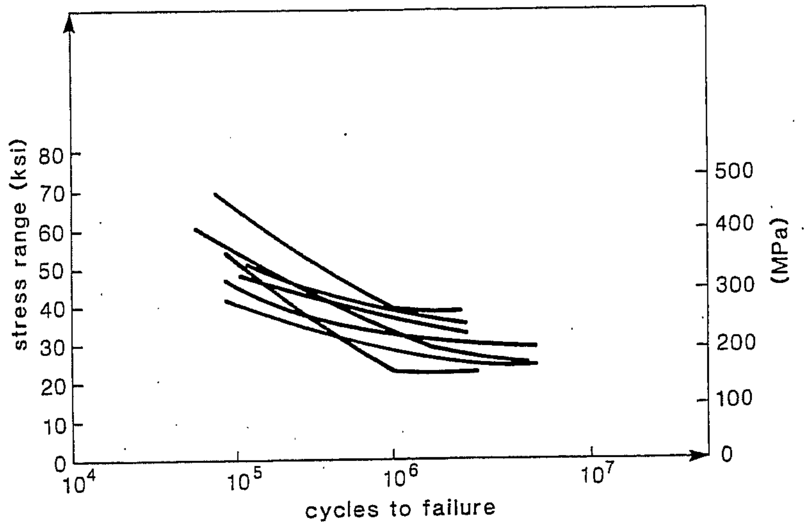


Figure 3-32 *S-N* curves for reinforcing bars. Adapted from Hanson, Somes, and Helgason (Ref. 3-48).

RELAXATION OF STRANDS

Movements over Time

creep – increased deflection under constant load

relaxation – decreased force to maintain a given deflection

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{10}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

log function: lots of early effects, long term is less

if the stress level is less than 55% of yield, don't worry

f_{pi} = initial load stress in prestressed strand

f_p = final load stress

measure relaxation using a modified E

$$E_{Peff} = \frac{f_p}{f_{pi}} E_P$$

low relaxation strands:

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{45}\left(\quad\right)$$

4.5x less than regular strands

creep, relaxation go in same direction – letting strain out of strands

assume creep reduces relaxation by 20%

$$E_{Peff} = \left[1 - \chi_r\left(1 - \frac{f_p}{f_{pi}}\right)\right]E_p$$

relaxation reduction coefficient ~ 0.8

REINFORCEMENT PROPERTIES

Thermal effects

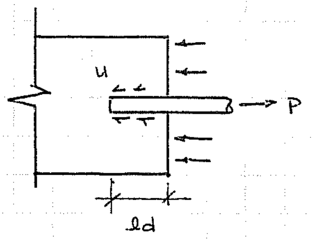
$$\varepsilon_{s_{th}} = \varepsilon_{p_{th}} = \alpha_s \Delta T$$

$\alpha_s = (11.0 \text{ to } 12.0 \times 10^{-6}/°C)$ or $6.5 \times 10^{-6}/°F$

we normally use $5.5 \text{ to } 6.0 \times 10^{-6}/°F$

so that $\alpha_s = \alpha_{conc}$. ← they're close, and it makes the numbers easier.

Bond characteristics



u = bond stresses

ℓd = development length

ACI code suggestions:

$\ell_t = 50 \cdot d$ strand

$= 100 \cdot d$ individual wires

Different than reinforcing bars — no deformations on surface, much higher strength

Not development length but <u>transfer length</u>

The length of tendon at the end of a pretensioned member over which the prestress must be developed.

Total development length:

$$\ell_d = \left(\frac{f_{se}}{3}\right) d_b + (f_{ps} - f_{se}) d_b$$

d_b: bar diameter

f_{se}: effective stress level

f_{ps}: force in prestress

corrosion and durability

↳ cracks and spalls cover
also occurs from cracking

] don't allow cracking
use admixtures to reduce / prevent corrosion

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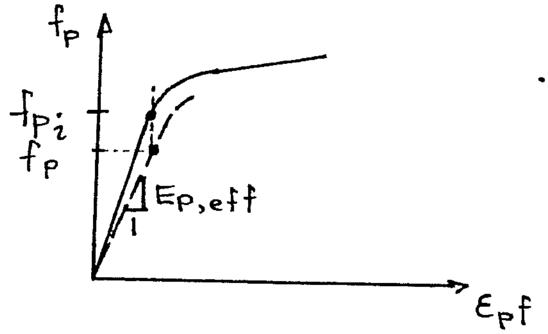
2.11

~~2.9~~ RELAXATION OF PRESTRESSING STEEL

- Force required to hold a strain in a highly stressed tendon will reduce with time; analogous to creep/relaxation in concrete.



- let f_{pi} be initial stress in p/s

For stress-relieved wires or strands:

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{10} \cdot \left(\frac{f_{pi}}{f_{py}} - 0.55 \right)$$

note: 1. relaxation is negligible for situations where $f_{pi}/f_{py} < 0.55$; do not use above

2. most of relaxation occurs over first few hours; rate drops off quickly

3. t is in 'hours'

For example: After 10 hours ($\log t = 1$) the loss $= \frac{1}{6}$ of the loss that will occur after 1 million hours (114 years)

For low-relaxation strand or prestressing bars:

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{45} \left(\frac{f_{pi}}{f_{py}} - 0.55 \right)$$

note: 1. much less relaxation

As with concrete, can account for creep/relaxation by using a reduced stiffness:

$$E_{p,eff} = \frac{f_p}{f_{pi}} E_p$$

where f_p is given by either of above expressions

Complication: strain in prestressing tendons does not stay constant (due to creep and shrinkage of concrete with time)

→ will <u>reduce</u> relaxation loss (think of it as starting with a lower f_{pi} / f_{py})

- reduction of relaxation expressed by

  χ_r - relaxation reduction coefficient

  χ_r = variable (see Figure 3-31)
  $\simeq 0.8$ good ~~enough~~ for us

  (ie 20% reduction in relaxation)

- therefore, effective steel stiffness becomes

$$E_{p,eff} = \left[1 - \chi_r\left(1 - \frac{f_p}{f_{pi}}\right)\right] E_p$$

2.11 THERMAL PROPERTIES OF REINFORCEMENT

- $$\varepsilon_{s_{th}} = \varepsilon_{p_{th}} = \alpha_s \cdot \Delta T$$
- in actual fact, $\alpha_s \simeq (11.0 \text{ to } 12.0 \times 10^{-6}/°C)$ $6.5 \times 10^{-6}/°F$
- <u>however</u>, normally use $\alpha_s = 10 \times 10^{-6}/°C$ $5.5 - 6.0 \times 10^{-6}/°F$

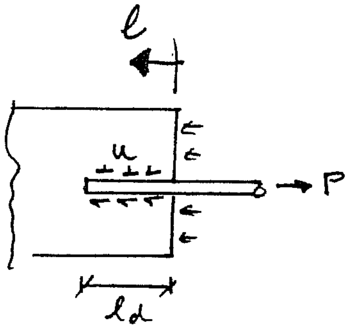
  so that $\alpha_s = \alpha_c$ (much more convenient)
- high temperatures also affect strength and stiffness of reinforcement

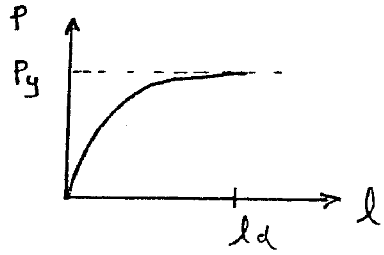
- @ T ≃ ~~400°C~~ 750°F , f_{py} reduced by 50%
- in a fire situation, temperatures (> 1000°c) may be 1500°F or More

∴ must be considered when there is function thermal load

2.~~12~~13 BOND CHARACTERISTICS OF REINFORCEMENT

- Bond stresses are active in transferring stresses from reinforcement to concrete and vice versa
- In previous CE 331 courses, studied bond stress mechanism for passive deformed reinforcing bars
-





$P_y = A_b \cdot f_y$

u = bond stresses ld = development length

l_d = fn (f'_c, f_y, A_b, bar type, bar location) See § 12.2.2

- Bond characteristics of prestressing tendons are of particular importance in pretensioned members where the tendons are anchored by bond alone

consider :
- high stresses to be transferred $f_p \simeq (1500\ MPa)$ 200-220 ksi
- smooth wires (no deformations)

- transfer length : length of tendon at the end of a pretensioned member over which the prestress must be developed (l_t)

(ACI) Code Suggests : (Same as CSA)

l_t = 50 x dia for strand
= 100 x dia for individual wires

CEB - FIP Suggests:

rapid vs. gradual release

l_t = (45 to 90) x dia for strand
= (100 to 140) x dia for wires

$$\ell_d = \underbrace{\left(\frac{\overset{\text{effective stress level}}{f_{se}}}{3}\right)}_{\text{transfer length}} \overset{}{\underset{\text{bar diam.}}{d_b}} + (f_{ps} - f_{se})\, d_b$$

Internally, development length somewhat different.

suggests: (ℨ 12.9.1)

ACI $\ell_d = \overset{0.333}{\cancel{0.048}}\, f_{se}\, d_b + \cancel{0.145}\,(f_{ps} - f_{se})\, d_b$ ksi and inches [~~MPa~~, mm]

where f_{se} = effective prestress in strand (after all losses)

f_{ps} = stress in strand at the critical section at nominal strength

d_b = ~~bar~~ wire diameter (or strand diameter)

ℓ_d = required development length beyond the critical section

this will ensure bond stresses okay

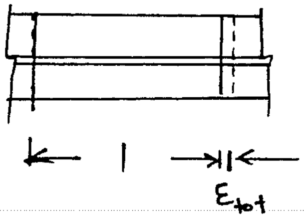
2.~~13~~ 15 CORROSION AND DURABILITY OF REINFORCEMENT

- major concern
- corrosion → cracking of cover → loss of prestress
  → collapse
- amount of cover is critical, tightly controlled
- also control amount of chlorides in concrete, to slow down corrosion process
- also control amount of cracking

(Crack Widths: see Table 3-14
Cover: see Table ~~3-13~~

- ~~read Collins/ Mitchell.~~)

2.14 COMPATIBILITY RELATION

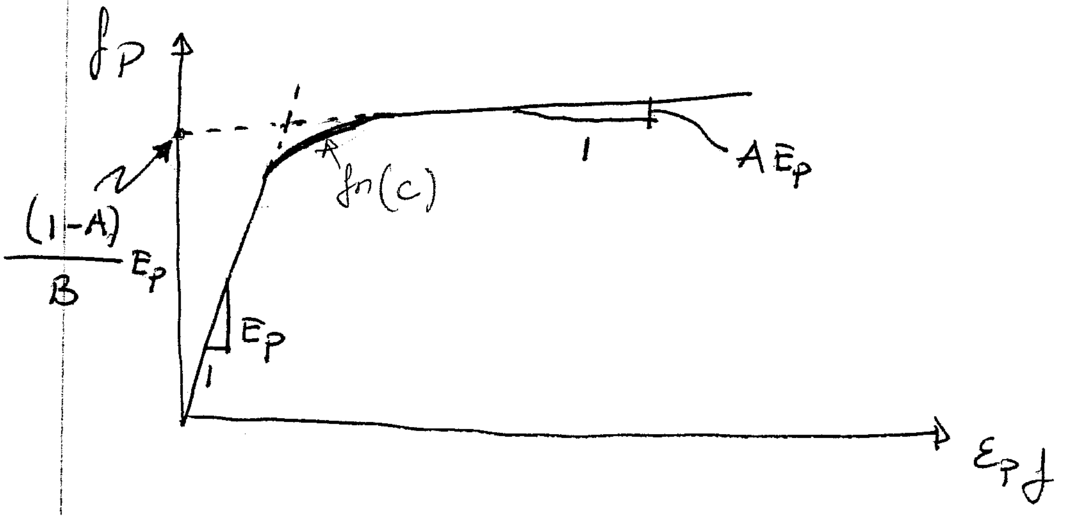


$\varepsilon_{s_{tot}} = \varepsilon_{tot}$ passive reinf

$\varepsilon_{p_{tot}} = \varepsilon_{tot} + \Delta\varepsilon_p$ p/s reinf

$= \varepsilon_{pf} + \varepsilon_{p_{th}}$

$\varepsilon_{pf} = \varepsilon_{tot} + \Delta\varepsilon_p - \varepsilon_{p_{th}}$



For low relaxation strand with $f_{pu} = 270$ ksi (1860 MPa)

$$f_p = 29 * 10^3 \, \varepsilon_{pf} \left\{ 0.025 + \frac{0.975}{\left[1 + (118\,\varepsilon_{pf})^{10}\right]^{0.10}} \right\} \leq 270 \text{ ksi}$$

For stress relieved strands with $f_{pu} = 270$ ksi (1860 MPa)

$$f_p = 29 \times 10^3 * \varepsilon_{pf} \left\{ 0.03 + \frac{0.97}{\left[1 + (121\,\varepsilon_{pf})^{6}\right]^{0.167}} \right\} \leq 270 \text{ ksi}$$

The actual $f_p : \varepsilon_{pf}$ curve lies somewhat above this curve.

We will mostly use $f_p = E_p \cdot \varepsilon_{pf} \leq f_{py}$ for prestressing reinforcement

and $f_s = E_s \cdot \varepsilon_{sf} \leq f_y$ for non-prestressed reinforcement

PRESTRESSING TECHNOLOGY

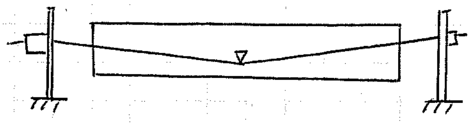
Prestress vs. post-tensioning

post-tensioning → unique projects, cast-in-place (segmental, slabs ...)
can be used (maybe?) for curved girders

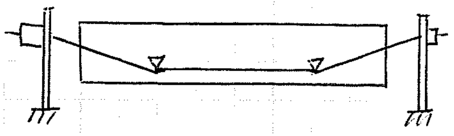
Prestress → repeated elements, easy turnaround
common elements

★ Purchase books!

strand harping



single harping point



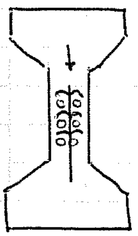
Double harping points

construction process

- stress strands to 90%, straight, then pull harp into place
- or, feed strand through rollers at harp point

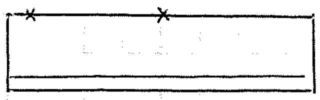
Bayrak says "dangerous" — rub on strands
dirt, etc. in rollers...

generally a dangerous part of the casting process



push down really hard
puts a big kink in the strands, very dangerous

harping and debonding are also used to reduce top fiber stresses near the supports

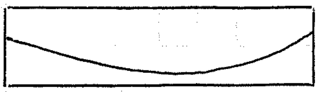


$$\frac{P}{A} \pm \frac{Mc}{I} = f$$

at midspan, dead load counters prestress force
at ends, reduce P — debonding
or reduce eccentricity of force — harping

PRESTRESSING TECHNOLOGY

Post-tensioning operations

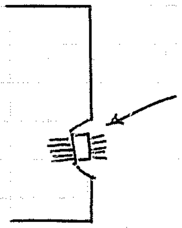


smoothly curving duct

ducts need to be continuous – good splices (not duct-taped), no gaps (concrete could get in)

include vents along length for air to escape when grout comes out, generally good

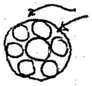
End blocks / anchorages



needs to be waterproofed

if anchor is lost, PT system is lost

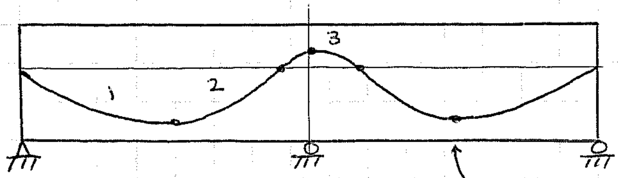
capillary action can pull water through interstitial space



air pockets suck water into tendon

CORROSION!

Duct / tendon profiles



transition point between parabolic profiles

$$\text{slope} = \frac{2e_c}{l_c}$$

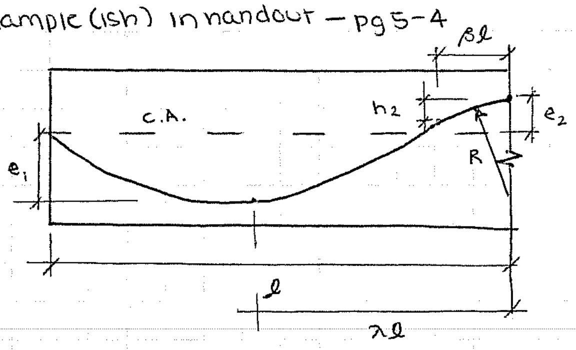
eccentricity

length

need the slope at transition points to be the same

duct geometry (diameter) determines amount of curvature allowable $R = \frac{\lambda\beta l^2}{2(e_1+e_2)}$

See example(ish) in handout – pg 5-4

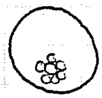


$$h_2 = \frac{\beta}{\lambda}(e_1+e_2)$$

PRESTRESSING TECHNOLOGY

Post-tensioning construction

strands do not stay in center of duct



around negative curvature

table in handout gives appropriate eccentricity

Elongation vs. jacking force

using statics, $\Delta = \frac{PL}{EA}$, 8 in elongation expected

when jacking to correct force, only measure 4 in

why?

generally, there's a jam somewhere; only half the beam is being stressed

code requirements say it all must be within 5% of calculations

To fix:

1. try stressing from the other side, add elongations
2. try to remove strands and rethread
3. throw out beam :(

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3. PRESTRESSING TECHNOLOGY

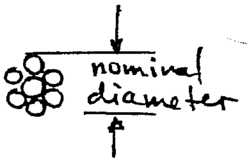
- techniques and technology associated with prestressing is all important in making the concept viable
- two types of prestressing operations
  - pretensioning
  - post-tensioning
- main difference: in pretensioning, tendon is tensioned prior to casting the concrete; in post-tensioning, the tendon is tensioned after the concrete has been cast

* READ CHAPTER 2 OF COLLINS / MITCHELL *

- only some key points follow:

3.1 PRESTRESSING TENDONS

- 7-wire strand is most widely used type of prestressed reinf



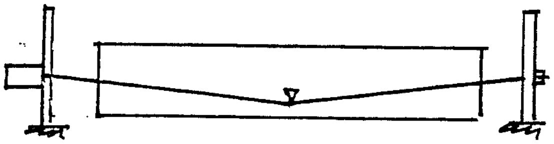
- standard nominal diameters used worldwide are given in inches (originated in States)
- most popular sizes are:
  - 3/8 in (9 mm)
  - → 1/2 in (13 mm) ← current standard
  - 0.6 in (15 mm)
  - 7/10 in ↑ Canadian nomenclature but no different
- $f_{pu} \simeq$ 270 ksi ~~1860 MPa~~ industry standard (ult. tensile strength)
- used in both pretensioning and post-tensioning

Deformed Prestressing Bars
- useful in some post-tensioning applications
- looks like conventional deformed bar except deformations slightly different

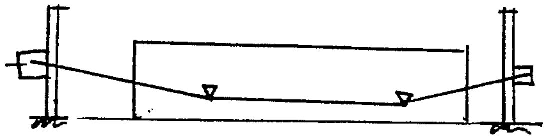
- Also:
  - multi-strand tendons
  - multi-wire tendons
  - individual wires

PRETENSIONING OPERATIONS

- see Collins / Mitchell
- to achieve better performance, tendons are often 'harped'



Single-Harping Point



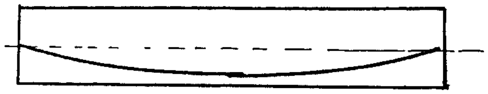
Double-Harping Points

- discuss internal force implications
- turn-around time 16-24 hours

POST-TENSIONING OPERATIONS

- post-tensioning ducts placed in formwork together with (passive) reinforcement cage
- after casting and curing of concrete, the tendons are tensioned and anchored
- then either
  - grout tendons (bonded tendons)
  - grease tendons (unbonded tendons)

  note: above two behave substantially different
- sequence of tensioning is important
- tendons are often 'draped' for better performance

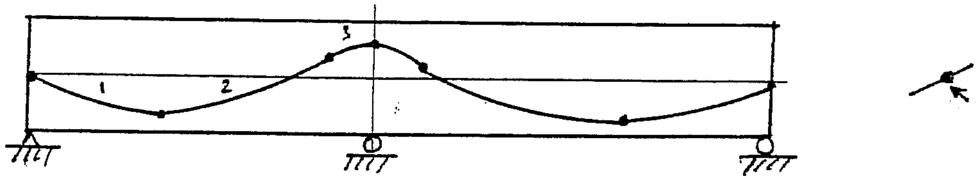


- typically parabolic profile
- no sharp corners because → problems with post-tensioning

- many proprietary systems and hardware for post-tensioning and anchoring
- anchor heads important
- grouting important (venting, corrosion protection)

3.2 PROFILES OF POST-TENSIONED TENDONS

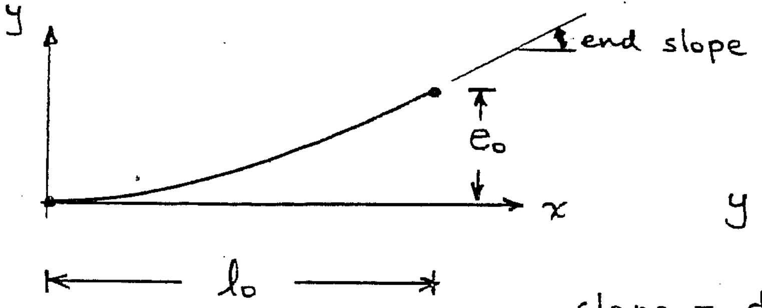
- tendon profiles are usually of a parabolic shape or can be series of parabolic segments
- in simply supported beam : one parabolic curve with maximum eccentricity at midspan
- in continuous beams, tendon shape consists of series of parabolic segments with concave segments in midspan regions and convex segments over the supports



denotes transition pt between parabolic profiles

- at a transition point, there should be continuity of slopes (ie no discontinuities or slopes ie no sharp corners)

- consider geometry of parabolic profile



cable has total eccentrici
e_o over length l_o

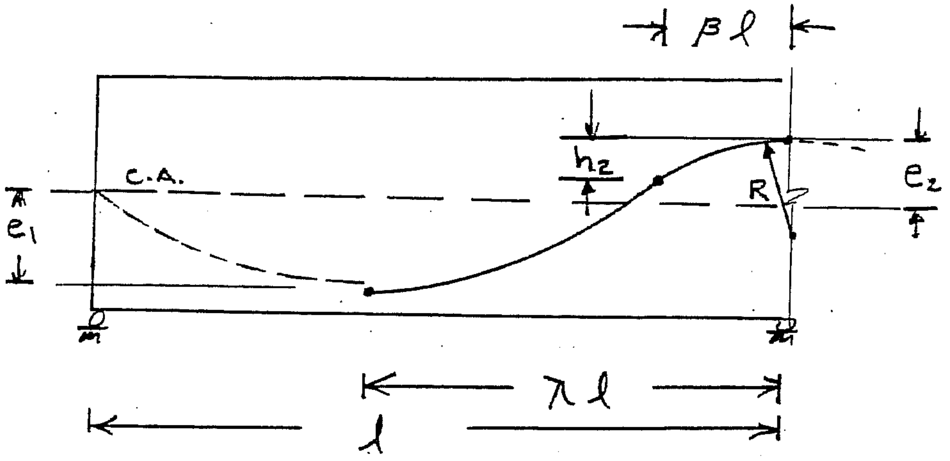
$$y = e_o\left(\frac{x}{l_o}\right)^2$$

$$\text{slope} = \frac{dy}{dx} = \frac{2e_o}{l_o^2}x$$

at $x = 0$ slope $= 0$ $\&$ $y = 0$

at $x = l_o$ slope $= \dfrac{2e_o}{l_o}$ $\&$ $y = e_o$

- consider above sketch again, must find location of transition point between segments 2 and 3 such that slopes are same



let e_1 = max eccentricity of segment 2 } relative to centroidal axes
e_2 = max eccentricity of segment 3

Start by selecting longitudinal location of transition points

i.e. select λ, β

- now, problem in descriptive geometry to find transverse location of transition point (ie find h_2) such that slope is continuous

turns out that $h_2 = \dfrac{\beta}{\lambda}(e_1 + e_2)$

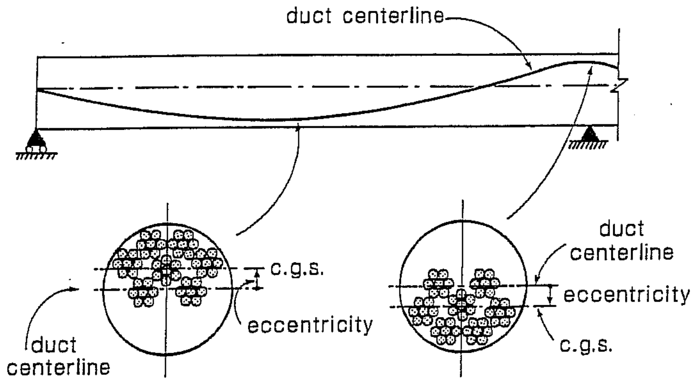
- <u>one more consideration</u>, the radius of curvature R must not be too small otherwise problems with friction losses, fraying, tensioning etc
- turns out that $R = \dfrac{\lambda \beta l^2}{2(e_1 + e_2)}$

- min radius governed primarily by duct diameter

Sheath Inside Diameter (in.)	1.8-2.5	2.6-3.2	3.3-3.8	3.9-4.3
Minimum Radius of Curvature (ft)	12	15	16	23

- if $R < R_{min}$, must modify λ, β (primarily β)

- <u>one final consideration</u>:
  when tendon is stressed, centroid of tendon (c.g.s. - centre of gravity of steel) will not coincide with centreline of duct because



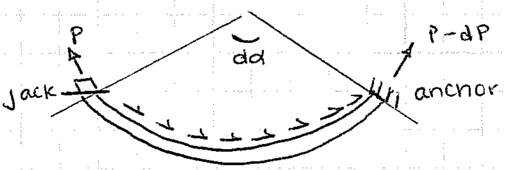
strands at top of duct

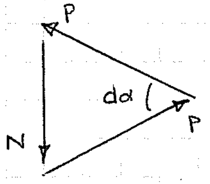
strands at bottom of duct

tendon size	sheath diameter in. (mm)	eccentricity in. (mm)
no. of 1/2 in. dia. (13mm) strands		
3	1.25 (32)	0.28 (7)
4	1.63 (41)	0.28 (7)
7	2.00 (51)	0.32 (8)
12	2.50 (64)	0.43 (11)
19	3.13 (80)	0.51 (13)
22	3.38 (86)	0.47 (12)
31	4.00 (102)	0.55 (14)
55	5.50 (140)	0.90 (23)
no. of 0.6 in. dia. (15mm) strands		
3	1.50 (38)	0.20 (5)
4	2.00 (38)	0.20 (5)
7	2.25 (57)	0.40 (10)
12	3.00 (76)	0.50 (13)
19	3.75 (95)	0.70 (18)
31	5.00 (127)	0.90 (23)
55	6.50 (165)	1.20 (30)

POST·TENSIONING

Losses during post·tensioning
- curvature (friction)
- wobble



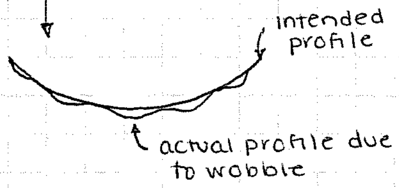


length of tendon dx
direction changes by angle dα

$N = 2P \cdot \sin(d\alpha/2)$

$dP = \mu \cdot N$ ← coefficient of friction

$\underline{dP_c = \mu P d\alpha}$, if small angles are assumed



wobble explains difference between calculations and experiments

wobble depends on:
- rigidity of duct
- diameter of duct
- spacing of duct supports
- tendon type
- form of construction

$dP_w = KPdx$ ← empirical wobble coefficient

Total losses:

$$dP = dP_w + dP_c = \mu P d\alpha + KPdx$$

change in tendon force between A and B (ends)

$$\frac{dP}{P} = \mu d\alpha + Kdx$$

$$\int \frac{dP}{P} = \int \mu d\alpha + \int Kdx$$

$$\boxed{P_B = P_A e^{-(\mu\alpha + Kx)}}$$

K = wobble coefficient
μ = friction coefficient
α = total angle change, in radians
x = tendon length A-B

CONSTANTS GIVEN IN HANDOUT

17 September 07

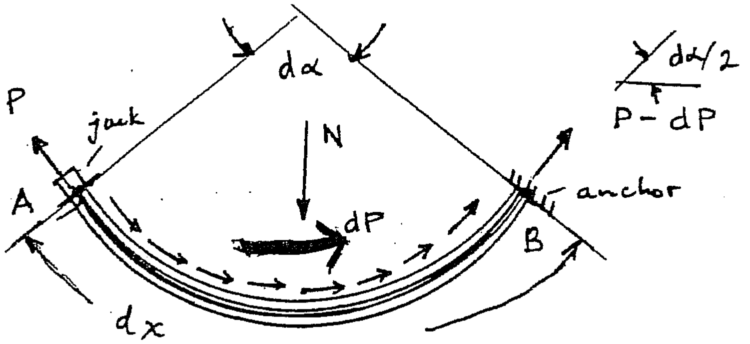
3.3 LOSSES DURING POST-TENSIONING

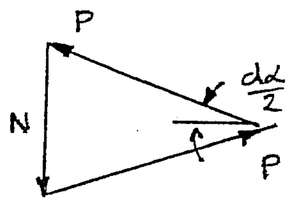
- When tendon is jacked → friction between tendon and ~~jack~~ duct
  → losses in tendon force

- Two components to losses : – curvature
  – wobble

CURVATURE FRICTIONAL LOSSES:

- result from change of angle of the tendon profile





jacked at pt A
anchored at pt B

- assume length of tendon dx
  direction changes by angle dα

- assume no losses for time being; from geometry, a normal force N developes

$$N = 2P \cdot \sin(d\alpha/2)$$

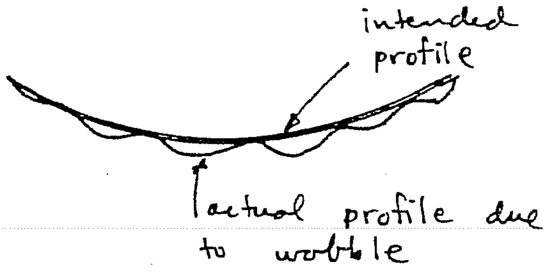
- let μ be coefficient of friction between the tendon and the duct

- total friction loss in length dx will be $\mu \cdot N$

$$\therefore \quad dP_c = \mu \cdot N$$
$$= \mu \cdot 2P \sin(d\alpha/2)$$
$$\simeq \mu P \, d\alpha \qquad \text{since } \sin(d\alpha/2) \simeq d\alpha/2$$

WOBBLE FRICTIONAL LOSSES:

wobble :



magnitude of wobble depends on:

- rigidity of duct
- diameter of duct
- spacing of duct supports
- tendon type
- form of construction

wobble losses expressed as:

$$dP_w = K P dx$$

where K = empirical wobble coefficient

TOTAL FRICTIONAL LOSSES:

$$dP = dP_c + dP_w$$

$$= \mu P d\alpha + K P dx$$

- change in tendon force between points A and B:

$$\frac{dP}{P} = \mu d\alpha + k dx$$

$$\int_{P_B}^{P_A} \frac{dP}{P} = \mu \int_0^\alpha d\alpha + k \int_0^x dx$$

Solving: $$P_B = P_A \cdot e^{-(\mu\alpha + kx)}$$

where P_A = tendon force at A
P_B = tendon force at B
μ = friction coefficient
K = wobble coefficient per metre of tendon
α = total angle change between A + B, in radians
x = tendon length between A + B, in metres

- range of values for μ and K are given in Table 2-2
- note μ depends on surface characteristics of tendon and duct
- note K depends on μ and on duct rigidity
- K values differ by an order of magnitude; need accurate info

Table 2-2 Range of friction coefficients recommended by ACI. From Ref. 2-15.

Type of Tendon	Curvature Coefficient, μ	Wobble Coefficient, K	
		per foot	per meter
Tendons in flexible metal sheathing			
Wire tendons	0.15–0.25	0.0010–0.0015	0.0033–0.0049
7-wire strand	0.15–0.25	0.0005–0.0020	0.0016–0.0066
High-strength bars	0.08–0.30	0.0001–0.0006	0.0003-0.0020
Tendons in rigid metal duct – 7-wire strand	0.15–0.25	0.0002	0.00066
Unbonded pregreased tendons – wires and 7-wire strand	0.05–0.15	0.0003–0.0020	0.0010–0.0066
Unbonded mastic-coated tendons – wires and 7-wire strand	0.05–0.15	0.0010–0.0020	0.0033–0.0066

Table 2-3 Representative friction values* recommended by CEB-FIP for tendons with radii of curvature not less than 6 m (20 ft). From Ref. 2-16.

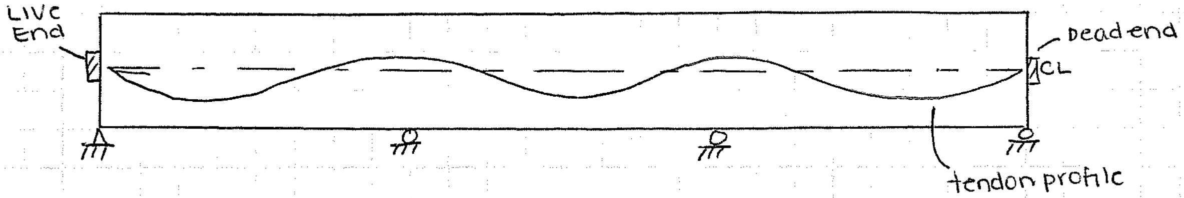
Type of Tendon	Curvature Coefficient, μ	Wobble Coefficient, K†	
		per foot	per meter
Cables in concrete ducts	0.50	0.0015	0.0050
Tendons in metal sheathing			
Drawn wires	0.20	0.0006	0.0020
Strand	0.20	0.0006	0.0020
Smooth rolled wires	0.25	0.0008	0.0025
Deformed wire	0.30	0.0009	0.0030

*Multiply values by 0.90 if tendon is slightly lubricated.

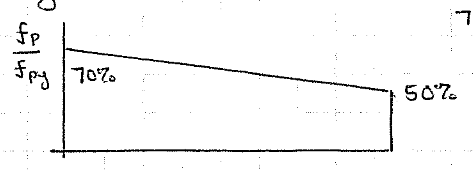
†In the CEB-FIP Model Code, K is expressed as 0.01μ per meter (0.003μ per foot).

POST-TENSIONING LOSSES

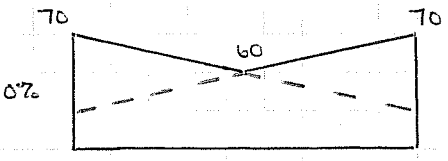
Stressing losses



- when stressing from one end, the forces are smaller at the dead end (more loss).
- stress from both ends - less loss, more symmetry



A only

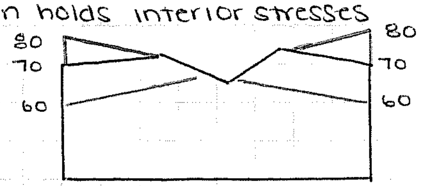


A and B

Temporary overstressing

overstress to 80%, then drop at ends

friction holds interior stresses



(not drawn well)

Elongations

$$\Delta = \frac{P_{av} \cdot L}{A_{ps} \cdot E_p}$$

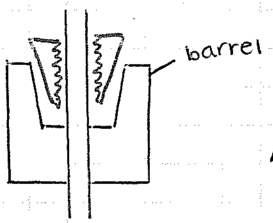
avg. cable force

length of tendon

Young's modulus

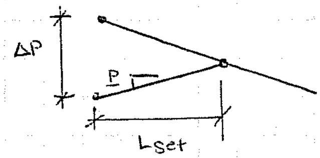
area of tendon

Anchoring losses



loss is generally ~ 1/4 in

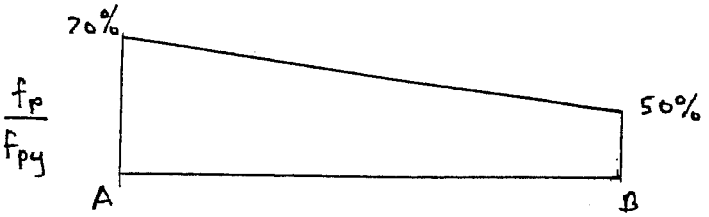
$\Delta_{set} = 1/4$ in



$L_{set} = \sqrt{\frac{\Delta_{set} \cdot A_{ps} \cdot E_p}{\underline{p}}}$, $\Delta P = 2\underline{p} \cdot L_{set}$

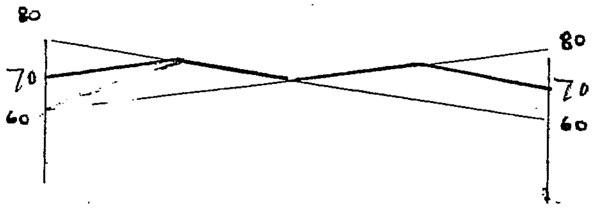
$\underline{p}$ = friction loss expressed as change in force per unit length calculated from tendon force diagram prior to anchorage

- If a reasonably long tendon is stressed from one end only, considerable loss in force along length
- losses reduced if member is stressed from both ends
- temporary over-stressing also helpful (normally stress at anchored limited to 0.70 fpy ; stress to 0.80 fpy then relax



stressed at A only

stressed at A & B



Overstressed to 80% at A & B ; then relaxed to 70% at A & B

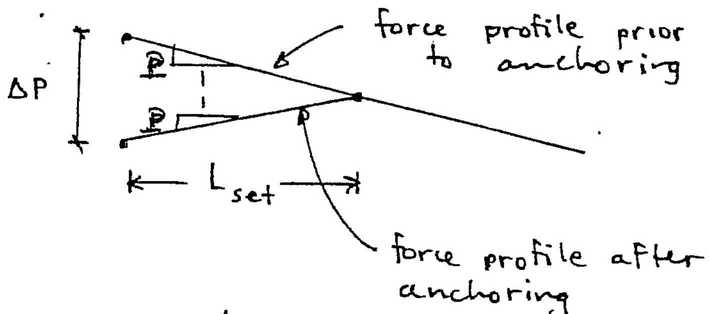
- During tensioning, tendon elongations are recorded and compared against expected elongations ; if difference > 5%, then problem ; must be checked out (possible jammed cable, etc)

Expected elongation:

$$\boxed{\Delta = \frac{P_{av} \cdot L}{A_{ps}\ E_p}}$$

Δ = elongation
P_{av} = avg force in cable
L = length of tendon
A_{ps} = area of tendon
E_p = Young's modulus of tendon

- Anchoring process also results in losses.
  Seating / deformation of anchor. shims results in a set loss of approx 1/4 inches.



$\therefore\ \Delta_{set} = 1/4$ inches.

define p = friction loss expressed as change in force per unit length calculated from tendon force diagram prior to anchorage

can solve :

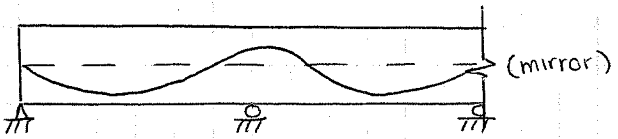
$$\boxed{L_{set} = \sqrt{\frac{\Delta_{set} \cdot A_{ps} \cdot E_p}{p}}}$$

lower case p

$$\boxed{\Delta P = 2p \cdot L_{set}}$$

POST-TENSIONING LOSSES

Example of loss calculations



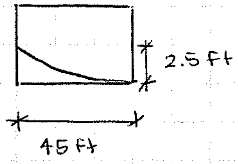
$A_{ps} = 4.30\,in^2$

$E_p = 28{,}200$ ksi

$\mu = 0.20$

$K = 0.0006/ft$

$\Delta_{set} = 0.25$ in



$$\alpha = \frac{2(2.5\,ft)}{45\,ft} = 0.111$$

$$\mu\alpha + Kx = (0.20)(0.111) + (0.0006)(45) = 0.049$$

sum $\mu\alpha + Kx$, then use as exponent

$e^{+\Sigma(\mu\alpha + kx)}$ – used when force is increasing – anchorage zone

Calculating average force

- sum the area under the tendon force variation diagram
- divide by length

Elongation

$$\Delta = \frac{P_{av} \cdot L \cdot (12\,in/ft)}{A_{ps} \cdot E_{ps}} = \frac{(745\,k)(224\,ft)}{(4.30\,in^2)(28200\,ksi)} = 16.5\,in$$ FROM EACH END!

Anchorage set

$$L_{set} = \left[\frac{\Delta_{set} \cdot A_s \cdot E}{\underline{p}}\right]^{\frac{1}{2}} = \left[\frac{(0.25\,in)(4.30\,in^2)(28200\,ksi)}{0.078\,k/in}\right]^{\frac{1}{2}} = 52\,ft$$

$$\underline{p} = \frac{\Delta P_{first\ segment}}{L_{first\ segment}} = \frac{(871\,k - 829\,k)}{45\,ft}$$

close enough to assumption of < 45 ft

$$\Delta P = 2\,\underline{p} \cdot L_{set}$$
$$= 2\,(0.078\,k/in)(52\,ft) = 97.2\,k$$

After anchoring $P_{end} = 871k - 97.2k = 773.8\,k$

$0.67 f_{pu} < 0.70 f_{pu}$, allowable stress limit

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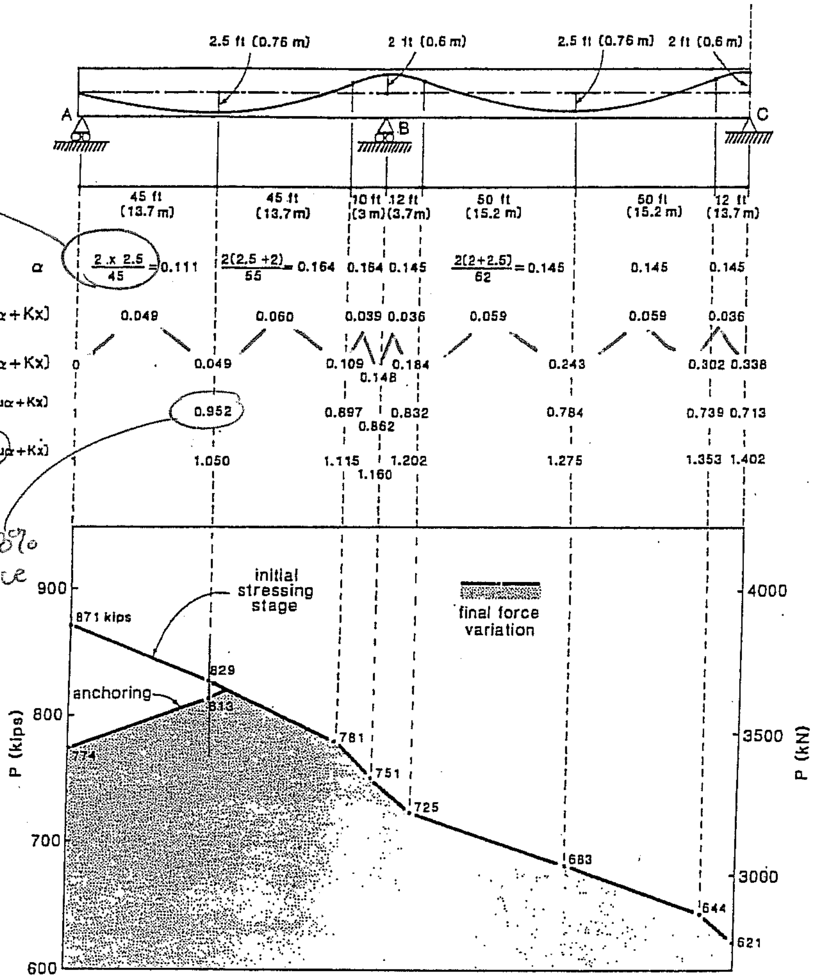
2.10 EXAMPLE OF FRICTION LOSS CALCULATIONS

The four-span continuous bridge girder shown in Fig. 2-29 is post-tensioned with tendons consisting of twenty 0.6 in. (15 mm) diameter strands with f_{pu} = 270 ksi (1860 MPa). The symmetrical tendons are simultaneously stressed to 75% f_{pu}, that is, 871 kips (3870 kN), from both ends and then anchored.



$\frac{2 \times H_{diff}}{length} = \sigma$

Used for anchorage zone.

4.8% of force lost

Figure 2-29 Example of frictional loss calculation.

For this application the tendon properties are:

$$
\begin{aligned}
A_{ps} &= 4.30 \text{ in}^2 \quad (2800 \text{ mm}^2) \\
E_p &= 28,200 \text{ ksi} \quad (195\,000 \text{ MPa}) \\
\mu &= 0.20 \\
K &= 0.0006/\text{ft} \quad (0.0020\,/\text{m}) \\
\Delta_{set} &= 0.25 \text{ in.} \quad (6 \text{ mm})
\end{aligned}
$$

1. Calculate the expected elongation due to the stressing operation.

2. Calculate the tendon force variation after anchorage.

(a) Determination of Tendon Force Variation

The $\mu\alpha + Kx$ values for each parabolic segment are first determined. Since each parabolic segment has one end which is horizontal (zero slope) the angular change, α, within each segment is equal to the slope at the inclined end. The equation in Fig. 2-23a and Eq. (2-2) are used to find these slopes.

During stressing, the force at location x along the tendon is given by

$$P_x = P_A e^{-\sum(\mu\alpha+Kx)}$$

The tendon force variation after jacking is shown in Fig. 2-29.

(b) Calculation of Elongation

The average force in the tendon can be approximated as

$$\begin{aligned} P_{av} = [\ &\frac{1}{2}(871+829) \times 45 + \frac{1}{2}(829+781) \times 45 \\ &+ \frac{1}{2}(781+751) \times 10 + \frac{1}{2}(751+725) \times 12 \\ &+ \frac{1}{2}(725+683) \times 50 + \frac{1}{2}(683+644) \times 50 \\ &+ \frac{1}{2}(644+621) \times 12\ ]/224 = 745 \text{ kips (3315 kN)} \end{aligned}$$

The calculation above assumes a linear force variation between the ends of each parabolic segment. From Eq. (2-4), the expected elongation is

$$\Delta = \frac{745 \times 224 \times 12}{4.30 \times 28{,}200} = 16.5 \text{ in. (419 mm)}$$

(c) Anchorage Set

Within the first 45 ft (13.7 m) of the tendon the friction loss per inch is (871 − 829)/(45 × 12) = 0.078 kips/in. (13.6 N/mm).

The length of tendon affected by anchorage set is given by Eq. (2-7):

$$\ell_{set} = \sqrt{\frac{0.25 \times 4.30 \times 28{,}200}{0.078}} = 623 \text{ in.} = 52 \text{ ft (15.8 m)}$$

Because ℓ_{set} exceeds 45 ft (13.7 m), the friction loss per inch, p, could be recalculated if a more accurate answer is desired. However, in this case the small difference will be neglected.

From Eq. (2-6),

$$\Delta P = 2 \times 0.078 \times 623 = 97.2 \text{ kips (432 kN)}$$

Hence after anchoring, the force at the end of the tendon is

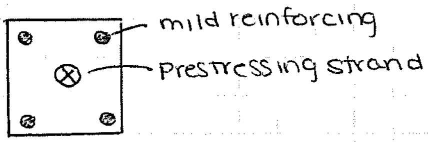
$$871 - 97.2 = 773.8 \text{ kips (3442 kN)}$$

This force in the tendon corresponds to a stress of $0.67 f_{pu}$, which is less than the stress limit of $0.70 f_{pu}$ permitted by the ACI Code (Ref. 2-19) at end anchorages. The force variation after anchoring is shown in Fig. 2-29.

AXIALLY LOADED MEMBERS

introduction

Not as common as flexural members



compatibility

$$\varepsilon_{tot} = \varepsilon_c = \frac{\Delta}{L} = \varepsilon_{ct} = \varepsilon_{cb} = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth}$$

engineering strain

"top" and "bottom" concrete strain

assume a perfect bond between concrete and steel

$\Delta\varepsilon_p$ is constant after casting / anchoring

Equilibrium

$$A_c f_c + A_s f_s + A_p f_p = N$$

externally applied loads

prestress

steel

concrete

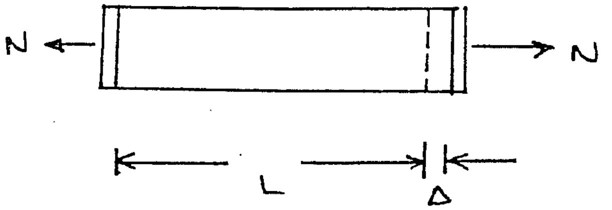
19 september07

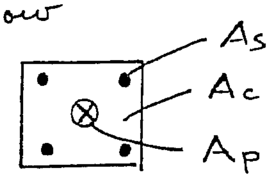
4. RESPONSE OF MEMBERS TO AXIAL LOAD

4.1 INTRODUCTION

- P/s members subjected to axial load not common in practice
- will examine, however, because it introduces fundamentals

- consider axial member shown below

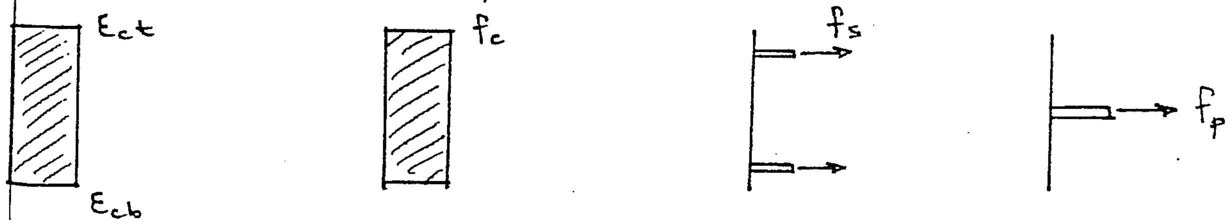




assume CGC = CGS (centres of gravity of conc + steel)

assume N collinear with GG

- strain and stress profiles



strain — stress in concrete — stresses in reinf — stresses in p/s reinf

assuming uniform strain and stress conditions

4.2 COMPATIBILITY CONDITION:

- assume concrete is initially unstrained and unstressed
- when subjected to force N → strain ε_c

$$(\varepsilon_{tot} =)\ \varepsilon_c = \frac{\Delta}{L} \qquad (= \varepsilon_{ct} = \varepsilon_{cb})$$

- allowing for shrinkage, thermal etc

$$\varepsilon_{tot} = \overset{\text{concrete stress}}{\varepsilon_{cf}} + \overset{\text{shrinkage}}{\varepsilon_{sh}} + \overset{\text{thermal}}{\varepsilon_{cth}}$$

- assume that the concrete, reinforcing bars and prestressing tendons are rigidly connected at ends; "perfect bond"

- non-prestressed reinforcement has zero strain at time of casting; undergoes identical deformation as surrounding concrete

$$\therefore \quad \varepsilon_{s_{tot}} = \varepsilon_{tot}$$

$$\varepsilon_{s_{tot}} = \varepsilon_{sf} + \varepsilon_{s_{th}} \quad (= \varepsilon_c)$$

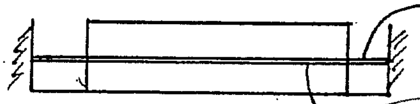
- prestressed reinforcement <u>does not</u> have same strain as the surrounding concrete; due to pre-tensioning or post-tensioning, there will be strain differential

let $\Delta\varepsilon_p$ = strain differential

$$\varepsilon_{p_{tot}} = \varepsilon_{tot} + \Delta\varepsilon_p$$

$$= \varepsilon_c + \Delta\varepsilon_p$$

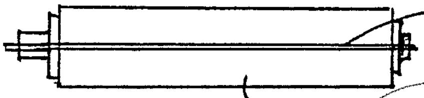
pretensioning:



$\varepsilon_p = \varepsilon_{p_i}$

$\varepsilon_c = 0 = \varepsilon_{ci}^{0}$

post-tensioning



$\varepsilon_p = \varepsilon_{p_i}$

$\varepsilon_c = \varepsilon_{ci} \; (\neq 0)$

strain differential

concrete

$$\boxed{\Delta\varepsilon_p = \varepsilon_{p_i} - \varepsilon_{c_i}}$$

strain in strand

now given $\varepsilon_{p_{tot}}$

$$\varepsilon_{p_{tot}} = \varepsilon_{pf} + \overset{\text{thermal}}{\varepsilon_{p_{th}}}$$

(note: relaxation taken into account in stress-strain relation)

after casting / anchoring, perfect bond assumed

$\Delta\varepsilon_p$ constant thereafter

4.3 EQUILIBRIUM CONDITIONS

- internal stresses must balance applied load

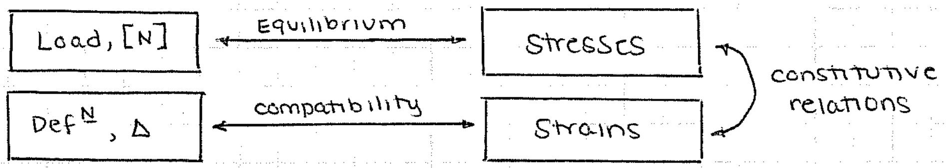
$$\int_A f\,dA = N \quad \Rightarrow \quad \int_{A_c} f_c\,dA_c + \int_{A_s} f_s\,dA_s + \int_{A_p} f_p\,dA_p = N$$

however, since stresses are uniform over section

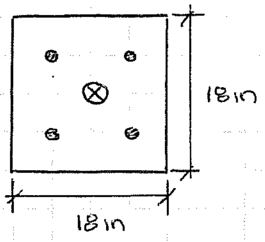
$$A_c f_c + A_s f_s + A_p f_p = N$$

AXIALLY LOADED MEMBERS

Predicting response



Example problem



$A_p = 1.224\ in^2$ (8 - ½" strands)
$A_s = 4-\#9$ bars $= 4.0 in^2$
$A_c = 318.8\ in^2$
$f_c' = 7$ ksi
$\varepsilon_c' = \varepsilon_o = 2.55\times10^{-3}$
$L = 20$ ft

Assume Ap is post-tensioned to a stress of 175 ksi.
$\Delta_{set} = 0$, anchored.

1. Find initial stresses and strains
2. Find stresses and strains at N=225 K (tension)
3. Find stresses and strains at N=-1000K (compression)

Solution:

First find $\Delta\varepsilon_p$ (strain differential)

$E_c = 57\sqrt{7000} = 4768$ ksi ← secant modulus (~10% less)

however, use E_{ct} instead ← initial tangent

$$E_{ct} = \frac{2\,(7000\,psi)}{2.55\times10^{-3}} = 5490\ ksi$$

$E_s = E_p = 29000$ ksi

$$\varepsilon_{pi} = \frac{175\ ksi}{29000\ ksi} = 6.03\times10^{-3}\ in/in = \frac{\sigma_p}{E_p}$$

$\varepsilon_{pi} \neq \Delta\varepsilon_p$ because $\varepsilon_{ci} \neq 0$ — POST-TENSIONED

$F_p = A_p f_p = (1.224 in^2)(175 ksi) = 214.2$ K

Initially,

$A_c f_c + A_s f_s + A_p f_p = 0$

Assume linear f_c, ε_c relationship (good assumption for now)

$(318.8 in^2)(5490 ksi)\varepsilon_{ci} + (4.0 in^2)(29000 ksi)\varepsilon_{ci} + 214.2 K = 0$

solving, $\varepsilon_{ci} = -0.115\times10^{-3}$ in/in

$\Delta_i = \varepsilon_{ci}\cdot L = (-0.115\times10^{-3})(20 ft) = -0.025$ in
$f_{ci} = E_{ct}\,\varepsilon_{ci} = (5490 ksi)(-0.115\times10^{-3}) = -0.63$ ksi
$f_{si} = E_s\,\varepsilon_{si} = (29000 ksi)(-0.115\times10^{-3}) = -3.32$ ksi

AXIALLY LOADED MEMBERS

Example solution, cont'd

Initially,

$F_c = f_{ci} A_c = (-0.63\,\text{KSI})(318.8\,\text{in}^2) = -200.8\,\text{K}$

$F_s = f_{si} A_s = (-3.32\,\text{KSI})(4.0\,\text{in}^2) = -13.3\,\text{K}$

$F_p = 214.2\,\text{K}$

Sum to ~0

$$\Delta\varepsilon_p = \varepsilon_{pi} - \varepsilon_{ci}$$

$$= 6.03\times10^{-3} - (-0.115\times10^{-3}) = \underline{6.145\times10^{-3}\ \text{in/in}}$$

Part 2. N = 225 K tension

$f_{ct} = 4\sqrt{f'_c} = 335\ \text{psi}$

does concrete crack? Assume no,
check (and fix) later as necessary.

$$A_c f_c + A_s f_s + A_p f_p = N$$

$$(318.8\,\text{in}^2)(5490\,\text{KSI})\varepsilon_c + (4.0\,\text{in}^2)(29000\,\text{KSI})\varepsilon_c + (1.224\,\text{in}^2)(29000\,\text{KSI})(\varepsilon_c + \underbrace{\Delta\varepsilon_p}_{6.145\times10^{-3}}) = \underbrace{N}_{225\,\text{K}}$$

solve for $\varepsilon_c = 3.6\times10^{-6}$ in/in

tension, very small — assumption of no cracking was appropriate

$f_c = 0.02\ \text{ksi} \longrightarrow 6.3\,\text{K}$

$f_s = 0.105\ \text{ksi} \longrightarrow 0.4\,\text{K}$

$f_p = 178.3\ \text{ksi} \longrightarrow \underline{218.3\,\text{K}}$

$\sim 225\,\text{K}$

Part 3. N = 1000 K compression

does this stay in the linear-elastic range?

see attached spreadsheet for nonlinear calculations

A = 318.8 in^2

A = 4.0 in^2

A = 1.224 in^2

	Concrete			Reinforcement			Prestressing			
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N
(x10^{-3})	(x10^{-3})	(ksi)	(kips)	(x10^{-3})	(ksi)	(kips)	(x10^{-3})	(ksi)	(kips)	(kips)
-0.115	-0.115	-0.6171	-196.74	-0.115	-3.335	-13.34	6.03	174.87	214.041	3.95801
-0.2	-0.2	-1.055	-336.33	-0.2	-5.8	-23.2	5.945	172.405	211.024	-148.5
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96
-0.7	-0.7	-3.3156	-1057	-0.7	-20.3	-81.2	5.445	157.905	193.276	-944.95
-0.8	-0.8	-3.7032	-1180.6	-0.8	-23.2	-92.8	5.345	155.005	189.726	-1083.7
-0.73	-0.73	-3.4342	-1094.8	-0.73	-21.17	-84.68	5.415	157.035	192.211	-987.28
-0.74	-0.74	-3.4732	-1107.3	-0.74	-21.46	-85.84	5.405	156.745	191.856	-1001.3
-0.7391	-0.7391	-3.4697	-1106.2	-0.7391	-21.434	-85.736	5.4059	156.771	191.888	-1000

ε_{ci}

ε_c

total strain

Change until final N value equals desired value (-1000 k here)

strain due to stress

using parabola

$f_c A_c$

same as concrete

$\varepsilon_{sf} E_s$ until > 60 ksi

$f_s A_s$

col 1 + $\Delta\varepsilon_p$

modified Ramburg-Osbourne (?) here, $\varepsilon_{pf} E_p$

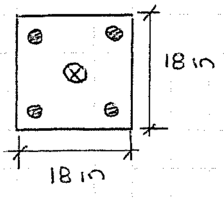
$f_p A_p$

Σ F_c, F_s, F_p

Concrete stressed to 3.5 ksi — half of f_c'. Linear approximation would be okay, but secant mod. better than tangent (use tangent for 10-20% f_c').

AXIALLY LOADED MEMBERS

Long-term effects / complete load-deformation response
can generate response curve by doing calculations
for a number of different values of ε_c



8-½" strands, $A_p = 1.224 \text{ in}^2$

4-#9 bars, $A_r = 4.0 \text{ in}^2$

$A_c = 318.8 \text{ in}^2$

$f_c' = 7 \text{ ksi}$

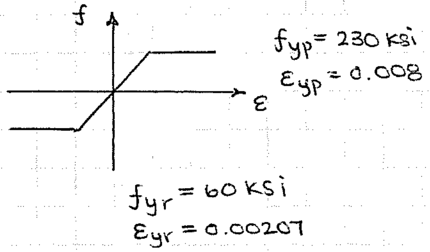
$\varepsilon_o = 2.55 \times 10^{-3}$

$\varepsilon_{cr} = 0.06 \times 10^{-3}$

$f_{cr} = 335 \text{ psi}$

$E_{ct} = 5490 \text{ ksi}$

$\Delta\varepsilon_p = 6.145 \times 10^{-3}$



Influence of prestressing

1. P·Δ curve is offset from origin
2. P/S does not influence tensile capacity
   As, Ap, fy, fpy, NOT $\Delta\varepsilon_p$
3. P/S increases cracking load
4. P/S reduces concrete tensile stress/strains at service loads
5. P/S reduces compression capacity and stiffness

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8-2

use parabola

$F_c = A_c f_c$

If statement included, max set to 60ksi

$\Delta\varepsilon_p = 6.145 \times 10^{-3}$

	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	$\varepsilon_{sf} = \varepsilon_{cf}$	f_s	F_s	ε_{pf}	f_p	F_p	N	
($x10^{-3}$)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	(kips)	
-3	-3	-6.782	-2162.1	-3	-60	-240	3.145	91.205	111.635	-2290.5	
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	3.395	98.455	120.509	-2337.4	
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	3.595	104.255	127.608	-2344	$\varepsilon_c = \varepsilon_o$
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	3.895	112.955	138.257	-2302.5	
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	4.075	118.175	144.646	-2247.9	$\varepsilon_c = \varepsilon_y$
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	4.395	127.455	156.005	-2059	
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	4.645	134.705	164.879	-1862.4	
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	4.895	141.955	173.753	-1622.9	
-1	-1	-4.4137	-1407.1	-1	-29	-116	5.145	149.205	182.627	-1340.5	
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96	
0	0	0	0	0	0	0	6.145	178.205	218.123	218.123	
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	6.205	179.945	220.253	332.225	$\varepsilon_c = \varepsilon_{cr-}$ Just prior to cracking
0.06	0.06	0	0	0.06	1.74	6.96	6.205	179.945	220.253	227.213	$\varepsilon_c = \varepsilon_{cr+}$ Just after cracking
1.855	1.855	0	0	1.855	53.795	215.18	8	230	281.52	496.7	$\varepsilon_p = \varepsilon_{py}$
2.07	2.07	0	0	2.07	60	240	8.215	230	281.52	521.52	$\varepsilon_s = \varepsilon_{sy}$
3	3	0	0	3	60	240	9.145	230	281.52	521.52	

chosen somewhat arbitrarily

compression side

tension side

0 in tension

steel has yielded

yield of strands

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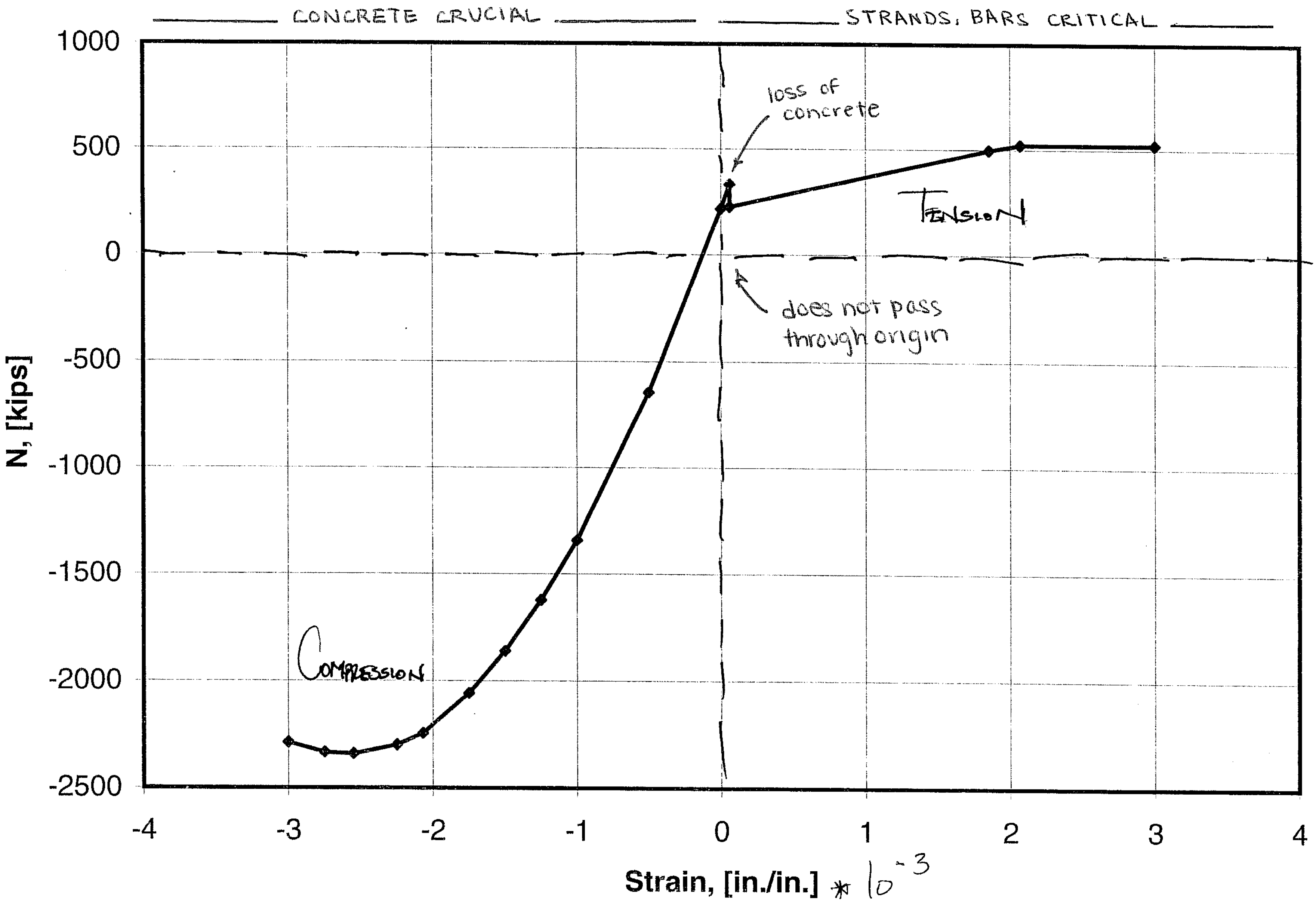
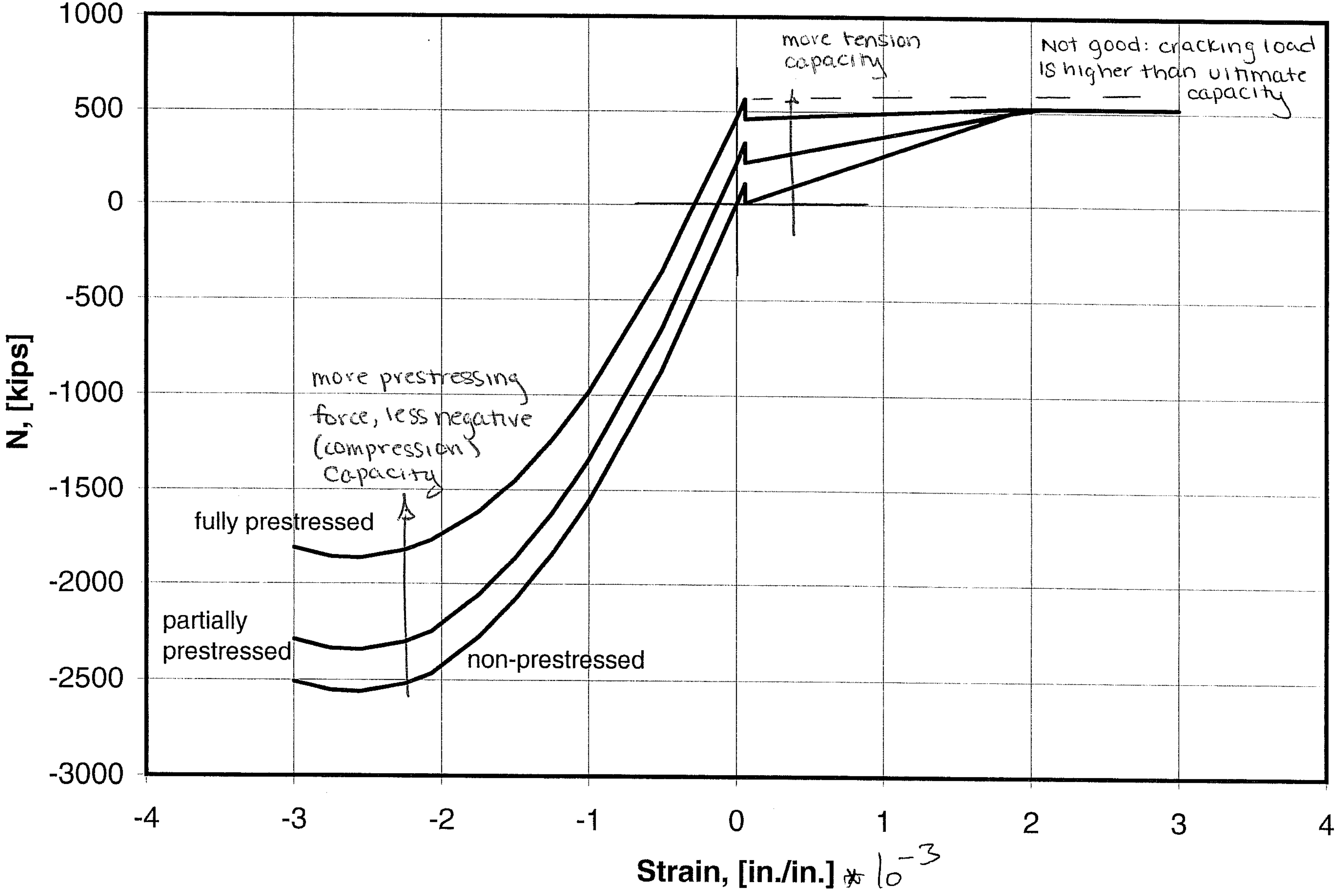



more tension capacity
Not good: cracking load is higher than ultimate capacity
more prestressing force, less negative (compression) capacity
fully prestressed
partially prestressed
non-prestressed
N, [kips]
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
-4
-3
-2
-1
0
1
2
3
4
Strain, [in./in.] * 10^-3

Part.	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
$(x10^{-3})$	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	(kips)	
-3	-3	-6.782	-2162.1	-3	-60	-240	3.145	91.205	111.635	-2290.5	
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	3.395	98.455	120.509	-2337.4	
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	3.595	104.255	127.608	-2344	$\varepsilon_c = \varepsilon_o$
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	3.895	112.955	138.257	-2302.5	
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	4.075	118.175	144.646	-2247.9	$\varepsilon_c = \varepsilon_y$
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	4.395	127.455	156.005	-2059	
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	4.645	134.705	164.879	-1862.4	
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	4.895	141.955	173.753	-1622.9	
-1	-1	-4.4137	-1407.1	-1	-29	-116	5.145	149.205	182.627	-1340.5	
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96	
0	0	0	0	0	0	0	6.145	178.205	218.123	218.123	
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	6.205	179.945	220.253	332.225	$\varepsilon_c = \varepsilon_{cr-}$
0.06	0.06	0	0	0.06	1.74	6.96	6.205	179.945	220.253	227.213	$\varepsilon_c = \varepsilon_{cr+}$
1.855	1.855	0	0	1.855	53.795	215.18	8	230	281.52	496.7	$\varepsilon_p = \varepsilon_{py}$
2.07	2.07	0	0	2.07	60	240	8.215	230	281.52	521.52	$\varepsilon_s = \varepsilon_{sy}$
3	3	0	0	3	60	240	9.145	230	281.52	521.52	

reinforcing is stressed, too (pure theoretical)

Full	Concrete			Reinforcement			Prestressing			
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N
($x10^{-3}$)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	(kips)
-3	-3	-6.782	-2162.1	3.145	60	240	3.145	91.205	111.635	-1810.5
-2.75	-2.75	-6.9569	-2217.9	3.395	60	240	3.395	98.455	120.509	-1857.4
-2.55	-2.55	-7	-2231.6	3.595	60	240	3.595	104.255	127.608	-1864
-2.25	-2.25	-6.9031	-2200.7	3.895	60	240	3.895	112.955	138.257	-1822.5
-2.07	-2.07	-6.752	-2152.5	4.075	60	240	4.075	118.175	144.646	-1767.9
-1.75	-1.75	-6.311	-2012	4.395	60	240	4.395	127.455	156.005	-1616
-1.5	-1.5	-5.8131	-1853.2	4.645	60	240	4.645	134.705	164.879	-1448.4
-1.25	-1.25	-5.1807	-1651.6	4.895	60	240	4.895	141.955	173.753	-1237.9
-1	-1	-4.4137	-1407.1	5.145	60	240	5.145	149.205	182.627	-984.46
-0.5	-0.5	-2.476	-789.34	5.645	60	240	5.645	163.705	200.375	-348.96
0	0	0	0	6.145	60	240	6.145	178.205	218.123	458.123
0.06	0.06	0.3294	105.013	6.205	60	240	6.205	179.945	220.253	565.265
0.06	0.06	0	0	6.205	60	240	6.205	179.945	220.253	460.253
1.855	1.855	0	0	8	60	240	8	230	281.52	521.52
2.07	2.07	0	0	8.215	60	240	8.215	230	281.52	521.52
3	3	0	0	9.145	60	240	9.145	230	281.52	521.52

Not really possible To put strands into compression

Reinf.	Concrete			Reinforcement			Prestressing			
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N
($x10^{-3}$)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	(kips)
-3	-3	-6.782	-2162.1	-3	-60	-240	-3	-87	-106.49	-2508.6
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	-2.75	-79.75	-97.614	-2555.5
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	-2.55	-73.95	-90.515	-2562.1
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	-2.25	-65.25	-79.866	-2520.6
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	-2.07	-60.03	-73.477	-2466
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	-1.75	-50.75	-62.118	-2277.1
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	-1.5	-43.5	-53.244	-2080.5
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	-1.25	-36.25	-44.37	-1841
-1	-1	-4.4137	-1407.1	-1	-29	-116	-1	-29	-35.496	-1558.6
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	-0.5	-14.5	-17.748	-865.09
0	0	0	0	0	0	0	0	0	0	0
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	0.06	1.74	2.12976	114.102
0.06	0.06	0	0	0.06	1.74	6.96	0.06	1.74	2.12976	9.08976
1.855	1.855	0	0	1.855	53.795	215.18	1.855	53.795	65.8451	281.025
2.07	2.07	0	0	2.07	60	240	2.07	230	281.52	521.52
3	3	0	0	3	60	240	3	230	281.52	521.52

AXIALLY LOADED MEMBERS

Accounting for relaxation, creep, shrinkage, and thermal
- before, discussing short-term response
- long term considers factors above

concrete:

shrinkage and thermal strains can be accounted for directly in the compatibility equation

$$\varepsilon_{ctot} = \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth}$$

creep is accounted for by using modified σ-ε relationship for concrete

$$f_c = -f_c' \left[2\left(\frac{\varepsilon_{cf}}{\varepsilon'_{ceff}}\right) - \left(\frac{\varepsilon_{cf}}{\varepsilon'_{ceff}}\right)^2 \right] \text{ in compression}$$

$$f_c = E_{ceff} \cdot \varepsilon_{cf} \left[\qquad \right] \text{ in tension}$$

where

$$E_{ceff} = \frac{E_{ci}}{1+\phi(t,t_1)}$$

$$\varepsilon'_{ceff} = \frac{2 f_c'}{E_{ceff}}$$

Reinforcing

Non P/S: account for thermal strains

$$\varepsilon_{stot} = \varepsilon_{sf} + \varepsilon_{sth}$$

P/S:

$$\varepsilon_{ptot} = \varepsilon_{pf} + \varepsilon_{pth} = \varepsilon_{ctot} + \Delta\varepsilon_p$$

but, must account for relaxation as well

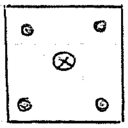
$$E_{peff} = \frac{f_p}{f_{pi}} \cdot E_p$$

see appropriate relationship for stress-relieved or low relaxation (lolax) strands

or lowlax

AXIALLY LOADED BEAMS

Return to example problem



creep coefficient = 2.7 = ϕ

shrinkage of concrete $\varepsilon_{sh} = -0.4 \times 10^{-3}$

strand relaxation = 5%

could be calculated, but takes time, etc.

Find conditions for N = −1000 K

$$E_{ceff} = \frac{E_{ci}}{1+\phi} = \frac{5490\text{ ksi}}{1+2.7} = 1484\text{ ksi}$$

$$\varepsilon'_{ceff} = \frac{2f_c'}{E_{ceff}} = \frac{2(7\text{ ksi})}{1484\text{ ksi}} = -9.0 \times 10^{-3}$$

$$E_{Peff} = 0.95 E_P = 0.95(29000\text{ ksi}) = 27550\text{ ksi}$$

For concrete,

$$\varepsilon_{cf} = \varepsilon_{cTOT} - \varepsilon_{sh}$$

Proceed using trial and error.

Recall that for short-term loading, N = −1000 K,

$$\varepsilon_c = -0.739 \times 10^{-3}$$

$\varepsilon_{c\,tot} - \varepsilon_{sh}$

"softer" parabola

No thermal strain, so $\varepsilon_{cf} = \varepsilon_{sf}$

L.T.	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
($\times 10^{-3}$)	($\times 10^{-3}$)	(ksi)	(kips)	($\times 10^{-3}$)	(ksi)	(kips)	($\times 10^{-3}$)	(ksi)	(kips)	(kips)	
-0.7391	-0.3391	-0.5176	-165	-0.7391	-21.434	-85.736	5.4059	148.933	182.293	-68.438	N.G.
-1.5	-1.1	-1.6065	-512.17	-1.5	-43.5	-174	4.645	127.97	156.635	-529.53	
-2	-1.6	-2.2677	-722.93	-2	-58	-232	4.145	114.195	139.774	-815.15	
-2.1	-1.7	-2.3947	-763.43	-2.1	-60	-240	4.045	111.44	136.402	-867.03	
-2.2	-1.8	-2.52	-803.38	-2.2	-60	-240	3.945	108.685	133.03	-910.35	
-2.3	-1.9	-2.6436	-842.77	-2.3	-60	-240	3.845	105.93	129.658	-953.12	
-2.4	-2	-2.7654	-881.62	-2.4	-60	-240	3.745	103.175	126.286	-995.33	
-2.412	-2.012	-2.7799	-886.24	-2.412	-60	-240	3.733	102.844	125.881	-1000.4	O.K.

short-term load value

change in strain $= 1.7 \times 10^{-3}$

change in load ~940 k

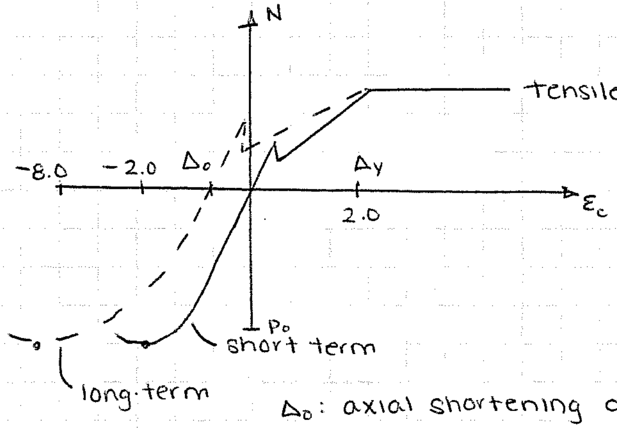
	Short-Term	Long Term	L.T./S.T.
ε_c	-0.7391×10^{-3}	-2.412×10^{-3}	3.26
f_c	-3.47 ksi	-2.78 ksi	0.80
f_s	-21.434 ksi	-60 ksi ← taken to yield!	2.80
f_p	156.8 ksi	102.8 ksi	0.66 ← 40% losses!

lots of change!

bars are going to yield. prevent from buckling and just accept it

LONG·TERM RESPONSE

Response curve



tensile capacity remains unchanged
determined by f_y, f_{py}

Δ_o: axial shortening occurs

$f_s \rightarrow -f_y$: under long·term loading, yielding of rebar in compression occurs sooner

P_o increases as prestressing force is lost
however, cannot be achieved because LT curve is ficticious

concrete cracking can occur when overall strain is negative — strain due to stress is positive and causes cracking

Linear elastic uncracked response

N_o: decompression force, or force required to produce zero strain in concrete

$$N_o = A_p E_p \Delta\varepsilon_p - (A_c E_c \varepsilon_{sh} + A_c E_c \varepsilon_{cth} + A_s E_s \varepsilon_{sth} + A_p E_p \varepsilon_{pth})$$

use transformed section,

$$A_{trans} = A_c + \underset{\uparrow \text{ long·term, use effective modulus}}{\frac{E_s}{E_c}} A_s + \overset{\leftarrow \text{ long·term, relaxation occurs}}{\frac{E_p}{E_c}} A_p$$

long term, A_{trans} will increase

$$\boxed{N = A_{trans} \cdot E_c \cdot \varepsilon_c + N_o}$$

$f_c = E_c \cdot \varepsilon_c$

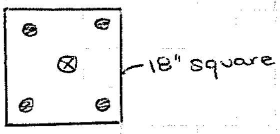
use for: $-0.5 f'_c < f_c < f'_t$

linear elastic range

- rebar shouldn't yield
- strands shouldn't yield

LONG-TERM RESPONSE

Calculation of short-term response



$f'_c = 7\text{ ksi}$ $A_c = 318.8\text{ in}^2$

$f'_t = 335\text{ psi}$ $A_s = 4.0\text{ in}^2$

$f_y = 60\text{ ksi}$ $A_p = 1.224\text{ in}^2$

$f_{py} = 230\text{ ksi}$ $\Delta\varepsilon_p = 6.145\times10^{-3}$

$E_{ct} = 5490\text{ ksi}$

$E_{cs} = 0.9E_{ct} = 4941\text{ ksi}$

Solve and compare σ, ε for $N = 225\text{ k}, -1000\text{ k}$

Decompression Force:

$\boxed{ST,}\ (\ ) = 0,\quad N_o = A_pE_p\Delta\varepsilon_p = 218.1\text{ k}$

$$A_{trans} = 318.8\text{ in}^2 + \frac{29000\text{ ksi}}{4941\text{ ksi}}\,4.0\text{ in}^2 + \frac{29000\text{ ksi}}{4941\text{ ksi}}\,1.224\text{ in}^2$$

$$= 349.5\text{ in}^2$$

$$N = A_{trans}E_c\varepsilon_c + N_o$$

or,

$$\frac{N-N_o}{A_{trans}\cdot E_c} = \frac{225\text{ k} - 218.1\text{ k}}{(349.5\text{ in}^2)(4941\text{ ksi})} = 0.004\times10^{-3}$$

non-linear calculations, $\varepsilon_c = 0.0036\times10^{-3}$

calculate σs

$f_c = E_c\varepsilon_c = 0.018\text{ ksi}$

$f_s = E_s\varepsilon_s = 0.105\text{ ksi}$

$f_p = E_p\underset{\uparrow\ \varepsilon_c+\Delta\varepsilon_p}{\varepsilon_p} = 178.3\text{ ksi}$

match non-linear answers, much faster calculation (see pg 22)

ST, $N = -1000\text{ k}$:

$N_o = 218.1\text{ k}$, $A_{trans} = 349.5\text{ in}^2$ still. yay.

$$\varepsilon_c = \frac{N-N_o}{A_{trans}E_c} = \frac{-1000\text{ k} - 218.1\text{ k}}{(4941\text{ ksi})(349.5\text{ in}^2)} = -0.705\times10^{-3}$$

compare to -0.739×10^{-3} (pg 22a)

$f_c = -3.48\text{ ksi}$

$f_s = -20.4\text{ ksi}$

$f_p = 157.8\text{ ksi}$

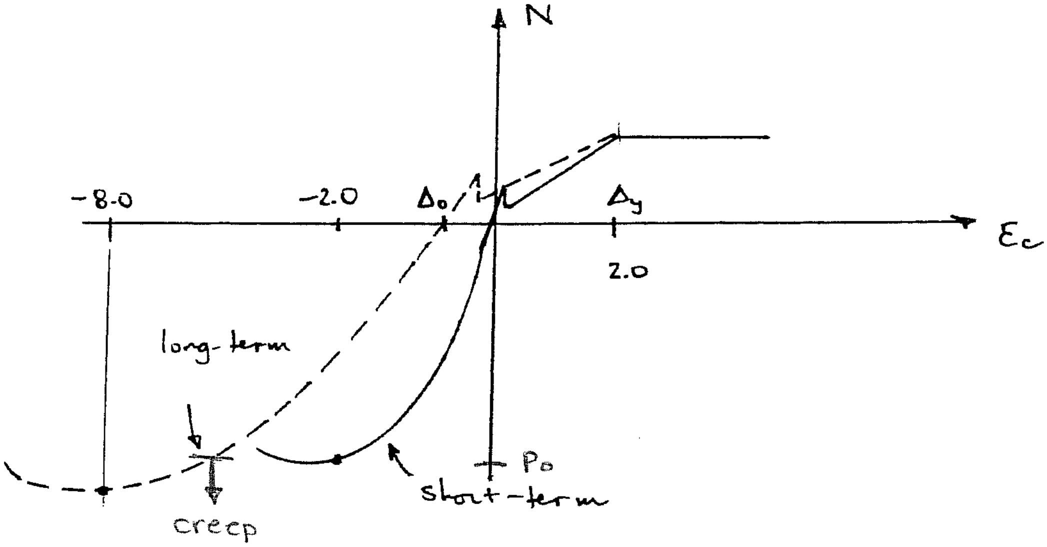
→ again, quite close (pg 22a) though less close than first calc., <u>not significant</u>

Note: $f_c/f'_c = .0.5 = 3.48\text{ ksi}/7\text{ ksi}$

24 September 07

4.7 LONG-TERM RESPONSE CURVES

- as we did in predicting short-term response curves, can generate complete response by computing axial loads for a range of ε_c
- will generally find significant differences compared to short-term response curves



The following general observations can be made:

i) Δ_o : Under zero applied load (N=0), there is substanti change in length with time. Axial shortening must be accounted for in design.

ii) $f_s \rightarrow -f_y$: Under LT loading, [yielding of rebars in compression occurs much sooner]; creep and shrinkage cause the compression load to be transferred to the reinforcing bars

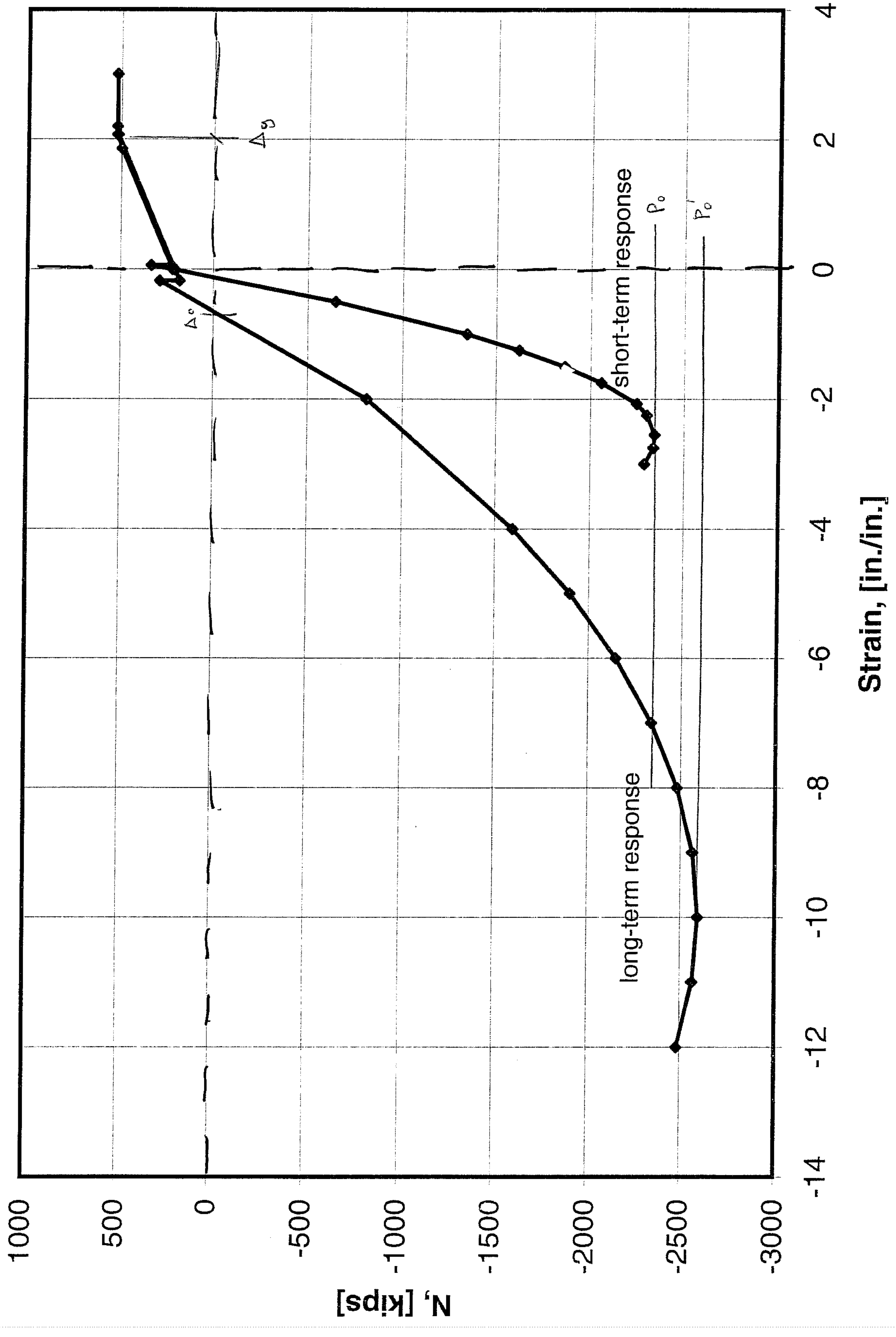
iii) P_o increase: LT compression capacity is slightly improved because some of the prestressing stresses have been lost

iv) N_{cr} depressed: Under LT loading, concrete cracking occurs when total strain is negative!, P/s structures can crack even though they've shortened
strain due to stress = positive $\varepsilon_{cr} + \varepsilon_{sh} > \varepsilon_{cf}$

v) N_y : Tensile capacity not influenced significantly; determined by yield strengths f_y + f_{py} ∴ N_y + Δ_y not affected

vi) E_c : Obviously, stiffness in compression for long-term loading much lower; however, E for superimposed loads unaffected

9-1-1


N, [kips]
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
Strain, [in./in.]
-14
-12
-10
-8
-6
-4
-2
0
2
4
short-term response
long-term response
P_o
P_o'
Δ_o
Δ_g

remove shrinkage $= 0.4 \times 10^{-3}$

$\varepsilon_o \quad \frac{E}{1+\phi}$ use parabola

same as $\varepsilon_{c_{TOT}}$ because there are no thermal strains

$5.145 - 12 = -5.855$

$E = 27550$, reduced by 5%

L.T.	Concrete			Reinforcement			Prestressing				
$\varepsilon_{c_{Total}}$	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
($x10^{-3}$)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	(kips)	
-12	-11.6	-6.4158	-2045.3578	-12	-60	-240	-5.855	-161.31	-197.44	-2482.8	
-11	-10.6	-6.7788	-2161.0704	-11	-60	-240	-4.855	-133.76	-163.72	-2564.8	
-10	-9.6	-6.9689	-2221.6818	-10	-60	-240	-3.855	-106.21	-130	-2591.7	$\varepsilon_c = \varepsilon_o$
-9	-8.6	-6.9862	-2227.1919	-9	-60	-240	-2.855	-78.655	-96.274	-2563.5	
-8	-7.6	-6.8306	-2177.6008	-8	-60	-240	-1.855	-51.105	-62.553	-2480.2	
-7	-6.6	-6.5022	-2072.9084	-7	-60	-240	-0.855	-23.555	-28.832	-2341.7	
-6	-5.6	-6.001	-1913.1149	-6	-60	-240	0.145	3.99475	4.88957	-2148.2	
-5	-4.6	-5.3269	-1698.22	-5	-60	-240	1.145	31.5448	38.6108	-1899.6	
-4	-3.6	-4.48	-1428.224	-4	-60	-240	2.145	59.0948	72.332	-1595.9	
-2	-1.6	-2.2677	-722.9282	-2	-58	-232	4.145	114.195	139.774	-815.15	
-0.178	0.222	0.32945	105.02802	-0.178	-5.162	-20.648	5.967	164.391	201.214	285.594	$\varepsilon_c = \varepsilon_{cr-}$
-0.178	0.222	0	0	-0.178	-5.162	-20.648	5.967	164.391	201.214	180.566	$\varepsilon_c = \varepsilon_{cr+}$
0	0.4	0	0	0	0	0	6.145	169.295	207.217	207.217	
2.069	2.469	0	0	2.069	60	240	8.214	226.296	276.986	516.986	$\varepsilon_s = \varepsilon_{sy}$
2.195	2.595	0	0	2.195	60	240	8.34	230	281.52	521.52	$\varepsilon_p = \varepsilon_{py}$
3	3.4	0	0	3	60	240	9.145	230	281.52	521.52	

linear elastic range of concrete

capture cracking point of concrete

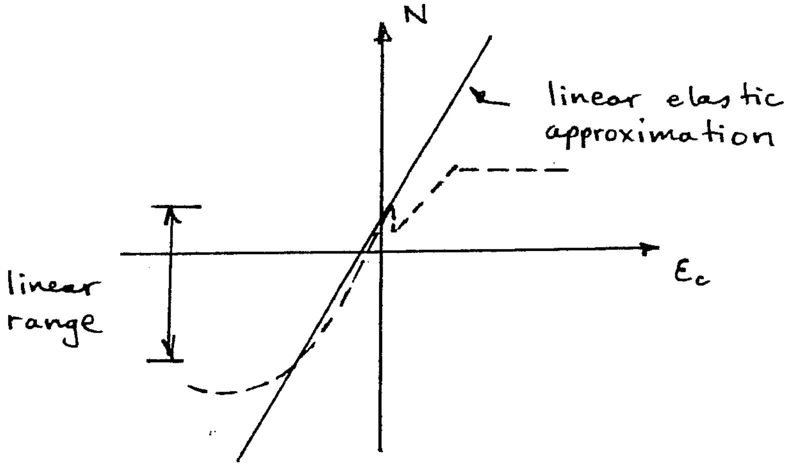
$0.222 \, E_{c_{eff}} = 0.32945$

4.8 LINEAR ELASTIC UNCRACKED RESPONSE

- As we've seen, nonlinear nature of N-Δ diagram requires iterative procedure to solve for given N

- For cases where:
  - concrete not cracked in tension
  - within 'linear' region of compressive respon

  can use linear approximation.



Equilibrium Eq^ns:

$$N = A_c f_c + A_s f_s + A_p f_p$$

COMPATIBILITY Eqns:

$$\varepsilon_c = \varepsilon_s$$

$$\varepsilon_p = \varepsilon_c + \Delta\varepsilon_p$$

where

$$\left[\begin{aligned} \varepsilon_c &= \varepsilon_{cf} + \varepsilon_{sh} + \varepsilon_{cth} \\ \varepsilon_s &= \varepsilon_{sf} + \varepsilon_{sth} \\ \varepsilon_p &= \varepsilon_{pf} + \varepsilon_{pth} \end{aligned}\right]$$

what you want to use in σ/ε relationships

CONSTITUTIVE RELATIONS:

$$f_c = E_c \cdot \varepsilon_{cf} \quad \text{where} \quad E_c = E_{cs} = \cancel{\left(\underset{\text{MPa}}{5000}\sqrt{f'_c}\right) = 0.9E_{ct}}$$

$$f_s = E_s \cdot \varepsilon_{sf}$$

$$f_y = E_p \cdot \varepsilon_{pf}$$

For linear-elastic response

$\underset{\text{psi}}{57000}\sqrt{f'_c}$ = secant modulus or 0.9 E_{ct}

Combining above three sets of relations, can get closed-form relation between N and ε_c.

Somewhat messy. Can simplify format somewhat by defining following:

N_o - decompression force, that is, force required to produce zero strain in concrete

$$N_o = A_p E_p \Delta\varepsilon_p - (A_c E_c \varepsilon_{sh} + A_c E_c \varepsilon_{cth} + A_s E_s \varepsilon_{sth} + A_p E_p \varepsilon_{pth})$$

A_{trans} - transformed concrete cross-sectional area

$$A_{trans} = A_c + \frac{E_s}{E_c} \cdot A_s + \frac{E_p}{E_c} \cdot A_p$$

Using this notation, can write

$$\boxed{N = A_{trans} \cdot E_c \cdot \varepsilon_c + N_o}$$

where $f_c = E_c \cdot \varepsilon_c$ $\quad \sim 0.5 f'_c < f_c < f'_t$

Can be used for both short-term and long-term responses.

LONG-TERM EFFECTS

Calculations of stresses

Previous work: $\phi = 2.7$

shrinkage $\varepsilon_{sh} = -0.4 \times 10^{-3}$

relaxation $= 5\%$

Same relationships, except use E_{ceff} instead of E_c

E_{peff} instead of E_p

$$E_{ceff} = \frac{E_{cs}}{1+\phi}$$

$$E_{peff} = 0.95 E_p$$

E_{ct} will be more accurate over a smaller range; E_{cs} will be softer but more applicable

Approximation:

$$N_0 = A_p E_{peff} \Delta\varepsilon_p - A_c E_{ceff} \left(\varepsilon_{ch} + \cancel{\varepsilon_{cth}} - A_s E_s \cancel{\varepsilon_{sth}} - A_p E_{peff} \cancel{\varepsilon_{pth}} \right)$$

no thermal strains

$$= (1.224\,in^2)(27550\,ksi)(6.145 \times 10^{-3}) - (318.8\,in^2)(1335\,ksi)(-0.4 \times 10^{-3})$$

$$= 377.5\,kip$$

$$A_{trans} = A_c + \frac{E_s}{E_{ceff}} A_s + \frac{E_{peff}}{E_{ceff}} A_p$$

$$= 318.8\,in^2 + \frac{29000\,ksi}{1335\,ksi}(4.01\,in^2) + \frac{27550\,ksi}{1335\,ksi}(1.224\,in^2)$$

$$= 430.9\,in^2 \longleftarrow \text{short term } A_{tr} = 349.5\,in^2$$

Apply loads:

$N = 225\,k$

$$\varepsilon_c = \frac{N - N_0}{E_{ceff} A_{trans}} = \frac{225\,k - 377.5\,k}{(1335\,ksi)(430.9\,in^2)} = -0.265 \times 10^{-3}$$

concrete is in compression

Calculate stresses / loads:

$$f_c = E_{c_{eff}}(\varepsilon_c - \varepsilon_{ch} - \varepsilon_{th}) = (1335\,ksi)(-0.265 \times 10^{-3} + 0.4 \times 10^{-3})$$

$$= 0.180\,ksi < f_{cr}$$

$$f_s = E_s(\varepsilon_s) = 29000\,ksi\,(-0.265 \times 10^{-3}) = -7.69\,ksi$$

$$f_p = E_{p_{eff}}(\varepsilon_c + \Delta\varepsilon_p) = 27550\,ksi\,(-0.265 \times 10^{-3} + 6.145 \times 10^{-3})$$

$$= 162\,ksi$$

f_c: If stress is greater than cracking, linear approx. is no longer appropriate

Apply $N = -1000\,k$

$\varepsilon_c = -2.394 \times 10^{-3}$

$f_c = -2.66\,ksi$

$f_s = -69.4\,ksi \longleftarrow f_s > f_y$, linear approximation is no longer a valid method for calculation

LONG-TERM CALCULATIONS

Comparison of methods

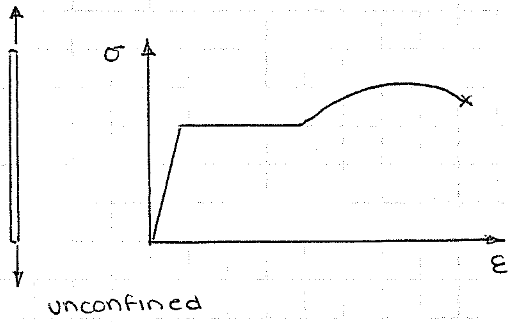
Linear Elastic	Nonlinear	
-2.394×10^{-3}	-2.412×10^{-3}	ε_c
-2.66 ksi	-2.78 ksi	f_c
$-69.4 > -60$ ksi Not good!	-60 ksi	f_s
103.3 ksi	102.8 ksi	f_p

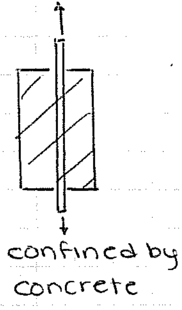
Numbers appear to be close, however, solution is not valid

Can't violate the assumptions of the analysis, including $f < f_y$

TENSION BEHAVIOR

Steel material response





overall deformations are reduced

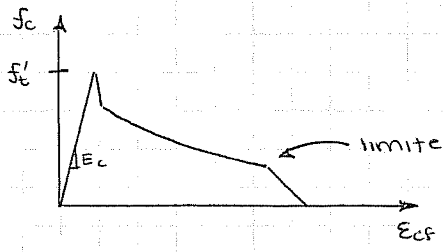
difference in response is attributed to tension stiffening, which considers the effect of the surrounding concrete

If $\varepsilon_{cf} > \varepsilon_t'$,

tensile strength

$$f_c = \frac{\alpha_1 \alpha_2 f_t'}{1+\sqrt{500\,\varepsilon_{cf}}}$$

Vecchio · Collins method



α_1 – bond characteristics

α_2 – load type

limited by yielding at a crack

ultimately: reduces deformations

Local crack conditions

At crack, $f_c = 0$

reinforcement at crack picks up total load

$$f_c = 0$$

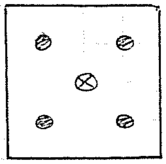
$$f_{s_{cr}} = E_s\,(\varepsilon_c + \Delta\varepsilon_{c_{cr}} - \varepsilon_{sth})$$

additional strain at crack

$$f_{p_{cr}} = E_p\,(\varepsilon_c + \Delta\varepsilon_p + \Delta\varepsilon_{ccr} - \varepsilon_{pth})$$

TENSION BEHAVIOR

Example, using same x-section



short-term behavior!

L = 20 ft

ΔL = 1/8 in forced on section

calculate:

- axial force
- average stresses
- local stresses at crack

$$\varepsilon_c = \frac{1/8 \text{ in}}{(20\text{ft})(12 \text{ in/ft})} = 0.52 \times 10^{-3}$$

huge tensile strain,

far above ε_t' or ε_{cr}

$\varepsilon_{cr} = 0.06 \times 10^{-3}$, given or calculated

$= \dfrac{f_t'}{E_c}$

$$\alpha_1 = \frac{(4.0 \text{in}^2)(1.0) + (1.224 \text{in}^2)(0.7)}{5.224 \text{in}^2} = 0.93$$

weighted average from strands and reinforcing bars — $\alpha_{1\,bars} = 1.0$,
$\alpha_{1\,strand} = 0.7$

$\alpha_2 = 1.0$

Average stresses

$\varepsilon_{cf} = \varepsilon_c - \varepsilon_{cth} - \varepsilon_{sh} = 0.52 \times 10^{-3}$

$$f_c = \frac{\alpha_1 \alpha_2 f_t'}{1 + \sqrt{500\,\varepsilon_{cf}}} = \frac{(0.93)(1.0)(335 \text{ psi})}{1 + (500 \times 0.52 \times 10^{-3})^{1/2}} = 206 \text{ psi}$$

$f_s = E_s \varepsilon_{sf} = 15.1$ ksi

same as ε_{cf}

$f_p = E_p \varepsilon_{pf} = 193.3$ ksi

$\varepsilon_{cf} + \Delta\varepsilon_p$

Axial force

$N = A_c f_c + A_s f_s + A_p f_p = 362.7$ k

Local stresses

let $\Delta\varepsilon_{ccr}$ = incremental strain at crack

$N = \cancel{A_c f_c} + A_s f_{scr} + A_p f_{pcr}$

$362.7 \text{ k} = (4.0 \text{in}^2)(29000 \text{ksi})(0.52 \times 10^{-3} + \Delta\varepsilon_{ccr}) + (1.224 \text{in}^2)(29000 \text{ksi})$
$\cdot (0.52 \times 10^{-3} + \Delta\varepsilon_p + \Delta\varepsilon_{ccr})$

$\Delta\varepsilon_{ccr} = 0.434 \times 10^{-3}$

then calculate f_{scr}, f_{pcr}

$f_{scr} = 27.7$ ksi — double previous case ⟵ goes into stress range calculations for fatigue

$f_{pcr} = 205.8$ ksi — 5% increase

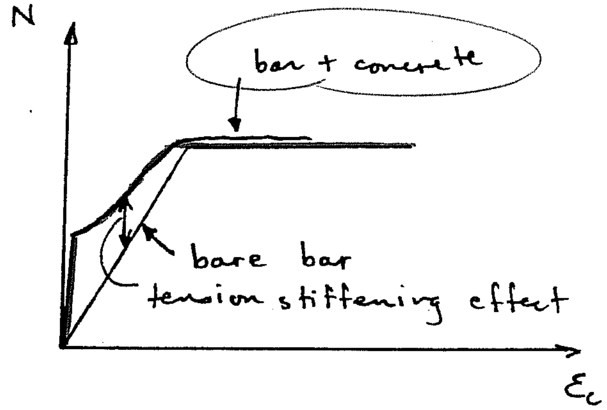
small stress range, no fatigue issues

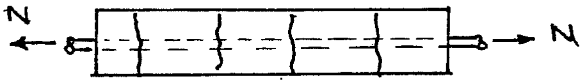
4.9 POST-CRACKING TENSILE STRESSES IN CONCRETE

- Previously assumed that when $\varepsilon_{cf} > \varepsilon_{cr}$, $f_c \rightarrow 0$

  equivalent to assuming that concrete disappears.

- In reality, concrete continues to make a contribution, even after cracking, due to bond between concrete and reinforcement.





The effect is commonly referred to as "tension stiffening"; quite distinct from tension softening

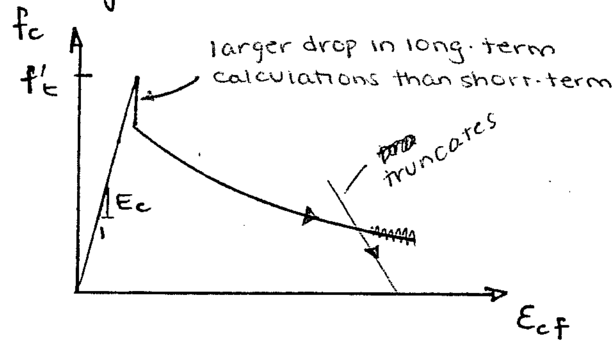
At all times : $N = N_c + N_s + N_p$

AVERAGE CONDITIONS:

Given a net elongation over length, can compute average strain:

$$\varepsilon_c = \Delta / L$$ less thermal and shrinkage

average stress in concrete:



If $\varepsilon_{cf} > \varepsilon_t'$

$$f_c = \frac{\alpha_1 \alpha_2 \cdot f_t'}{1 + \sqrt{500\, \varepsilon_{cf}}}$$

$\varepsilon_{cf} = \varepsilon_c - \varepsilon_{cth} - \varepsilon_{sh}$

where

α_1 depends on bond characteristi

α_2 depends on load type

$\alpha_1 = 1.0$ for deformed bars
$\alpha_1 = 0.7$ for plain bars, wires or bonded strands
$\alpha_1 = 0.0$ for unbonded reinforcement

$\alpha_2 = 1.0$ for short-term monotonic loading
$\alpha_2 = 0.7$ for sustained or repeated loads

average stress in reinforcement:

$$f_s = E_s \cdot \varepsilon_{sf} \qquad \varepsilon_{sf} = \varepsilon_c - \varepsilon_{sth}$$

$$f_p = E_p \cdot \varepsilon_{pf} \qquad \varepsilon_{pf} = \varepsilon_c - \varepsilon_{pth} + \Delta\varepsilon_p$$

Internal forces must balance applied load:

$$N = A_c f_c + A_s f_s + A_p f_p$$

LOCAL CONDITIONS AT CRACK

- assume that, at crack, $f_c = 0$
- ∴ reinforcement crossing crack must pick up entire load

let $\Delta\varepsilon_{c,cr}$ be additional strain at crack

$$f_c = 0$$

$$f_{s,cr} = E_s \cdot (\varepsilon_c + \Delta\varepsilon_{c,cr} - \varepsilon_{sth})$$

$$f_{p,cr} = E_p \cdot (\varepsilon_c + \Delta\varepsilon_p + \Delta\varepsilon_{c,cr} - \varepsilon_{pth})$$

again $N = A_c f_c + A_s f_s + A_p f_p$

but ε_c known from average stress calculations
can solve for incremental strain $\Delta\varepsilon_{c,cr}$
then can solve for $f_{s,cr}$ and $f_{p,cr}$

TENSION STIFFENING

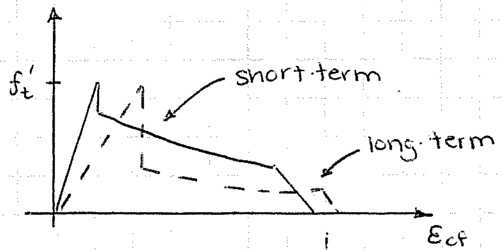
Load-deformation response

long-term response

$E_c = E_{ceff}$ – creep

$\alpha_2 = 0.7$

$E_p = E_{peff}$ – ~~shrinkage~~, relaxation



capacity is not necessarily the same; either could be larger

Not included in capacity calculations

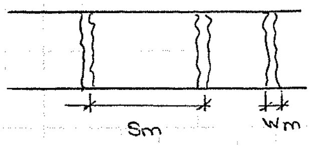
provides no values

used in deflection, crack width calculations

theory came from an inability to predict response

crack widths and crack spacing

$\varepsilon_{cf} = \frac{\Delta}{L}$



mean crack spacing

mean crack width

$n = L/s_m$

$w_m = \frac{\Delta}{n} = \frac{\varepsilon_{cf} L}{L/s_m}$, $\boxed{w_m = \varepsilon_{cf} \cdot s_m}$

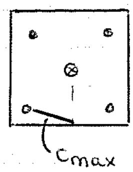
including tension stiffening – strain at any point gives widths, spacing at that point

Equation is still undefined

Declare:

$s_m = 3c_{max}$

maximum distance from concrete fibre in tension to nearest rebar or strand



ignores deformation between cracks

all estimation – +/- 70%, easy

4.10 LOAD - DEFORMATION RESPONSE ALLOWING FOR T.S.

- Essentially, as before.
- Find total axial force for range of ε_c
- Remember, yielding of the reinforcement at the cracks will govern the maximum load.

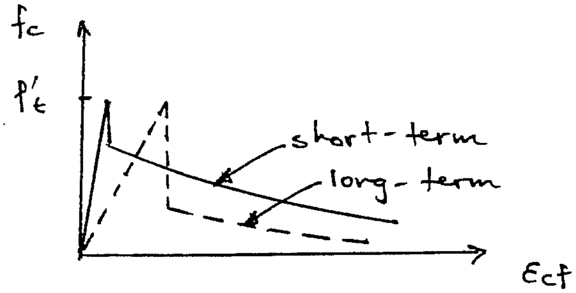
LONGTERM RESPONSE

Make note that: i) $E_c = E_{c,eff}$ ∴ cracking strain is higher

ii) $\alpha_2 = 0.7$

iii) $E_p = E_{p,eff}$

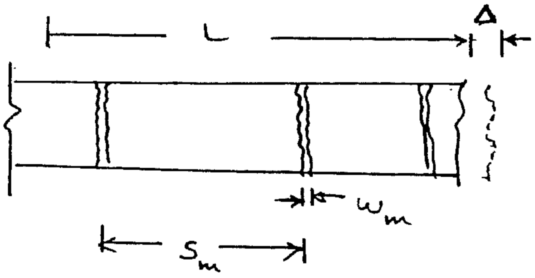
otherwise, same procedure



4.11 CRACK WIDTHS AND CRACK SPACINGS

- In cracked reinforced concrete, elongation made up of: i) discontinuous separation at cracks
  ii) straining between cracks.

First is much larger than second; latter is often ignore in crack width calculations



consider length L deforms distance Δ

∴ strain $\varepsilon_{cf} = \Delta / L$

let S_m = mean crack spacing
w_m = mean crack width

no. of cracks over distance L:

$n = L / S_m$

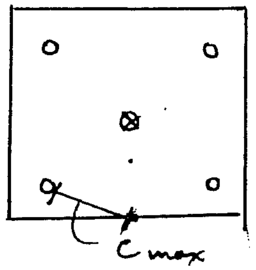
total deformation accounted for by cracks ∴ $\Delta = n \cdot w_m$

$$\therefore \quad w_m = \frac{\Delta}{n} = \frac{\varepsilon_{cf} \cdot L}{L / S_m} \Rightarrow \boxed{w_m = \varepsilon_{cf} \cdot S_m}$$

Still need to estimate crack spacing.

Simple answer: $S_m = 3\, c_{max}$

where c_{max} = the maximum distance from concrete fibre in tension to nearest reinforcement



More accurate expressions for crack widths / crack spacings

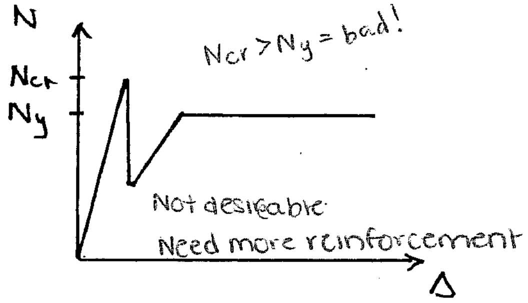
i) CEB-FIP formulations

ii) Gergely-Lutz formulations

} see Collins / Mitchell if interested

4.12 MINIMUM REINFORCEMENT FOR CRACK CONTROL

- If $(A_s + A_p)$ insufficient, all reinforcement yields upon first cracking. No Good! Cracking is not controlled



Recall, for RC:

$$\rho_{min} = \frac{f'_t}{f_y}$$

calculates values close to ACI code levels

overestimates flexural need

- For a member in which reinforcement is prestressed

$$\rho_{P_{min}} = \frac{f'_t}{f_{py} - f_{pcr}}$$

where $\rho_{p\,min}$ = minimum percentage of prestressing steel

f_{pcr} = stress in P/S steel just prior to cracking

Local Conditions

Part.	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
($x10^{-3}$)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	($x10^{-3}$)	(ksi)	(kips)	(kips)	
-3	-3	-6.782	-2162.1	-3	-60	-240	3.145	91.205	111.635	-2290.5	
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	3.395	98.455	120.509	-2337.4	
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	3.595	104.255	127.608	-2344	$\varepsilon_c = \varepsilon_o$
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	3.895	112.955	138.257	-2302.5	
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	4.075	118.175	144.646	-2247.9	$\varepsilon_c = \varepsilon_y$
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	4.395	127.455	156.005	-2059	
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	4.645	134.705	164.879	-1862.4	
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	4.895	141.955	173.753	-1622.9	
-1	-1	-4.4137	-1407.1	-1	-29	-116	5.145	149.205	182.627	-1340.5	
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96	
0	0	0	0	0	0	0	6.145	178.205	218.123	218.123	
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	6.205	179.945	220.253	332.225	$\varepsilon_c = \varepsilon_{cr-}$
0.06	0.06	0	0	0.06	1.74	6.96	6.205	179.945	220.253	227.213	$\varepsilon_c = \varepsilon_{cr+}$
1.855	1.855	0	0	1.855	53.795	215.18	8	230	281.52	496.7	$\varepsilon_p = \varepsilon_{py}$
2.07	2.07	0	0	2.07	60	240	8.215	230	281.52	**521.52**	$\varepsilon_s = \varepsilon_{sy}$
3	3	0	0	3	60	240	9.145	230	281.52	**521.52**	

Note: Max Axial Load (Tension) = 521.52 kips

same as pg 8-5

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*Average Conditions **Incorrect***

Part.	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
$(x10^{-3})$	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	(kips)	
-3	-3	-6.782	-2162.1	-3	-60	-240	3.145	91.205	111.635	-2290.5	
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	3.395	98.455	120.509	-2337.4	
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	3.595	104.255	127.608	-2344	$\varepsilon_c = \varepsilon_o$
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	3.895	112.955	138.257	-2302.5	
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	4.075	118.175	144.646	-2247.9	$\varepsilon_c = \varepsilon_y$
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	4.395	127.455	156.005	-2059	
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	4.645	134.705	164.879	-1862.4	
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	4.895	141.955	173.753	-1622.9	
-1	-1	-4.4137	-1407.1	-1	-29	-116	5.145	149.205	182.627	-1340.5	
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96	
0	0	0	0	0	0	0	6.145	178.205	218.123	218.123	
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	6.205	179.945	220.253	332.225	$\varepsilon_c = \varepsilon_{cr-}$
0.06	0.06	0.26555	84.6588	0.06	1.74	6.96	6.205	179.945	220.253	311.871	$\varepsilon_c = \varepsilon_{cr+}$
1.855	1.855	0.15871	50.5954	1.855	53.795	215.18	8	230	281.52	**547.295**	$\varepsilon_p = \varepsilon_{py}$
2.07	2.07	0.15444	49.234	2.07	60	240	8.215	230	281.52	**570.754**	$\varepsilon_s = \varepsilon_{sy}$
3	3	0.14004	44.6443	3	60	240	9.145	230	281.52	**566.164**	

Compare to 521.5 k from pg before

$$f_c = \frac{\alpha_1 \alpha_2 f_t'}{1+\sqrt{500\varepsilon_{cf}}}$$

Capacity is determined at a crack — no tension stiffening

no physical meaning

10-3-2

*Average Conditions **Correct***

Part.	Concrete			Reinforcement			Prestressing				
ε_c	ε_{cf}	f_c	F_c	ε_{sf}	f_s	F_s	ε_{pf}	f_p	F_p	N	
$(x10^{-3})$	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	$(x10^{-3})$	(ksi)	(kips)	(kips)	
-3	-3	-6.782	-2162.1	-3	-60	-240	3.145	91.205	111.635	-2290.5	
-2.75	-2.75	-6.9569	-2217.9	-2.75	-60	-240	3.395	98.455	120.509	-2337.4	
-2.55	-2.55	-7	-2231.6	-2.55	-60	-240	3.595	104.255	127.608	-2344	$\varepsilon_c = \varepsilon_o$
-2.25	-2.25	-6.9031	-2200.7	-2.25	-60	-240	3.895	112.955	138.257	-2302.5	
-2.07	-2.07	-6.752	-2152.5	-2.07	-60	-240	4.075	118.175	144.646	-2247.9	$\varepsilon_c = \varepsilon_y$
-1.75	-1.75	-6.311	-2012	-1.75	-50.75	-203	4.395	127.455	156.005	-2059	
-1.5	-1.5	-5.8131	-1853.2	-1.5	-43.5	-174	4.645	134.705	164.879	-1862.4	
-1.25	-1.25	-5.1807	-1651.6	-1.25	-36.25	-145	4.895	141.955	173.753	-1622.9	
-1	-1	-4.4137	-1407.1	-1	-29	-116	5.145	149.205	182.627	-1340.5	
-0.5	-0.5	-2.476	-789.34	-0.5	-14.5	-58	5.645	163.705	200.375	-646.96	
0	0	0	0	0	0	0	6.145	178.205	218.123	218.123	
0.06	0.06	0.3294	105.013	0.06	1.74	6.96	6.205	179.945	220.253	332.225	$\varepsilon_c = \varepsilon_{cr-}$
0.06	0.06	0.26555	84.6588	0.06	1.74	6.96	6.205	179.945	220.253	311.871	$\varepsilon_c = \varepsilon_{cr+}$
1.66	1.66	0.16303	51.9727	1.66	48.14	192.56	7.805	226.345	277.046	**521.52**	—capped
1.855	*1.855*	*0.0779*	*24.835*	1.855	53.795	215.18	8	230	281.52	**521.52**	$\varepsilon_p = \varepsilon_{py}$
2.07	2.07	0	0	2.07	60	240	8.215	230	281.52	**521.52**	$\varepsilon_s = \varepsilon_{sy}$
3	3	0	0	3	60	240	9.145	230	281.52	**521.52**	

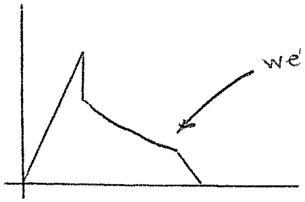
uses $f_c = \alpha_1 \alpha_2 \ldots$

not strain → stress → force

calculated backwards

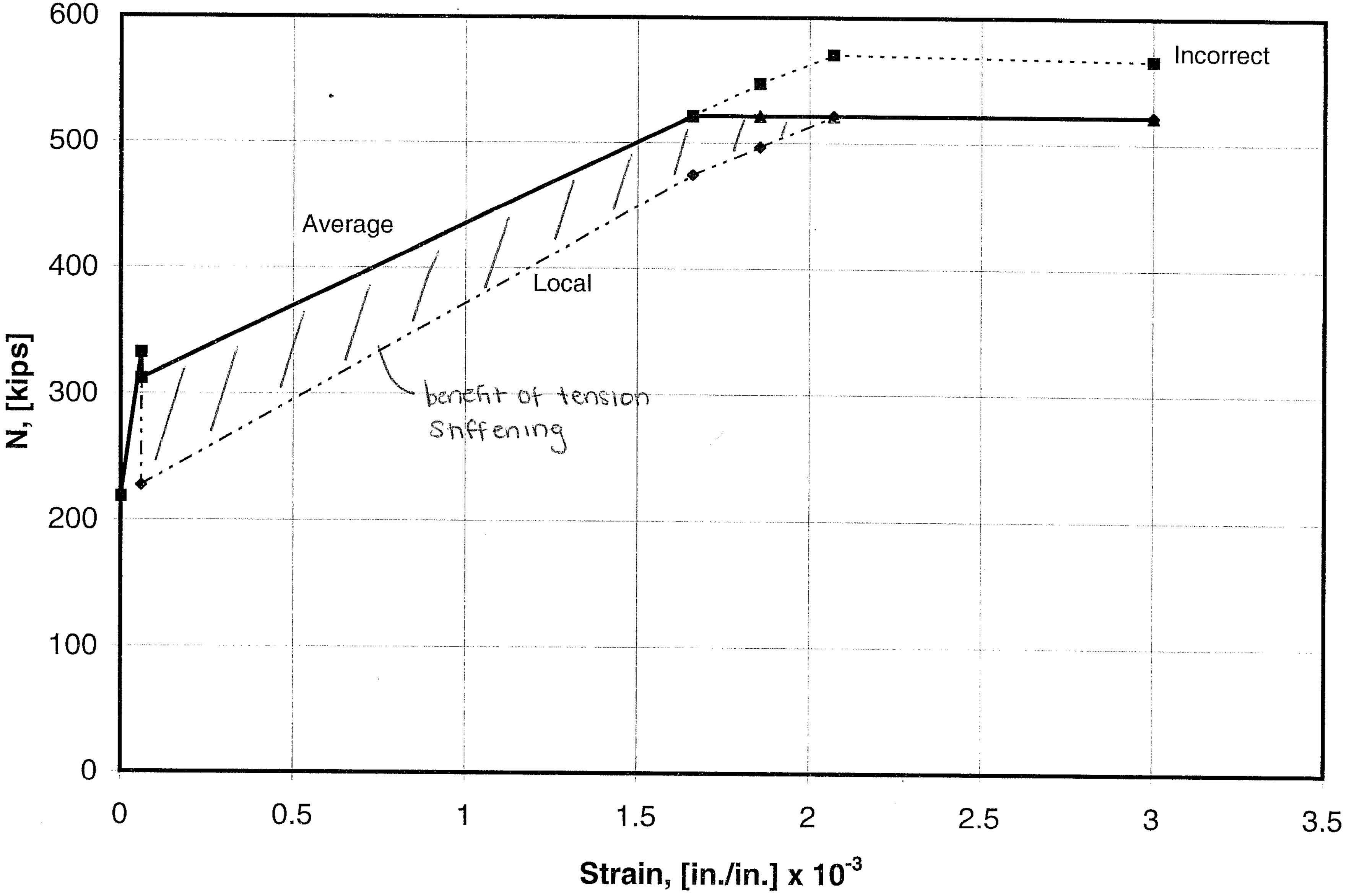
matches

Total – strands – reinforce.



we're doing this part here

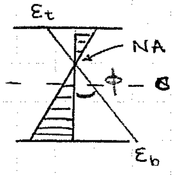
10-3.3


600
500
400
300
200
100
0
N, [kips]
0
0.5
1
1.5
2
2.5
3
3.5
Strain, [in./in.] x 10^{-3}
Incorrect
Average
Local
benefit of tension stiffening

10-3-4

FLEXURAL MEMBERS

compatibility conditions

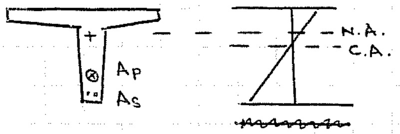


— C.A., or rather, centroidal axis (CA)

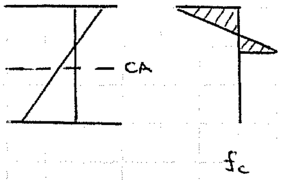
$$\boxed{\begin{aligned}\varepsilon_c &= \varepsilon_{cen} - \phi y\\ \varepsilon_s &= \varepsilon_{cen} - \phi y_s\\ \varepsilon_p &= \varepsilon_{cen} - \phi y_p + \Delta\varepsilon_p\end{aligned}}$$

defined as such to allow decompression loads to be calculated (later) easily

forces and moments are calculated and given around centroidal axis of gross x-section



or



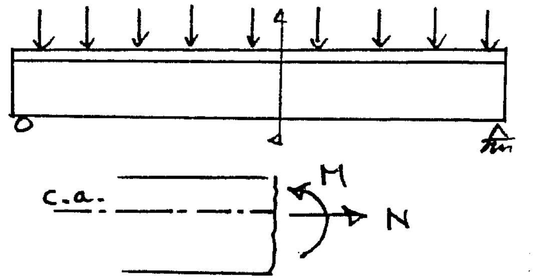
should line up; drawings aren't awesome here

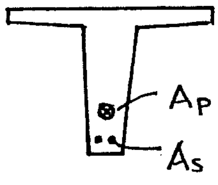
use C.A. not N.A. for loads because analysis would be set up with beams and columns defined by centroidal axis

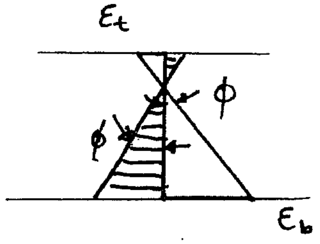
positive ϕ is concave up (sag)

5. RESPONSE OF MEMBERS SUBJECTED TO FLEXURE

5.1 INTRODUCTION

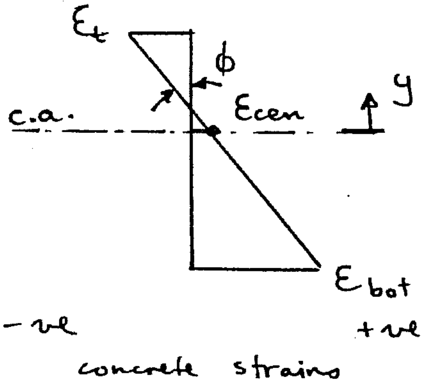
- flexural members (beams, girders, slabs) are the most widely used prestressed concrete elements
- often $N=0$, but will develop procedures for general case
- 





- note : - cross-sections may contain both A_s and A_p
  - section loads are wrt centroidal axis (c.a.) of gross cross section

5.2 COMPATIBILITY CONDITIONS

- need two variables to define strain profile, if we assume "plane sections remain plane"
- 

- can use
  - ε_t & ε_b or ϕ, ε_{cen} ←
- concrete strain at any distance y from c.a. is given by:

$$\varepsilon_c = \varepsilon_{cen} - \phi \cdot y$$

- strains in rebar : $\varepsilon_s = \varepsilon_{cen} - \phi \cdot y_s$

since $\varepsilon_s = \varepsilon_c$, y_s is location of rebar

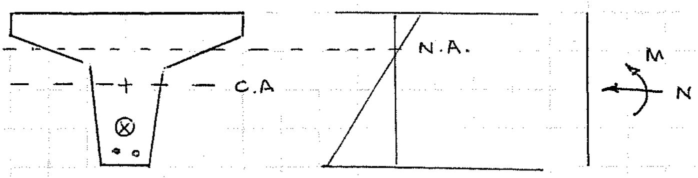
- strains in prestressing tendons :

$$\varepsilon_p = \varepsilon_{cen} - \phi \cdot y_p + \Delta \varepsilon_p$$

since $\varepsilon_p = \varepsilon_c + \Delta \varepsilon_p$.

FLEXURAL MEMBERS

Equilibrium

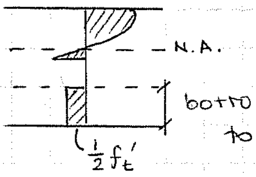


$$N = \int_{A_c} f_c \cdot dA_c + \int_{A_s} f_s \cdot dA_s + \int_{A_p} f_p \, dA_p$$

$$M = -\int_{A_c} f_c y \, dA_c - \int_{A_s} f_s y \, dA_s - \int_{A_p} f_p y \, dA_p$$

Tension stiffening used for Δ, θ calculations
ignored in strength considerations } applied in effective embedment zone

$h = 7.5 d_b$



do not consider in strength calculations! only deflection and curvature.

Predicting response

$N = f_n(\varepsilon_{ct}, \varepsilon_{cb})$ top and bottom strains
$M = f_n(\varepsilon_{ct}, \varepsilon_{cb})$

Now, use trial and error

1. Given N, M (external)
2. Select value for top fibre strain, ε_{ct}
3. By trial and error, find ε_{cb} such that $N' \rightarrow N$
4. Calculate corresponding M'
5. If $M' \neq M$, return to step 2

Note: above relations assume ϕ is positive when beam concave upwards

5.3 EQUILIBRIUM CONDITIONS

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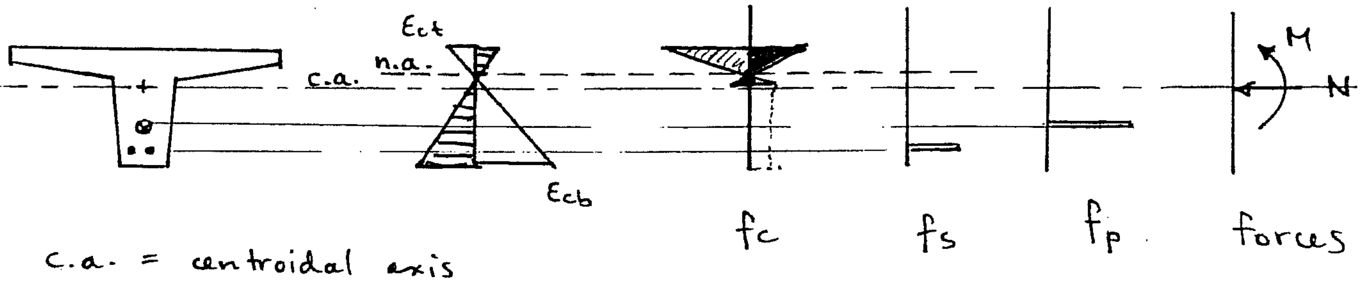


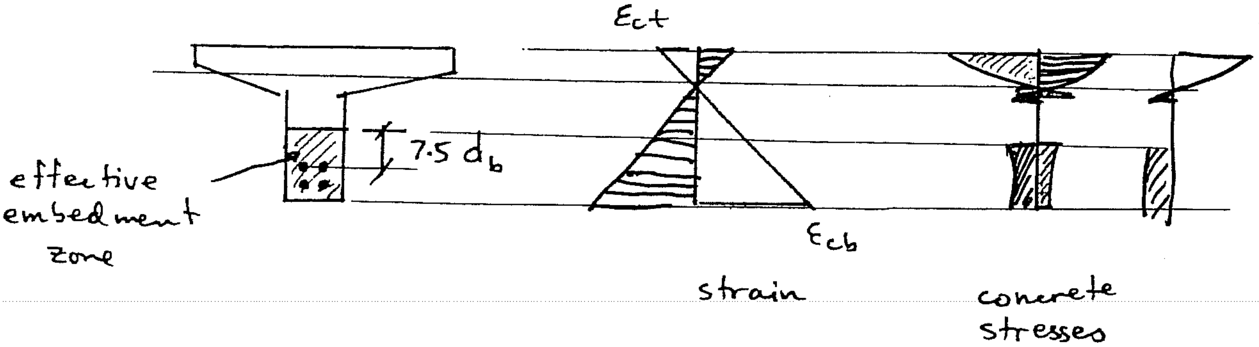
c.a. = centroidal axis
n.a. = neutral axis

- at any section, stresses integrated over area, must balance applied forces

hence
$$N = \int_{A_c} f_c \cdot dA_c + \int_{A_s} f_s \cdot dA_s + \int_{A_p} f_p dA_p$$
$$M = -\int_{A_c} f_c \cdot y \, dA_c - \int_{A_s} f_s \, y \, dA_s - \int_{A_p} f_p \cdot y \, dA_p$$

- note that compression in concrete is nonlinear
- also, post-cracking tensile stresses in concrete may be considered → tension stiffening considered or may be neglected

- TENSION STIFFENING CONSIDERED
  - post-cracking tensile stresses calulated according to
$$f_c = \frac{\alpha_1 \alpha_2 f'_t}{1 + \sqrt{500 \, \varepsilon_{cf}}}$$
  - these stresses only present in "effective embedment zone" ie. area of concrete within 7.5 d_b from any reinf.



5.4 PREDICTING RESPONSE

- Once again, use equilibrium + compatibility + constitutive relations
- Essentially get down to two equations

$$\left.\begin{aligned} N &= fn(\varepsilon_{ct}, \varepsilon_{cb}) \\ M &= fn(\varepsilon_{ct}, \varepsilon_{cb}) \end{aligned}\right\}$$

use trial and error approach;

i) given N, M

ii) select value of top fibre strain, ε_{ct}

iii) by trial and error, find ε_{cb} such that $N' \rightarrow N$

iv) calculate~~d~~ corresponding M'

v) if $M' \neq M$, go to (ii)

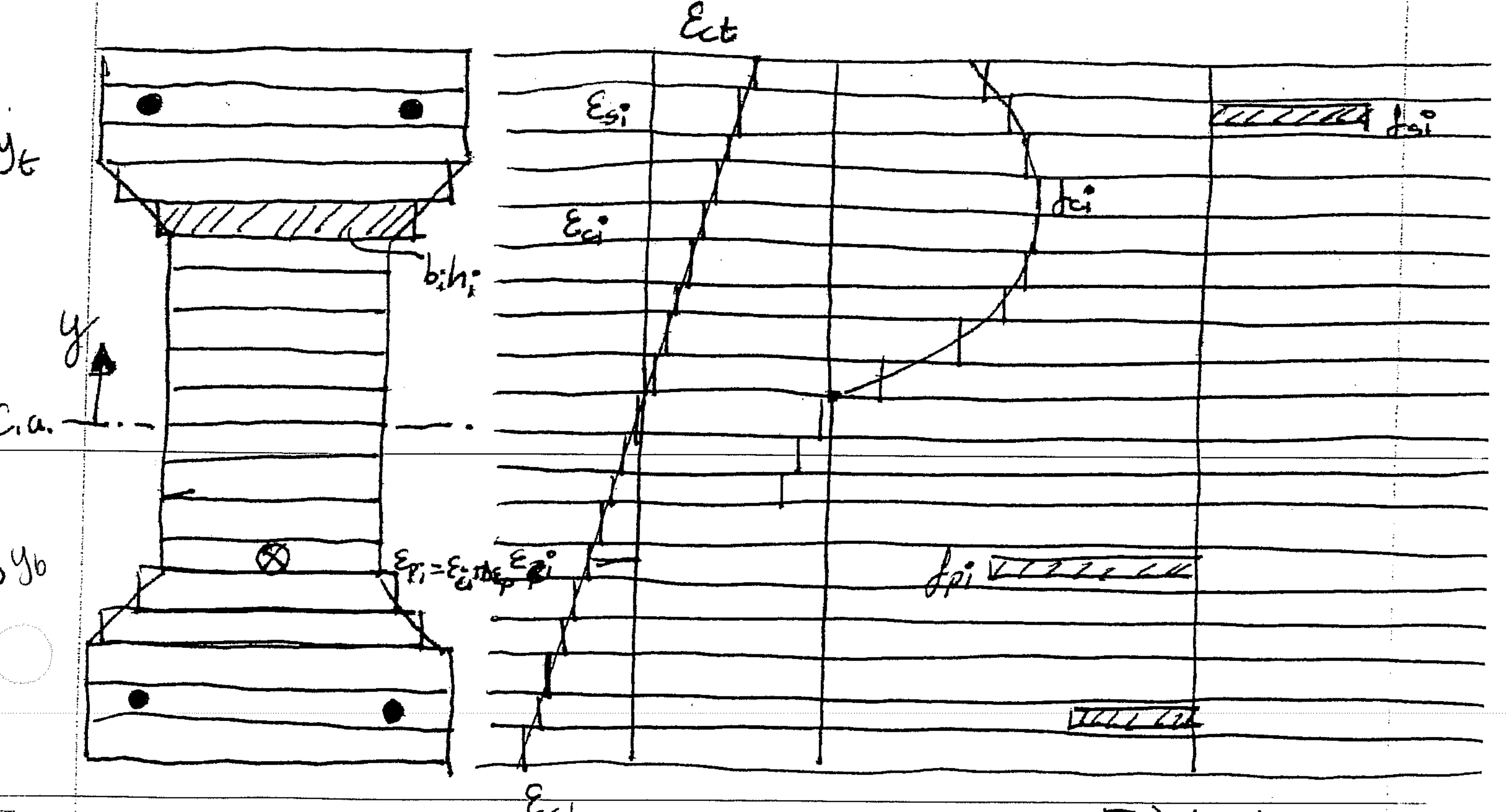
Difficult to work with closed-form equations and integrals; two alternate approaches:

i) layered - section analysis (on next pg – trapezoidal / rectangular approximation)

ii) rectangular stress blocks

5.5 LAYERED - SECTION ANALYSIS

- expedient to idealize cross-section as a series of rectangular layers.
- assume that ~~the~~ ~~section~~ strain in each layer is uniform and equal to the actual strain at centre of layer.
- → uniform force in layer; easy to integrate stresses



Using plane sections remain plane

$$\left.\begin{aligned} \varepsilon_{ci} &= \varepsilon_{cen} - \phi y_{ci} \\ \varepsilon_{si} &= \varepsilon_{cen} - \phi y_{si} \\ \varepsilon_{pi} &= \varepsilon_{cen} - \phi y_{pi} \end{aligned}\right\} \quad \text{where} \quad \phi = \frac{(\varepsilon_{cb} - \varepsilon_{ct})}{H}$$

$\boxed{+\Delta\varepsilon_p}$

$$\varepsilon_{cen} = \varepsilon_{ct} + \phi \cdot y_t$$

$$H = (y_b + y_t)$$

given

$$\left.\begin{aligned} \varepsilon_{ci} &\longrightarrow f_{ci} \\ \varepsilon_{si} &\longrightarrow f_{si} \\ \varepsilon_{pi} &\longrightarrow f_{pi} \end{aligned}\right\} \quad \text{Using appropriate constitutive relations}$$

then, integrating

$$N' = \sum_{i=1}^{l} f_{ci} b_i h_i + \sum_{J=1}^{m} f_{sJ} A_{sJ} + \sum_{k=1}^{n} f_{pk} A_{pk}$$

$$M' = \sum_{i=1}^{l} -f_{ci} b_i h_i y_{ci} + \sum_{J=1}^{m} -f_{sJ} A_{sJ} y_{sJ} + \sum_{k=1}^{n} -f_{pk} A_{pk} y_{pk}$$

where i = no. of concrete layers (~16-20 layers)

J = no. of rebar layers

k = no. of p/s layers

---

obviously a tedious process; even more so when we introduce shear effects (later) don't try with hand calcs.

Programs available
- ~~RESPONSE~~
- SMAL
- SECREC (by Bayrak)

Use ▭ stress block

RESPONSE PROGRAM

Various notes

- main menu comes up with slash key (forward)
- tension stiffening coefficient = α_1, α_2
  or, factor
  TS only away from cracks
  may use 2 concrete types to capture cracked and uncracked
- default Ramberg-Osgood coefficients are for low relaxation strands. only change for stress relieved, etc.
- F8 toggles screens
- can calculate ε_c' for use — type in zero, go to graph (F8), read value and re-input
- "distance to moment axis": measured from the bottom (centroidal axis)
- concrete layers: define 1, then change number
  program will propogate other layers accordingly
- F9 shows cross-sectional appearance
- option for ~~[illegible]~~ finer analysis may be needed

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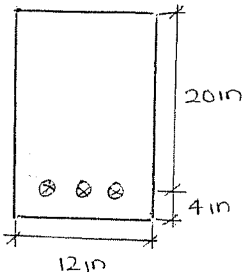
Typical output from program RESPONSE

```
RESPONSE          OUTPUT    FILE
ANALYSIS:         Axial-Load
SECTION           NAME      :          [ Rect        Sec ]
CONCRETE          MODEL:    Parabolic    Material    Factors:      ϕ =  C:1.000    S:1.000   P:1.000  - all the
TENSION-STIFF.:   No        FACTORED:    No          ACCURACY:          well_done  }                   same in the US
Axial-Load:       0.00kip   Moment:      0.00kft     Shear:             0.00kip    options: rare, medium,
                  dN/dM:               0 dN/dV:                       0 dM/dV:           0         well_done
                  Axial-Load  Moment     Curvature   @-Axis              Bottom     Top      ~~Iter.~~ Line No.
                  kips        ft*kip     rad/10^6in  [-------Milli-Strain------]
                                                                         STRAINS
                        0.02     52.63          0              -0.063    -0.063    -0.063        1
                       -0.01    123.27      33.33               0.046     0.446    -0.354        2
                       -0.01    133.35      66.67               0.292     1.092    -0.508        3
                       -0.01    143.82        100               0.557     1.757    -0.643        4
                           0    152.92     133.33               0.834     2.434    -0.766        5
                       -0.27    160.45     166.67                1.12      3.12     -0.88        6
                       -0.21    165.88        200               1.415     3.815    -0.985        7
                       -0.16     169.7     233.33               1.719     4.519    -1.081        8
                       -0.13    172.35     266.67               2.029     5.229    -1.171        9
                        -0.1    174.21        300               2.345     5.945    -1.255       10
                       -0.07    175.58     333.33               2.665     6.665    -1.335       11
                       -0.05    176.65     366.67               2.988     7.388    -1.412       12
                       -0.04    177.52        400               3.313     8.113    -1.487       13
                       -0.03    178.28     433.33                3.64      8.84     -1.56       14
                       -0.02    178.94     466.67               3.969     9.569    -1.631       15
                       -0.01    179.55        500               4.299    10.299    -1.701       16
                       -0.01    180.11     533.33                4.63     11.03     -1.77       17
                       -0.01    180.64     566.67               4.961    11.761    -1.839       18
                       -0.23    181.33        600               5.291    12.491    -1.909       19
                       -0.12    181.71     633.33               5.624    13.224    -1.976       20
                       -0.02    182.08     666.67               5.958    13.958    -2.042       21
                        0.08    182.45        700               6.292    14.692    -2.108       22
                        0.13    182.82     733.33               6.626    15.426    -2.174       23
                        0.19    183.18     766.67                6.96     16.16     -2.24       24
                        0.26    183.52        800               7.294    16.894    -2.306       25
                           0    184.12     833.33               7.622    17.622    -2.378       26
                       -0.01    184.49     866.67               7.954    18.354    -2.446       27
                       -0.01    184.85        900               8.285    19.085    -2.515       28
                       -0.01     185.2     933.33               8.616    19.816    -2.584       29
                       -0.01    185.54     966.67               8.946    20.546    -2.654       30
                       -0.02    185.86       1000               9.274    21.274    -2.726       31
                       -0.02    186.18    1033.33               9.601    22.001    -2.799       32
                       -0.03    186.48    1066.67               9.927    22.727    -2.873       33
                       -0.03    186.76       1100              10.251    23.451    -2.949       34
                       -0.03    187.03    1133.33              10.574    24.174    -3.026       35
                       -0.04    187.28    1166.67              10.893    24.893    -3.107       36
                       -0.04    187.51       1200               11.21     25.61     -3.19       37
                       -0.05    187.72    1233.33              11.523    26.323    -3.277       38
                       -0.06    187.91    1266.67              11.832    27.032    -3.368       39
                       -0.07    188.06       1300              12.135    27.735    -3.465       40
                       -0.09    188.17    1333.33              12.432    28.432    -3.568       41
                        -0.1    188.22    1366.67              12.718    29.118    -3.682       42
                        -0.1    188.17       1400              12.991    29.791    -3.809       43
```



$f_c' = 5 ksi$

$\varepsilon_o = 2.25 \times 10^{-3}$

$f_t' = 530 psi$

$\Delta\varepsilon_p = 6.0 \times 10^{-3}$

$A_p = 3 - 1/2"$ 7wire $= 0.153 in^2 \times 3 = 0.459 in^2$

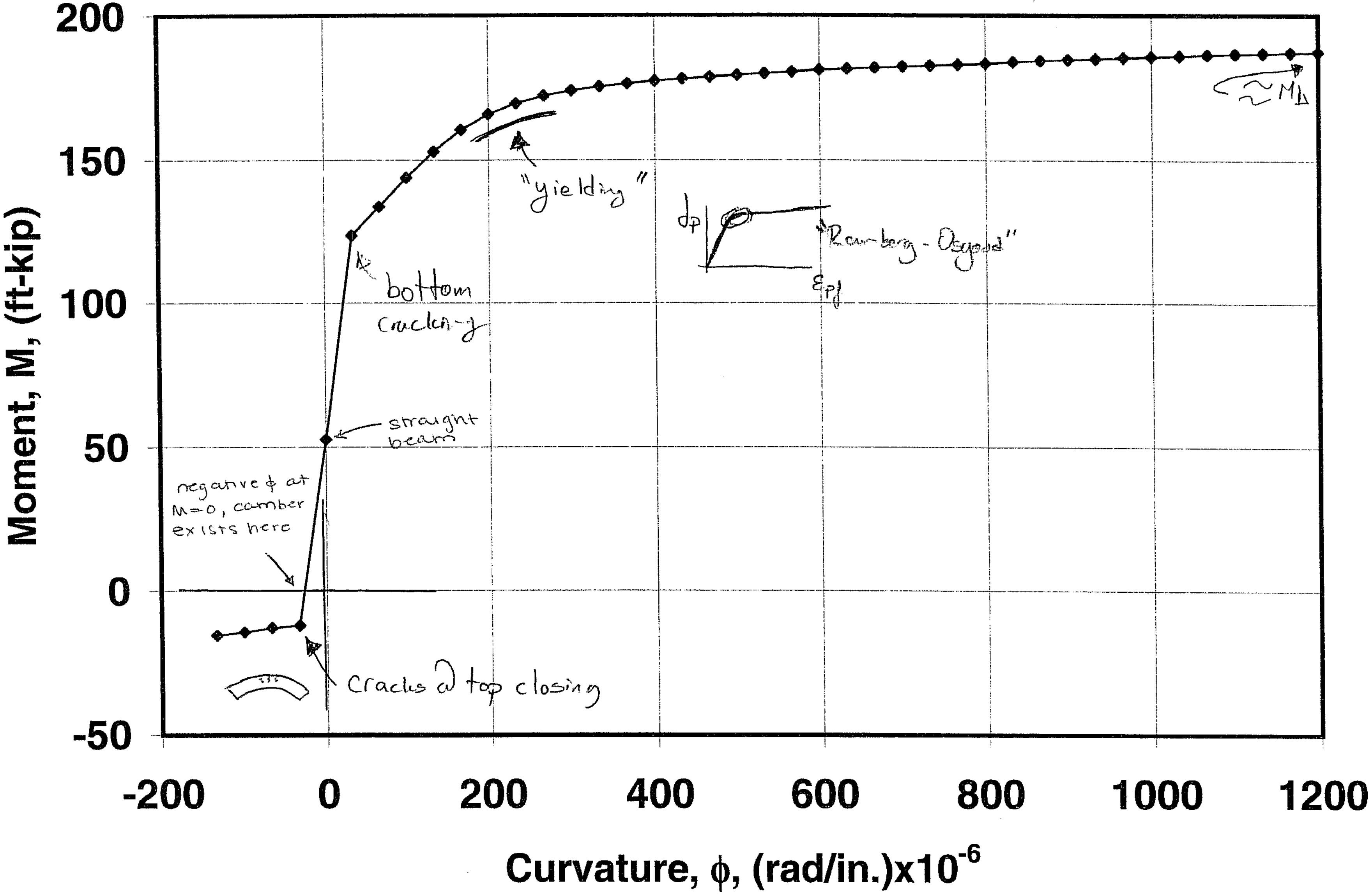
A = 0.025, B = 118, C = 10
Ramberg-Osgood eq.
low relaxation coefficients

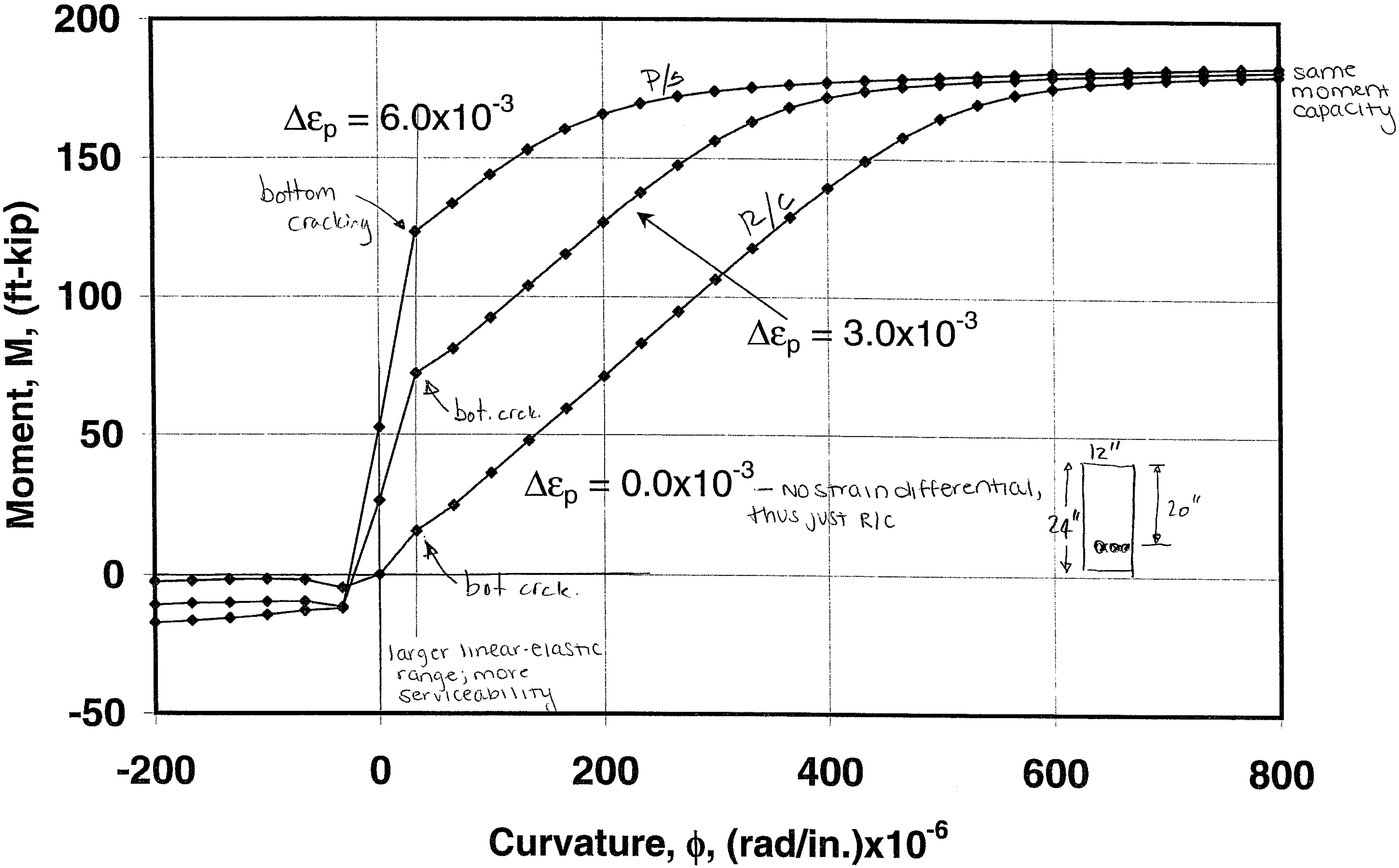
use 16 layers in concrete

-0.11	187.98	1433.33	13.241	30.441	-3.959	44
-0.09	187.43	1466.67	13.442	31.042	-4.158	45
0.07	171.66	1500	11.542	29.542	-6.458	46
0.13	115.72	1533.33	3.642	22.042	-14.758	47
0.02	64.23	1566.67	-4.258	14.542	-23.058	48
0.02	52.63	0	-0.063	-0.063	-0.063	1
-0.04	-12.18	-33.33	0.058	-0.342	0.458	2
-0.03	-13.07	-66.67	0.327	-0.473	1.127	3
-0.02	-14.6	-100	0.622	-0.578	1.822	4
-0.01	-15.76	-133.33	0.93	-0.67	2.53	5
-0.01	-16.67	-166.67	1.245	-0.755	3.245	6
0	-17.42	-200	1.567	-0.833	3.967	7
0	-18.05	-233.33	1.892	-0.908	4.692	8
-0.21	-18.78	-266.67	2.219	-0.981	5.419	9
-0.14	-19.27	-300	2.549	-1.051	6.149	10
-0.11	-19.72	-333.33	2.881	-1.119	6.881	11
-0.08	-20.13	-366.67	3.214	-1.186	7.614	12
-0.06	-20.53	-400	3.548	-1.252	8.348	13
-0.08	-20.93	-433.33	3.883	-1.317	9.083	14
-0.05	-21.28	-466.67	4.219	-1.381	9.819	15
-0.04	-21.62	-500	4.556	-1.444	10.556	16
-0.03	-21.95	-533.33	4.893	-1.507	11.293	17
-0.02	-22.26	-566.67	5.23	-1.57	12.03	18
-0.02	-22.57	-600	5.567	-1.633	12.767	19
-0.01	-22.86	-633.33	5.904	-1.696	13.504	20
-0.01	-23.14	-666.67	6.242	-1.758	14.242	21
-0.01	-23.41	-700	6.579	-1.821	14.979	22
-0.25	-23.88	-733.33	6.914	-1.886	15.714	23
-0.12	-24.02	-766.67	7.252	-1.948	16.452	24
-0.16	-24.3	-800	7.588	-2.012	17.188	25
-0.12	-24.5	-833.33	7.925	-2.075	17.925	26
-0.02	-24.63	-866.67	8.263	-2.137	18.663	27
0.04	-24.79	-900	8.6	-2.2	19.4	28
0.09	-24.95	-933.33	8.936	-2.264	20.136	29
0.14	-25.1	-966.67	9.271	-2.329	20.871	30
0.19	-25.24	-1000	9.607	-2.393	21.607	31
0.23	-25.37	-1033.33	9.941	-2.459	22.341	32
0.27	-25.49	-1066.67	10.275	-2.525	23.075	33
0	-25.86	-1100	10.603	-2.597	23.803	34
0	-25.99	-1133.33	10.934	-2.666	24.534	35
-0.01	-26.11	-1166.67	11.265	-2.735	25.265	36
-0.01	-26.22	-1200	11.594	-2.806	25.994	37
-0.01	-26.31	-1233.33	11.922	-2.878	26.722	38
-0.01	-26.39	-1266.67	12.249	-2.951	27.449	39
-0.01	-26.45	-1300	12.574	-3.026	28.174	40
-0.02	-26.5	-1333.33	12.897	-3.103	28.897	41
-0.02	-26.53	-1366.67	13.217	-3.183	29.617	42
-0.02	-26.54	-1400	13.536	-3.264	30.336	43
-0.03	-26.53	-1433.33	13.851	-3.349	31.051	44
-0.03	-26.49	-1466.67	14.162	-3.438	31.762	45
-0.04	-26.42	-1500	14.469	-3.531	32.469	46
-0.05	-26.31	-1533.33	14.77	-3.63	33.17	47

-0.05	-26.16	-1566.67	15.064	-3.736	33.864	48
-0.06	-25.94	-1600	15.347	-3.853	34.547	49
-0.08	-25.63	-1633.33	15.615	-3.985	35.215	50
-0.07	-25.14	-1666.67	15.859	-4.141	35.859	51
-0.04	-24.28	-1700	16.05	-4.35	36.45	52
-0.22	-22.26	-1733.33	16.05	-4.75	36.85	53
0.01	-20.24	-1766.67	16.1	-5.1	37.3	54
0.01	-18.51	-1800	16.153	-5.447	37.753	55
0.23	-16.95	-1833.33	16.253	-5.747	38.253	56
0	-15.66	-1866.67	16.324	-6.076	38.724	57
-0.09	-14.49	-1900	16.424	-6.376	39.224	58
0	-13.38	-1933.33	16.553	-6.647	39.753	59
0	-12.4	-1966.67	16.681	-6.919	40.281	60
0	-11.5	-2000	16.818	-7.182	40.818	61
0	-10.68	-2033.33	16.962	-7.438	41.362	62
-0.01	-9.93	-2066.67	17.111	-7.689	41.911	63
-0.01	-9.24	-2100	17.266	-7.934	42.466	64
-0.01	-8.6	-2133.33	17.426	-8.174	43.026	65
-0.11	-8.05	-2166.67	17.582	-8.418	43.582	66
0.04	-7.44	-2200	17.761	-8.639	44.161	67
-0.01	-7	-2233.33	17.94	-8.86	44.74	68
-0.01	-6.47	-2266.67	18.098	-9.102	45.298	69
-0.01	-5.99	-2300	18.26	-9.34	45.86	70
0	-5.54	-2333.33	18.426	-9.574	46.426	71
-0.01	-5.13	-2366.67	18.596	-9.804	46.996	72
0	-4.74	-2400	18.771	-10.029	47.571	73
-0.01	-4.4	-2433.33	18.948	-10.252	48.148	74
-0.01	-4.07	-2466.67	19.129	-10.471	48.729	75
-0.01	-3.76	-2500	19.313	-10.687	49.313	76
-0.01	-3.48	-2533.33	19.499	-10.901	49.899	77
-0.01	-3.21	-2566.67	19.688	-11.112	50.488	78
-0.01	-2.96	-2600	19.88	-11.32	51.08	79
-0.02	-2.74	-2633.33	20.074	-11.526	51.674	80
-0.02	-2.52	-2666.67	20.269	-11.731	52.269	81
-0.01	-2.32	-2700	20.469	-11.931	52.869	82
0.02	-2.11	-2733.33	20.672	-12.128	53.472	83
-0.01	-1.95	-2766.67	20.872	-12.328	54.072	84
-0.03	-1.8	-2800	21.077	-12.523	54.677	85
-0.01	-1.63	-2833.33	21.283	-12.717	55.283	86
-0.01	-1.49	-2866.67	21.49	-12.91	55.89	87
-0.01	-1.35	-2900	21.7	-13.1	56.5	88
-0.01	-1.22	-2933.33	21.911	-13.289	57.111	89
-0.01	-1.1	-2966.67	22.123	-13.477	57.723	90
-0.01	-0.99	-3000	22.337	-13.663	58.337	91
-0.01	-0.89	-3033.33	22.552	-13.848	58.952	92
-0.01	-0.79	-3066.67	22.769	-14.031	59.569	93
-0.01	-0.7	-3100	22.986	-14.214	60.186	94
-0.01	-0.61	-3133.33	23.206	-14.394	60.806	95
-0.01	-0.52	-3166.67	23.426	-14.574	61.426	96
-0.01	-0.45	-3200	23.647	-14.753	62.047	97
-0.01	-0.38	-3233.33	23.869	-14.931	62.669	98
-0.01	-0.31	-3266.67	24.093	-15.107	63.293	99

```
                     -0.02        -0.25     -3300             24.317   -15.283   63.917     100
END              OF         RESPONSE  OUTPUT   FILE
```

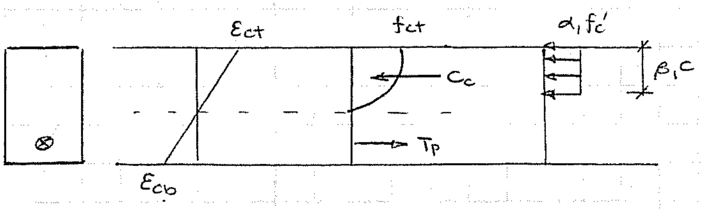

200
150
100
50
0
-50
-200
0
200
400
600
800
1000
1200
Moment, M, (ft-kip)
Curvature, ϕ, (rad/in.)x10^-6
≈ MΔ
"yielding"
f_p
ε_{pf}
"Ramberg - Osgood"
bottom cracking
straight beam
negative ϕ at M=0, camber exists here
Cracks @ top closing



Low P/S forces mean cracking, Δ, curvature concerns, but no strength concerns.

FLEXURAL MEMBERS

Rectangular stress block



$\varepsilon_{ct} \neq 0.003$

1. calculate the area under the parabola, needs to equal the area of the rectangle

$$\int_0^c \sigma_c \, b \, dy = \alpha_1 f_c' \beta_1 c b$$

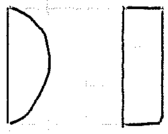
2. The centroid of the parabola matches the centroid of the rectangle

$$\bar{y} = \frac{\int_0^c \sigma_c \, b y \, dy}{\int_0^c \sigma_c \, b \, dy} = c - 0.5\beta_1 c$$

solve two equations simultaneously

$$\boxed{\begin{aligned} \beta_1 &= \frac{4 - \varepsilon_{ct}/\varepsilon_c'}{6 - 2\varepsilon_{ct}/\varepsilon_c'} \\ \alpha_1\beta_1 &= \frac{\varepsilon_{ct}}{\varepsilon_c'} - \frac{1}{3}\left(\frac{\varepsilon_{ct}}{\varepsilon_c'}\right)^2 \\ C_c &= \alpha_1 \beta_1 f_c' c b \end{aligned}}$$

consider $\varepsilon_{ct}/\varepsilon_c' = 2.0$ — full parabola



$\varepsilon_{ct}/\varepsilon_c' = 1.5 = .003/0.002$

- $\alpha_1\beta_1 = 0.750$ using these equations
- $\alpha_1\beta_1 = 0.725$ from ACI (0.85×0.85)

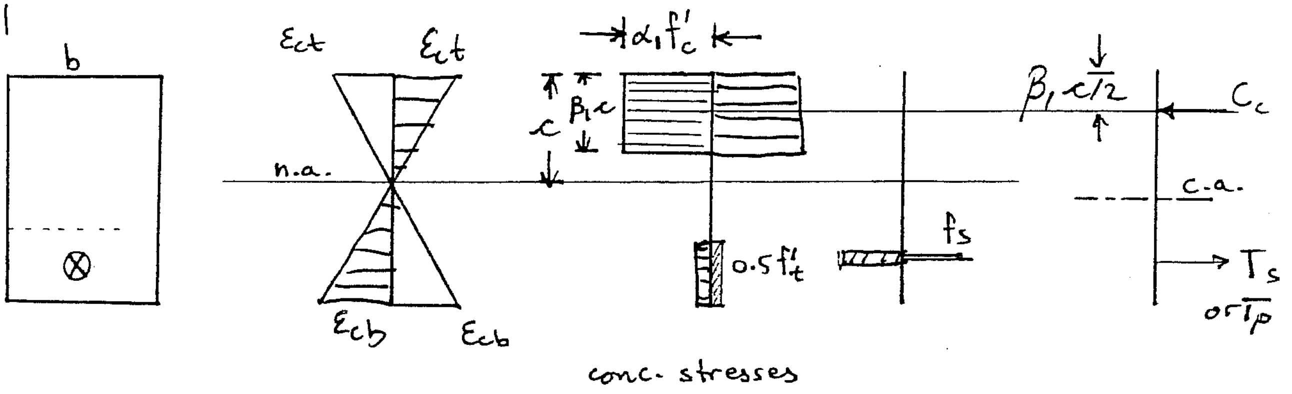
Not bad — conservative

ONLY APPLLES WHEN HOGNASTAD'S PARABOLA WORKS

$f_c' < 8000$ psi, generally

5.6 RECTANGULAR STRESS BLOCK APPROACH

- Layered section analysis can be time consuming, particularly by hand
- Stress-block more efficient for cross sections having essentially constant widths
- Recall



- nonlinear distribution of stresses replaced by rectangular stress-block
- for a parabolic stress-strain curve and a constant section width b,

$$\beta_1 = \frac{4 - (\varepsilon_{ct}/\varepsilon'_c)}{6 - 2(\varepsilon_{ct}/\varepsilon'_c)} \quad \text{- - - - - -}$$

$$\alpha_1 = \frac{1}{\beta_1}\left[\left(\frac{\varepsilon_{ct}}{\varepsilon'_c}\right) - \frac{1}{3}\left(\frac{\varepsilon_{ct}}{\varepsilon'_c}\right)^2\right] \quad \text{- - - -}$$

Then $C_c = \alpha_1 \beta_1 f'_c c b$

- Then take moments about centroidal axis
- Even though not exactly correction, also applied to cases where cross section width varies; typically conservative
- For tension stiffening, can consider a stress of

$$f_c = 0.5 f'_t$$

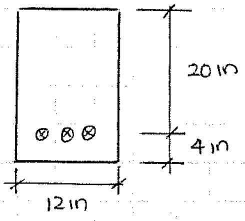
Full parabola

acting over part of embedment zone in tension.

$\varepsilon_{ct}/\varepsilon'_c$	0.25	0.5	0.75	1.0	1.25	1.50	1.75	2.0
α_1	0.336	0.595	0.779	0.888	0.928	0.900	0.810	0.67

FLEXURAL MEMBERS

Example problem



$f_c' = 5\text{ ksi}$

$\varepsilon_o = 2.25 \times 10^{-3}$

$f_t' = 530\text{ psi}$

$A_p = 0.459\text{ in}^2$

$\Delta\varepsilon_p = 6.0 \times 10^{-3}$

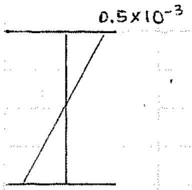
Ramberg-Osgood:

A = 0.025 B = 118 C = 10

tension stiffening: 7.5 d_b up and down from strands = 3.75 in — treat as 4 in below

Find M-ϕ point corresponding to $\varepsilon_{ct} = 0.5 \times 10^{-3}$

trial and error process involving guessing c until sectional equilibrium is achieved (N=0)



(ignore tension stiffening)

$$\varepsilon_{pf} = -\varepsilon_{ct}\frac{d-c}{c} + \Delta\varepsilon_p\text{, } \varepsilon_{ct} \text{ is negative}$$

great assumption

try $c = 7.38\text{ in}$, $\varepsilon_{pf} = 6.86 \times 10^{-3}$

$\varepsilon_{cb} = 1.12 \times 10^{-3}$ (cracked)

$$\phi = -\frac{\varepsilon_{ct}}{c} = \underline{67.8 \times 10^{-6}\text{ rad/in}}$$

$f_p = E_p \varepsilon_{pf} = 198.8\text{ ksi}$ — linear-elastic, but approaching yield

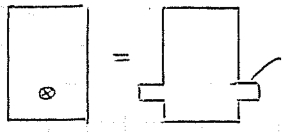
$T_p = A_p f_p = 91.0\text{ kip}$

$C_c = \alpha_1\beta_1 f_c' cb = (0.303)(0.680)(-5\text{ ksi})(7.38\text{ in})(12\text{ in}) = -91.1\text{ k}$

hey look, guess of c was good! fancy that.

calculate the moment

take contributions about the centroidal axis



$A = (0.459\text{ in}^2)(n-1) = 2.84\text{ in}^2$

$n = \frac{E_s}{E_c} = 7.2$

$\bar{y} = 12.07\text{ in}$ — not very different from $\bar{y} = 12\text{ in}$

[for lightly reinforced sections, C.G. of transformed section is very close to C.G. of gross section]

$$M = C_c\left[\beta_1 c/2 - 12\text{ in}\right] + T_p\left[d - 12\text{ in}\right] = 1593\text{ k-in}$$

$$= \underline{132.8\text{ k-ft}}$$

FLEXURAL MEMBERS

comparing example to Response

	▭ Stress Block	Response
ε_{ct}	-0.5×10^{-3}	-0.508×10^{-3}
ε_{cb}	1.12×10^{-3}	1.092×10^{-3}
ϕ	67.8×10^{-6} rad/in	66.67×10^{-6} rad/in
M	132.8 k·ft	133.35 k·ft

wow, that's pretty close ☺

in this case, remember: tension stiffening was ignored

include tension stiffening

$$T_c = 0.5 f_t' A_t = 0.5\,(530\text{ psi})(7.5\text{ in})(2\text{ in}) = 23.85\text{ k}$$

↑ effective embedment zone

$$M = C_c\left(\frac{\beta_1 c}{2} - 12\text{ in}\right) + T_p\,(20\text{ in} - 12\text{ in}) + T_c(20\text{ in} - 12\text{ in})$$

$$= 1889\text{ k·in} = 157\text{ k·ft}$$

$$\phi = \frac{-\varepsilon_{ct}}{c} = \frac{+0.5 \times 10^{-3}}{9.04\text{ in}} = 55.31 \times 10^{-6}\text{ rad/in}$$

$$\varepsilon_{cb} = -\varepsilon_{ct}\left(\frac{h-c}{c}\right) = 0.827 \times 10^{-3}\text{ in/in}$$

compare to response solution

	Hand calcs	Response
ε_{ct}	-0.5×10^{-3}	-0.5×10^{-3}
ε_{cb}	0.827×10^{-3}	0.751×10^{-3}
ϕ	55.3×10^{-6} rad/in	52.0×10^{-6} rad/in
M	157.4 k·ft	157.8 k·ft

again, hand calcs are very similar to Response output

10 October 07

With Tension Stiffening

-Coarse Analysis-

```
RESPONSE        OUTPUT    FILE
ANALYSIS:       Axial-Load
SECTION         NAME      :           New-Section
CONCRETE        MODEL:    Parabolic   Material     Factors:    C:1.000    S:1.000  P:1.000
TENSION-STIFF.: Yes       FACTORED:   No           ACCURACY:   well_done
Axial-Load:     0.00kip   Moment:     0.00kft      Shear:      0.00kip
                dN/dM:             0  dN/dV:                0  dM/dV:           0
                Axial-Load Moment     Curvature    @-Axis      Bottom     Top      Iter.
                kips       ft*kip     rad/10^6in   [-------Milli-Strain------]
                  0.02      52.55           0       -0.062     -0.062    -0.062      1
                  0.09     154.77       33.33       -0.001      0.399    -0.401      2
                 -0.01     161.29       66.67        0.235      1.035    -0.565      3
                     0     169.53         100        0.496      1.696    -0.704      4
                     0     177.05      133.33        0.768      2.368    -0.832      5
                 -0.23     183.37      166.67         1.05       3.05     -0.95      6
                 -0.17     187.88         200        1.342      3.742    -1.058      7
                 -0.13     190.93      233.33        1.642      4.442    -1.158      8
                  -0.1     192.89      266.67        1.949      5.149    -1.251      9
                 -0.07     194.12         300        2.262      5.862    -1.338     10
                 -0.05     194.91      333.33        2.579      6.579    -1.421     11
                 -0.04     195.43      366.67        2.899      7.299    -1.501     12
                 -0.03     195.81         400        3.221      8.021    -1.579     13
                 -0.02      196.1      433.33        3.545      8.745    -1.655     14
                 -0.01     196.34      466.67        3.871      9.471    -1.729     15
                 -0.01     196.54         500        4.198     10.198    -1.802     16
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                 -0.19     197.07      566.67        4.853     11.653    -1.947     18
                 -0.07     197.14         600        5.184     12.384    -2.016     19
                  0.03     197.23      633.33        5.514     13.114    -2.086     20
                  0.13     197.31      666.67        5.845     13.845    -2.155     21
                  0.24     197.38         700        6.176     14.576    -2.224     22
                     0     197.74      733.33        6.503     15.303    -2.297     23
                     0     197.89      766.67        6.832     16.032    -2.368     24
                     0     198.05         800        7.162     16.762    -2.438     25
                 -0.01      198.2      833.33         7.49      17.49     -2.51     26
                 -0.01     198.35      866.67        7.818     18.218    -2.582     27
                 -0.01     198.49         900        8.145     18.945    -2.655     28
                 -0.02     198.63      933.33        8.471     19.671    -2.729     29
                 -0.02     198.76      966.67        8.795     20.395    -2.805     30
                 -0.03     198.88        1000        9.118     21.118    -2.882     31
                 -0.03        199     1033.33        9.439     21.839    -2.961     32
                 -0.04      199.1     1066.67        9.758     22.558    -3.042     33
                 -0.04     199.19        1100       10.074     23.274    -3.126     34
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                 -0.06     199.31     1166.67       10.697     24.697    -3.303     36
                 -0.07     199.33        1200       11.001     25.401    -3.399     37
                 -0.08     199.32     1233.33       11.299     26.099    -3.501     38
                 -0.09     199.25     1266.67       11.588     26.788    -3.612     39
                 -0.11     199.11        1300       11.866     27.466    -3.734     40
                 -0.12     198.84     1333.33       12.126     28.126    -3.874     41
                 -0.11     198.33     1366.67       12.353     28.753    -4.047     42
                  -0.2     196.96        1400       12.453     29.253    -4.347     43
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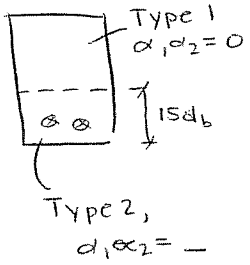
different from last analysis

use two types of concrete



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0	-26.23	-666.67	6.124	-1.876	14.124	21
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-0.07	-26.95	-766.67	7.123	-2.077	16.323	24
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0.14	-27.36	-866.67	8.123	-2.277	18.523	27
0.19	-27.48	-900	8.455	-2.345	19.255	28
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0	-27.94	-966.67	9.114	-2.486	20.714	30
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-0.02	-28.6	-1300	12.347	-3.253	27.947	40
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0	-21.75	-1666.67	14.952	-5.048	34.952	51
0.23	-19.95	-1700	15.052	-5.348	35.452	52

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      0      -8.91   -2066.67     16.527    -8.273   41.327      63
      0      -8.33      -2100     16.694    -8.506   41.894      64
      0      -7.79   -2133.33     16.865    -8.735   42.465      65
      0      -7.29   -2166.67     17.039    -8.961   43.039      66
      0      -6.82      -2200     17.216    -9.184   43.616      67
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      0      -5.26   -2333.33      17.95    -10.05    45.95      71
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  -0.01      -4.64      -2400     18.331   -10.469   47.131      73
      0      -4.35   -2433.33     18.524   -10.676   47.724      74
      0      -4.08   -2466.67      18.72    -10.88    48.32      75
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   0.02      -3.16      -2600     19.522   -11.678   50.722      79
  -0.01      -2.99   -2633.33     19.724   -11.876   51.324      80
      0       -2.8   -2666.67      19.93    -12.07    51.93      81
      0      -2.63      -2700     20.137   -12.263   52.537      82
      0      -2.47   -2733.33     20.345   -12.455   53.145      83
      0      -2.33   -2766.67     20.555   -12.645   53.755      84
      0      -2.19      -2800     20.766   -12.834   54.366      85
      0      -2.05   -2833.33     20.979   -13.021   54.979      86
      0      -1.93   -2866.67     21.193   -13.207   55.593      87
      0      -1.81      -2900     21.407   -13.393   56.207      88
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  -0.03      -1.62   -2966.67     21.838   -13.762   57.438      90
  -0.03      -1.52      -3000     22.056   -13.944   58.056      91
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   0.02      -1.24      -3100     22.721   -14.479   59.921      94
   0.02      -1.16   -3133.33     22.943   -14.657   60.543      95
   0.02      -1.09   -3166.67     23.166   -14.834   61.166      96
  -0.01      -1.04      -3200      23.39    -15.01    61.79      97
  -0.04      -0.99   -3233.33     23.612   -15.188   62.412      98
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END          OF        RESPONSE  OUTPUT     FILE
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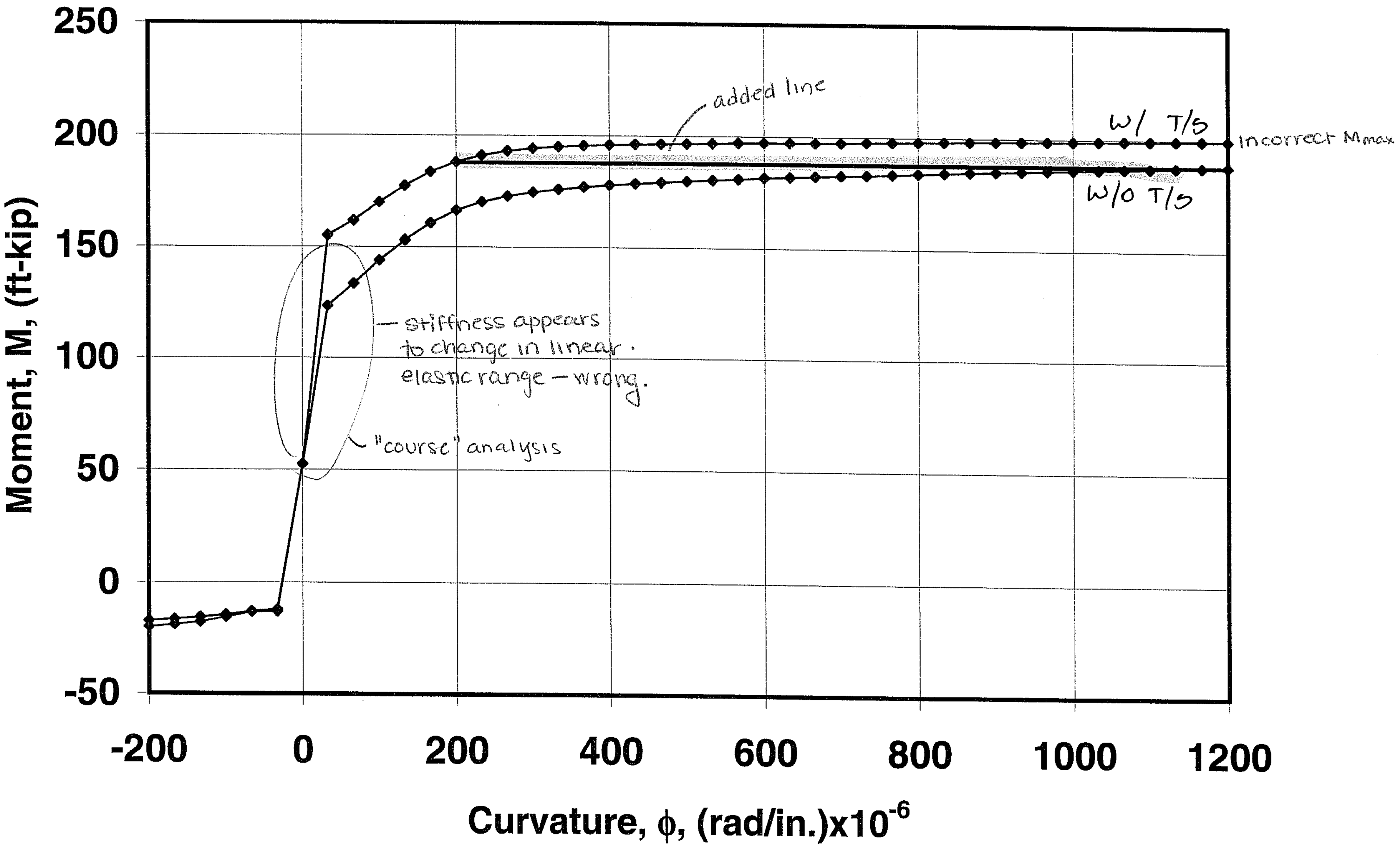
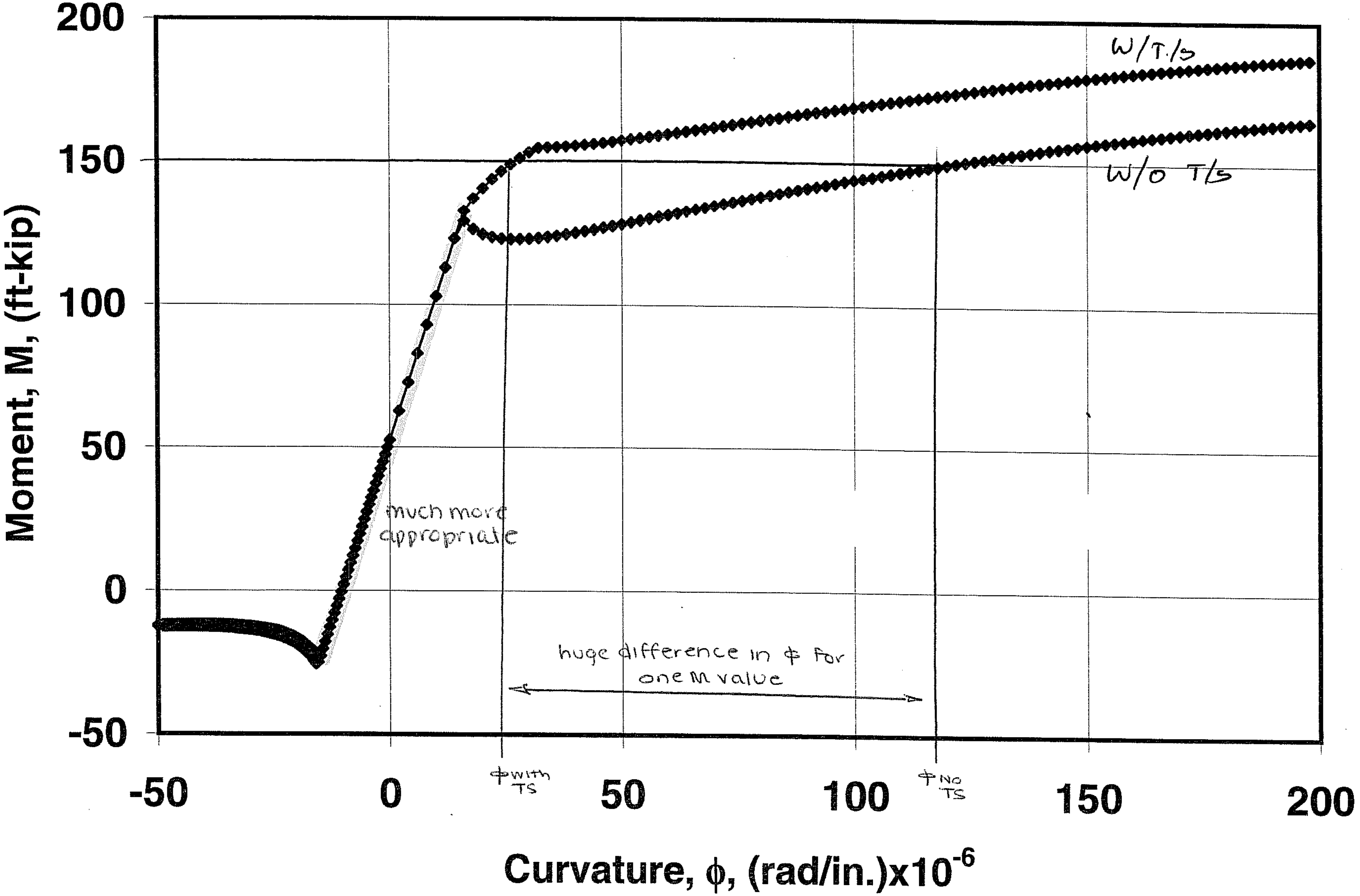

added line
w/ T/S
Incorrect Mmax
w/o T/S
— stiffness appears to change in linear-elastic range — wrong.
"course" analysis
Moment, M, (ft-kip)
250
200
150
100
50
0
-50
-200
0
200
400
600
800
1000
1200
Curvature, ϕ, (rad/in.)x10-6



"View Plot Finer" to get finer mesh

WITH TENSION STIFFENING

- FINE ANALYSIS -

adds points between
linear-elastic
range

```
RESPONSE        OUTPUT    FILE
ANALYSIS:       Axial-Load
SECTION         NAME      :            New-Section
CONCRETE        MODEL:    Parabolic    Material     Factors:    C:1.000   S:1.000  P:1.000
TENSION-STIFF.: No        FACTORED:    No           ACCURACY:   well_done
Axial-Load:     0.00kip   Moment:      0.00kft      Shear:      0.00kip
                dN/dM:              0  dN/dV:                 0 dM/dV:             0
                Axial-Load Moment      Curvature    @-Axis      Bottom    Top      Iter.
                kips      ft*kip       rad/10^6in   [-------Milli-Strain------]
                     0.02       52.55            0      -0.062    -0.062   -0.062      1
                     0.01       62.64            2      -0.063    -0.039   -0.087      2
                     0.01       72.72            4      -0.063    -0.015   -0.111      3
                     0.01       82.81            6      -0.063     0.009   -0.135      4
                    -0.05       92.87            8      -0.064     0.032    -0.16      5
                    -0.15      102.89           10      -0.064     0.056   -0.184      6
                    -0.25      112.87           12      -0.065     0.079   -0.209      7
                     0.23      122.82           14      -0.065     0.103   -0.233      8
                        0      128.84           16      -0.063     0.129   -0.255      9
                     0.03      126.02           18      -0.052     0.164   -0.268     10
                     0.03      124.31           20       -0.04       0.2    -0.28     11
                     0.02       123.3           22      -0.028     0.236   -0.292     12
                     0.02      122.76           24      -0.015     0.273   -0.303     13
                     0.02      122.55           26      -0.003     0.309   -0.315     14
                     0.02      122.55           28        0.01     0.346   -0.326     15
                     0.02      122.73           30       0.024     0.384   -0.336     16
                     0.02      123.02           32       0.037     0.421   -0.347     17
                     0.01       123.4           34       0.051     0.459   -0.357     18
                     0.01      123.84           36       0.064     0.496   -0.368     19
                     0.01      124.34           38       0.078     0.534   -0.378     20
                     0.01      124.88           40       0.093     0.573   -0.387     21
                     0.01      125.45           42       0.107     0.611   -0.397     22
                     0.01      126.04           44       0.121     0.649   -0.407     23
                     0.01      126.65           46       0.136     0.688   -0.416     24
                     0.01      127.27           48        0.15     0.726   -0.426     25
                     0.01       127.9           50       0.165     0.765   -0.435     26
                     0.01      128.54           52        0.18     0.804   -0.444     27
                     0.01      129.19           54       0.195     0.843   -0.453     28
                     0.01      129.84           56        0.21     0.882   -0.462     29
                     0.01       130.5           58       0.225     0.921   -0.471     30
                     0.01      131.15           60        0.24      0.96    -0.48     31
                     0.01      131.81           62       0.256         1   -0.488     32
                     0.01      132.46           64       0.271     1.039   -0.497     33
                     0.01      133.12           66       0.287     1.079   -0.505     34
                     0.01      133.77           68       0.302     1.118   -0.514     35
                        0      134.43           70       0.318     1.158   -0.522     36
                        0      135.08           72       0.333     1.197   -0.531     37
                        0      135.72           74       0.349     1.237   -0.539     38
                        0      136.37           76       0.365     1.277   -0.547     39
                        0      137.01           78        0.38     1.316   -0.556     40
                        0      137.65           80       0.396     1.356   -0.564     41
                        0      138.28           82       0.412     1.396   -0.572     42
                        0      138.92           84       0.428     1.436    -0.58     43
                        0      139.54           86       0.444     1.476   -0.588     44
                        0      140.17           88        0.46     1.516   -0.596     45
                        0      140.79           90       0.476     1.556   -0.604     46
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0	141.4	92	0.492	1.596	-0.612	47
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0	142.62	96	0.525	1.677	-0.627	49
0	143.22	98	0.541	1.717	-0.635	50
0	143.81	100	0.557	1.757	-0.643	51
0.27	144.22	102	0.575	1.799	-0.649	52
0.25	144.82	104	0.591	1.839	-0.657	53
0.25	145.4	106	0.607	1.879	-0.665	54
0.24	145.98	108	0.624	1.92	-0.672	55
0.24	146.55	110	0.64	1.96	-0.68	56
0.23	147.12	112	0.657	2.001	-0.687	57
0.23	147.68	114	0.673	2.041	-0.695	58
0.22	148.23	116	0.69	2.082	-0.702	59
0.22	148.78	118	0.706	2.122	-0.71	60
0.22	149.33	120	0.723	2.163	-0.717	61
0.21	149.86	122	0.74	2.204	-0.724	62
0.21	150.39	124	0.756	2.244	-0.732	63
0.2	150.92	126	0.773	2.285	-0.739	64
0.2	151.44	128	0.79	2.326	-0.746	65
0.2	151.95	130	0.807	2.367	-0.753	66
0.19	152.45	132	0.824	2.408	-0.76	67
0.19	152.95	134	0.841	2.449	-0.767	68
0.18	153.44	136	0.858	2.49	-0.774	69
0.18	153.93	138	0.875	2.531	-0.781	70
0.18	154.41	140	0.892	2.572	-0.788	71
0.18	154.88	142	0.909	2.613	-0.795	72
0.17	155.35	144	0.926	2.654	-0.802	73
0.17	155.8	146	0.943	2.695	-0.809	74
0.17	156.26	148	0.96	2.736	-0.816	75
0.17	156.7	150	0.977	2.777	-0.823	76
0.16	157.14	152	0.994	2.818	-0.83	77
0.16	157.57	154	1.011	2.859	-0.837	78
0.16	157.99	156	1.029	2.901	-0.843	79
0.16	158.41	158	1.046	2.942	-0.85	80
0.16	158.82	160	1.063	2.983	-0.857	81
0.15	159.22	162	1.081	3.025	-0.863	82
0.15	159.62	164	1.098	3.066	-0.87	83
0.15	160.01	166	1.116	3.108	-0.876	84
0.14	160.4	168	1.133	3.149	-0.883	85
0.14	160.77	170	1.15	3.19	-0.89	86
0.14	161.14	172	1.168	3.232	-0.896	87
0.14	161.51	174	1.186	3.274	-0.902	88
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0.13	163.22	184	1.274	3.482	-0.934	93
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0.12	165.06	196	1.381	3.733	-0.971	99
0.12	165.35	198	1.399	3.775	-0.977	100
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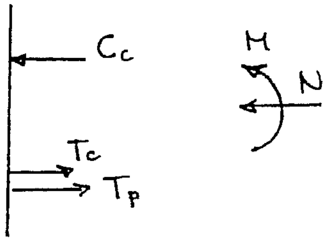
-0.07	19.74	-6.5	-0.062	-0.14	0.016	14
-0.04	17.22	-7	-0.062	-0.146	0.022	15
0	14.7	-7.5	-0.062	-0.152	0.028	16
0.05	12.19	-8	-0.062	-0.158	0.034	17
0.1	9.67	-8.5	-0.062	-0.164	0.04	18
0.16	7.16	-9	-0.062	-0.17	0.046	19
0.22	4.65	-9.5	-0.062	-0.176	0.052	20
-0.07	2.15	-10	-0.063	-0.183	0.057	21
-0.01	-0.36	-10.5	-0.063	-0.189	0.063	22
0.06	-2.86	-11	-0.063	-0.195	0.069	23
0.14	-5.36	-11.5	-0.063	-0.201	0.075	24
0.22	-7.86	-12	-0.063	-0.207	0.081	25
-0.16	-10.35	-12.5	-0.063	-0.213	0.087	26
-0.07	-12.84	-13	-0.063	-0.219	0.093	27
0.02	-15.33	-13.5	-0.063	-0.225	0.099	28
0.11	-17.82	-14	-0.063	-0.231	0.105	29
0.21	-20.31	-14.5	-0.063	-0.237	0.111	30
-0.26	-22.78	-15	-0.064	-0.244	0.116	31
0	-23.21	-15.5	-0.062	-0.248	0.124	32
0	-22.15	-16	-0.059	-0.251	0.133	33
0	-21.2	-16.5	-0.056	-0.254	0.142	34
0	-20.34	-17	-0.053	-0.257	0.151	35
0	-19.57	-17.5	-0.05	-0.26	0.16	36
0	-18.87	-18	-0.047	-0.263	0.169	37
0	-18.24	-18.5	-0.044	-0.266	0.178	38
0	-17.67	-19	-0.04	-0.268	0.188	39
0	-17.15	-19.5	-0.037	-0.271	0.197	40
0	-16.68	-20	-0.034	-0.274	0.206	41
0	-16.25	-20.5	-0.031	-0.277	0.215	42
0	-15.86	-21	-0.028	-0.28	0.224	43
0	-15.5	-21.5	-0.024	-0.282	0.234	44
0	-15.18	-22	-0.021	-0.285	0.243	45
0	-14.88	-22.5	-0.018	-0.288	0.252	46
0	-14.61	-23	-0.014	-0.29	0.262	47
0	-14.36	-23.5	-0.011	-0.293	0.271	48
0	-14.13	-24	-0.008	-0.296	0.28	49
0	-13.92	-24.5	-0.004	-0.298	0.29	50
0	-13.73	-25	-0.001	-0.301	0.299	51
0	-13.56	-25.5	0.003	-0.303	0.309	52
0	-13.4	-26	0.006	-0.306	0.318	53
0	-13.25	-26.5	0.01	-0.308	0.328	54
0	-13.12	-27	0.013	-0.311	0.337	55
0	-13	-27.5	0.017	-0.313	0.347	56

```
                 0      -12.88        -28     0.02   -0.316   0.356      57
                 0      -12.78      -28.5    0.024   -0.318   0.366      58
                 0      -12.69        -29    0.027   -0.321   0.375      59
                 0      -12.61      -29.5    0.031   -0.323   0.385      60
                 0      -12.53        -30    0.035   -0.325   0.395      61
                 0      -12.46      -30.5    0.038   -0.328   0.404      62
                 0       -12.4        -31    0.042    -0.33   0.414      63
                 0      -12.34      -31.5    0.045   -0.333   0.423      64
                 0      -12.29        -32    0.049   -0.335   0.433      65
                 0      -12.24      -32.5    0.053   -0.337   0.443      66
                 0       -12.2        -33    0.057   -0.339   0.453      67
                 0      -12.16      -33.5     0.06   -0.342   0.462      68
              0.28      -11.96        -34    0.065   -0.343   0.473      69
              0.24      -11.96      -34.5    0.068   -0.346   0.482      70
              0.24      -11.93        -35    0.072   -0.348   0.492      71
              0.23      -11.91      -35.5    0.076    -0.35   0.502      72
              0.23      -11.89        -36     0.08   -0.352   0.512      73
              0.23      -11.88      -36.5    0.083   -0.355   0.521      74
              0.23      -11.87        -37    0.087   -0.357   0.531      75
              0.22      -11.86      -37.5    0.091   -0.359   0.541      76
              0.22      -11.85        -38    0.095   -0.361   0.551      77
              0.22      -11.85      -38.5    0.099   -0.363   0.561      78
              0.22      -11.85        -39    0.102   -0.366    0.57      79
              0.21      -11.85      -39.5    0.106   -0.368    0.58      80
              0.21      -11.85        -40     0.11    -0.37    0.59      81
              0.21      -11.85      -40.5    0.114   -0.372     0.6      82
              0.21      -11.86        -41    0.118   -0.374    0.61      83
               0.2      -11.86      -41.5    0.122   -0.376    0.62      84
               0.2      -11.87        -42    0.126   -0.378    0.63      85
               0.2      -11.88      -42.5     0.13    -0.38    0.64      86
               0.2      -11.89        -43    0.134   -0.382    0.65      87
               0.2       -11.9      -43.5    0.138   -0.384    0.66      88
              0.19      -11.91        -44    0.142   -0.386    0.67      89
              0.19      -11.93      -44.5    0.146   -0.388    0.68      90
              0.19      -11.94        -45    0.149   -0.391   0.689      91
              0.19      -11.96      -45.5    0.153   -0.393   0.699      92
              0.19      -11.97        -46    0.157   -0.395   0.709      93
              0.18      -11.99      -46.5    0.161   -0.397   0.719      94
              0.18      -12.01        -47    0.165   -0.399   0.729      95
              0.18      -12.02      -47.5     0.17     -0.4    0.74      96
              0.18      -12.04        -48    0.174   -0.402    0.75      97
              0.18      -12.06      -48.5    0.178   -0.404    0.76      98
              0.18      -12.08        -49    0.182   -0.406    0.77      99
              0.17      -12.12        -50     0.19    -0.41    0.79       1
END        OF     RESPONSE  OUTPUT   FILE
```

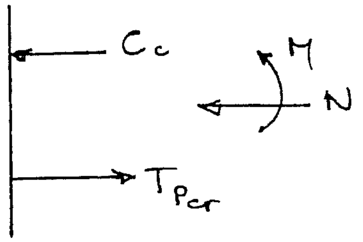
10 october 07

5.8 MOMENT - CURVATURE AT A CRACK

- Consider what is happening in cross section

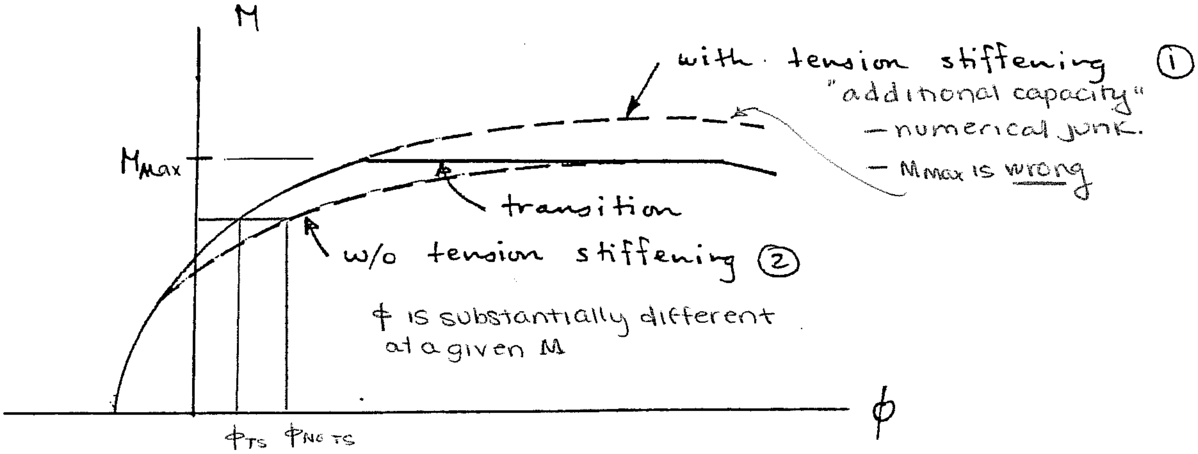


Avg Conditions
deformation calculations



At Crack NO T's at crack
capacity calculations

- At crack, local concrete tensile stresses → 0
- Must ensure that $\Sigma (T_c + T_p) \not> T_{p\,yield}$
- Can take into account by generating second M-ϕ diagram, but ignoring contribution from tensile stresses in concrete



∴ · M_{Max} dictated by curve generated assuming no tension stiffening

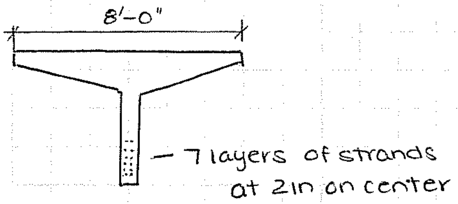
- ① gives average conditions; ② gives maximum local conditions

- program SMAL generates only one curve; at each M-ϕ point, check is made that stresses can be transmitted across crack, or f_c^+ are reduced.

FLEXURAL MEMBERS

Moment · Curvature response

consider 8ST36



$A_p = 2.14 \text{ in}^2$
$A_c = 567.2 \text{ in}^2$

$f_{cr} = 300$ psi
$f_c' = 5$ KSI
$f_{pu} = 270$ KSI

strain rupture occurs when
$\varepsilon_p = 0.040$ in/in
long term value typically equals 0.054 in/in

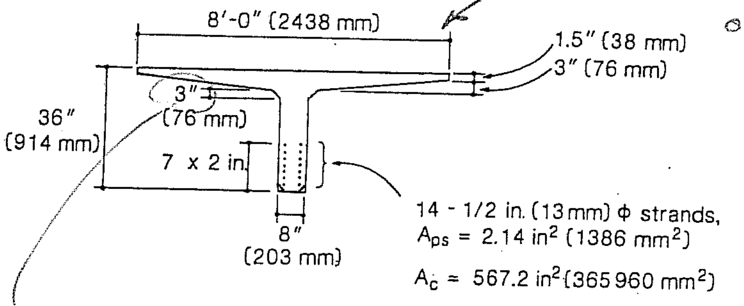
Long term:
- use E_{peff}, ε'_{ceff}, E_{ceff}, ε'_{teff}
- shrinkage in concrete
  $\varepsilon_{cf} = \varepsilon_c - \varepsilon_{sh}$
- change strand rupture value

5.9 DETERMINING COMPLETE MOMENT-CURVATURE RESPONSE

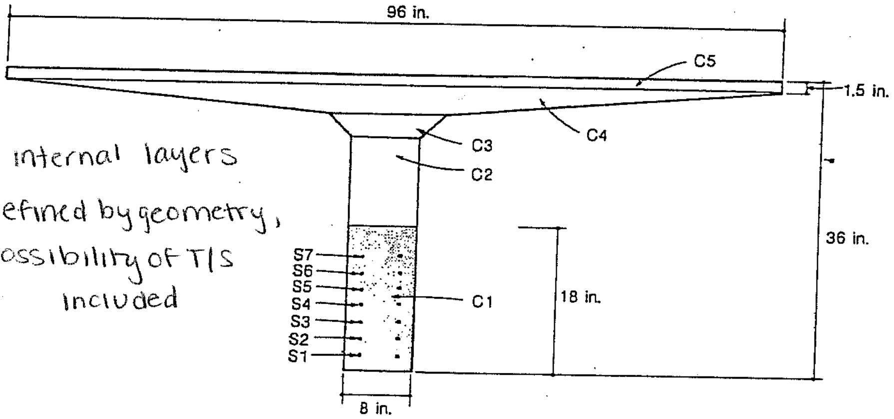
- Consider precast, pretensioned tee-beam shown
- Note that low-relaxation strands were tensioned to a stress of 200 ksi ($\Delta \varepsilon_p = \frac{200}{29000} = 0.0069$)
- Analysis done using Response

design is "to be topped" — gets a slab on top of beam, as opposed to those that have built-in tops

8ST36

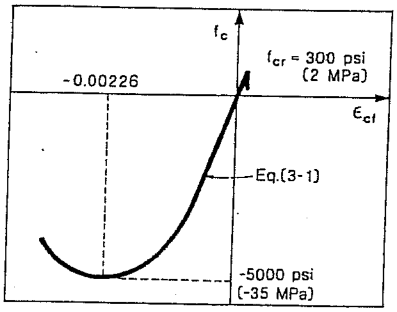


always avoid snap corners - use chamfers

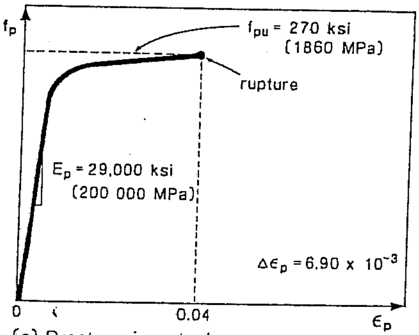


internal layers defined by geometry, possibility of T/S included

(a) Discretization of cross section



(b) Concrete short-term



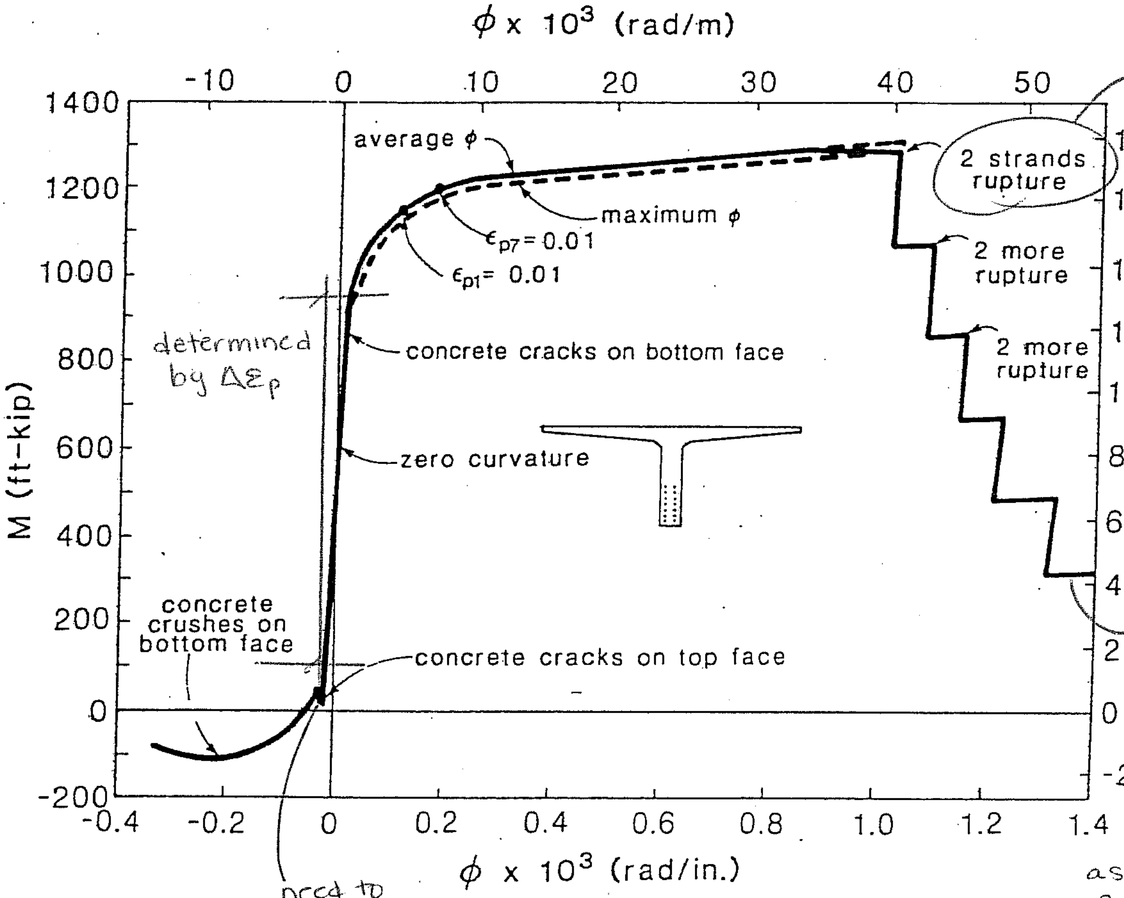
(c) Prestressing steel

Average Conditions

ϵ_t $\times 10^3$	ϵ_b $\times 10^3$	$\phi \times 10^3$		M		Comments
		rad/in.	rad/m	ft-kips	kNm	
7.51	−4.00	−0.320	−12.58	− 89	−121	Bottom concrete crushing
4.84	−2.70	−0.210	− 8.27	−109	−148	Maximum negative moment
0.52	−1.10	−0.044	− 1.73	0	0	Zero moment
0.20	−0.88	−0.030	− 1.18	47	63	
0.077	−0.85	−0.026	− 1.01	22	30	Top concrete cracking
−0.19	−0.19	0	0	620	841	Zero curvature
−0.30	0.077	0.010	0.41	866	1174	Bottom concrete cracking
−0.50	1.60	0.058	2.30	1083	1469	
−0.75	5.28	0.168	6.60	1198	1624	
−1.00	10.33	0.315	12.39	1228	1665	
−1.25	16.30	0.488	19.20	1249	1694	
−1.50	22.87	0.677	26.65	1269	1721	
−1.95	35.15	1.031	40.57	1305	1769	
−1.76	35.16	1.026	40.38	1079	1463	Bottom strands ruptured

Conditions At Crack

ϵ_t $\times 10^3$	ϵ_b $\times 10^3$	$\phi \times 10^3$		M		Comments
		rad/in.	rad/m	ft-kips	kNm	
−0.50	1.76	0.063	2.47	1059	1436	
−0.75	5.50	0.174	6.84	1174	1591	
−1.00	10.59	0.322	12.67	1208	1637	
−1.25	16.61	0.496	19.53	1231	1670	
−1.50	23.22	0.687	27.03	1254	1700	
−1.93	35.15	1.030	40.58	1290	1750	Maximum moment
−1.75	35.16	1.025	40.36	1065	1444	2 strands ruptured
−1.83	37.45	1.091	42.95	1071	1451	
−1.62	37.45	1.085	42.73	858	1163	4 strands ruptured
−1.69	40.05	1.159	45.65	863	1170	
−1.48	40.03	1.153	45.40	663	899	6 strands ruptured
−1.55	42.99	1.237	48.70	667	904	
−1.31	42.94	1.229	48.40	479	650	8 strands ruptured
−1.37	46.34	1.325	52.18	483	654	



rupture as a layer, not as a strand

determined by $\Delta\varepsilon_p$

strand rupture controls this capacity (vs. concrete crushing) desireable response; same as R/C, where concre failure is bad, yield of bars is good.

need to reduce e; harp or debond strands

Note: strands are assumed to rupture at a strain of 0.040

Note the following observations

i-) Concrete remains uncracked ~~uncracked~~ for $22 \leq M \leq 866$ kip-ft

Fairly Large Range; can use linear, elastic, uncracked behavior assumptions

ii-) Cracking moments signal change in slopes of $M-\phi$ response

iii-) Because of eccentric prestress, top face will crack if beam is not loaded; if simply supported and if tendons are straight, will crack at ends; reason why tendons are draped, and at C.G. at ends.

iv-) Rounded nature of $M-\phi$ due in part to rounded $f_p : \varepsilon_p$ relation (modelled using Ramberg-Osgood formulation,

v-) Because of large width of the top flange, failure of the beam occurs by rupturing of the strand nearest bottom rather than by crushing of concrete

5.6 LONG-TERM MOMENT-CURVATURE RESPONSE

As before, except include following modifications.

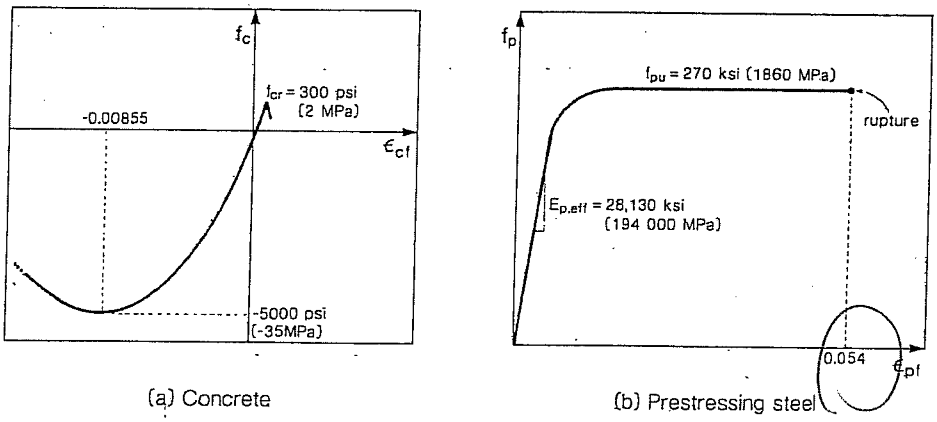
$$E_{c,eff} = \frac{E_{ci}}{1+\phi}$$

$E_p \rightarrow E_{p,eff}$ allowing for relaxation in strands.

$$\varepsilon'_{c,eff} = \frac{2f_c'}{E_{c,eff}}$$

$$\varepsilon'_{t,eff} = \frac{f_{cr}}{E_{c,eff}}$$

- account for shrinkage in concrete : $\varepsilon_{cf} = \varepsilon_c - \varepsilon_{sh}$
- also rupture strain in strands changes ($\rightarrow$ 0.054)



(a) Concrete

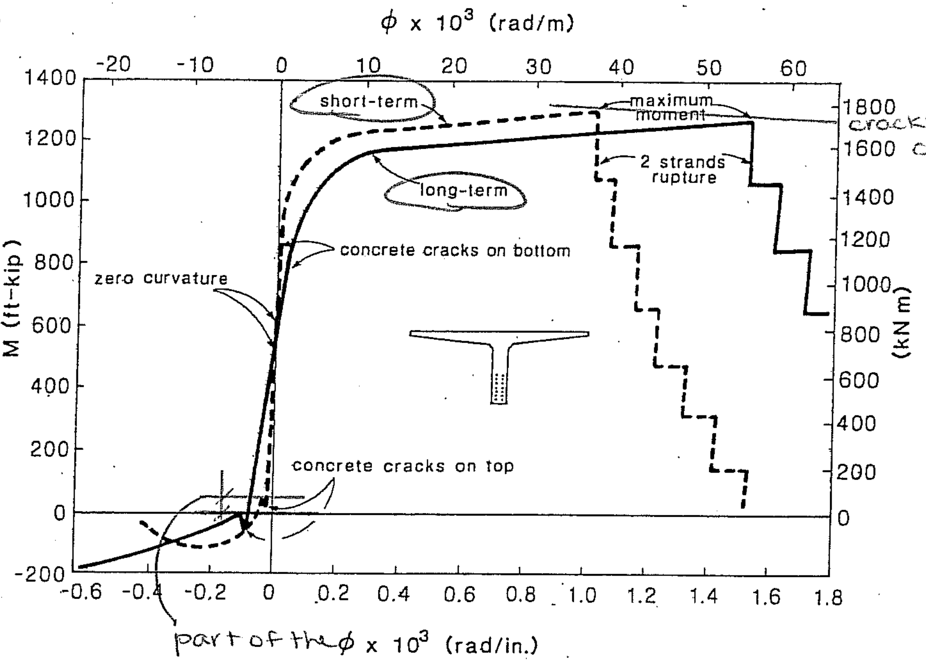
(b) Prestressing steel

1. Average conditions:

ϵ_t	ϵ_b	$\phi \times 10^3$		M		
$\times 10^3$	$\times 10^3$	rad/in.	rad/m	ft-kips	kNm	Comments
16.39	−8.00	−0.678	−26.67	−142	−193	
12.35	−7.00	−0.538	−21.16	−126	−171	
5.05	−5.00	−0.279	−10.99	− 77	−105	
0.32	−3.25	−0.099	− 3.90	12	16	
−0.15	−3.22	−0.085	− 3.36	− 45	− 61	Top concrete cracking
−0.23	−3.05	−0.078	− 3.09	0	0	Zero moment
−1.15	−1.15	0	0	522	708	Zero curvature
−1.64	−0.15	0.041	1.62	797	1080	Bottom concrete cracking
−2.00	1.10	0.086	3.39	946	1283	
−2.50	4.18	0.186	7.31	1104	1497	
−3.00	10.58	0.377	14.85	1160	1573	
−4.00	27.73	0.881	34.70	1218	1651	
−5.00	47.12	1.448	57.00	1276	1730	

At crack:

ϵ_t	ϵ_b	$\phi \times 10^3$		M		
$\times 10^3$	$\times 10^3$	rad/in.	rad/m	ft-kips	kNm	Comments
−2.00	1.24	0.090	3.54	937	1270	
−3.00	10.98	0.388	15.29	1147	1556	
−4.00	28.11	0.892	35.11	1208	1639	
−5.00	47.53	1.459	57.45	1269	1721	
−5.13	50.16	1.536	60.46	1276	1730	Maximum moment
−4.71	50.16	1.524	60.01	1052	1426	2 strands ruptured
−4.86	53.58	1.624	63.92	1060	1437	
−4.42	53.55	1.610	63.40	848	1150	4 strands ruptured
−4.57	57.42	1.722	67.80	855	1159	
−4.09	57.35	1.707	67.19	656	889	6 strands ruptured



cracking moment changes, max moment is the same

part of the PS force is lost to creep and shrinkage

Note following:

i.) LT response uncracked over wide range of moments

ii.) uncracked LT response has lower flexural stiffness

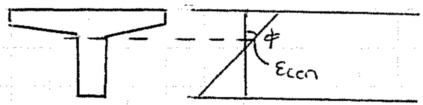
iii.) L.T. cracking moment ≈ 10% lower than ST M_{cr}

iv.) M_u essentially ~~unchanged~~ unchanged

v.) ϕ_u is considerably higher under LT response

FLEXURAL MEMBERS

Elastic uncracked response



$$\varepsilon_c = \varepsilon_{cen} - \phi y$$
$$\varepsilon_s = \varepsilon_{cen} - \phi y$$
$$\varepsilon_p = \varepsilon_{cen} - \phi y + \Delta\varepsilon_p$$

approximate using linear elastic uncracked response

$$N_o = \int_{A_p} E_p \Delta\varepsilon_p \, dA_p \underbrace{- \int_{A_c} E_c \varepsilon_{co} \, dA_c - \int_{A_s} E_s \varepsilon_{so} \, dA_s - \int_{A_p} E_p \varepsilon_{po} \, dA_p}_{\text{long-term effects, not included in short-term analysis}}$$

application of N_o makes $\varepsilon_{cen} = 0$

M_o makes $\phi = 0$

$$M_o = - \int_{A_p} E_p \Delta\varepsilon_p \, y \, dA_p + \underbrace{\int_{A_s} E_s \varepsilon_{so} y \, dA_s + \int_{A_c} E_c \varepsilon_{co} y \, dA_c + \int_{A_p} E_p \varepsilon_{po} y \, dA_p}_{\text{long-term}}$$

$$A_{trans} = A_c + \frac{E_s}{E_c} A_s + \frac{E_p}{E_c} \cdot A_p$$

using transformed properties:

$$\varepsilon_{cen} = \frac{N - N_o}{E_c \cdot A_{trans}}$$

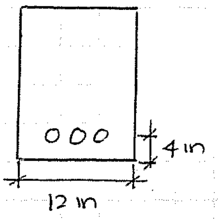
then find f, ε for materials

check limits – no yield!

$$\phi = \frac{M - M_o}{E_c \cdot I_{trans}}$$

FLEXURAL MEMBERS

Example



$f_c' = 5$ ksi

$\varepsilon_o = 2.25 \times 10^{-3}$

$f_t' = 530$ psi

$\Delta\varepsilon_p = 6.0 \times 10^{-3}$

$A_p = 0.459$ in^2

$d = 20$ in

Find f, ε for $N = 0, M = 53$ k·ft

$$E_c = E_{cs} = 0.9 \frac{2 f_c'}{\varepsilon_o} = 4000 \text{ ksi}$$

secant modulus increases the range over which solution is applicable

$$n_i = \frac{E_P}{E_{cs}} = 7.25$$

$$A_{trans} = (12 \text{ in})(24 \text{ in}) + (7.25 - 1)(0.459 \text{ in}^2) = 290.9 \text{ in}^2$$

$$\bar{y}_{trans} = \frac{(12 \text{ in})(24 \text{ in})(12 \text{ in}) + 6.25\,(0.459 \text{ in}^2)(20 \text{ in})}{290.9 \text{ in}^2} = 12.08 \text{ in, or about}$$

$y_c = 12$ in

$$I_{trans} = \overset{*}{1/12}\,(12 \text{ in})(24 \text{ in})^3 + (12 \text{ in})(24 \text{ in})(0.08 \text{ in})^2 + 6.25\,(0.459 \text{ in}^2)(\overset{7.92}{\cancel{8.08}} \text{ in})^2$$

$$= 14005 \text{ in}^4 \quad \text{vs. } 13824 \text{ in}^4 \text{ gross section}$$

Decompression forces

$$N_o = \int_{A_p} E_p \Delta\varepsilon_p \, dA_p \quad , \; \varepsilon_{co} = \varepsilon_{so} = \varepsilon_{po} = 0 \text{, short-term calcs}$$

$$= (29000 \text{ ksi})(6.0 \times 10^{-3})(0.459 \text{ in}^2) = 79.7 \text{ k}$$

$$M_o = -E_p \Delta\varepsilon_p y A_p = -79.7 \text{ k}\,(12.08 \text{ in} - 20 \text{ in}) = 631 \text{ k·in}$$

Calculate ε_{cen}, ϕ:

$$\varepsilon_{cen} = \frac{-N_c}{E_c A_{trans}} = \frac{-79.7 \text{ k}}{(4000 \text{ ksi})(290.9 \text{ in}^2)} = -0.068 \times 10^{-3} \text{ comp.}$$

$$\phi = \frac{53 \text{ k·ft} - 52.6 \text{ k·ft}}{(4000 \text{ ksi})(14005 \text{ in}^4)} = 0.09 \times 10^{-6} \text{ rad/in} \sim 0$$

matches well with Response output

check linear elastic:

$$f_{tc} = f_{cb} = 0.068 \times 10^{-3}\,(4000 \text{ ksi}) = 272 \text{ psi}$$

no cracking, woo!

$$f_p = 29000 \text{ ksi}\,(-0.068 \times 10^{-3} + 6.0 \times 10^{-3}) = 172 \text{ ksi} < f_{yp} \quad ✓$$

good assumption, good answers

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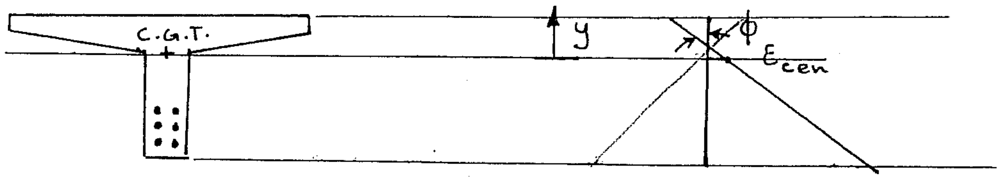
5.11 ELASTIC UNCRACKED RESPONSE

- prestressed members remain uncracked over a large range of moments
- in this range, can get reasonably accurate predictions of response assuming <u>linear elastic uncracked</u> response

COMPATIBILITY

- use centroidal axis of transformed cross section



as before

$$\left.\begin{aligned} \varepsilon_c &= \varepsilon_{cen} - \phi \cdot y \\ \varepsilon_s &= \varepsilon_{cen} - \phi \cdot y \\ \varepsilon_p &= \varepsilon_{cen} - \phi \cdot y + \Delta \varepsilon_p \end{aligned}\right\} \text{plus shrinkage and thermal effects}$$

STRESS - STRAIN RELATIONSHIPS

let

$$\left.\begin{aligned} \varepsilon_{co} &= \varepsilon_{sh} + \varepsilon_{cth} \\ \varepsilon_{so} &= \varepsilon_{sth} \\ \varepsilon_{po} &= \varepsilon_{pth} \end{aligned}\right\} \text{initial offsets in stress-strain relations}$$

$$\therefore \quad \begin{aligned} \varepsilon_{cf} &= \varepsilon_c - \varepsilon_{co} &&= \varepsilon_{cen} - \phi \cdot y - \varepsilon_{co} \\ \varepsilon_{sf} &= \varepsilon_s - \varepsilon_{so} &&= \varepsilon_{cen} - \phi \cdot y - \varepsilon_{so} \\ \varepsilon_{pf} &= \varepsilon_p - \varepsilon_{po} &&= \varepsilon_{cen} - \phi \cdot y + \Delta \varepsilon_p - \varepsilon_{po} \end{aligned}$$

thus, stresses are

strain due to stress

$$\begin{aligned} f_c &= E_c \cdot \varepsilon_{cf} &&= E_c (\ \ldots \ ) \\ f_s &= E_s \cdot \varepsilon_{sf} &&= E_s (\ \ldots \ ) \\ f_p &= E_p \cdot \varepsilon_{pf} &&= E_p (\ \ldots \ ) \end{aligned}$$

EQUILIBRIUM

as before

$$N = \int_{A_c} f_c \, dA_c + \int_{A_s} f_s \, dA_s + \int_{A_p} f_p \, dA_p$$

$$M = \int_{A_c} -f_c \cdot y \, dA_c + \int_{A_s} -f_s \cdot y \, dA_s + \int_{A_p} -f_p \, y \, dA_p$$

now, can make appropriate substitutions $\Rightarrow$ big mess

To simplify somewhat, define

- decompression forces:

$$N_o = \int_{A_p} E_p \cdot \Delta \varepsilon_p \, dA_p - \int_{A_c} E_c \cdot \varepsilon_{co} \, dA_c - \int E_s \varepsilon_{so} \, dA_s$$

$$- \int E_p \varepsilon_{po} \, dA_p$$

$$M_o = - \int_{A_p} E_p \cdot \Delta \varepsilon_p \, y \, dA_p + \int_{A_c} E_c \varepsilon_{co} \, y \, dA_c$$

$$+ \int_{A_s} E_s \varepsilon_{so} \, y \, dA_s + \int_{A_p} E_p \varepsilon_{po} \, y \, dA_p$$

note: these are forces that, when applied, give $\varepsilon_{cen} = 0$, $\phi = 0$

- transformed section properties:

$$A_{trans} = A_c + \frac{E_s}{E_c} A_s + \frac{E_p}{E_c} A_p$$

I_{trans}: · transform all rebar + p/s area into equivalent concrete areas ie. $n_i A_{s_i}$ where $n_i = E_{si} / E_c$

· find <u>new</u> centroidal axis, $\bar{y}_t$

· $I_{trans} = I_g + A_g (\bar{y}_t - \bar{y}_g)^2 + \sum I_{s_i} \cdot n_i$
$+ \sum n_i A_{s_i} (\bar{y}_t - y_i)^2$

note A_{s_i}, I_{s_i} are transformed area

- given these, then:

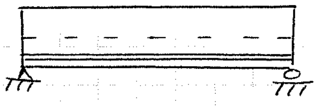
$$\varepsilon_{cen} = \frac{N - N_o}{E_c \cdot A_{trans}}$$

$$\phi = \frac{M - M_o}{E_c \cdot I_{trans}}$$

- then find strains / stresses
- make sure no limiting conditions violated

FLEXURAL MEMBERS

Camber and deflections



How to calculate:

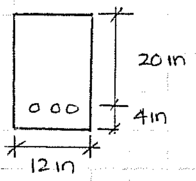
1. Generate $M \cdot \phi$ diagram
2. Generate moment diagram along the length of the member
3. make ϕ diagram along the length of the member
4. Integrate ϕ to get slopes, $\theta = \int \phi\, dx$
5. Integrate θ to get deflections, $\delta = \int \theta\, dx$ remember boundaries!

A beam with draped strands ∴ just sucks. Needs $M \cdot \phi$ calculations at each point

Camber: if SW=0, M=0 along length, ϕ is constant

$$\Delta = \phi_c \cdot \frac{L}{2} \cdot \frac{L}{4} = \frac{\phi_o L^2}{8}$$

Example



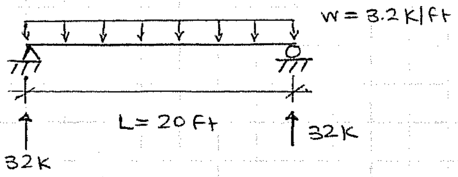
$f_c' = 5$ ksi

$f_t = 530$ psi

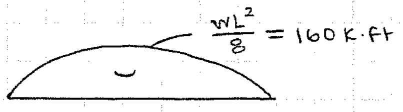
$\Delta\varepsilon_p = 6.0 \times 10^{-3}$

$\varepsilon_o = 2.25 \times 10^{-3}$

$A_p = 0.459\,in^2$



What is midspan δ?



$M(x) = 32x - 3.2x^2/2$

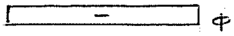
consider 10 segments on one half of the beam

See attached spreadsheet

Camber: make the unreasonable assumption that dead load/self-weight = 0

in truth, $w_D = 0.3$ k/lf

$\phi(M=0) =$ -10.43×10^{-6}



$$\Delta = \phi \frac{L^2}{8} = \frac{1}{8}(-10.43 \times 10^{-6})(120\,in)^2 = -0.08\,in$$

or 0.10 in upwards

add in self-weight in same manner as distributed load above

Goal: DL (SW + deck) causes a straight bridge

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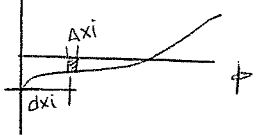
42a 1.1

W/ T.S. better calculation for deflections

From Response results



Average/segment

x, [ft]	M, [ft-kips]	φ, [*10-6 rad/in.]	φ, [*10-6 rad/in.]	(i)	ϕ_i, [*10-6 rad/in.]	Δxi, [in.]	dxi, [in.]	(ϕ_i)(Δxi)(dxi)
0	0	-10.43						
1	30.4	-4.39	-7.41	1	-7.41	12	6	-0.001
2	57.6	1	-1.695	2	-1.695	12	18	0.000
3	81.6	5.76	3.38	3	3.38	12	30	0.001
4	102.4	9.9	7.83	4	7.83	12	42	0.004
5	120	13.43	11.665	5	11.665	12	54	0.008
6	134.4	16.84	15.135	6	15.135	12	66	0.012
7	145.6	23.4	20.12	7	20.12	12	78	0.019
8	153.6	30.55	26.975	8	26.975	12	90	0.029
9	158.4	54.69	42.62	9	42.62	12	102	0.052
10	160	61.39	58.04	10	58.04	12	114	0.079
				Total				0.203

$M(x) = 32x - 3.2x^2/2$

δmid = 0.2 in.

w/o T.S.

Average/segment

x, [ft]	M, [ft-kips]	φ, [*10-6 rad/in.]	φ, [*10-6 rad/in.]	(i)	ϕ_i, [*10-6 rad/in.]	Δxi, [in.]	dxi, [in.]	(ϕ_i)(Δxi)(dxi)
0	0	-10.43						
1	30.4	-4.39	-7.41	1	-7.41	12	6	-0.001
2	57.6	1	-1.695	2	-1.695	12	18	0.000
3	81.6	5.76	3.38	3	3.38	12	30	0.001
4	102.4	9.9	7.83	4	7.83	12	42	0.004
5	120	13.43	11.665	5	11.665	12	54	0.008
6	134.4	69.61	41.52	6	41.52	12	66	0.033
7	145.6	106.08	87.845	7	87.845	12	78	0.082
8	153.6	136.1	121.09	8	121.09	12	90	0.131
9	158.4	157.39	146.745	9	146.745	12	102	0.180
10	160	165.37	161.38	10	161.38	12	114	0.221
				Total				0.658

δmid = 0.66 in. very different. This one no good.

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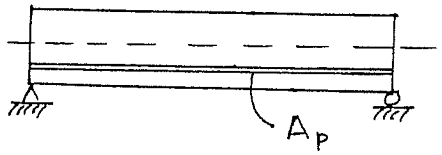
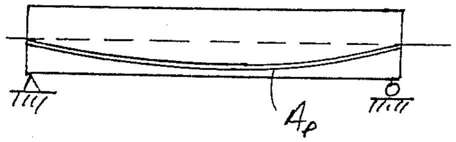
5.12 CAMBER AND DEFLECTIONS

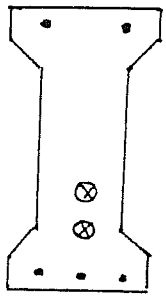


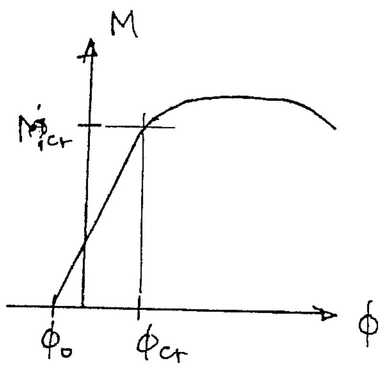
- due to eccentric / draped prestressing tendons, prestressed concrete beams typically are curved upward
- resulting deflection is called 'camber'



- superimposed loads will cause downward deflections of the member
- both camber } deflections } must be checked
- How to Find?

i) Given cross section, generate moment-curvature relation.

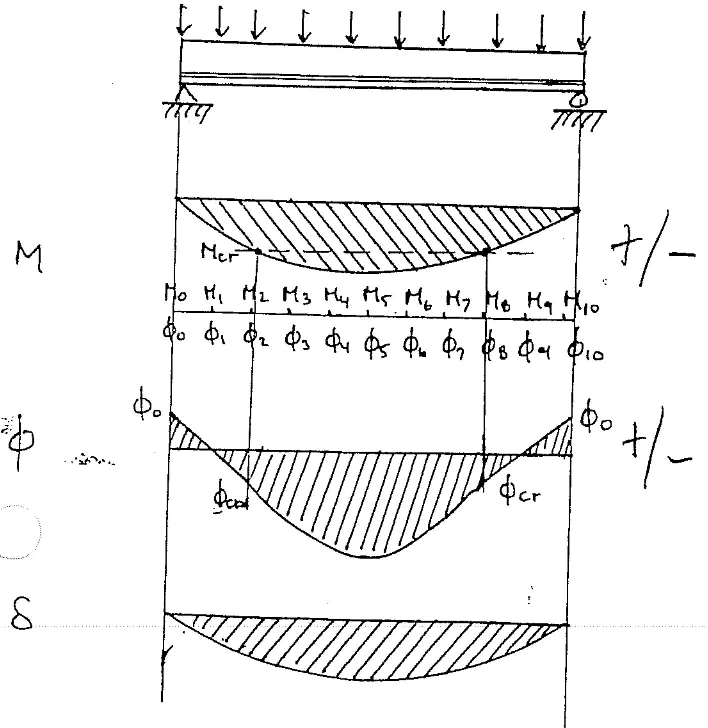




Problem:

If cross section changes along length (ie as with draped tendon), then $M-\phi$ response changes.

ii) Given loading etc, generate M-diag along length of member



iii) From $M-\phi$ diagram(s), or using SMAL/RESPONSE, find corresponding ϕ-diag along length of member

iv) Integrate ϕ to get slopes

$$\theta = \int \phi \, dx$$

v) Integrate ~~θ~~ θ to get deflections

$$\delta = \int \theta \, dx$$

5.13 CRACK WIDTHS AND CRACK SPACING

- fully prestressed members shouldn't crack

partially prestressed do crack initially

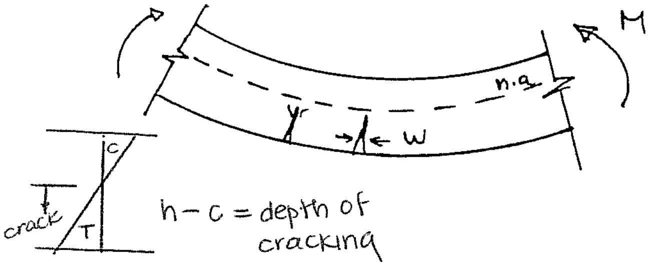
- for partially prestressed members, it is necessary to check that crack widths are not unacceptably wide
- ignore the small elastic tensile strains in concrete
- at any level y

cracks are wider at the bottom of the beam

$$\boxed{w = \varepsilon_{cf} \cdot S_m}$$

where ε_{cf} = avg strain due to stress at level y

run analysis using T/S to get average strain values

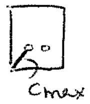


h − c = depth of cracking

- again, spacing S_m is influenced by many parameters
- again, we'll keep it simple ; $\boxed{S_m = 3\,C_{max}}$

C_{max} = distance from strands to most heavily strained concrete

- if really concerned with crack widths, should use more realistic formulation



5.14 MEMBERS WITH UNBONDED TENDONS

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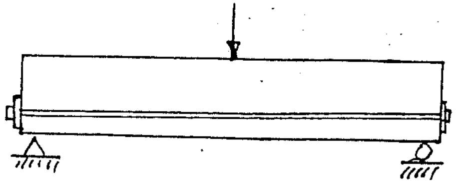
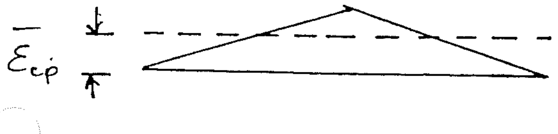
- so far, dealt with members with bonded tendons ; change in P/s strain equal to change in surrounding concrete

ie. $\varepsilon_p = \varepsilon_c + \Delta\varepsilon_p$

- not so with unbonded tendons
-



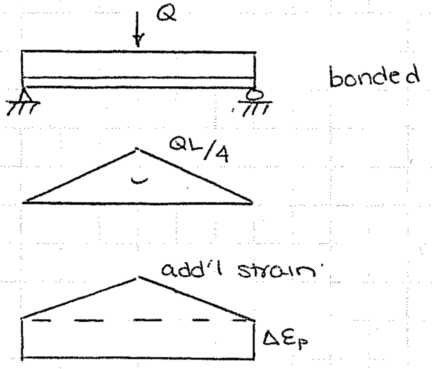


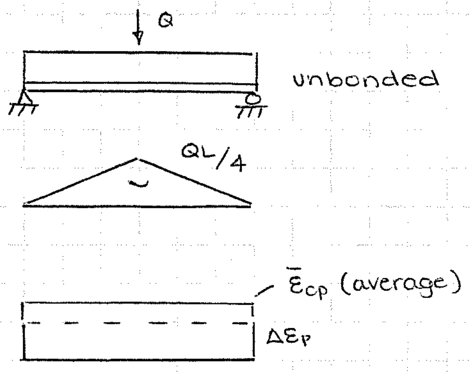
ε_{cp} : strain in concrete <u>at</u> level of P/s reinforcement ; varies along length of beam

$\bar{\varepsilon}_{cp}$: average value of ε_{cp}

BEAM DESIGN

Unbonded tendons





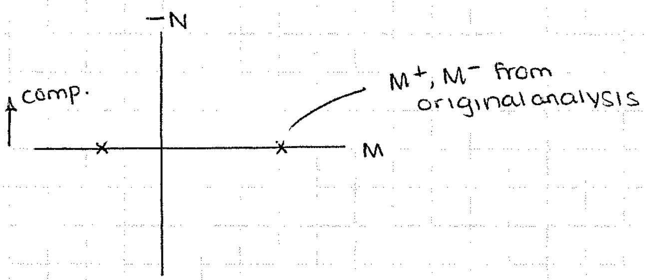
$\bar{\varepsilon}_{cp} = \frac{1}{2}\varepsilon_{max}$ (beyond $\Delta\varepsilon_p$)

Axial load plus flexure

$N \neq 0$ — not the P/S force!

in Response

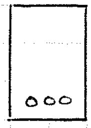
axial load = 0



now, change N to another value (−100k)

NOT I/S! capacity calcs

Example



same as previous

ratio of balance to maximum ~ 3

max M = 1400 k·ft, balance M = 500 k·ft

by hand:

$N = -500k$, $M = 393.75\,k\cdot ft$

$\varepsilon_t = -3.125\times10^{-3}$

$\varepsilon_b = 2.475\times10^{-3}$

$\varepsilon_c' = -2.25\times10^{-3}$

$\varepsilon_t/\varepsilon_c' = 1.389$, $\beta_1 = 0.81$

$\alpha_1 = 0.92$

$$c = -\varepsilon_t \cdot \frac{24\,in}{-\varepsilon_t + \varepsilon_b} = 13.4\,in$$

$$\varepsilon_p = \frac{-\varepsilon_t}{c}\cdot 20\,in + \varepsilon_t + \Delta\varepsilon_p = 7.54\times10^{-3}$$

α_1, rather (0.91)

$C_c = 0.85 f_c' \beta_1 c b = 598\,K$

$f_p = 219$ ksi — bordering on non-linear behavior

$F_p = 100\,k$

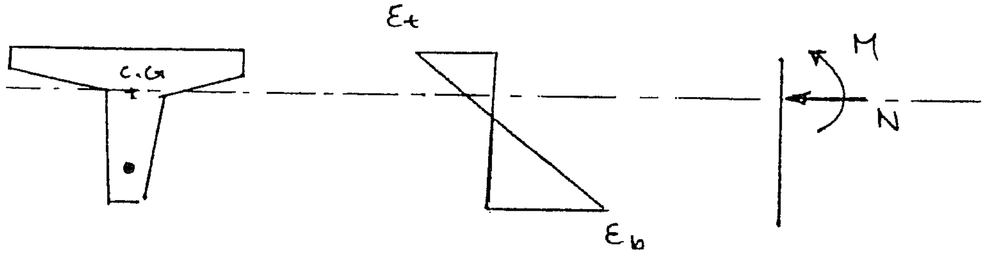
$N = F_p - C_c = -498.5\,K$ — axial load acting on the section

$M_{midheight} = 395\,k\cdot ft = c(h/2 - \beta_1 c/2) + T(d - h/2)$, T, C both (+)

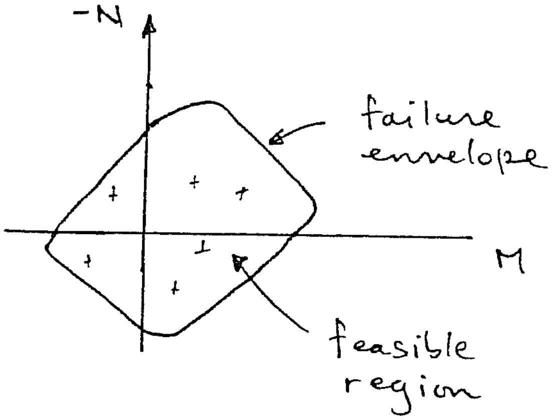
Response = ✓ good.

5.15 MEMBERS SUBJECTED TO COMBINED AXIAL LOAD & FLEXURE

- so far, have dealt with N=0
- more general case of N ≠ 0 can be treated using the same procedures
- again, assume "plane sections remain plane"; strain distribution defined by two variables



- for a particular set of $\{\varepsilon_t, \varepsilon_b\}$, will get a set $\{N, M\}$
- repeating for many different combos of ε_b & ε_t will get a "feasible range"



typically, most concerned with quadrant where
N is −ve
M is +ve
but can consider all combos

the boundary of this feasible region is called "failure envelope" or "interaction diagram"

FAILURE ENVELOPE

- easiest to set N to fixed value, and find highest and lowest values of M that can be resisted
- because capacity is controlled by conditions at crack, can ignore tension stiffening
- however, for load deformation responses, should include tension stiffening

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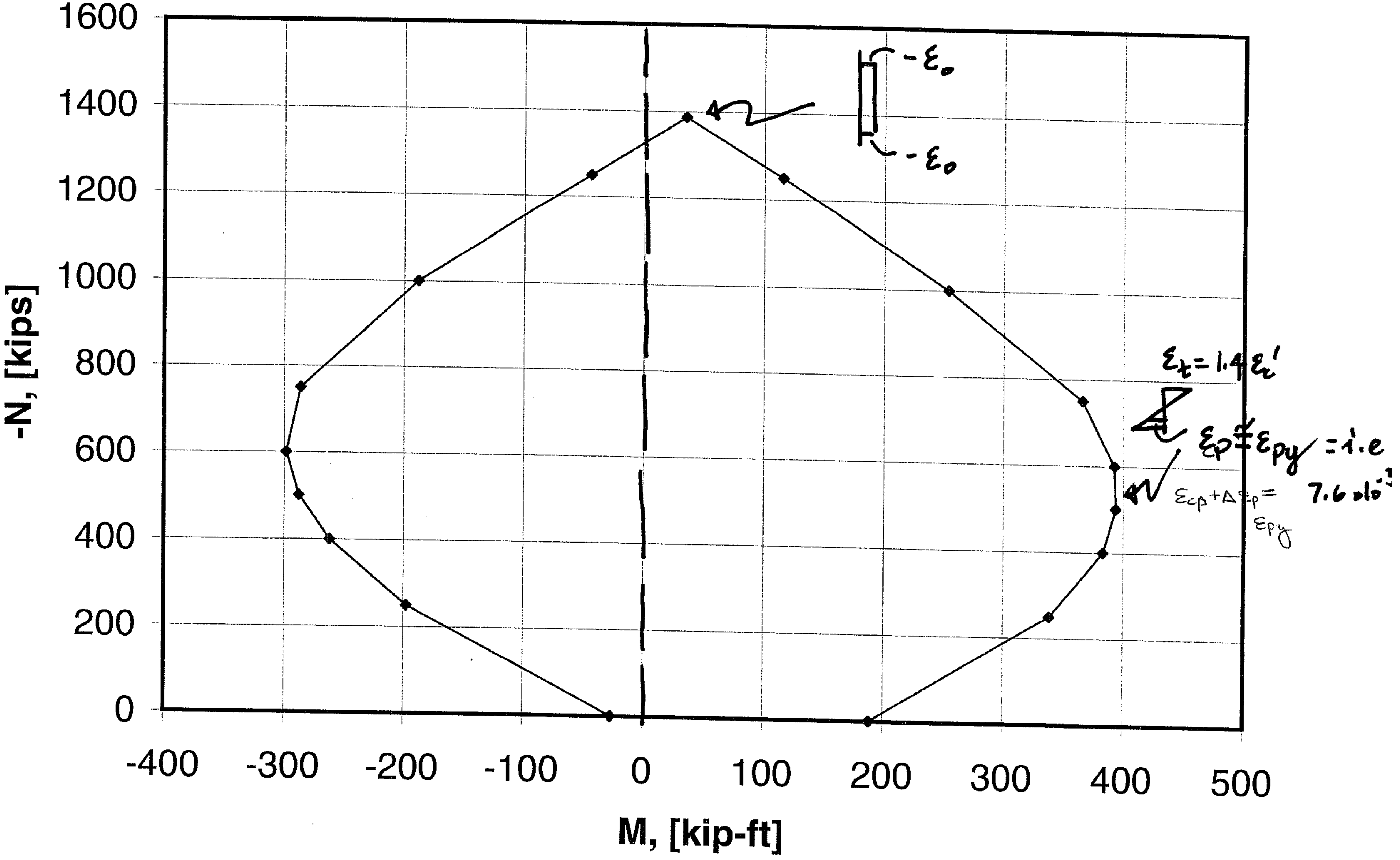
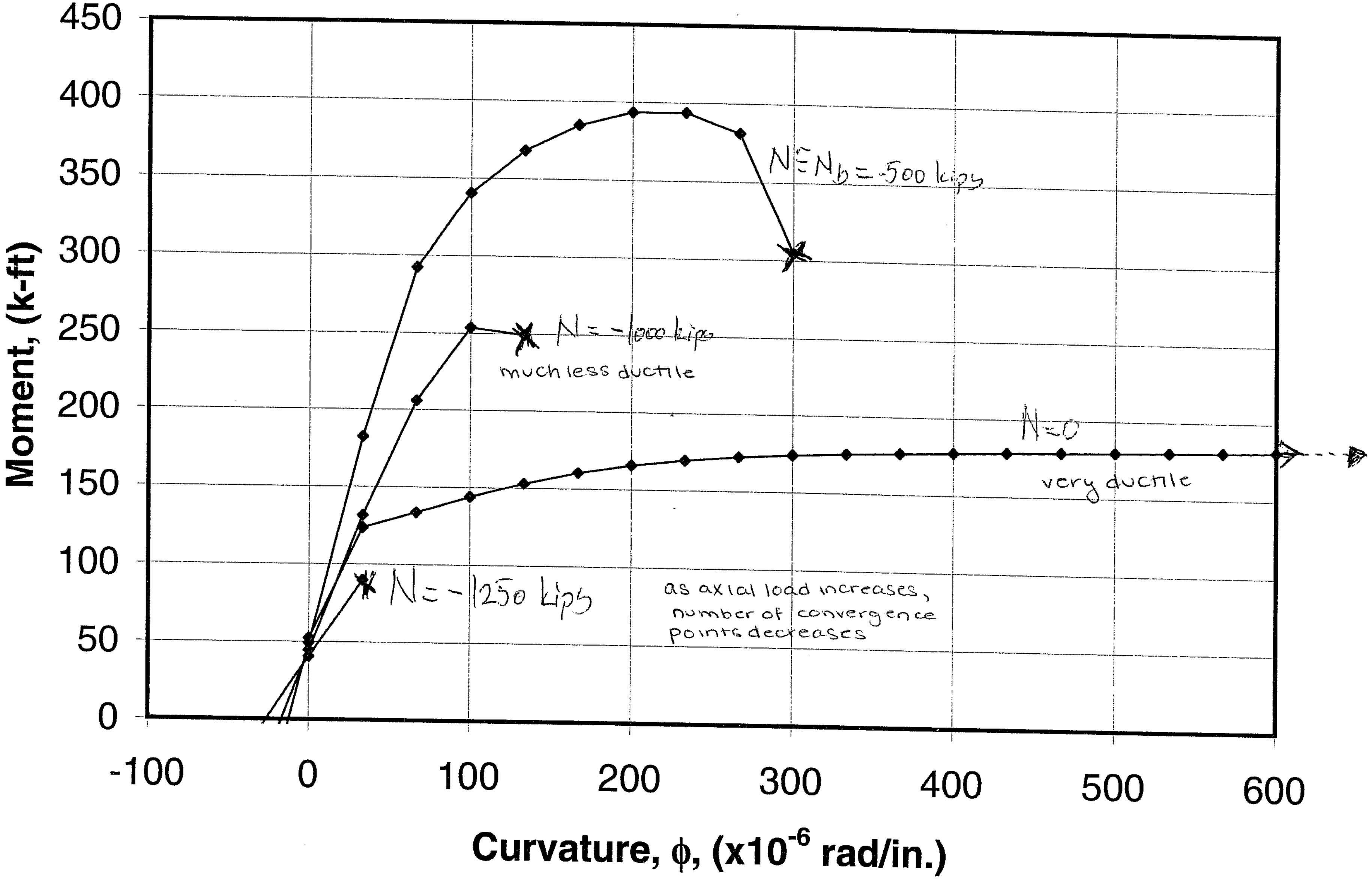
higher than ACI's strain limit of 0.003

N	Positive Flexure				Negative Flexure			
N	M	ϕ	ε_b	ε_t	M	ϕ	ε_b	ε_t
kips	kip-ft	$x10^{-6}$ rad/in.	$x10^{-3}$	$x10^{-3}$	kip-ft	$x10^{-6}$ rad/in	$x10^{-3}$	$x10^{-3}$
0	188.22	1366.67	29.118	-3.682	-26.54	-1400	-3.264	30.336
-250	338.58	366.67	5.804	-2.996	-197.6	-400	-2.871	6.729
-400	383.2	266.67	3.371	-3.029	-262.3	-266.67	-2.806	3.594
-500	393.75	233.33	2.475	-3.125	-287.88	-233.33	-2.938	2.662
-600	392.39	200	1.701	-3.099	-298.51	-200	-2.958	1.842
-750	365.84	166.67	0.889	-3.111	-286.44	-166.67	-3.01	0.99
-1000	253.6	100	-0.151	-2.551	-188.31	-133.33	-3.125	0.075
-1250	115.51	66.67	-0.975	-2.575	-44.38	-66.67	-2.529	-0.929
-1388	34.91	0	-2.238	-2.238				

balance pointish

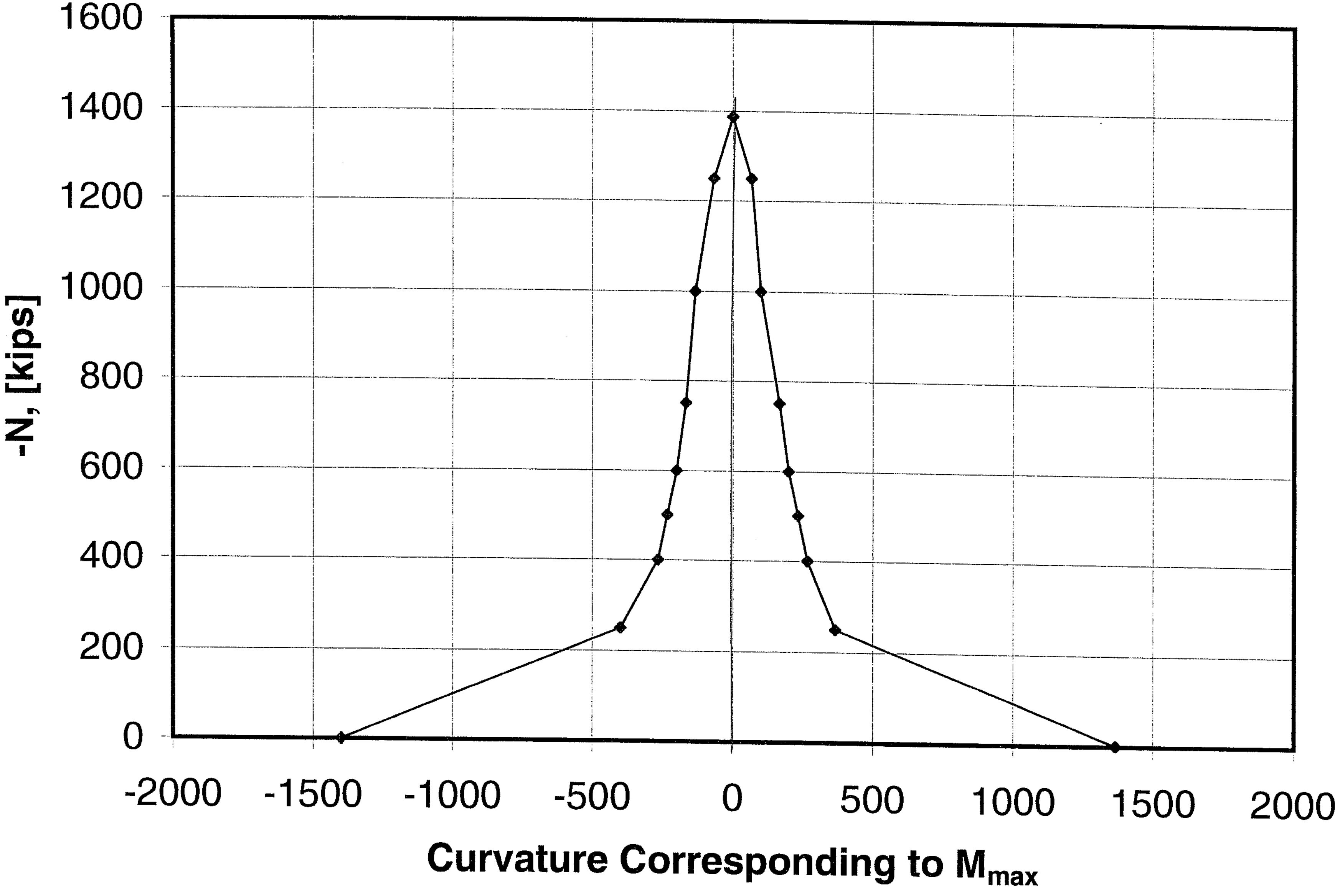
ACI value is just a limit.
This solution was found using equilibrium, etc.
0.003 would return similar results


1600
1400
1200
1000
800
600
400
200
0
-N, [kips]
-400
-300
-200
-100
0
100
200
300
400
500
M, [kip-ft]
$-\varepsilon_o$
$-\varepsilon_o$
$\varepsilon_t = 1.4\varepsilon_c'$
$\varepsilon_p \neq \varepsilon_{py}$ = i.e
$\varepsilon_{cp} + \Delta\varepsilon_p = \varepsilon_{py}$
7.6×10^{-3}



N≅N_b = -500 kips
N = -1000 kips
much less ductile
N=0
very ductile
N = -1250 kips
as axial load increases, number of convergence points decreases
Moment, (k-ft)
450
400
350
300
250
200
150
100
50
0
-100
0
100
200
300
400
500
600
Curvature, ϕ, (x10^-6 rad/in.)

how reinforced concrete columns behave



FLEXURAL DESIGN

Design checks

- P/S transfer : initial σ in concrete and reinforcing
- service loads : σ, Δ, loads
- transportation
- ultimate load : failure mode
- long-term service : creep, shrinkage, relaxation

Tendon stresses

Need to avoid:
- tendon fracture
- anchorage failure
- inelastic tendon deformation
- excessive relaxation losses

Concrete stresses

initial stresses from P/S

$f'_{ci} \neq f'_c$

f'_{ci} = strength at release

$$A_{s_{req}} = \frac{N_c}{0.6 f_y} = \frac{\frac{1}{2}\,\overset{\text{top fiber stress}}{f_{ct}} \cdot \overset{\text{N.A. depth}}{c} \cdot b \leftarrow \text{width beam}}{0.6\,\underset{\leq 30{,}000 \text{ psi}}{f_y}}$$

final stage

include volume changes, all loads

$f_c < 0.45 f'_c$ sustained loads — runaway creep to failure

$< 0.6 f'_c$ total loads

Notice: No Load Factors
No safety
Relies heavily on f'_c ~~[illegible]~~
being known

[Note: river gravel is hard,
crushed limestone is not]

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6. DESIGN FOR FLEXURE

- in introducing basic concepts of design, we will restrict our attention to simply supported beams
- later, will consider indeterminate structures
- will be applying our concepts of behaviour
- moreso that other members, P/S members must be checked at various stages of life:
    - at prestress transfer (initial stresses in conc, reinf)
    - under service loads (stresses, defl$^{\underline{ns}}$, cracks)
    - under ultimate load (failure mode, capacity)
    - long-term service (creep, shrinkage, relaxation)

6.1 PERMISSIBLE STRESSES IN TENDONS

- max stresses in tendons (during jacking and transfer) is limited to avoid:
    - tendon fracture
    - anchorage failure
    - inelastic tendon deformation
    - excessive relaxation losses

§ 18.5 in Code:
tensile stresses permitted depend on tendon type, whether prestressed or post-tensioned, at jack or transfer, etc.

additional sheet on this, 18-5

18.5.1 — Tensile stress in prestressing steel shall not exceed the following:

(a) Due to prestressing steel jacking force ... **0.94f_{py}**

but not greater than the lesser of **0.80f_{pu}** and the maximum value recommended by the manufacturer of prestressing steel or anchorage devices.

(b) Immediately after prestress transfer........ **0.82f_{py}**

but not greater than **0.74f_{pu}**.

(c) Post-tensioning tendons, at anchorage devices and couplers, immediately after force transfer........ **0.70f_{pu}**

6.2 PERMISSIBLE STRESSES IN CONCRETE

- concrete stresses investigated at different stages in life
- checked assuming elastic, uncracked response

INITIAL STAGE

- re stresses in concrete immediatly after prestress transfer (ie before time dependent losses have occurred)
- meant to prevent crushing or cracking of young concrete

18.4.1 — Stresses in concrete immediately after pr stress transfer (before time-dependent prestress losses) shall not exceed the following:

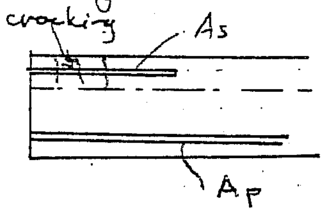
(a) Extreme fiber stress in compression .... $0.60f'_{ci}$

(b) Extreme fiber stress in tension except as permitted in (c) .... $3\sqrt{f'_{ci}}$

(c) Extreme fiber stress in tension at ends of simply supported members .... $6\sqrt{f'_{ci}}$

R18.4.1 — The concrete stresses at this stage are caused by the force in the prestressing steel at transfer reduced by the losses due to elastic shortening of the concrete, relaxation of the prestressing steel, seating at transfer, and the stresses due to the weight of the member. Generally, shrinkage and creep effects are not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer.

- tensile stress limits can be exceeded if As provided to control the resulting cracking
- this is particularly a concern in members with straight off-set tendons



$$As_{req'd} = \frac{\overset{\text{total tensile force}}{N_c}}{\underbrace{0.6 f_y}_{\lesssim 30000 psi}} = \frac{1/2\, f_{ct} \cdot c \cdot b}{0.6\, f_y}$$

c = depth n.a

b = width be

fct = top fibre stress

FINAL STAGE

- stresses in concrete investigated assuming:
  - all prestress losses have occurred
  - full specified loads are applied
- tensile zones investigated are those that are pre-compressed under prestressing only, but go into tension when specified loads are applied ∴ excludes end splitting cracks

*

- compressive stresses < 0.45 f'c due to sustained loads

  < 0.60 f'c due to total loads

- tensile stress limits may be exceeded under certain conditions

uncracked transition cracked R/C beams

CODE **COMMENTARY**

TABLE R18.3.3 — SERVICEABILITY DESIGN REQUIREMENTS

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\le 7.5\sqrt{f_c'}$	$7.5\sqrt{f_c'} < f_t \le 12\sqrt{f_c'}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times$ lever arm), or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

using $6\sqrt{f'_{ci}}$ for release stresses.

changes driven by precasters who want to make more money

(treated as R/C beam)

29 october 07

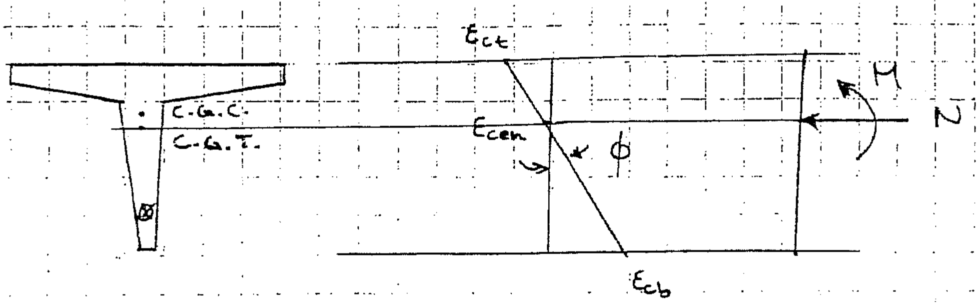
6.3 CALCULATING STRESSES IN CONCRETE

Apparently, two approaches available:

i) the Strain Compatibility Approach
ii) the Force-In-Tendon Approach

STRAIN COMPATIBILITY APPROACH:

- assume concrete remains uncracked and that material stress-strain relationships are linear
- assume tendons are bonded to concrete
- hence, use relationships previously developed



- total strains in concrete found by calculating ε_{cen} and ϕ using transformed section properties

- recall $\varepsilon_{cen} = \dfrac{N - N_o}{E_c \cdot A_{trans}}$ A_{trans}, I_{trans} are transformed section properties

  $\phi = \dfrac{M - M_o}{E_c \cdot I_{trans}}$ N_o, M_o are decompression forces.

- once ε_{cen} and ϕ are determined

$$\varepsilon_c = \varepsilon_{cen} - \phi \cdot y$$

$$\varepsilon_{cf} = \varepsilon_c - \varepsilon_{sh} - \varepsilon_{cth}$$

$$f_c = E_c \cdot \varepsilon_{cf}$$

check top and bottom fibre stresses

- when considering long-term response, don't forget $E_c \rightarrow E_{c,eff}$

- UNBONDED TENDONS:

  - remember, difficult problem; strain in the tendon will not be directly related to strain in surrounding concrete

  - conservative assumption is that strain in the prestressing tendon remains at $\Delta\varepsilon_p$ thru-out service life

*

$$\varepsilon_p = \Delta \varepsilon_p$$

* also, <u>must not</u> include A_p in calculating A_{trans}, I_{trans}

- realize that in post-tensioned construction, tendons are not bonded during stressing stage; procedures for unbonded tendons must be used in determining stresses in this stage

6.2 PERMISSIBLE STRESSES IN PRESTRESSING TENDONS: §18.5

a.) Due to tendon jacking force: 0.94 f_{py} but not greater than 0.80 f

- low relax. wire and strands (f_{py}=0.90 f_{pu}) - - - - - 0.80 f_{pu}
- stress-relieved wire and strands, and plain bars (ASTM A722) (f_{py}= 0.85 f_{pu}) - - - - - - - - 0.80 f_{pu}
- deformed bars (ASTM A722) (f_{py} = 0.80 f_{pu}) - - - - 0.75 f

b.) Immediately after prestress transfer: 0.82 f_{py} but not greater than 0.74 f

- Low relax. wire and strands (f_{py}=0.90 f_{pu}) - - - - 0.74 f_{pu}
- stress-relieved wire and strand, an plain bars (f_{py} = 0.85 f_{pu}) - - - - - - - 0.70 f_{pu}
- deformed bars (f_{py} = 0.80 f_{pu}) - - - - - - 0.66 f_{pu}

c.) Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage - - - 0.70 f_{pu}

* Note that the permissible stresses given in §18.5.1 a & b apply to both pretensioned and post-tensioned tendons.

* These stress limits are imposed to avoid |
  * tendon fracture
  * anchorage failure
  * inelastic tendon deformation
  * excessive relaxation losses

CODE

Where computed tensile stresses, $\boldsymbol{f_t}$, exceed the limits in (b) or (c), additional bonded reinforcement (nonprestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in concrete computed with the assumption of an uncracked section.

18.4.2 — For Class U and Class T prestressed flexural members, stresses in concrete at service loads (based on uncracked section properties, and after allowance for all prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression due to prestress plus sustained load .... $\boldsymbol{0.45f_c'}$

(b) Extreme fiber stress in compression due to prestress plus total load .... $\boldsymbol{0.60f_c'}$

COMMENTARY

TABLE R18.3.3 — SERVICEABILITY DESIGN REQUIREMENTS

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\leq 7.5\sqrt{f_c'}$	$7.5\sqrt{f_c'} < f_t \leq 12\sqrt{f_c'}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times$ lever arm), or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

the permissible values, the total force in the tensile stress zone may be calculated and reinforcement proportioned on the basis of this force at a stress of $\boldsymbol{0.6f_y}$, but not more than 30,000 psi. The effects of creep and shrinkage begin to reduce the tensile stress almost immediately; however, some tension remains in these areas after allowance is made for all prestress losses.

R18.4.2(a) and (b) — The compression stress limit of $\boldsymbol{0.45f_c'}$ was conservatively established to decrease the probability of failure of prestressed concrete members due to repeated loads. This limit seemed reasonable to preclude excessive creep deformation. At higher values of stress, creep strains tend to increase more rapidly as applied stress increases.

The change in allowable stress in the 1995 code recognized that fatigue tests of prestressed concrete beams have shown that concrete failures are not the controlling criterion. Designs with transient live loads that are large compared to sustained live and dead loads have been penalized by the previous single compression stress limit. Therefore, the stress limit of $\boldsymbol{0.60f_c'}$ permits a one-third increase in allowable compression stress for members subject to transient loads.

Sustained live load is any portion of the service live load that will be sustained for a sufficient period to cause significant time-dependent deflections. Thus, when the sustained live and dead loads are a large percentage of total service load, the $\boldsymbol{0.45f_c'}$ limit of 18.4.2(a) may control. On the other hand, when a large portion of the total service load consists of a transient or temporary service live load, the increased stress limit of 18.4.2(b) may apply.

The compression limit of $\boldsymbol{0.45f_c'}$ for prestress plus sustained loads will continue to control the long-term behavior of prestressed members.

CODE

18.4.3 — Permissible stresses in 18.4.1 and 18.4.2 shall be permitted to be exceeded if shown by test or analysis that performance will not be impaired.

18.4.4 — For Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by 10.6.4.

For structures subject to fatigue or exposed to corrosive environments, special investigations and precautions are required.

18.4.4.1 — The spacing requirements shall be met by nonprestressed reinforcement and bonded tendons. The spacing of bonded tendons shall not exceed 2/3 of the maximum spacing permitted for nonprestressed reinforcement.

Where both reinforcement and bonded tendons are used to meet the spacing requirement, the spacing between a bar and a tendon shall not exceed 5/6 of that permitted by 10.6.4. See also 18.4.4.3.

18.4.4.2 — In applying Eq. (10-4) to prestressing tendons, Δf_{ps} shall be substituted for f_s, where Δf_{ps} shall be taken as the calculated stress in the prestressing steel at service loads based on a cracked section analysis minus the decompression stress f_{dc}. It shall be permitted to take f_{dc} equal to the effective stress in the prestressing steel f_{se}. See also 18.4.4.3.

18.4.4.3 — In applying Eq. (10-4) to prestressing tendons, the magnitude of Δf_{ps} shall not exceed 36,000 psi. When Δf_{ps} is less than or equal to 20,000 psi, the spacing requirements of 18.4.4.1 and 18.4.4.2 shall not apply.

18.4.4.4 — Where h of a beam exceeds 36 in., the area of longitudinal skin reinforcement consisting of reinforcement or bonded tendons shall be provided as required by 10.6.7.

COMMENTARY

R18.4.3 — This section provides a mechanism whereby development of new products, materials, and techniques in prestressed concrete construction need not be inhibited by code limits on stress. Approvals for the design should be in accordance with 1.4 of the code.

R18.4.4 — Spacing requirements for prestressed members with calculated tensile stress exceeding $12\sqrt{f_c'}$ were introduced in the 2002 edition of the code.

For conditions of corrosive environments, defined as an environment in which chemical attack (such as seawater, corrosive industrial atmosphere, or sewer gas) is encountered, cover greater than that required by 7.7.2 should be used, and tension stresses in the concrete reduced to eliminate possible cracking at service loads. The engineer should use judgment to determine the amount of increased cover and whether reduced tension stresses are required.

R18.4.4.1 — Only tension steel nearest the tension face need be considered in selecting the value of c_c used in computing spacing requirements. To account for prestressing steel, such as strand, having bond characteristics less effective than deformed reinforcement, a 2/3 effectiveness factor is used.

For post-tensioned members designed as cracked members, it will usually be advantageous to provide crack control by the use of deformed reinforcement, for which the provisions of 10.6 may be used directly. Bonded reinforcement required by other provisions of this code may also be used as crack control reinforcement.

R18.4.4.2 — It is conservative to take the decompression stress f_{dc} equal to f_{se}, the effective stress in the prestressing steel.

R18.4.4.3 — The maximum limitation of 36,000 psi for Δf_{ps} and the exemption for members with Δf_{ps} less than 20,000 psi are intended to be similar to the code requirements before the 2002 edition.

CODE

18.5 — Permissible stresses in prestressing steel

18.5.1 — Tensile stress in prestressing steel shall not exceed the following:

(a) Due to prestressing steel jacking force ... $\mathbf{0.94}f_{py}$

but not greater than the lesser of $\mathbf{0.80}f_{pu}$ and the maximum value recommended by the manufacturer of prestressing steel or anchorage devices.

(b) Immediately after prestress transfer........ $\mathbf{0.82}f_{py}$

but not greater than $\mathbf{0.74}f_{pu}$.

(c) Post-tensioning tendons, at anchorage devices and couplers, immediately after force transfer........ $\mathbf{0.70}f_{pu}$

18.6 — Loss of prestress

18.6.1 — To determine effective stress in the prestressing steel, f_{se}, allowance for the following sources of loss of prestress shall be considered:

COMMENTARY

R18.4.4.4 — The steel area of reinforcement, bonded tendons, or a combination of both may be used to satisfy this requirement.

R18.5 — Permissible stresses in prestressing steel

The code does not distinguish between temporary and effective prestressing steel stresses. Only one limit on prestressing steel stress is provided because the initial prestressing steel stress (immediately after transfer) can prevail for a considerable time, even after the structure has been put into service. This stress, therefore, should have an adequate safety factor under service conditions and cannot be considered as a temporary stress. Any subsequent decrease in prestressing steel stress due to losses can only improve conditions and no limit on such stress decrease is provided in the code.

R18.5.1 — With the 1983 code, permissible stresses in prestressing steel were revised to recognize the higher yield strength of low-relaxation wire and strand meeting the requirements of ASTM A 421 and A 416. For such prestressing steel, it is more appropriate to specify permissible stresses in terms of specified minimum ASTM yield strength rather than specified minimum ASTM tensile strength. For the low-relaxation wire and strands, with f_{py} equal to $\mathbf{0.90}f_{pu}$, the $\mathbf{0.94}f_{py}$ and $\mathbf{0.82}f_{py}$ limits are equivalent to $\mathbf{0.85}f_{pu}$ and $\mathbf{0.74}f_{pu}$, respectively. In the 1986 supplement and in the 1989 code, the maximum jacking stress for low-relaxation prestressing steel was reduced to $\mathbf{0.80}f_{pu}$ to ensure closer compatibility with the maximum prestressing steel stress value of $\mathbf{0.74}f_{pu}$ immediately after prestress transfer. The higher yield strength of the low-relaxation prestressing steel does not change the effectiveness of tendon anchorage devices; thus, the permissible stress at post-tensioning anchorage devices and couplers is not increased above the previously permitted value of $\mathbf{0.70}f_{pu}$. For ordinary prestressing steel (wire, strands, and bars) with f_{py} equal to $\mathbf{0.85}f_{pu}$, the $\mathbf{0.94}f_{py}$ and $\mathbf{0.82}f_{py}$ limits are equivalent to $\mathbf{0.80}f_{pu}$ and $\mathbf{0.70}f_{pu}$, respectively, the same as permitted in the 1977 code. For bar prestressing steel with f_{py} equal to $\mathbf{0.80}f_{pu}$, the same limits are equivalent to $\mathbf{0.75}f_{pu}$ and $\mathbf{0.66}f_{pu}$, respectively.

Because of the higher allowable initial prestressing steel stresses permitted since the 1983 code, final stresses can be greater. Designers should be concerned with setting a limit on final stress when the structure is subject to corrosive conditions or repeated loadings.

R18.6 — Loss of prestress

R18.6.1 — For an explanation of how to compute prestress losses, see References 18.3 through 18.6. Lump sum values of prestress losses for both pretensioned and post-tensioned members that were indicated before the 1983 commentary

CODE

18.3.2.1 — Strains vary linearly with depth through the entire load range.

18.3.2.2 — At cracked sections, concrete resists no tension.

18.3.3 — Prestressed flexural members shall be classified as Class U, Class T, or Class C based on f_t, the computed extreme fiber stress in tension in the precompressed tensile zone calculated at service loads, as follows:

(a) Class U: $f_t \leq 7.5\sqrt{f_c'}$;
(b) Class T: $7.5\sqrt{f_c'} < f_t \leq 12\sqrt{f_c'}$;
(c) Class C: $f_t > 12\sqrt{f_c'}$;

Prestressed two-way slab systems shall be designed as Class U with $f_t \leq 6\sqrt{f_c'}$.

18.3.4 — For Class U and Class T flexural members, stresses at service loads shall be permitted to be calculated using the uncracked section. For Class C flexural members, stresses at service loads shall be calculated using the cracked transformed section.

18.3.5 — Deflections of prestressed flexural members shall be calculated in accordance with 9.5.4

18.4 — Serviceability requirements — Flexural members

18.4.1 — Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression........**$0.60f'_{ci}$**

(b) Extreme fiber stress in tension except as permitted in (c).. **$3\sqrt{f'_{ci}}$**

(c) Extreme fiber stress in tension at ends of simply supported members....................................... **$6\sqrt{f'_{ci}}$**

COMMENTARY

R18.3.3 — This section defines three classes of behavior of prestressed flexural members. Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in Table R18.3.3. For comparison, Table R18.3.3 also shows corresponding requirements for nonprestressed members.

These classes apply to both bonded and unbonded prestressed flexural members, but prestressed two-way slab systems must be designed as Class U.

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

R18.3.4 — A method for computing stresses in a cracked section is given in Reference 18.1.

R18.3.5 — Reference 18.2 provides information on computing deflections of cracked members.

R18.4 — Serviceability requirements — Flexural members

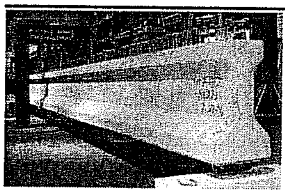
Permissible stresses in concrete address serviceability. Permissible stresses do not ensure adequate structural strength, which should be checked in conformance with other code requirements.

R18.4.1 — The concrete stresses at this stage are caused by the force in the prestressing steel at transfer reduced by the losses due to elastic shortening of the concrete, relaxation of the prestressing steel, seating at transfer, and the stresses due to the weight of the member. Generally, shrinkage and creep effects are not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer.

R18.4.1(b) and (c) — The tension stress limits of $3\sqrt{f'_{ci}}$ and $6\sqrt{f'_{ci}}$ refer to tensile stress at locations other than the precompressed tensile zone. Where tensile stresses exceed

Effects of Increasing the Allowable Compressive Release Stress of Pretensioned Girders

TxDOT Project 5197



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Prestressed Concrete Class, CE 383P
November 5, 2007

University of Texas at Austin
D. Birrcher
O. Bayrak

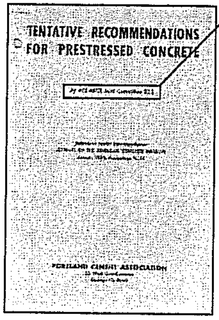
Outline

Allowable Compressive Stress at Prestress Transfer

- ☐ History of $0.60f'_{ci}$
  - ➢ Code Developments
  - ➢ Recent research projects and publicity
- ☐ Benefits of Increasing the Limit
- ☐ TxDOT Project 5197
- ☐ Current Conclusions

History of $0.60f'_{ci}$ - 1958

Prestressed concrete is born!



TENTATIVE RECOMMENDATIONS FOR PRESTRESSED CONCRETE

ACI-ASCE Joint Committee 323
(known as Committee 423 today)

(1) Temporary stresses before losses due to creep and shrinkage:

Compression

Pretensioned members...... $\underline{0.60\ f'_{ci}}$

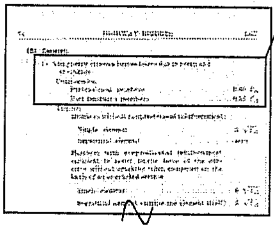
Post-tensioned members.... $0.55f'_{ci}$

"...production had preceded design recommendations and the stress of $0.60f'_{ci}$ had already been widely established in the pretensioning industry. No ill effect had been reported in regard to strength and performance."

History of $0.60f'_{ci}$ - 1961

AASHTO

AASHTO Standard Specs (1961)



(1) Temporary stresses before losses due to creep and shrinkage:

Compression

Pretensioned members... $\underline{0.60\ f'_{ci}}$

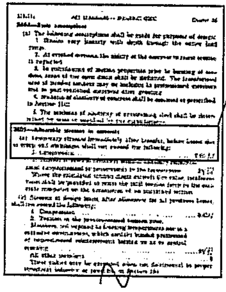
Post-tensioned members... $0.55f'_{ci}$

Unchanged from 1961 to LRFD of present except stress in post-tensioned members was increased to $0.60f'_{ci}$ in LRFD 2007

History of $0.60f'_{ci}$ - 1963

ACI

ACI 318 Building Code (1963)



2605 – Allowable stresses in concrete

(a) Temporary stresses immediately after transfer, before losses due to creep and shrinkage, shall not exceed the following:

1. Compression......... $0.60\ f'_{ci}$

Unchanged from 1963 to 2005, but in 2008......

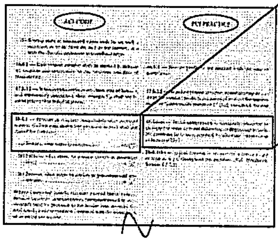
Compression

At ends..................... $0.70\ f'_{ci}$

At midspan................ $0.60\ f'_{ci}$

History of $0.60f'_{ci}$ - 1997

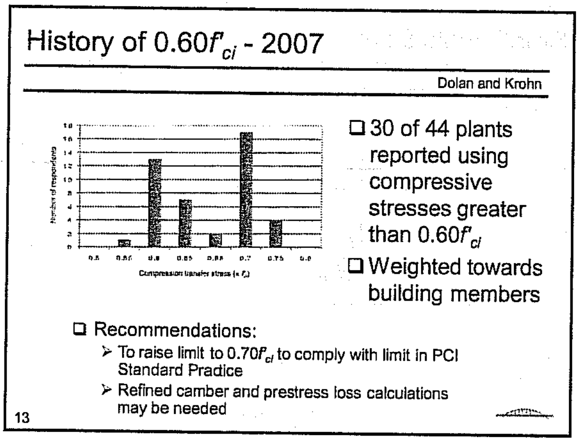
PCI Standard Practice



18.4.1 – Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression......... $0.60\ f'_{ci}$

18.4.1(a) – Initial compression is frequently permitted to go higher in order to avoid debonding or depressing strands. No problems have been reported by allowing compression as high as $0.75f'_{ci}$



History of 0.60f'_{ci} - 2007

Dolan and Krohn

- ☐ 30 of 44 plants reported using compressive stresses greater than 0.60f'_{ci}
- ☐ Weighted towards building members
- ☐ Recommendations:
  - ➢ To raise limit to 0.70f'_{ci} to comply with limit in PCI Standard Pradice
  - ➢ Refined camber and prestress loss calculations may be needed

History of 0.60f'_{ci} - Summary

What have we learned?

- ☐ 0.60f'_{ci} was determined empirically
- ☐ Remained unchanged since 1958
- ☐ Heavy interest in industry to increase limit in last decade
- ☐ Lack of comprehensive research

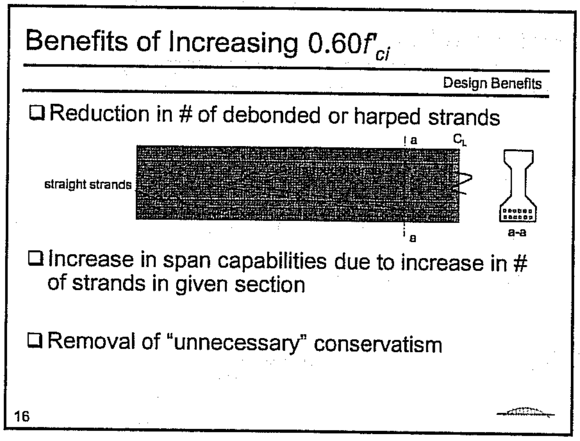
- ☐ Why?
  - ➢ What are the benefits to increasing the limit?

Benefits of Increasing 0.60f'_{ci}

Fabrication Benefits

- ☐ Reduction in cycle time at precast yard

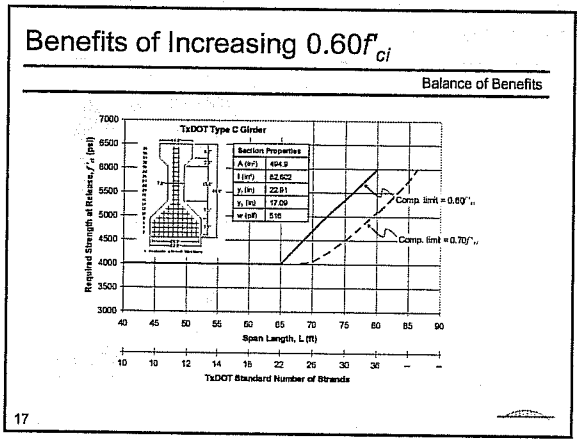
- ☐ Reduction in external curing costs
- ☐ Reduction of overall cement content



Benefits of Increasing 0.60f'_{ci}

Design Benefits

- ☐ Reduction in # of debonded or harped strands
- ☐ Increase in span capabilities due to increase in # of strands in given section
- ☐ Removal of "unnecessary" conservatism



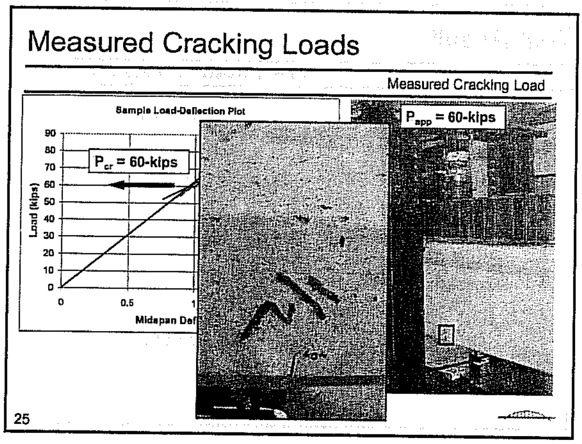
Benefits of Increasing 0.60f'_{ci}

Balance of Benefits

TxDOT Project 5197

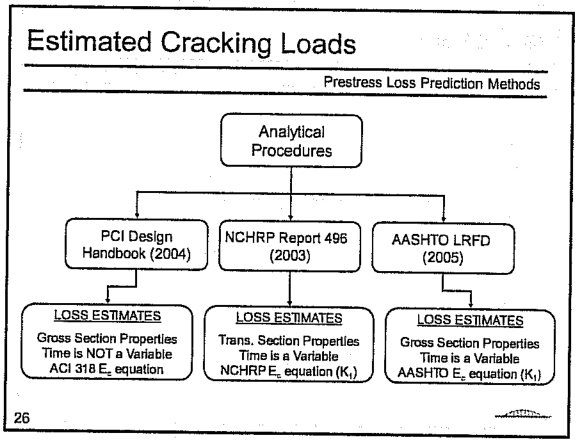
Overview

- ☐ Test Specimens
  - ☐ Small-scale girders fabricated at UT (24)
  - ☐ Full-scale girders fabricated at Texas precast facility (12)
- ☐ Experimental Program
  - ☐ Static test setup and procedure
- ☐ Measured Cracking Loads
- ☐ Estimated Cracking Loads
- ☐ Results
- ☐ Conclusions



Measured Cracking Loads

Measured Cracking Load



Estimated Cracking Loads

Prestress Loss Prediction Methods

Analytical Procedures

PCI Design Handbook (2004)

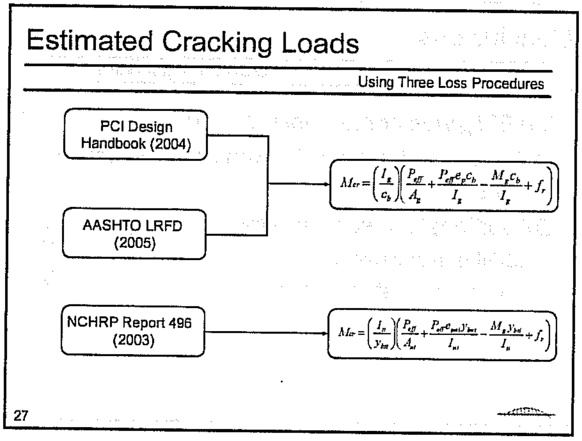
NCHRP Report 496 (2003)

AASHTO LRFD (2005)

LOSS ESTIMATES
Gross Section Properties
Time is NOT a Variable
ACI 318 E_c equation

LOSS ESTIMATES
Trans. Section Properties
Time is a Variable
NCHRP E_c equation (K_1)

LOSS ESTIMATES
Gross Section Properties
Time is a Variable
AASHTO E_c equation (K_1)



Estimated Cracking Loads

Using Three Loss Procedures

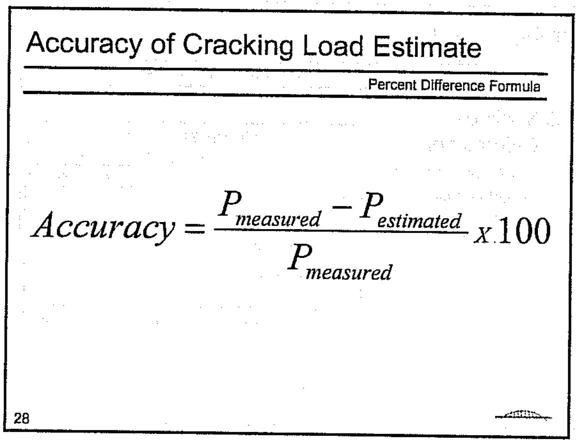
PCI Design Handbook (2004)

AASHTO LRFD (2005)

$$M_{cr} = \left(\frac{I_g}{c_b}\right)\left(\frac{P_{eff}}{A_g} + \frac{P_{eff}e_p c_b}{I_g} - \frac{M_g c_b}{I_g} + f_r\right)$$

NCHRP Report 496 (2003)

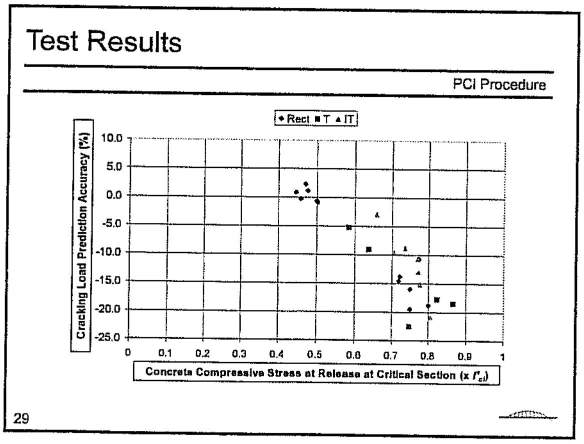
$$M_{cr} = \left(\frac{I_n}{y_{bnt}}\right)\left(\frac{P_{eff}}{A_{nt}} + \frac{P_{eff}e_{pnt}y_{bnt}}{I_{nt}} - \frac{M_g y_{bd}}{I_n} + f_r\right)$$



Accuracy of Cracking Load Estimate

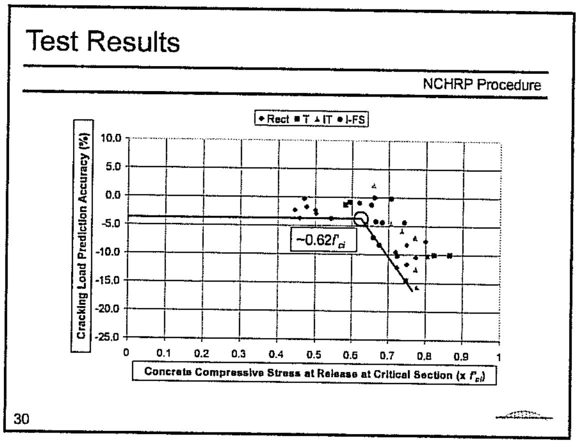
Percent Difference Formula

$$Accuracy = \frac{P_{measured} - P_{estimated}}{P_{measured}} x100$$



Test Results

PCI Procedure

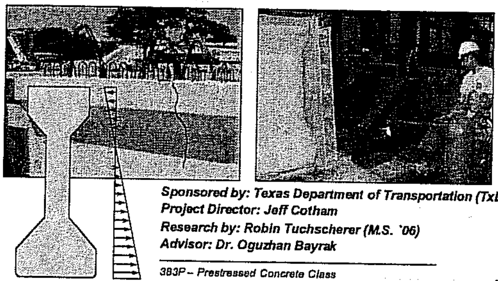


Test Results

NCHRP Procedure

Tensile Stress Limit for Prestressed Concrete at Release

THE UNIVERSITY OF TEXAS AT AUSTIN
FERGUSON STRUCTURAL ENGINEERING LABORATORY

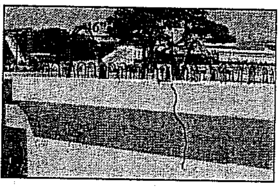


***Sponsored by:** Texas Department of Transportation (TxDOT)*
***Project Director:** Jeff Cotham*
***Research by:** Robin Tuchscherer (M.S. '06)*
***Advisor:** Dr. Oguzhan Bayrak*

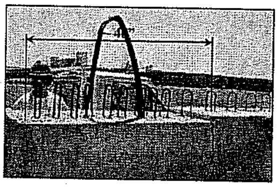
383P – Prestressed Concrete Class
November 5, 2007

INTRODUCTION

- The scope of this project is to identify the source of concrete cracking in short, Type IV girders with highly eccentric strand pattern and recommend solutions to reduce or eliminate cracking.



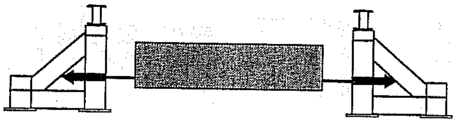
Without Blockout



With Blockout

INTRODUCTION

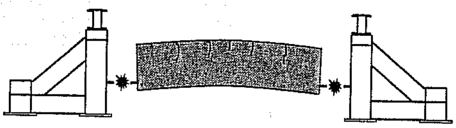
Example Prestressing Operation



Prestressing Bed

INTRODUCTION

Example Prestressing Operation

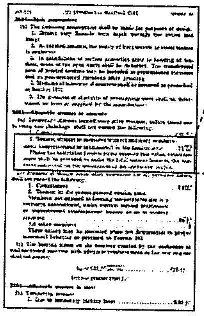


Prestressing Bed

BACKGROUND: ACI 318 PROVISIONS

- From 1963 → 1977

ACI 318 – 63



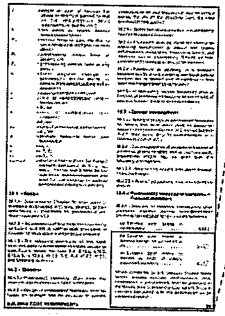
Allowable tensile stress in members without auxiliary reinforcement (unprestressed or prestressed) in the tension zone	$3\sqrt{f'_{ci}}$
Where the calculated tension stress exceeds this value, reinforcement shall be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section	

Note:
$3 \cdot \sqrt{4000 \text{ psi}} = 190$ psi

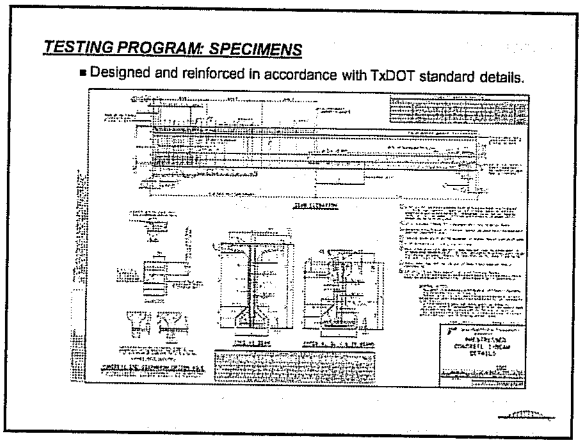
BACKGROUND: ACI 318 PROVISIONS

- From 1977 → Present

ACI 318 – 77

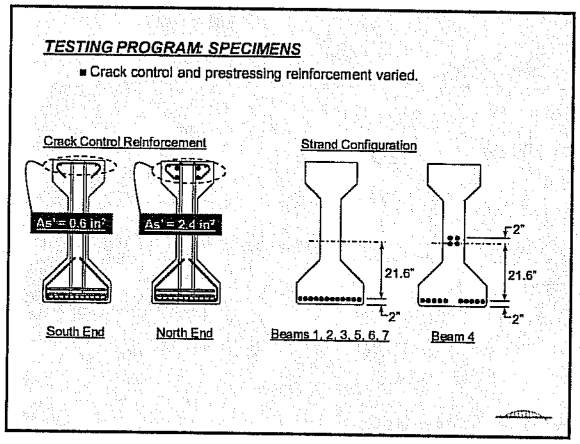


b) Extreme fiber stress in tension except as permitted in c)	$3\sqrt{f'_{ci}}$
c) Extreme fiber stress in tension at ends of simply supported members	$6\sqrt{f'_{ci}}$
Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (non-prestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in concrete computed with the assumption of an uncracked section.	



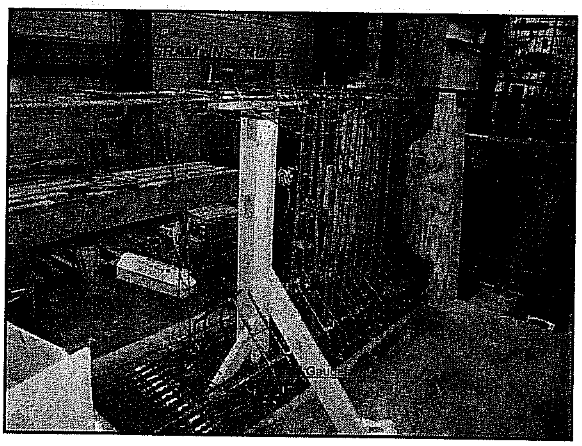
TESTING PROGRAM: SPECIMENS

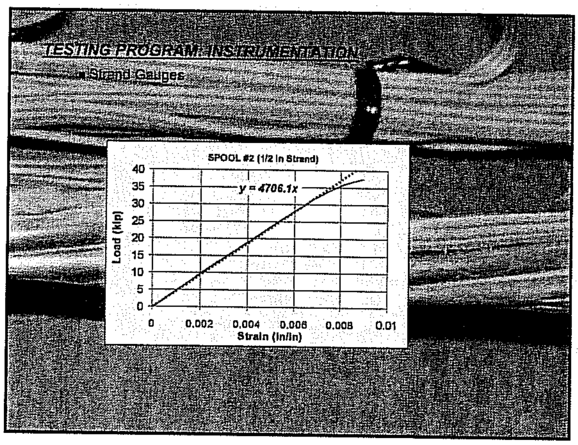
- Designed and reinforced in accordance with TxDOT standard details.



TESTING PROGRAM: SPECIMENS

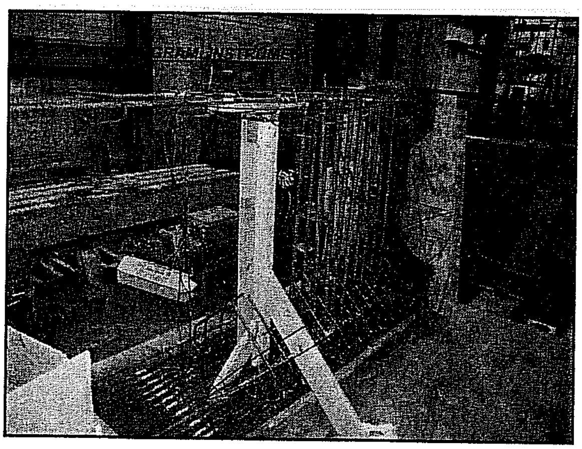
- Crack control and prestressing reinforcement varied.

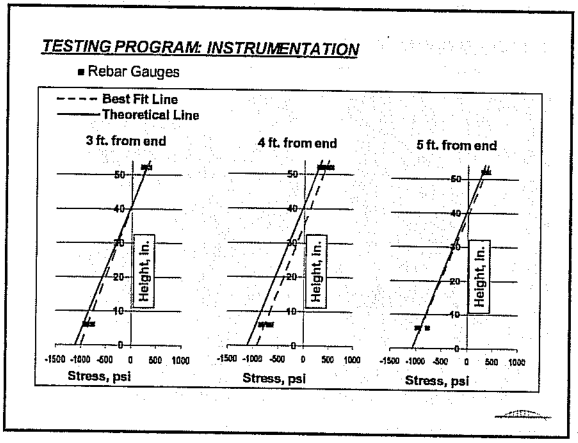




TESTING PROGRAM: INSTRUMENTATION

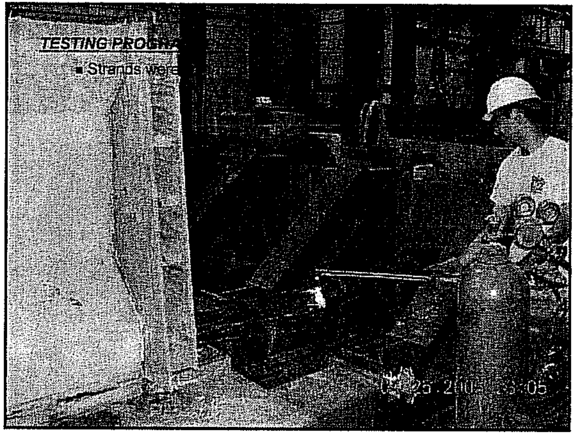
- Strand Gauges





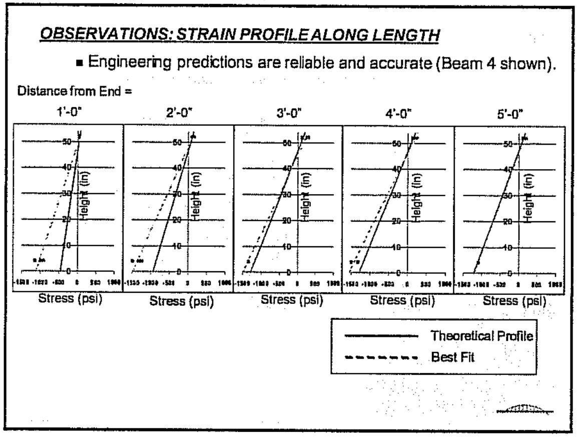
TESTING PROGRAM: INSTRUMENTATION

- Rebar Gauges



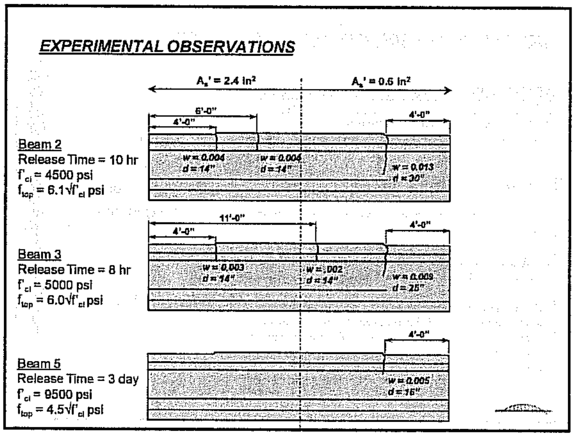
TESTING PROG

- Strands were



OBSERVATIONS: STRAIN PROFILE ALONG LENGTH

- Engineering predictions are reliable and accurate (Beam 4 shown).

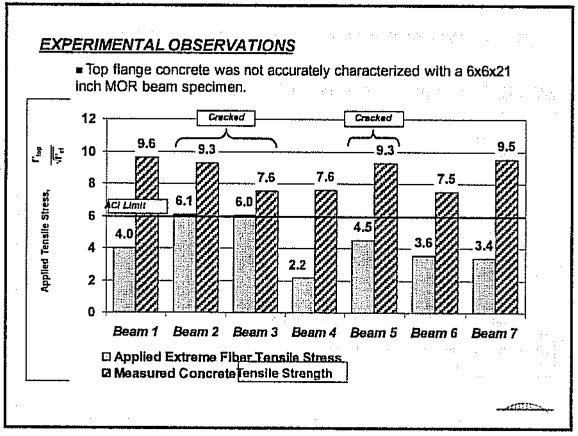


EXPERIMENTAL OBSERVATIONS

Beam 2
Release Time = 10 hr
f'_{ci} = 4500 psi
$f_{top} = 6.1\sqrt{f'_{ci}}$ psi

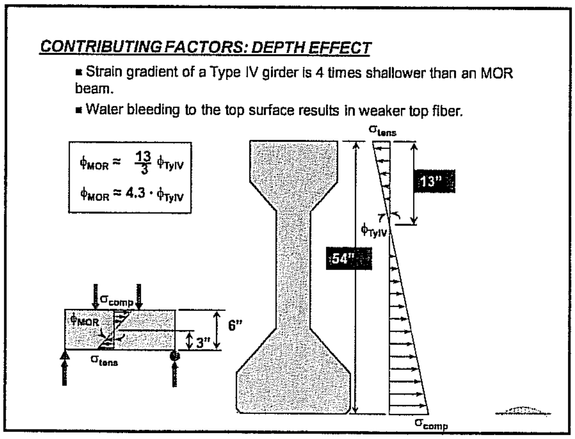
Beam 3
Release Time = 8 hr
f'_{ci} = 5000 psi
$f_{top} = 6.0\sqrt{f'_{ci}}$ psi

Beam 5
Release Time = 3 day
f'_{ci} = 9500 psi
$f_{top} = 4.5\sqrt{f'_{ci}}$ psi



EXPERIMENTAL OBSERVATIONS

- Top flange concrete was not accurately characterized with a 6x6x21 inch MOR beam specimen.

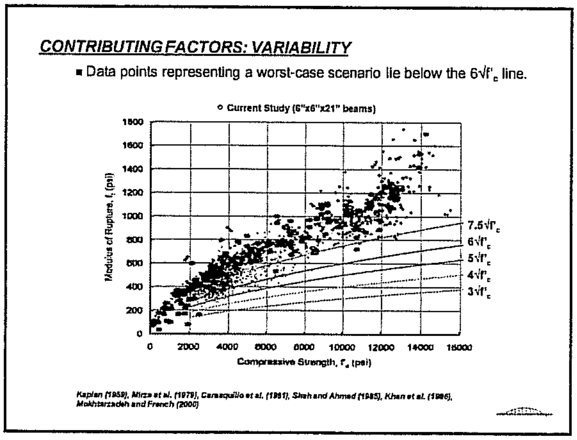


CONTRIBUTING FACTORS: DEPTH EFFECT

- Strain gradient of a Type IV girder is 4 times shallower than an MOR beam.
- Water bleeding to the top surface results in weaker top fiber.

$$\phi_{MOR} \approx \frac{13}{3}\,\phi_{TyIV}$$

$$\phi_{MOR} \approx 4.3 \cdot \phi_{TyIV}$$



CONTRIBUTING FACTORS: VARIABILITY

- Data points representing a worst-case scenario lie below the $6\sqrt{f'_c}$ line.

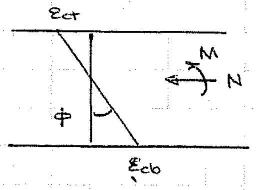
Kaplan (1959), Mirza et al. (1979), Carrasquillo et al. (1981), Shah and Ahmed (1985), Khan et al. (1996), Mokhtarzadeh and French (2000)

FLEXURAL DESIGN

Stresses in concrete

strain compatibility approach

assume concrete remains uncracked



$$\varepsilon_{cen} = \frac{N - N_o}{E_c A_{trans}}$$

$$\phi = \frac{M - M_o}{E_c I_{trans}}$$

N_o, M_o: decompression force and moment

$\varepsilon_c = \varepsilon_{cen} - \phi y$

$\varepsilon_{cf} = \varepsilon_c - \varepsilon_{sh} - \varepsilon_{cth}$

$f_c = E_c \cdot \varepsilon_{cf}$ – check top and bottom stress values

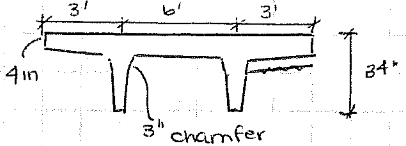
unbonded tendons

$\varepsilon_p = \Delta\varepsilon_p$, constant

A_p does not get included in A_{trans}, I_{trans}

→ strain in tendon does not directly relate to strain in surrounding concrete

Example of stress calcs



PCI Pretopped Double T: 12DT34 – 148 D1

- 12 ft wide
- double T
- 34" tall
- 14: # of strands
- 8: $\frac{8}{16}$" diam. strands
- D: depressed, or harped strands (vs. S = straight)
- 1: 1 harping pt

$A = 978 \text{ in}^2$

$I = 86072 \text{ in}^4$

$\bar{y}_{bot} = 25.77 \text{ in}$

$\bar{y}_{top} = 8.23 \text{ in}$

$S_b = 3340 \text{ in}^3$

$S_t = 10458 \text{ in}^3$

$w_t = 1019 \text{ plf} = 85 \text{ psf}$

$V/S = 2.39 \text{ in}$ (for creep and shrinkage)

$f'_{ci} = 5000 \text{ psi}$

$f'_c = 7500 \text{ psi}$

$f_{pu} = 270 \text{ ksi}$ tensioned to $0.75 f_{pu}$

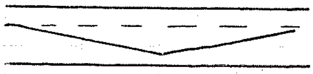
span = 60 ft

Loads:

live load = 30 psf

dead load = 10 psf + S.W.

Strand profile



$e_e = 12.91 \text{ in}$

$e_c = 22.27 \text{ in}$

$e_{0.4L} = 20.40 \text{ in}$

§2.2.2(b): critical point for service load moment is assumed at midspan for members with straight strands and at 0.4L for members depressed at midspan

more critical section!

FLEXURAL DESIGN

Example (cont'd)

Need to perform stress checks at 0.4L (or midspan if straight)

and at the ends (at transfer length)

└ $50d_s = 25\text{in}$, generally

focus in example is on 0.4L point

$x = 0.4L$

$M_D = (85\text{psf})(12\text{ft})(60\text{ft})^2(0.12) = 440.6\ \text{k}\cdot\text{ft} = 0.12wL^2$

$M_S = (10\text{ psf})(12\text{ft})(60\text{ft})^2(0.12) = 51.9\ \text{k}\cdot\text{ft}$

$M_L = (30\text{psf})(12\text{ft})(60\text{ft}^2)(0.12) = 155.5\ \text{k}\cdot\text{ft}$

Recall stress limits

	compressive	tensile
initial stage (after elastic shortening)	$0.6f'_{ci} = 3\text{ksi}$	$3\sqrt{f'_{ci}} = 212\text{ psi}$
Final stage (after all losses)	$0.45f'_c = 3.38\text{ksi}$ ↑ P/S + sustained $0.6f'_c = 4.5\text{ksi}$ ↑ P/S + total load	$7.5\sqrt{f'_c} = 650\text{psi}$

Use force in tendon approach

Allow for 7.5% reduction in lowlax strands for short term (elastic shortening)

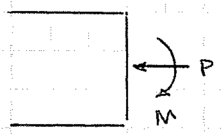
$f_{pi} = 187\text{ ksi}$

$F_{pi} = A_p f_{pi} = (14)(0.153\text{in}^2)(187\text{KSI})$

$P_i = 400.6\text{ K}$

$M_i = P_i e = (400.6\text{K})(20.40\text{in}) = 680.9\ \text{K}\cdot\text{ft}$

↑ at 0.4L



$$f = \frac{-P}{A} \pm \frac{Mc}{I}$$

$$f_{cb} = \frac{-400.6\text{K}}{978\text{ in}^2} - \frac{(-680.9\text{K}\cdot\text{ft})(-25.77\text{in})}{86072\text{ in}^4} = -2.86\text{ksi}$$

all from P/S, no SW included

$$f_{ct} = \frac{-400.6\text{K}}{978\text{ in}^2} - \frac{(-680.9\text{K}\cdot\text{ft})(8.23\text{in})}{86072\text{ in}^4} = 370\text{ psi}$$

> stress limit

Adding SW,

$$f_{cb} = \frac{+440.6\ \text{K}\cdot\text{ft}\ (25.77\text{in})}{86072\text{ in}^4} = 1.58\text{ksi}$$

$$f_{ct} = \frac{440.6\ \text{K}\cdot\text{ft}\ (8.23\text{in})}{86072\text{ in}^4} = -0.51\text{ksi}$$

combining,

$f_{cb} = -1.28\text{ksi}$ ✓

$f_{ct} = -0.14\text{ ksi}$ ✓

SELF-WEIGHT DOES A LOT! NO FLIPPING BEAMS.

FLEXURAL DESIGN

Example, cont'd

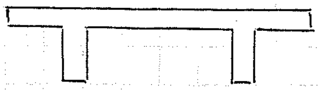
Final stage calculations: need to separate ST, LT loads

assume $\phi = 2.5$

30% of the live load is sustained

	net	short-term	long-term
E_c		$57\sqrt{f'_c} = 4936$ ksi	$E_c/1+\phi = 1410$ ksi
E_p		29000 ksi	27550 ksi
A_{trans}	978 in^2	988.45 in^2	1017.71 in^2
n_i		5.88	19.54
I_{trans}	86072 in^4	90375 in^4	101953 in^4
$\bar{y}_{btrans}$	25.77 in	25.55 in	24.97 in
$\bar{y}_{ttrans}$	8.23 in	8.45 in	9.03 in

$n_i = \frac{E_s}{E_c}$



$A_p = 14\,(0.153\,in^2) = 2.142\,in^2$

$A_t = 978\,in^2 + 4.88\,(2.142\,in^2) = 988.45\,in^2$

$$\bar{y}_{top} = \frac{978\,in^2\,(8.23\,in) + 10.45\,in^2\,(20.40\,in + 8.23\,in)}{988.45\,in^2} = 8.45\,in$$

$\bar{y}_{bot} = 34\,in - 8.45\,in = 25.55\,in$

$I_{trans} = 86072\,in^4 + 978\,in^2(8.45 - 8.23\,in)^2 + (10.45\,in^2)(20.18\,in)^2$
$= 90375\,in^4$

$P_f = f_{pf} \cdot A_p$, $f_{Pbed} = 200$ ksi

$f_{Prelease} = 187$ ksi (includes elastic shortening)

$f_{PFinal} = 157$ ksi (add relaxation, creep, shrinkage)

Not calculated as part of this example

$P_f = (157\,ksi)(14)(0.153\,in^2) = 336.3$ kip

Moments for stress calculations

due to sustained load

$M_S = M_{SD} + 0.3M_L = 51.84\,k\cdot ft + 0.3(155.52\,k\cdot ft) = 98.5\,k\cdot ft$

↑ superimposed dead load

self-equilibrating forces at release

$M_{SW} = M_D = 440.6\,k\cdot ft$ + P/S force

live load (short-term)

$M_L = 0.7\,(155.52\,k\cdot ft) = 108.9\,k\cdot ft$

FLEXURAL MEMBERS

Example (cont'd)

stress calculations

$$f_c = \underbrace{\frac{P_f}{A_{net}} \pm \frac{(P_f e)\,y}{I_{net}} \pm \frac{M_D y}{I_{net}}}_{\text{net section properties}} \pm \underbrace{\frac{M_S Y}{I_{trans_{L.T.}}}}_{\text{long-term transformed properties}} \pm \underbrace{\frac{M_L Y}{I_{trans_{ST}}}}_{\text{short-term transformed properties}}$$

valid if section remains uncracked.

calculate bottom fibre stress

$$f_b = \frac{-336.3\,k}{978\,in^2} - \frac{(336.3\,k)(20.4\,in)(25.8\,in)}{86072\,in^4} + \frac{(440.6\,k\cdot ft)(25.77\,in)}{86072\,in^4}$$

$$+ \frac{(98.5\,k\cdot ft)(24.97\,in)}{101953\,in^4} + \frac{(108.9\,k\cdot ft)(25.55\,in)}{90375\,in^4} = 0.16\ ksi$$

within stress limits

Now, top fibre stress

$$f_t = \frac{-336.3\,k}{978\,in^2} + \frac{(336.3\,k)(20.40)(8.23\,in)}{86072\,in^4} - \frac{(440.6\,k\cdot ft)(8.23\,in)}{86072\,in^4}$$

$$- \frac{(98.5\,k\cdot ft)(9.03\,in)}{101953\,in^4} - \frac{(108.9\,k\cdot ft)(8.45\,in)}{90375\,in^4} = -0.42\ ksi$$

also within limits

Now, insert giant book o' example calculations

Prestressed Concrete Beam Stress & Camber Calculations

University of Texas at Austin

Oguzhan Bayrak, Ph.D.
Associate Professor

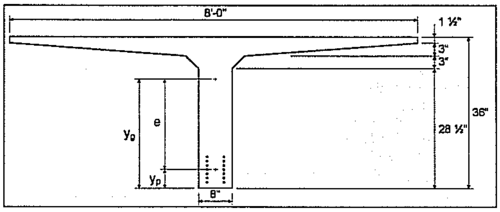
Nathan Dickerson
Graduate Research Assistant

Three Methods Available

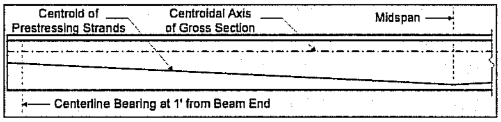
- Traditional "Force-in-the-Tendon" Approach
- Strain Compatibility Approach (Linear)
- Layered Section Approach (Non-linear)

Example Problem
Cross-Section

Single Tee Section With 14 - ½" Dia. Strands



Example Problem
Span & Loads



- L = 80 ft Simple Span with Single Harping Point
- Self-Weight: $w_{sw} = 0.600$ k/ft
- Superimposed Dead Load: $w_{sd} = 0.100$ k/ft
- Live Load: $w_L = 0.400$ k/ft (50 psf)

Example Problem
Prestressing Strand Profile

- Eccentricity of Prestressing Strands

End: e = 9.01 in (y_p = 17.00 in)

Transfer Length ($50d_b$): e = 9.63 in (y_p = 16.38 in)

0.4L: e = 18.87 in (y_p = 7.14 in)

Midspan: e = 21.26 in (y_p = 4.75 in)

Example Problem
Gross Section Properties

$A_g = 570$ in^2

$y_g = 26.01$ in

$I_g = 68917$ in^4

$A_p = 14(0.153) = 2.142$ in^2

Example Problem
Material Properties

Concrete (at 28 days)	Concrete (at Release)
$f'_c = 5000$ psi	$f'_{ci} = 3500$ psi
$E_c = 57\sqrt{f'_c} = 4030$ ksi	$E_{ci} = 57\sqrt{f'_{ci}} = 3370$ ksi

Prestressing Steel (Low Relaxation 7-Wire Strand)

$f_{pu} = 270$ ksi

$E_p = 29000$ ksi

$f_{pbed} = 0.74(f_{pu}) = 200$ ksi

Traditional Approach

- Prestress losses must be estimated at each stage of the service life under consideration
- Gross section or transformed section properties may be used
- Use superposition of forces and moments to calculate stresses and deflections
- Only valid in elastic range of material stress

Traditional Approach

- Easiest to use this approach with gross section properties as will be demonstrated with the example problem
- Losses only calculated at midspan and prestress force is assumed constant along the length of the beam, but this is just an approximation

Traditional Approach
Loss Calculations

- Use Gross Section Properties
- Elastic Shortening Loss:

  $ES = (E_p/E_{ci})\cdot f_{cir}$

  f_{cir} = concrete stress due to self-equilibrating loads (prestress and self-weight) at centroid of prestressing

  f_{pi} = stress in prestressing strand at release (after elastic shortening)

  Iterative Procedure: Assume f_{pi} to calculate f_{cir}

  $f_{pi} = 0.925(f_{pbed}) = 187.3$ ksi

  $P_i = f_{pi}A_p = 401.2$ k

  $f_{cir} = P_i/A_g + P_ie^2/I_g - M_{sw}e/I_g$

  $f_{cir} = 0.704 + 2.631 - 1.777 = 1.558$ ksi

  $ES = (29000/3370)(1.558) = 13.40$ ksi

  $f_{pi} = f_{pbed} - ES = 200 - 13.40 = 186.6$, which is close enough - OK

Traditional Approach
Loss Calculations

- Creep Loss: $CR = 2(E_p/E_c)(f_{cir} - f_{cds}) = 18.16$ ksi

  f_{cds} = concrete stress due to superimposed sustained loads at centroid of prestressing

  $f_{cds} = M_{sd}\cdot e/I_g = 0.296$ ksi
- Shrinkage Loss: $SH = \varepsilon_{sh}\cdot E_p = 6.59$ ksi

  $\varepsilon_{sh} = 8.2\cdot 10^{-6}(K_{sh})(K_{vs})(K_h) = 0.227\cdot 10^{3}$

  $K_{sh} = 1.0$, $K_{vs} = 0.86$, & $K_h = 32$
- Steel Relaxation Loss:

  $RE = [K_{re} - J(SH + CR + ES)]C = 3.47$ ksi

  $C = 1.0$, $J = 0.04$, & $K_{re} = 5$ ksi
- Total Losses: $ES + CR + SH + RE = 41.62$ ksi

Traditional Approach
Prestress Forces & Moments

Initial Prestress: $f_{pi} = 200 - 13.40 = 186.6$ ksi

$P_i = f_{pi}A_p = 399.7$ kips

Final Prestress: $f_{pf} = 200 - 41.62 = 158.4$ ksi

$P_f = f_{pf}A_p = 339.2$ kips

Midspan	0.4L	Transfer Length
$P_ie = 8498$ k-in	$P_ie = 7542$ k-in	$P_ie = 3849$ k-in
$P_fe = 7211$ k-in	$P_fe = 6400$ k-in	$P_fe = 3266$ k-in

Traditional Approach
Stresses at Release

Top Fiber Stress: $f_{top} = -P_i/A_g + P_ie(h-y_g)/I_g - M_{sw}(h-y_g)/I_g$
Bottom Fiber Stress: $f_{bot} = -P_i/A_g - P_iey_g/I_g + M_{sw}y_g/I_g$

Midspan
$f_{top} = -0.701 + 1.232 - 0.835 = -0.304$ ksi
$f_{bot} = -0.701 - 3.207 + 2.174 = -1.734$ ksi

0.4L
$f_{top} = -0.701 + 1.093 - 0.802 = -0.410$ ksi
$f_{bot} = -0.701 - 2.846 + 2.087 = -1.460$ ksi

Transfer Length
$f_{top} = -0.701 + 0.558 - 0.045 = -0.188$ ksi
$f_{bot} = -0.701 - 1.453 + 0.116 = -2.038$ ksi

Traditional Approach
Stresses Due to Sustained Loads

Top Fiber Stress: $f_{top} = -P_f/A_g + P_fe(h-y_g)/I_g - M_{sw}(h-y_g)/I_g - M_{sd}(h-y_g)/I_g$
Bottom Fiber Stress: $f_{bot} = -P_f/A_g - P_fey_g/I_g + M_{sw}y_g/I_g + M_{sd}y_g/I_g$

Midspan
$f_{top} = -0.595 + 1.045 - 0.835 - 0.139 = -0.524$ ksi
$f_{bot} = -0.595 - 2.722 + 2.174 + 0.362 = -0.781$ ksi

0.4L
$f_{top} = -0.595 + 0.928 - 0.802 - 0.134 = -0.603$ ksi
$f_{bot} = -0.595 - 2.415 + 2.087 + 0.348 = -0.575$ ksi

Transfer Length
$f_{top} = -0.595 + 0.473 - 0.045 - 0.007 = -0.174$ ksi
$f_{bot} = -0.595 - 1.233 + 0.116 + 0.019 = -1.693$ ksi

Traditional Approach
Stresses Due to Service Loads

Top Fiber Stress: $f_{bot} = -P_f/A_g + P_fe(h-y_g)/I_g - M_{sw}(h-y_g)/I_g - M_{sd}(h-y_g)/I_g - M_L(h-y_g)/I_g$
Bottom Fiber Stress: $f_{bot} = -P_f/A_g - P_fey_g/I_g + M_{sw}y_g/I_g + M_{sd}y_g/I_g + M_Ly_g/I_g$

Midspan
$f_{top} = -0.595 + 1.045 - 0.835 - 0.139 - 0.557 = -1.081$ ksi
$f_{bot} = -0.595 - 2.722 + 2.174 + 0.362 + 1.449 = 0.668$ ksi

0.4L
$f_{top} = -0.595 + 0.928 - 0.802 - 0.134 - 0.534 = -1.137$ ksi
$f_{bot} = -0.595 - 2.415 + 2.087 + 0.348 + 1.391 = 0.816$ ksi

Transfer Length
$f_{top} = -0.595 + 0.473 - 0.045 - 0.007 - 0.030 = -0.204$ ksi
$f_{bot} = -0.595 - 1.233 + 0.116 + 0.019 + 0.077 = -1.616$ ksi

Traditional Approach

- Using the gross section properties resulted in large bottom fiber tensile stress due to service loads
- Can take advantage of steel by using transformed section properties with the traditional method as will be demonstrated with the example problem – however, this increases the labor of calculations tremendously

Traditional Approach
Long Term Material Properties

- Assumptions to Estimate Long-Term Effects
  - Volume/Surface Ratio: V/S = 570/254.6 = 2.38 in
  - Relative Humidity: H = 68%
  - Loading Age: t_i = 1 day and $t - t_i$ = 10,000 days
- Creep Coefficient
  - $\phi(t,t_i) = 3.5(k_c)(k_f)(k_h)(k_l)(t - t_i)^{0.6}/\{10 + (t - t_i)^{0.6}\} = 2.50$
  - Volume/Surface Ratio Factor: $k_c = 0.90$
  - Concrete Strength Factor: $k_f = 1/(0.67 + f'_c/9000) = 0.82$
  - Relative Humidity Factor: $k_h = 1.58 - H/120 = 1.01$
  - Loading Age Factor: $k_l = t_i^{-0.118} = 1.00$

Traditional Approach
Long Term Material Properties

- Effective Modulus of Elasticity of Concrete
  - $E_{ceff} = E_c/(1 + \phi) = 4030/(1 + 2.50) = 1150$ ksi
- Prestressing Steel Relaxation
  - $f_p/f_{pbed} = 1 - \{(\log t)/45\}(f_{pbed}/f_{pu} - 0.55) = 0.98$
  - $E_{peff} = E_p(f_p/f_{pbed}) = 29000(0.98) = 28420$ ksi

Traditional Approach

Net Section Properties

Midspan	0.4L	Transfer Length
A_{net} = 567.9 in^2	A_{net} = 567.9 in^2	A_{net} = 567.9 in^2
y_{net} = 26.09 in	y_{net} = 26.08 in	y_{net} = 26.05 in
I_{net} = 67946 in^4	I_{net} = 68152 in^4	I_{net} = 68718 in^4

NOTE: In this case, the net section is the gross concrete section with prestressing strand area removed. However, when non-prestressed steel is present, then the net section would be the transformed section without the prestressing strands and would need to be calculated for both short term and long term. Used for calculating stresses and deflections due to prestress and self-weight.

Traditional Approach

Short Term Transformed Section Properties

$n = E_p/E_c = 7.20$

Midspan	0.4L	Transfer Length
A_{tr} = 583.3 in^2	A_{tr} = 583.3 in^2	A_{tr} = 583.3 in^2
y_{tr} = 25.53 in	y_{tr} = 25.58 in	y_{tr} = 25.79 in
I_{tr} = 74784 in^4	I_{tr} = 73539 in^4	I_{tr} = 70122 in^4

NOTE: Short term transformed section properties are used for calculating stresses and deflections due to live loads.

Traditional Approach

Long Term Transformed Section Properties

$n = E_{peff}/E_{ceff} = 24.70$

Midspan	0.4L	Transfer Length
A_{tr} = 620.8 in^2	A_{tr} = 620.8 in^2	A_{tr} = 620.8 in^2
y_{tr} = 24.27 in	y_{tr} = 24.47 in	y_{tr} = 25.22 in
I_{tr} = 89987 in^4	I_{tr} = 85516 in^4	I_{tr} = 73242 in^4

NOTE: Long term transformed section properties are used for calculating stresses and deflections due to sustained loads.

Traditional Approach

Stresses at Release

Top Fiber Stress: $f_{top} = -P_i/A_{net} + P_ie(h-y_{net})/I_{net} - M_{sw}(h-y_{net})/I_{net}$

Bottom Fiber Stress: $f_{bot} = -P_i/A_{net} - P_iey_{net}/I_{net} + M_{sw}y_{net}/I_{net}$

Midspan

$f_{top} = -0.704 + 1.239 - 0.840 = -0.305$ ksi

$f_{bot} = -0.704 - 3.263 + 2.212 = -1.755$ ksi

0.4L

$f_{top} = -0.704 + 1.098 - 0.805 = -0.411$ ksi

$f_{bot} = -0.704 - 2.886 + 2.116 = -1.474$ ksi

Transfer Length

$f_{top} = -0.704 + 0.557 - 0.044 = -0.191$ ksi

$f_{bot} = -0.704 - 1.459 + 0.116 = -2.047$ ksi

Traditional Approach

Stresses Due to Sustained Loads

Top Fiber Stress: $f_{top} = -P_f/A_{net} + P_fe(h-y_{net})/I_{net} - M_{sw}(h-y_{net})/I_{net} - M_{sd}(h-y_{tr})/I_{tr}$

Bottom Fiber Stress: $f_{bot} = -P_f/A_{net} - P_fey_{net}/I_{net} + M_{sw}y_{net}/I_{net} + M_{sd}y_{tr}/I_{tr}$

Midspan

$f_{top} = -0.597 + 1.052 - 0.840 - 0.125 = -0.510$ ksi

$f_{bot} = -0.597 - 2.769 + 2.212 + 0.259 = -0.895$ ksi

0.4L

$f_{top} = -0.597 + 0.932 - 0.805 - 0.124 = -0.594$ ksi

$f_{bot} = -0.597 - 2.449 + 2.116 + 0.264 = -0.666$ ksi

Transfer Length

$f_{top} = -0.597 + 0.473 - 0.044 - 0.008 = -0.176$ ksi

$f_{bot} = -0.597 - 1.238 + 0.116 + 0.018 = -1.701$ ksi

Traditional Approach

Stresses Due to Service Loads

Top Fiber Stress: $f_{top} = -P_f/A_{net} + P_fe(h-y_{net})/I_{net} - M_{sw}(h-y_{net})/I_{net} - M_{sd}(h-y_{tr})/I_{tr} - M_L(h-y_{tr})/I_{tr}$

Bottom Fiber Stress: $f_{bot} = -P_f/A_{net} - P_fey_{net}/I_{net} + M_{sw}y_{net}/I_{net} + M_{sd}y_{tr}/I_{tr} + M_Ly_{tr}/I_{tr}$

Midspan

$f_{top} = -0.597 + 1.052 - 0.840 - 0.125 - 0.538 = -1.048$ ksi

$f_{bot} = -0.597 - 2.769 + 2.212 + 0.259 + 1.311 = 0.416$ ksi

0.4L

$f_{top} = -0.597 + 0.932 - 0.805 - 0.124 - 0.522 = -1.116$ ksi

$f_{bot} = -0.597 - 2.449 + 2.116 + 0.264 + 1.282 = 0.616$ ksi

Transfer Length

$f_{top} = -0.597 + 0.473 - 0.044 - 0.008 - 0.030 = -0.206$ ksi

$f_{bot} = -0.597 - 1.238 + 0.116 + 0.018 + 0.075 = -1.626$ ksi

Traditional Approach
Initial Camber Calculation

Deflection Due to Prestress:

$\delta_p = (2e_c + e_e)(P_i)(L^2)/24(E_{ci})(I_g)$

$\delta_p = (2 \cdot 21.26 + 9.01)(-399.7)(80 \cdot 12)^2/24(3370)(68917)$

$\delta_p = -3.41$ in (up)

Deflection Due to Self-Weight:

$\delta_{sw} = (5/384)(w_{sw})(L^4)/(E_{ci})(I_g)$

$\delta_{sw} = (5/384)(0.6/12)(80 \cdot 12)^4/(3370)(68917)$

$\delta_{sw} = 2.38$ in (down)

Total Initial Camber:

$\delta_i = \delta_p + \delta_{sw} = -3.41 + 2.38 = -1.03$ in (up)

Traditional Approach
Final Camber Calculation

Deflection Due to Prestress:

$\delta_p = (2e_c + e_e)(P_f)(L^2)/24(E_{ceff})(I_g)$

$\delta_p = (2 \cdot 21.26 + 9.01)(-339.2)(80 \cdot 12)^2/24(1150)(68917)$

$\delta_p = -8.47$ in (up)

Deflection Due to Sustained Loads:

$\delta_{sus} = (5/384)(w_{sw} + w_{sd})(L^4)/(E_{ceff})(I_g)$

$\delta_{sus} = (5/384)(0.6/12+0.1/12)(80 \cdot 12)^4/(1150)(68917)$

$\delta_{sus} = 8.14$ in (down)

Total Final Camber:

$\delta_f = \delta_p + \delta_{sus} = -8.47 + 8.14 = -0.33$ in (up)

Traditional Approach
Service Load Deflection Calculation

Immediate Deflection Due to Live Load:

$\delta_L = (5/384)(w_L)(L^4)/(E_c)(I_g)$

$\delta_L = (5/384)(0.4/12)(80 \cdot 12)^4/(4030)(68917)$

$\delta_L = 1.33$ in (down)

Total Service Load Deflection:

$\Delta_f = \delta_f + \delta_L = -0.33 + 1.33 = 1.00$ in (down)

Strain Compatibility Approach

- At selected points along the beam for each stage of service life under consideration
  1. Calculate the "Decompression" Force and Moment
  2. Determine the Strain at the Centroid
  3. Determine the Curvature of the Section
  4. Calculate the Stresses from the Strain Profile given by the Curvature and the Strain at the Centroid
  6. Calculate Deflections using the Moment-Area Theorems along with the Curvature Diagram
- Transformed Section Properties are Required
- Only Valid in Elastic Range of Material Stress

Strain Compatibility Approach
Long Term Material Properties

- Assumptions to Estimate Long-Term Effects
  - Volume/Surface Ratio: V/S = 570/254.6 = 2.38 in
  - Relative Humidity: H = 68%
  - Loading Age: $t_i = 1$ day and $t - t_i = 10{,}000$ days
- Creep Coefficient

  $\phi(t,t_i) = 3.5(k_c)(k_f)(k_h)(k_l)(t - t_i)^{0.6}/\{10 + (t - t_i)^{0.6}\} = 2.50$

  - Volume/Surface Ratio Factor: $k_c = 0.90$
  - Concrete Strength Factor: $k_f = 1/(0.67 + f'_c/9000) = 0.82$
  - Relative Humidity Factor: $k_h = 1.58 - H/120 = 1.01$
  - Loading Age Factor: $k_l = t_i^{-0.118} = 1.00$

Strain Compatibility Approach
Long Term Material Properties

- Effective Modulus of Elasticity of Concrete

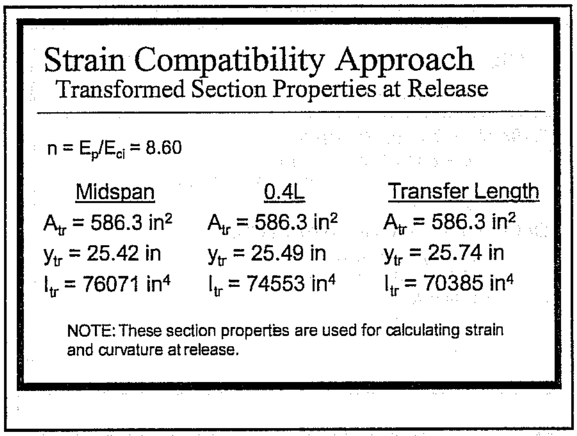
  $E_{ceff} = E_c/(1 + \phi) = 4030/(1 + 2.50) = 1150$ ksi
- Shrinkage

  $\varepsilon_{sh} = -0.51 \cdot 10^{-3}(k_s)(k_h)\{t/(35 + t)\} = -0.48 \cdot 10^{-3}$

  - Volume/Surface Ratio Factor: $k_s = 0.90$
  - Relative Humidity Factor: $k_h = 1.05$
  - Time: $t = 10{,}000$ days
- Prestressing Steel Relaxation

  $f_p/f_{pbed} = 1 - \{(\log t)/45\}(f_{pbed}/f_{pu} - 0.55) = 0.98$

  $E_{peff} = E_p(f_p/f_{pbed}) = 29000(0.98) = 28420$ ksi

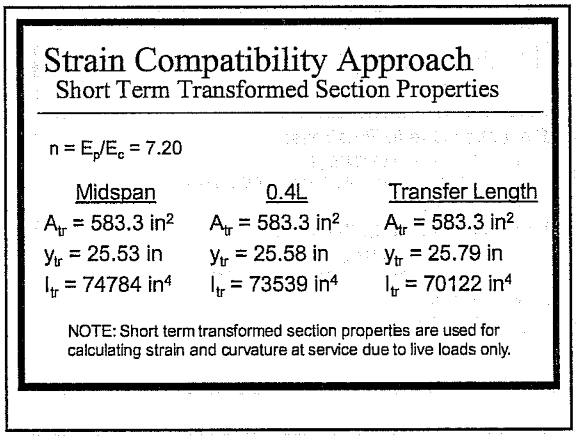


Strain Compatibility Approach
Transformed Section Properties at Release

$n = E_p/E_{ci} = 8.60$

Midspan	0.4L	Transfer Length
$A_{tr} = 586.3$ in^2	$A_{tr} = 586.3$ in^2	$A_{tr} = 586.3$ in^2
$y_{tr} = 25.42$ in	$y_{tr} = 25.49$ in	$y_{tr} = 25.74$ in
$I_{tr} = 76071$ in^4	$I_{tr} = 74553$ in^4	$I_{tr} = 70385$ in^4

NOTE: These section properties are used for calculating strain and curvature at release.

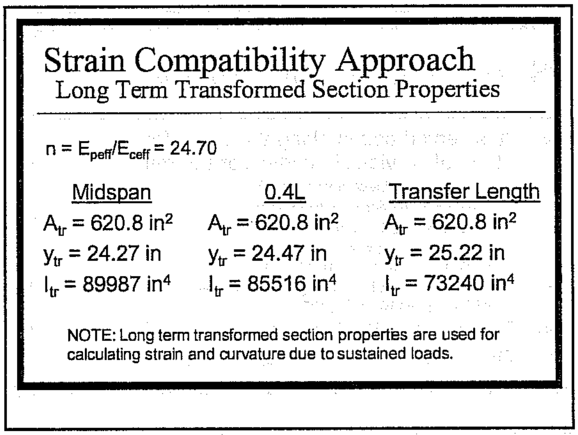


Strain Compatibility Approach
Short Term Transformed Section Properties

$n = E_p/E_c = 7.20$

Midspan	0.4L	Transfer Length
$A_{tr} = 583.3$ in^2	$A_{tr} = 583.3$ in^2	$A_{tr} = 583.3$ in^2
$y_{tr} = 25.53$ in	$y_{tr} = 25.58$ in	$y_{tr} = 25.79$ in
$I_{tr} = 74784$ in^4	$I_{tr} = 73539$ in^4	$I_{tr} = 70122$ in^4

NOTE: Short term transformed section properties are used for calculating strain and curvature at service due to live loads only.

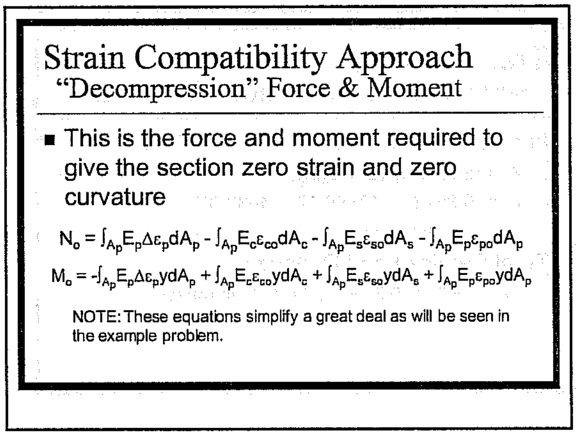


Strain Compatibility Approach
Long Term Transformed Section Properties

$n = E_{peff}/E_{ceff} = 24.70$

Midspan	0.4L	Transfer Length
$A_{tr} = 620.8$ in^2	$A_{tr} = 620.8$ in^2	$A_{tr} = 620.8$ in^2
$y_{tr} = 24.27$ in	$y_{tr} = 24.47$ in	$y_{tr} = 25.22$ in
$I_{tr} = 89987$ in^4	$I_{tr} = 85516$ in^4	$I_{tr} = 73240$ in^4

NOTE: Long term transformed section properties are used for calculating strain and curvature due to sustained loads.



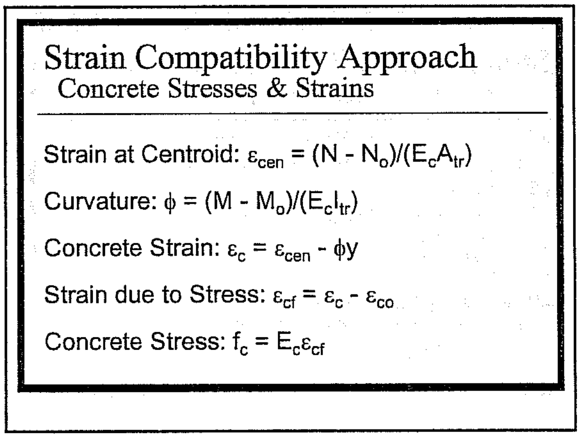
Strain Compatibility Approach
"Decompression" Force & Moment

- This is the force and moment required to give the section zero strain and zero curvature

$$N_o = \int_{A_p} E_p \Delta\varepsilon_p dA_p - \int_{A_p} E_c \varepsilon_{co} dA_c - \int_{A_p} E_s \varepsilon_{so} dA_s - \int_{A_p} E_p \varepsilon_{po} dA_p$$

$$M_o = -\int_{A_p} E_p \Delta\varepsilon_p y dA_p + \int_{A_p} E_c \varepsilon_{co} y dA_c + \int_{A_p} E_s \varepsilon_{so} y dA_s + \int_{A_p} E_p \varepsilon_{po} y dA_p$$

NOTE: These equations simplify a great deal as will be seen in the example problem.



Strain Compatibility Approach
Concrete Stresses & Strains

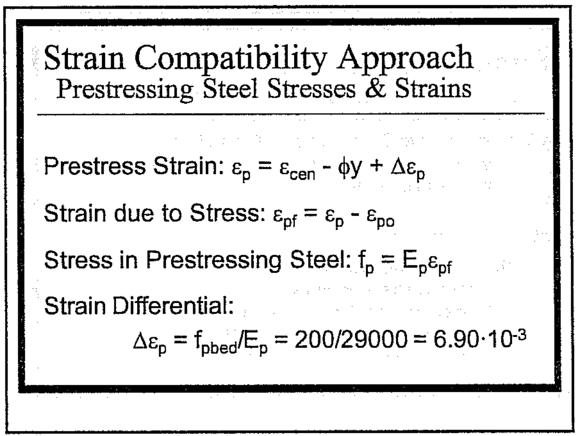
Strain at Centroid: $\varepsilon_{cen} = (N - N_o)/(E_c A_{tr})$

Curvature: $\phi = (M - M_o)/(E_c I_{tr})$

Concrete Strain: $\varepsilon_c = \varepsilon_{cen} - \phi y$

Strain due to Stress: $\varepsilon_{cf} = \varepsilon_c - \varepsilon_{co}$

Concrete Stress: $f_c = E_c \varepsilon_{cf}$



Strain Compatibility Approach
Prestressing Steel Stresses & Strains

Prestress Strain: $\varepsilon_p = \varepsilon_{cen} - \phi y + \Delta\varepsilon_p$

Strain due to Stress: $\varepsilon_{pf} = \varepsilon_p - \varepsilon_{po}$

Stress in Prestressing Steel: $f_p = E_p \varepsilon_{pf}$

Strain Differential:

$$\Delta\varepsilon_p = f_{pbed}/E_p = 200/29000 = 6.90 \cdot 10^{-3}$$

Strain Compatibility Approach
Decompression Force & Moment at Release

$N_o = E_p \Delta\varepsilon_p A_p$

$M_o = E_p \Delta\varepsilon_p A_p (y_{tr} - y_p)$

(simplification due to $\varepsilon_{co} = \varepsilon_{po} = \varepsilon_{so} = 0$)

Midspan	0.4L	Transfer Length
$N_o = 428.4$ k	$N_o = 428.4$ k	$N_o = 428.4$ k
$M_o = 8859$ k-in	$M_o = 7865$ k-in	$M_o = 4012$ k-in

Strain Compatibility Approach
Stresses at Release

Midspan

$M_{sw} = w_{sw}(L^2)/8 = 480$ k-ft

$\varepsilon_{cen} = (0 - 428.4)/(3370 \cdot 586.3) = -0.217 \cdot 10^{-3}$

$\phi = (480 \cdot 12 - 8859)/(3370 \cdot 76071) = -12.089 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.217 \cdot 10^{-3} + 12.089 \cdot 10^{-6}(36 - 25.42) = -0.089 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.217 \cdot 10^{-3} + 12.089 \cdot 10^{-6}(-25.42) = -0.524 \cdot 10^{-3}$

$f_{ctop} = E_{ci}\varepsilon_{ctop} = -0.300$ ksi

$f_{cbot} = E_{ci}\varepsilon_{cbot} = -1.766$ ksi

$\varepsilon_p = -0.217 \cdot 10^{-3} + 12.089 \cdot 10^{-6}(-25.42 + 4.75) + 6.9 \cdot 10^{-3} = 6.433 \cdot 10^{-3}$

$f_p = E_p \varepsilon_p = 186.6$ ksi (NOTE: This is the prestress after elastic shortening)

Strain Compatibility Approach
Stresses at Release

0.4L

$M_{sw} = 3w_{sw}(L^2)/25 = 460.8$ k-ft

$\varepsilon_{cen} = (0 - 428.4)/(3370 \cdot 586.3) = -0.217 \cdot 10^{-3}$

$\phi = (460.8 \cdot 12 - 7865)/(3370 \cdot 74553) = -9.295 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.217 \cdot 10^{-3} + 9.295 \cdot 10^{-6}(36 - 25.49) = -0.119 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.217 \cdot 10^{-3} + 9.295 \cdot 10^{-6}(-25.49) = -0.454 \cdot 10^{-3}$

$f_{ctop} = E_c\varepsilon_{ctop} = -0.401$ ksi

$f_{cbot} = E_c\varepsilon_{cbot} = -1.530$ ksi

$\varepsilon_p = -0.217 \cdot 10^{-3} + 9.295 \cdot 10^{-6}(-25.49 + 7.14) + 6.9 \cdot 10^{-3} = 6.512 \cdot 10^{-3}$

$f_p = E_p \varepsilon_p = 188.9$ ksi

Strain Compatibility Approach
Stresses at Release

Transfer Length

$M_{sw} = w_{sw}(L^2)/150 = 25.6$ k-ft

$\varepsilon_{cen} = (0 - 428.4)/(3370 \cdot 586.3) = -0.217 \cdot 10^{-3}$

$\phi = (25.6 \cdot 12 - 4012)/(3370 \cdot 70385) = -15.619 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.217 \cdot 10^{-3} + 15.619 \cdot 10^{-6}(36 - 25.74) = -0.057 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.217 \cdot 10^{-3} + 15.619 \cdot 10^{-6}(-25.74) = -0.619 \cdot 10^{-3}$

$f_{ctop} = E_{ci}\varepsilon_{ctop} = -0.192$ ksi

$f_{cbot} = E_{ci}\varepsilon_{cbot} = -2.086$ ksi

$\varepsilon_p = -0.217 \cdot 10^{-3} + 15.619 \cdot 10^{-6}(-25.74 + 16.38) + 6.9 \cdot 10^{-3} = 6.537 \cdot 10^{-3}$

$f_p = E_p \varepsilon_p = 189.6$ ksi

Strain Compatibility Approach
Long Term Decompression Force & Moment

$N_o = E_{peff} \Delta\varepsilon_p A_p - E_{ceff} \varepsilon_{co} A_g$

$(\varepsilon_{co} = \varepsilon_{sh} = -0.48 \cdot 10^{-3})$

$M_o = E_{peff} \Delta\varepsilon_p A_p (y_{tr} - y_p) + E_{ceff} \varepsilon_{co} A_g (y_g - y_{tr})$

Midspan	0.4L	Transfer Length
$N_o = 734.7$ k	$N_o = 734.7$ k	$N_o = 734.7$ k
$M_o = 7652$ k-in	$M_o = 6795$ k-in	$M_o = 3465$ k-in

Strain Compatibility Approach
Stresses Due to Sustained Loads

Midspan

$M_{sus} = (w_{sw} + w_{sd})(L^2)/8 = 560$ k-ft

$\varepsilon_{cen} = (0 – 734.7)/(1150 \cdot 620.8) = -1.029 \cdot 10^{-3}$

$\phi = (560 \cdot 12 - 7652)/(1150 \cdot 89987) = -9.006 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -1.029 \cdot 10^{-3} + 9.006 \cdot 10^{-6}(36 – 24.27) = -0.923 \cdot 10^{-3}$

$\varepsilon_{cbot} = -1.029 \cdot 10^{-3} + 9.006 \cdot 10^{-6}(-24.27) = -1.248 \cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.509$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -0.882$ ksi

$\varepsilon_p = -1.029 \cdot 10^{-3} + 9.006 \cdot 10^{-6}(-24.27 + 4.75) + 6.9 \cdot 10^{-3} = 5.695 \cdot 10^{-3}$

$f_{psus} = E_{peff}\varepsilon_p = 161.9$ ksi (NOTE: This is the prestress after all losses.)

Strain Compatibility Approach
Additional Stresses Due to Live Load

Midspan

$M_L = w_L(L^2)/8 = 320$ k-ft

$\phi_L = (320\cdot 12)/(4030\cdot 74784) = 12.741\cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -12.741\cdot 10^{-6}(36 - 25.53) = -0.133\cdot 10^{-3}$

$\varepsilon_{cbotL} = -12.741\cdot 10^{-6}(-25.53) = 0.325\cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.536$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 1.310$ ksi

$\varepsilon_{pL} = -12.741\cdot 10^{-6}(-25.53 + 4.75) = 0.265\cdot 10^{-3}$

$f_{pL} = E_p\varepsilon_{pL} = 7.7$ ksi

Strain Compatibility Approach
Stresses Due to Service Loads

Midspan

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.509 - 0.536 = -1.045$ ksi

$f_{cbot} = f_{cbotsus} + f_{cbotL} = -0.882 + 1.310 = 0.428$ ksi

$f_p = f_{psus} + f_{pL} = 161.9 + 7.7 = 169.6$ ksi

Strain Compatibility Approach
Stresses Due to Sustained Loads

0.4L

$M_{sus} = 3(w_{sw} + w_{sd})(L^2)/25 = 537.6$ k-ft

$\varepsilon_{cen} = (0 - 734.7)/(1150\cdot 620.8) = -1.029\cdot 10^{-3}$

$\phi = (537.6\cdot 12 - 6795)/(1150\cdot 85516) = -3.496\cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -1.029\cdot 10^{-3} + 3.496\cdot 10^{-6}(36 - 24.47) = -0.989\cdot 10^{-3}$

$\varepsilon_{cbot} = -1.029\cdot 10^{-3} + 3.496\cdot 10^{-6}(-24.47) = -1.115\cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.585$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -0.730$ ksi

$\varepsilon_p = -1.029\cdot 10^{-3} + 3.496\cdot 10^{-6}(-24.47 + 7.14) + 6.9\cdot 10^{-3} = 5.810\cdot 10^{-3}$

$f_{psus} = E_{peff}\varepsilon_p = 165.1$ ksi

Strain Compatibility Approach
Additional Stresses Due to Live Load

0.4L

$M_L = 3(w_L)(L^2)/25 = 307.2$ k-ft

$\phi_L = (307.2\cdot 12)/(4030\cdot 73539) = 12.439\cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -12.439\cdot 10^{-6}(36 - 25.58) = -0.130\cdot 10^{-3}$

$\varepsilon_{cbotL} = -12.439\cdot 10^{-6}(-25.58) = 0.318\cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.524$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 1.282$ ksi

$\varepsilon_{pL} = -12.439\cdot 10^{-6}(-25.58 + 7.14) = 0.229\cdot 10^{-3}$

$f_{pL} = E_p\varepsilon_{pL} = 6.6$ ksi

Strain Compatibility Approach
Stresses Due to Service Loads

0.4L

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.585 - 0.524 = -1.109$ ksi

$f_{cbot} = f_{cbotsus} + f_{cbotL} = -0.730 + 1.282 = 0.552$ ksi

$f_p = f_{psus} + f_{pL} = 165.1 + 6.6 = 171.7$ ksi

Strain Compatibility Approach
Stresses Due to Sustained Loads

Transfer Length

$M_{sus} = (w_{sw} + w_{sd})(L^2)/150 = 29.9$ k-ft

$\varepsilon_{cen} = (0 - 734.7)/(1150\cdot 620.8) = -1.029\cdot 10^{-3}$

$\phi = (29.9\cdot 12 - 3465)/(1150\cdot 73240) = -36.879\cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -1.029\cdot 10^{-3} + 36.879\cdot 10^{-6}(36 - 25.22) = -0.631\cdot 10^{-3}$

$\varepsilon_{cbot} = -1.029\cdot 10^{-3} + 36.879\cdot 10^{-6}(-25.22) = -1.959\cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.174$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -1.701$ ksi

$\varepsilon_p = -1.029\cdot 10^{-3} + 36.879\cdot 10^{-6}(-25.22 + 16.38) + 6.9\cdot 10^{-3} = 5.545\cdot 10^{-3}$

$f_{psus} = E_{peff}\varepsilon_p = 157.6$ ksi

Strain Compatibility Approach
Additional Stresses Due to Live Load

Transfer Length

$M_L = w_L(L^2)/150 = 17.1$ k-ft

$\phi_L = (17.1 \cdot 12)/(4030 \cdot 70121) = 0.726 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -0.726 \cdot 10^{-6}(36 - 25.79) = -0.007 \cdot 10^{-3}$

$\varepsilon_{cbotL} = -0.726 \cdot 10^{-6}(-25.79) = 0.019 \cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.028$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 0.077$ ksi

$\varepsilon_{pL} = -0.726 \cdot 10^{-6}(-25.79 + 16.38) = 0.007 \cdot 10^{-3}$

$f_{pL} = E_p\varepsilon_{pL} = 0.2$ ksi

Strain Compatibility Approach
Stresses Due to Service Loads

Transfer Length

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.174 - 0.028 = -0.202$ ksi

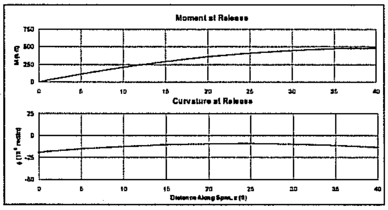
$f_{cbot} = f_{cbotsus} + f_{cbotL} = -1.701 + 0.077 = -1.624$ ksi

$f_p = f_{psus} + f_{pL} = 157.6 + 0.2 = 157.8$ ksi

Strain Compatibility Approach
Calculation of Deflection Using Curvature

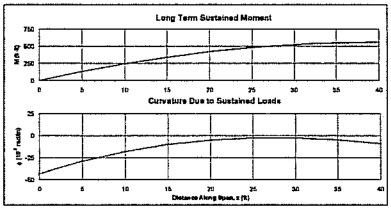
- Requires calculation of curvatures at several more points along the span

Strain Compatibility Approach
Initial Camber at Midspan



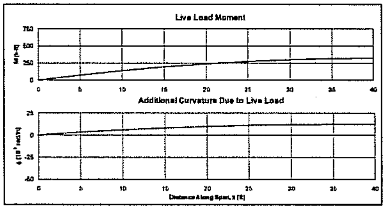
$\delta_i = -1.36$ in (up)

Strain Compatibility Approach
Final Camber at Midspan



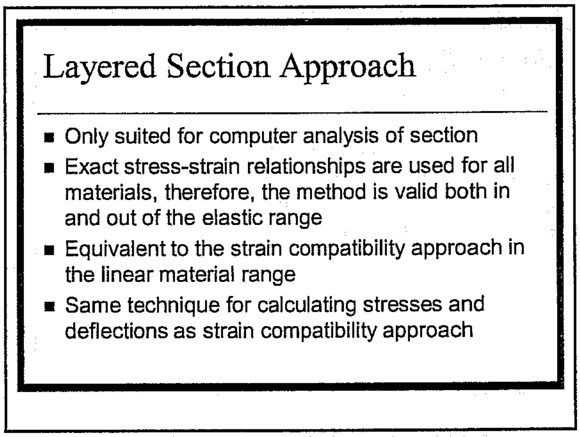
$\delta_f = -0.91$ in (up)

Strain Compatibility Approach
Service Load Deflection at Midspan



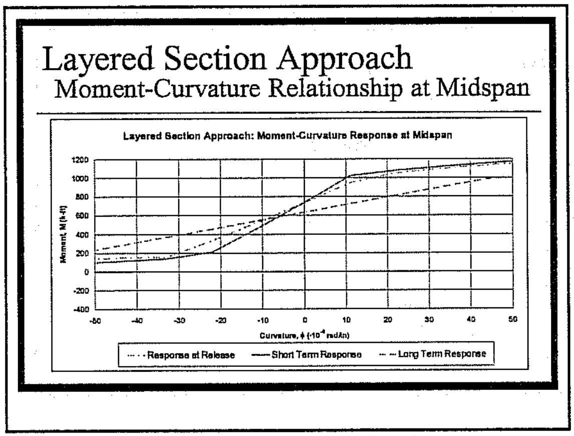
$\delta_L = 1.32$ in (down)

$\Delta_f = \delta_f + \delta_L = 0.41$ in (down)

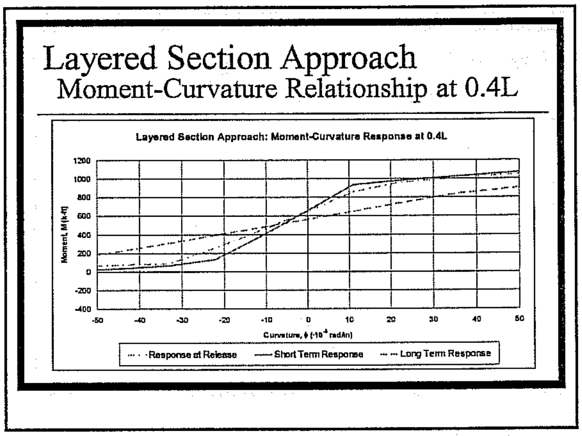


Layered Section Approach

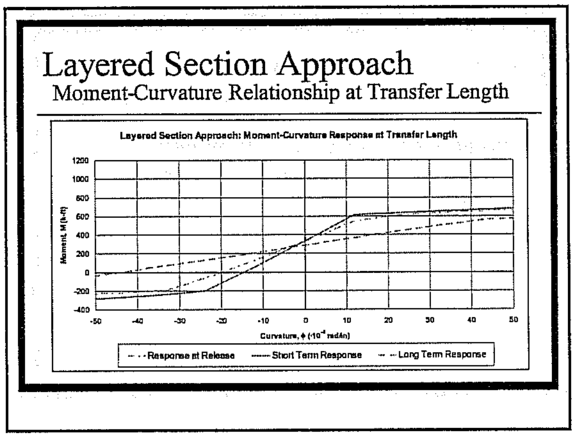
- Only suited for computer analysis of section
- Exact stress-strain relationships are used for all materials, therefore, the method is valid both in and out of the elastic range
- Equivalent to the strain compatibility approach in the linear material range
- Same technique for calculating stresses and deflections as strain compatibility approach



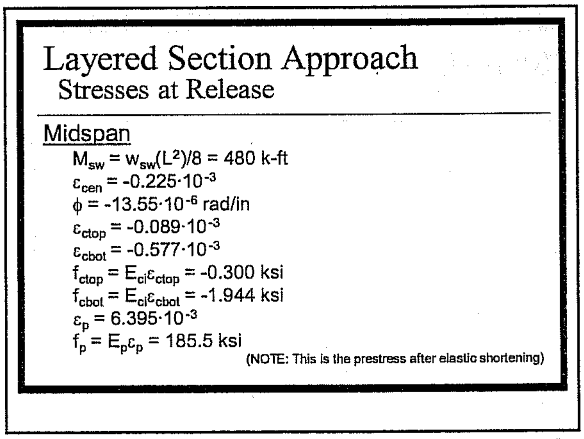
Layered Section Approach
Moment-Curvature Relationship at Midspan



Layered Section Approach
Moment-Curvature Relationship at 0.4L



Layered Section Approach
Moment-Curvature Relationship at Transfer Length



Layered Section Approach
Stresses at Release

Midspan

$M_{sw} = w_{sw}(L^2)/8 = 480$ k-ft

$\varepsilon_{cen} = -0.225 \cdot 10^{-3}$

$\phi = -13.55 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.089 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.577 \cdot 10^{-3}$

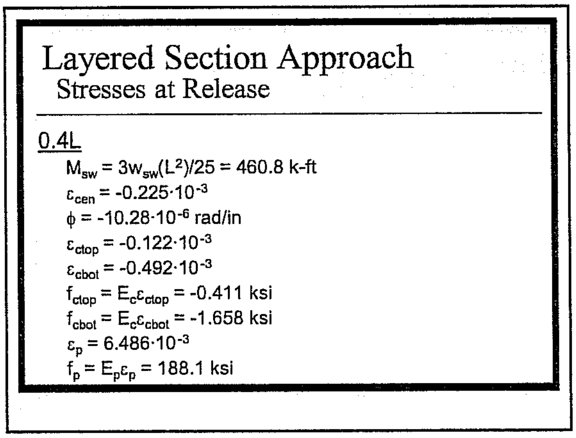
$f_{ctop} = E_{ci}\varepsilon_{ctop} = -0.300$ ksi

$f_{cbot} = E_{ci}\varepsilon_{cbot} = -1.944$ ksi

$\varepsilon_p = 6.395 \cdot 10^{-3}$

$f_p = E_p\varepsilon_p = 185.5$ ksi

(NOTE: This is the prestress after elastic shortening)



Layered Section Approach
Stresses at Release

0.4L

$M_{sw} = 3w_{sw}(L^2)/25 = 460.8$ k-ft

$\varepsilon_{cen} = -0.225 \cdot 10^{-3}$

$\phi = -10.28 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.122 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.492 \cdot 10^{-3}$

$f_{ctop} = E_c\varepsilon_{ctop} = -0.411$ ksi

$f_{cbot} = E_c\varepsilon_{cbot} = -1.658$ ksi

$\varepsilon_p = 6.486 \cdot 10^{-3}$

$f_p = E_p\varepsilon_p = 188.1$ ksi

Layered Section Approach
Stresses at Release

Transfer Length

$M_{sw} = w_{sw}(L^2)/150 = 25.6$ k-ft

$\varepsilon_{cen} = -0.233 \cdot 10^{-3}$

$\phi = -18.30 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.050 \cdot 10^{-3}$

$\varepsilon_{cbot} = -0.709 \cdot 10^{-3}$

$f_{ctop} = E_c \varepsilon_{ctop} = -0.169$ ksi

$f_{cbot} = E_c \varepsilon_{cbot} = -2.389$ ksi

$\varepsilon_p = 6.496 \cdot 10^{-3}$

$f_p = E_p \varepsilon_p = 188.4$ ksi

Layered Section Approach
Stresses Due to Sustained Loads

Midspan

$M_{sus} = (w_{sw} + w_{sd})(L^2)/8 = 560$ k-ft

$\varepsilon_{cen} = -1.028 \cdot 10^{-3}$

$\phi = -9.14 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.937 \cdot 10^{-3}$

$\varepsilon_{cbot} = -1.266 \cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.526$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -0.904$ ksi

$\varepsilon_p = 5.694 \cdot 10^{-3}$

$f_{psus} = E_{peff} \varepsilon_p = 161.8$ ksi

(NOTE: This is the prestress after all losses.)

Layered Section Approach
Additional Stresses Due to Live Load

Midspan

$M_L = w_L(L^2)/8 = 320$ k-ft

$\phi_L = 12.70 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -0.131 \cdot 10^{-3}$

$\varepsilon_{cbotL} = 0.326 \cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.528$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 1.314$ ksi

$\varepsilon_{pL} = 0.264 \cdot 10^{-3}$

$f_{pL} = E_p \varepsilon_{pL} = 7.7$ ksi

Layered Section Approach
Stresses Due to Service Loads

Midspan

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.526 - 0.528 = -1.053$ ksi

$f_{cbot} = f_{cbotsus} + f_{cbotL} = -0.904 + 1.314 = 0.410$ ksi

$f_p = f_{psus} + f_{pL} = 161.8 + 7.7 = 169.5$ ksi

Layered Section Approach
Stresses Due to Sustained Loads

0.4L

$M_{sus} = 3(w_{sw} + w_{sd})(L^2)/25 = 537.6$ k-ft

$\varepsilon_{cen} = -1.039 \cdot 10^{-3}$

$\phi = -3.30 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -1.006 \cdot 10^{-3}$

$\varepsilon_{cbot} = -1.124 \cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.605$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -0.741$ ksi

$\varepsilon_p = 5.804 \cdot 10^{-3}$

$f_{psus} = E_{peff} \varepsilon_p = 164.9$ ksi

Layered Section Approach
Additional Stresses Due to Live Load

0.4L

$M_L = 3(w_L)(L^2)/25 = 307.2$ k-ft

$\phi_L = 12.27 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -0.128 \cdot 10^{-3}$

$\varepsilon_{cbotL} = 0.313 \cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.516$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 1.261$ ksi

$\varepsilon_{pL} = 0.226 \cdot 10^{-3}$

$f_{pL} = E_p \varepsilon_{pL} = 6.6$ ksi

Layered Section Approach
Stresses Due to Service Loads

0.4L

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.605 - 0.516 = -1.121$ ksi

$f_{cbot} = f_{cbotsus} + f_{cbotL} = -0.741 + 1.261 = 0.521$ ksi

$f_p = f_{psus} + f_{pL} = 164.9 + 6.6 = 171.5$ ksi

Layered Section Approach
Stresses Due to Sustained Loads

Transfer Length

$M_{sus} = (w_{sw} + w_{sd})(L^2)/150 = 29.9$ k-ft

$\varepsilon_{cen} = -1.022 \cdot 10^{-3}$

$\phi = -39.89 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctop} = -0.623 \cdot 10^{-3}$

$\varepsilon_{cbot} = -2.060 \cdot 10^{-3}$

$f_{ctopsus} = E_{ceff}(\varepsilon_{ctop} - \varepsilon_{co}) = -0.164$ ksi

$f_{cbotsus} = E_{ceff}(\varepsilon_{cbot} - \varepsilon_{co}) = -1.817$ ksi

$\varepsilon_p = 5.525 \cdot 10^{-3}$

$f_{psus} = E_{peff}\varepsilon_p = 157.0$ ksi

Layered Section Approach
Additional Stresses Due to Live Load

Transfer Length

$M_L = w_L(L^2)/150 = 17.1$ k-ft

$\phi_L = 0.76 \cdot 10^{-6}$ rad/in

$\varepsilon_{ctopL} = -0.007 \cdot 10^{-3}$

$\varepsilon_{cbotL} = 0.020 \cdot 10^{-3}$

$f_{ctopL} = E_c(\varepsilon_{ctopL}) = -0.028$ ksi

$f_{cbotL} = E_c(\varepsilon_{cbotL}) = 0.081$ ksi

$\varepsilon_{pL} = 0.007 \cdot 10^{-3}$

$f_{pL} = E_p\varepsilon_{pL} = 0.2$ ksi

Layered Section Approach
Stresses Due to Service Loads

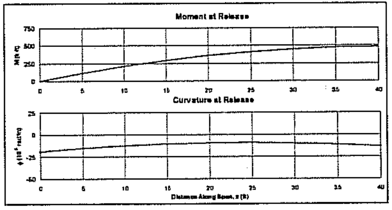
Transfer Length

$f_{ctop} = f_{ctopsus} + f_{ctopL} = -0.164 - 0.028 = -0.193$ ksi

$f_{cbot} = f_{cbotsus} + f_{cbotL} = -1.817 + 0.081 = -1.736$ ksi

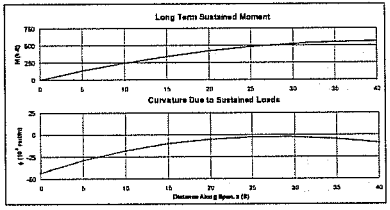
$f_p = f_{psus} + f_{pL} = 157.0 + 0.2 = 157.2$ ksi

Layered Section Approach
Initial Camber at Midspan

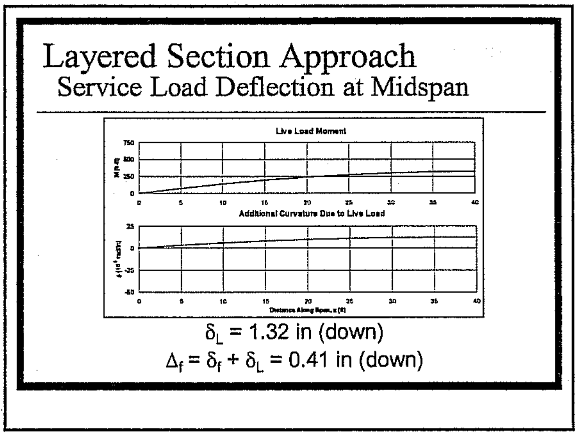


$\delta_i = -1.36$ in (up)

Layered Section Approach
Final Camber at Midspan



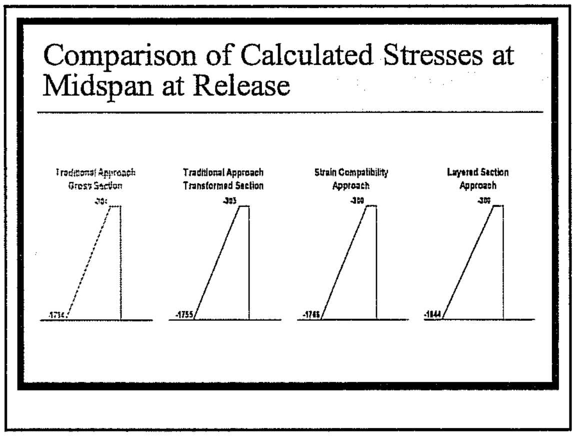
$\delta_f = -0.91$ in (up)



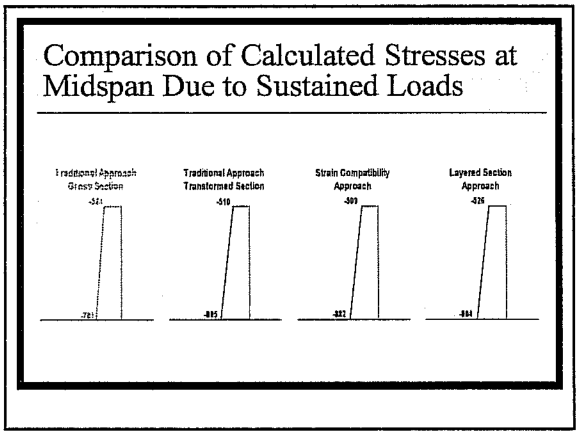
Layered Section Approach
Service Load Deflection at Midspan

$\delta_L = 1.32$ in (down)

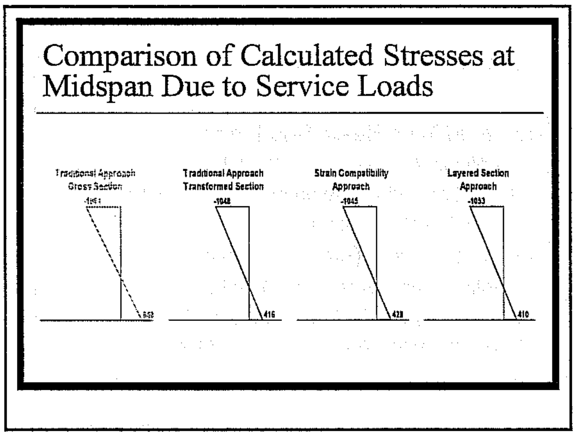
$\Delta_f = \delta_f + \delta_L = 0.41$ in (down)



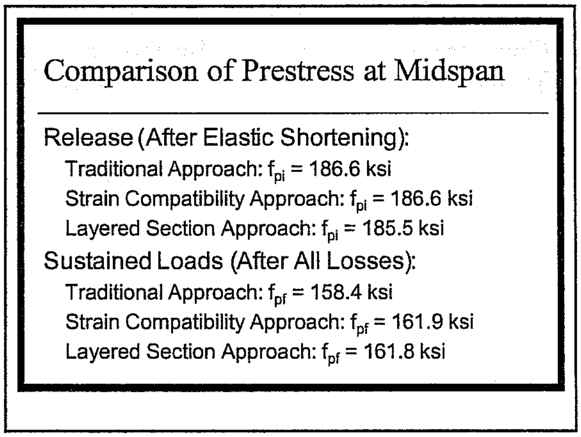
Comparison of Calculated Stresses at Midspan at Release



Comparison of Calculated Stresses at Midspan Due to Sustained Loads



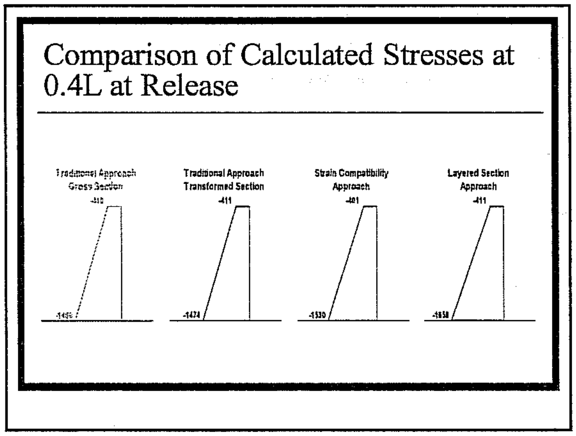
Comparison of Calculated Stresses at Midspan Due to Service Loads



Comparison of Prestress at Midspan

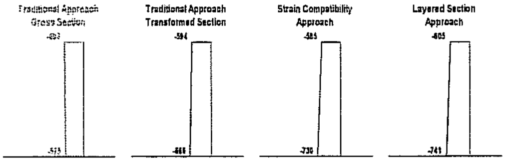
Release (After Elastic Shortening):
- Traditional Approach: f_{pi} = 186.6 ksi
- Strain Compatibility Approach: f_{pi} = 186.6 ksi
- Layered Section Approach: f_{pi} = 185.5 ksi

Sustained Loads (After All Losses):
- Traditional Approach: f_{pf} = 158.4 ksi
- Strain Compatibility Approach: f_{pf} = 161.9 ksi
- Layered Section Approach: f_{pf} = 161.8 ksi

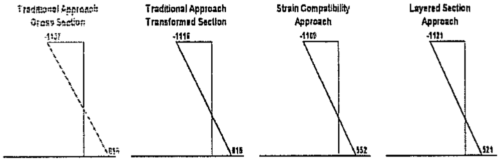


Comparison of Calculated Stresses at 0.4L at Release

Comparison of Calculated Stresses at 0.4L Due to Sustained Loads



Comparison of Calculated Stresses at 0.4L Due to Service Loads

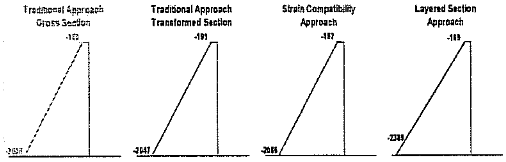


Comparison of Prestress at 0.4L

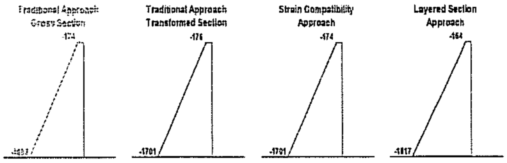
Release (After Elastic Shortening):
- Traditional Approach: f_{pi} = 186.6 ksi
- Strain Compatibility Approach: f_{pi} = 188.9 ksi
- Layered Section Approach: f_{pi} = 188.1 ksi

Sustained Loads (After All Losses):
- Traditional Approach: f_{pf} = 158.4 ksi
- Strain Compatibility Approach: f_{pf} = 165.1 ksi
- Layered Section Approach: f_{pf} = 164.9 ksi

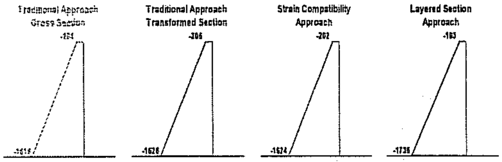
Comparison of Calculated Stresses at Transfer Length at Release

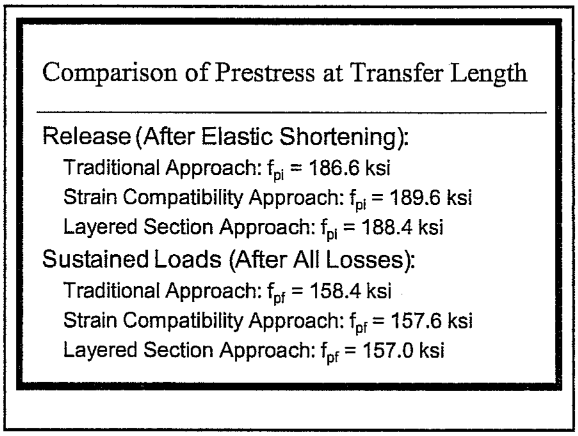


Comparison of Calculated Stresses at Transfer Length Due to Sustained Loads



Comparison of Calculated Stresses at Transfer Length Due to Service Loads

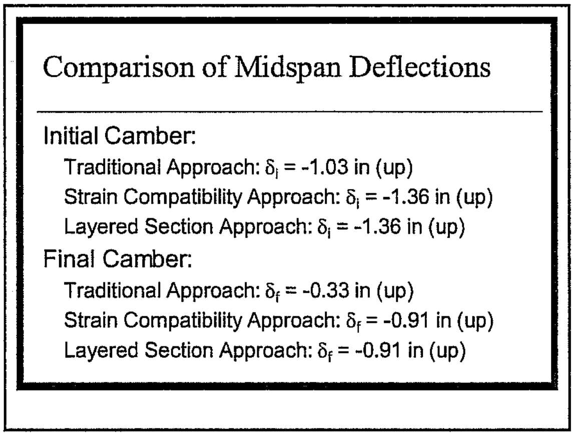




Comparison of Prestress at Transfer Length

Release (After Elastic Shortening):
- Traditional Approach: f_{pi} = 186.6 ksi
- Strain Compatibility Approach: f_{pi} = 189.6 ksi
- Layered Section Approach: f_{pi} = 188.4 ksi

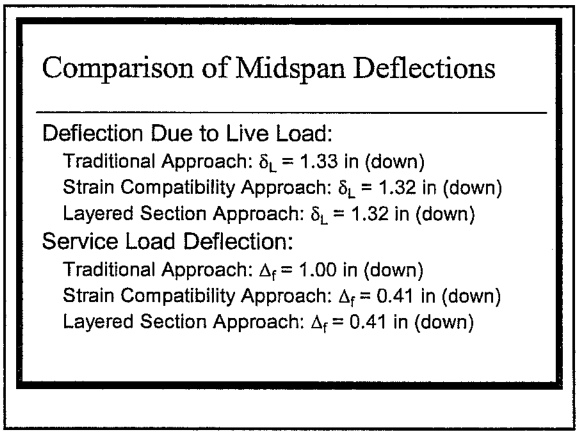
Sustained Loads (After All Losses):
- Traditional Approach: f_{pf} = 158.4 ksi
- Strain Compatibility Approach: f_{pf} = 157.6 ksi
- Layered Section Approach: f_{pf} = 157.0 ksi



Comparison of Midspan Deflections

Initial Camber:
- Traditional Approach: δ_i = -1.03 in (up)
- Strain Compatibility Approach: δ_i = -1.36 in (up)
- Layered Section Approach: δ_i = -1.36 in (up)

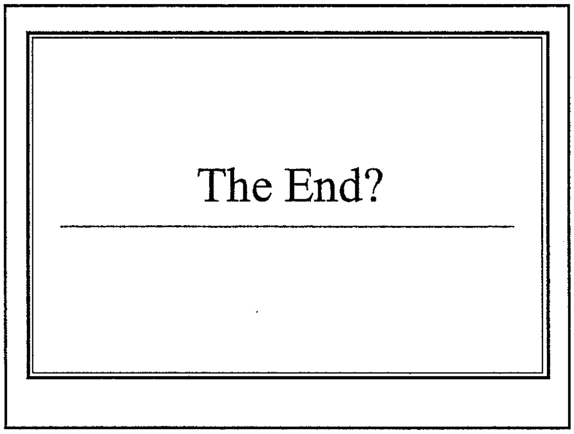
Final Camber:
- Traditional Approach: δ_f = -0.33 in (up)
- Strain Compatibility Approach: δ_f = -0.91 in (up)
- Layered Section Approach: δ_f = -0.91 in (up)



Comparison of Midspan Deflections

Deflection Due to Live Load:
- Traditional Approach: δ_L = 1.33 in (down)
- Strain Compatibility Approach: δ_L = 1.32 in (down)
- Layered Section Approach: δ_L = 1.32 in (down)

Service Load Deflection:
- Traditional Approach: Δ_f = 1.00 in (down)
- Strain Compatibility Approach: Δ_f = 0.41 in (down)
- Layered Section Approach: Δ_f = 0.41 in (down)



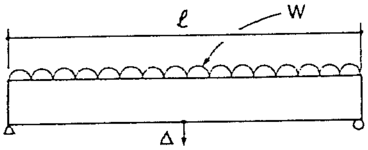
The End?

02 November 07

Maximum Permissible Computed Deflections

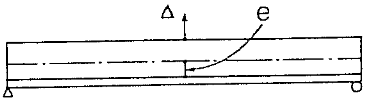
Type of Member	Deflection to Be Considered	Deflection Limitation
Roof member	Immediate deflection due to specified live load	ℓ/180
Floor member	Immediate deflection due to specified live load	ℓ/360
Roof or floor supporting or attached to nonstructural element likely to be damaged by large deflection	Sum of long-term deflection due to all sustained loads that occurs after attachment of nonstructural element and immediate deflection due to additional live load	ℓ/480
Roof or floor supporting or attached to nonstructural element not likely to be damaged by large deflection		ℓ/240

camber / deflections
from prestress



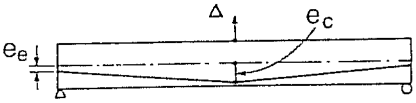
$$\Delta = \frac{5}{384}\frac{w\ell^4}{EI}$$

(a) Uniform loading



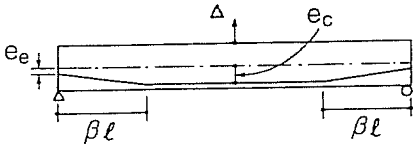
$$\Delta = \frac{1}{8}\frac{Pe\ell^2}{EI}$$

(b) Constant eccentricity



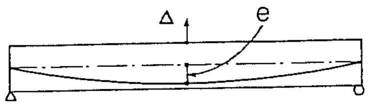
$$\boxed{\Delta = \frac{2e_c + e_e}{24}\frac{P\ell^2}{EI}}$$

(c) Single harping point



$$\Delta = \left[\frac{e_c}{8} - \frac{\beta^2}{6}(e_c - e_e)\right]\frac{P\ell^2}{EI}$$

(d) Double harping point



$$\Delta = \frac{5}{48}\frac{Pe\ell^2}{EI}$$

(e) Parabolic profile

FLEXURAL MEMBERS

crack control (ACI 318-05)

§ 10.6.4 – the spacing s of rebar closest to a surface in tension shall not exceed that given by

$$s = \frac{600}{f_s} - 2.5c_c \leq 12 \cdot \frac{40}{f_s}\quad , \quad c_c = \text{clear cover}$$

replaces the z-factor requirements
acceptable crack width: $w \leq 0.016$ in
– prior to 2002

if bars meet these spacing requirements, crack widths will not exceed acceptable values

ex. for beams with Gr. 60 rebar and 2" clear cover to the main reinforcement, with $f_s = 40$ ksi,

$$s_{max} = 10 \text{ in}$$

Research shows w is highly variable; current code for acceptable crack width ≤ that which is accepted in practice

$$f_s = \frac{M_{unfactored}}{A_s\,(jd)}, \text{ or } f_s = \tfrac{2}{3} f_y \text{ in 2005 } \left[0.6 f_y \text{ in 2002}\right]$$

↑ moment arm

§ 18.4.4 (.1 and .2)

For class C (cracked) prestressed flexural members not subject to fatigue or to aggressive exposure (parking garages, etc.), the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by §10.6.4 (above).

The spacing requirements shall be met by non-P/S and P/S reinforcement

$$\text{The spacing of bonded tendons} \leq \frac{2}{3}\left[\begin{array}{l}\text{max spacing permitted}\\ \text{for non-P/S reinforcing}\end{array}\right]$$

where both P/S and non-P/S reinforcing are used, the spacing between a bar and a tendon shall not exceed 5/6 the spacing ~~to~~ from §10.6.4

for f_s, use Δf_{ps} (take out $\Delta\varepsilon_p$)

Δf_{ps} : the difference between the stress computed in the tendons at service loads (based on a cracked section analysis) and decompression forces, f_{dc}

$\Delta f_{ps} \leq 36$ ksi (inconsistent w/ rebar)

if $\Delta f_{ps} \leq 20$ ksi, §18.4.4.1 and .2 do not apply

§18.4.4.4 the use of skin reinforcement as per §10.6.7 when $d \geq 36$ in

FLEXURAL MEMBERS

Camber and Deflections

ACI 318·05 requires that
- short·term deflections under live loads
- long·term deflections under dead and sustained loads

be checked and must comply.

Attachment to non·structural elements changes requirements

In calculating long·term camber and deflections, include:
- creep
- shrinkage of concrete
- relaxation of P/S reinforcement

For class C members, cracking must be considered

Computed deflections must be smaller than code·permissible values

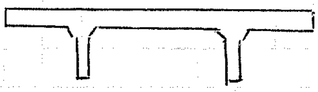
Deflection calculations

can use strain compatibility and force·in·tendon approaches

$\rightarrow \phi = \frac{M - M_c}{E_c I_{trans}}$ provided that section remains uncracked

$\rightarrow \phi = \phi_{tendon} + \phi_{loading}$ in calculating deflections due to P·e, $I = I_{net}$; for other deflections, use $I = I_{trans}$

Example



12DT34 – 148·D1 (pg 45)

- check initial camber
- long·term camber
- Δ under service loads

$f'_{ci} = 5$ ksi
$f'_c = 7.5$ ksi
$f_{pu} = 270$ ksi
span = 60 ft

$e_e = 12.91$ in, $e_c = 22.27$ in
LL = 30 psf, S.DL = 10 psf
SW = 85 psf

Roof member unlikely to cause damage to non·structural elements

$P_i = 400.55$ k, $P_F = 336.3$ k, $I_{net} = 86072$ in^4
$I_{trans_{ST}} = 90375$ in^4, $I_{trans_{LT}} = 101953$ in^4
$E_{c_{ST}} = 4936$ ksi, $E_{c_{LT}} = 1410$ ksi

Difference from handout example:
use PCI multipliers here (given in handout)

Sign convention:
(-) is upwards

Initial camber:

$$\Delta = \frac{-P\ell^2}{24EI}(2e_c + e_e) + \frac{5wL^4}{384EI}$$

$$= \frac{-(400.55k)(60ft)^2}{24(4936ksi)(86072in^4)} \overset{\text{adds}}{\downarrow} + \frac{5\,(85psf)\overset{\times 12}{(60ft)^4}}{384(4936ksi)(86072in^4)} = -0.5in \text{ (upward)}$$

Final camber:

$$\Delta_{mid} = \frac{-(336.3k)(60ft)^2}{24(1410ksi)(86072in^4)}\left[2(22.27in) + 12.91in\right] + \frac{5}{384}\cdot\frac{(85psf)(60ft)^4}{(1410ksi)(86072in^4)}$$

$$= -1in \text{ (upward)}$$

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9-[illegible]

Table 4.8.2 Suggested simple span multipliers to be used as a guide in estimating long-term cambers and deflections for typical prestressed members

	Without Composite Topping	With Composite Topping
At erection:		
(1) Deflection (downward) component—apply to the elastic deflection due to the member weight at release of prestress	1.85	1.85
(2) Camber (upward) component—apply to the elastic camber due to prestress at the time of release of prestress	1.80	1.80
Final:		
(3) Deflection (downward) component—apply to the elastic deflection due to the member weight at release of prestress	2.70	2.40
(4) Camber (upward) component—apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
(5) Deflection (downward)—apply to elastic deflection due to superimposed dead load only	3.00	3.00
(6) Deflection (downward)—apply to elastic deflection caused by the composite topping	—	2.30

FLEXURAL MEMBERS

Deflection calculations

under service loads

$$\left[(\Delta_{mid})_{LT}\right]_{\text{due to }(SW+P/S)} + \Delta_{\substack{\text{sustained}\\ \text{loads}}} + \underset{\text{short-term}}{\Delta_{S.T.}} = \Delta_{service}$$

$$\left[w_{sustained}\right] = 10\text{ psf} + 0.3(30\text{ psf}) = 19\text{ psf}, \quad w_{ST} = 21\text{ psf} = 30\text{ psf} - 9\text{ psf}$$

$$\Delta_{service} = -0.99\text{ in} + \Delta_{sust} + \Delta_{S.T.}$$

$$= -0.99\text{ in} + \frac{5}{384}\frac{(19\text{ psf})(12\text{ ft})(60\text{ ft})^4}{(1410\text{ ksi})(101953\text{ in}^4)} + \frac{5}{384}\frac{(21\text{ psf})(12\text{ ft})(60\text{ ft})^4}{(4936\text{ ksi})(90375\text{ in}^4)}$$

$$= -0.35\text{ in (upward)}$$

check: $L/240 = \frac{60\text{ ft}}{240} = 3\text{ in}, \quad \Delta_{service} < \text{limit}$

Note that immediate deflection to specified live load $< L/180$

$$\Delta_{imm.} = \frac{5}{384}\frac{(30\text{ psf})(12\text{ ft})(60\text{ ft})^4}{(4936\text{ ksi})(90375\text{ in}^4)} = 0.24\text{ in} < L/180 = 4\text{ in}$$

Alternatively, PCI provides a table (Table 4.8.2) that includes suggested simple span multipliers to be used as a guide in estimating long-term cambers and deflections for typical P/S members.

Case	Multiplier
self-weight	2.70
camber from P/S	2.45
superimposed DL	3.0

Using these,

initial camber $\Delta_{mid} = \overset{\text{camber due to P/S}}{-1.17\text{ in}} + 0.70\text{ in} = -0.47\text{ in}$

final camber $\Delta_{mid} = -1.17\text{ in} \times 2.45 + 0.7'' \times 2.70 = -0.97\text{ in}$

we calculated this as −0.99 in through rigorous calcs.

Final deflection:

$$\Delta = -0.97\text{ in} + \frac{5}{384}\frac{(10\text{ psf})(12\text{ ft})(60\text{ ft})^4}{(4936\text{ ksi})(90375\text{ in}^4)} + \frac{5}{384}\frac{(30\text{ psf})(12\text{ ft})(60\text{ ft})^4}{(4936\text{ ksi})(90375\text{ in}^4)}$$

$$= -0.49\text{ in (vs. } -0.35\text{ in)}$$

reasonably close, but this is not always the case

FLEXURAL MEMBERS

Flexural strength

Loading methods are similar to RC concrete

load = 1.2dead + 1.6live (from 1.4dead + 1.7live)

same ϕ factor; change based on economy

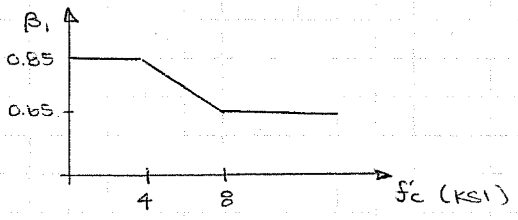
AASHTO prestressed concrete $\phi = 1.0$

└ P/S concrete is perfect, without any variability

$\phi M_n \geq M_u$

$\alpha_1 = 0.85, \beta_1 \longrightarrow$

$\varepsilon_{cu} = 0.003$ (negative)



Bonded tendons:

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}(w - w')\right]\right]$$

A_{ps}/bd_p

distance from extreme compression fiber to centroid of non-P/S reinforcement

factor accounting for the shape of the stress-strain relationship of the P/S steel

$\frac{\rho' f_y}{f'_c}$, ρ' = ratio of comp. reinforce.

$\frac{\rho f_y}{f'_c}$, $\rho = \frac{A_c}{bd}$

distance from extreme compression fiber to centroid of P/S.

$\gamma_p = 0.4$ for $f_{py}/f_{pu} \geq 0.85$

stress-relieved wire and strands, rebar

$\gamma_p = 0.28$ for $f_{py}/f_{pu} \geq 0.90$

low-lax wire, strands

$\gamma_p = 0.55$ for $f_{py}/f_{pu} \geq 0.80$

deformed bars

without non-~~RM~~ P/S reinforcing,

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f'_c}\right]$$

can only be used when P/S is in the tension zone. if there is compression steel, use strain compatibility

$$\underline{\underline{M_n = A_{ps} f_{ps}\left(d_p - a/2\right)}}$$

$$a = \frac{A_{ps} f_{ps}}{0.85 b f'_c}$$

$\phi M_n \geq 1.2 M_{cr}$ to ensure ductility

M_{cr} calculated by elastic theory with $f'_r = 7.5\sqrt{f'_c}$

change in strand strain ≥ 0.005

FLEXURAL STRENGTH

The flexural strength of prestressed members can be calculated using the same assumptions as for nonprestressed members

$\varepsilon_{cu} = -0.003$ (maximum comp. concrete strain)

$\phi M_n \geq M_u$

nominal flexural strength

$\phi = 0.90$

$M_u = M_f = \overset{1.2}{\cancel{1.4}} M_D + \overset{1.6}{\cancel{1.7}} M_L$

$\alpha_1 = 0.85$

$\beta_1 = 0.85$ (for $f_c' \leq 4000$ psi)

$= 0.85 - (f_c' - 4000)0.00005 \geq 0.65$ for $f_c' > 4000$p

$\varepsilon_{cu} = -0.003$

f_{ps} will vary depending on the amount of prestressing

f_{ps} can be obtained from strain compatibility analysis.

To avoid lengthy calculations, the code allows f_{ps} to be obtained by the approximate Equation 18.3

For members with Bonded Tendons

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\underbrace{\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]}_{> 0.17 \ \& \ d' < 0.15 d_p}\right)$$

γ_p = factor accounting for the shape of the stress-strain relationship of the prestressing steel

$\gamma_p = 0.4$ for $f_{py}/f_{pu} \geq 0.85$

i.e. stress-relieved wire and strands and plain bars

$\gamma_p = 0.28$ for $f_{py}/f_{pu} \geq 0.90$

i.e. low-relaxation wire and strands

$\gamma_p = 0.55$ for $f_{py}/f_{pu} \geq 0.80$

i.e. deformed bars

ρ_p = ratio of prestressed reinforcement $= \dfrac{A_{ps}}{b d_p}$

d = distance from extreme compression fiber to centroid of non-prestressed reinforcement

d_p = distance from extreme compression fiber to centroid of prestressed reinforcement

$w = \frac{\rho f_y}{f_c'}$, where ρ is the ratio of non-prestressed reinforcement.

(i.e., $\rho = A_s/bd$)

$w' = \frac{\rho' f_y}{f_c'}$, where ρ' is the ratio of compression reinforcement

For a fully prestressed member (with no nonprestressed tension or compression reinforcement), Eq. (18-3) reduces to

$$\boxed{f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f_c'}\right)}$$

This can further be written in nondimensional form as follows:

$$w_p = w_{pu}\left(1 - \frac{\gamma_p}{\beta_1} w_{pu}\right)$$

$$w_p = \frac{A_{ps} f_{ps}}{b\, d_p\, f_c'}$$

$$w_{pu} = \frac{A_{ps} f_{pu}}{b\, d_p\, f_c'}$$

* The moment strength of a prestressed member with bonded tendons may be computed using Eq 18.3 only when all the prestressed reinforcement is located in the tension zone.

* When part of the prestressed reinforcement is located in the compression zone of a cross-section involving d_p is not valid. For such a case use strain compatibility

The nominal strength of a fully prestressed bonded beam.

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

$$a = \frac{A_{ps} f_{ps}}{0.85 \, b f_c'}$$

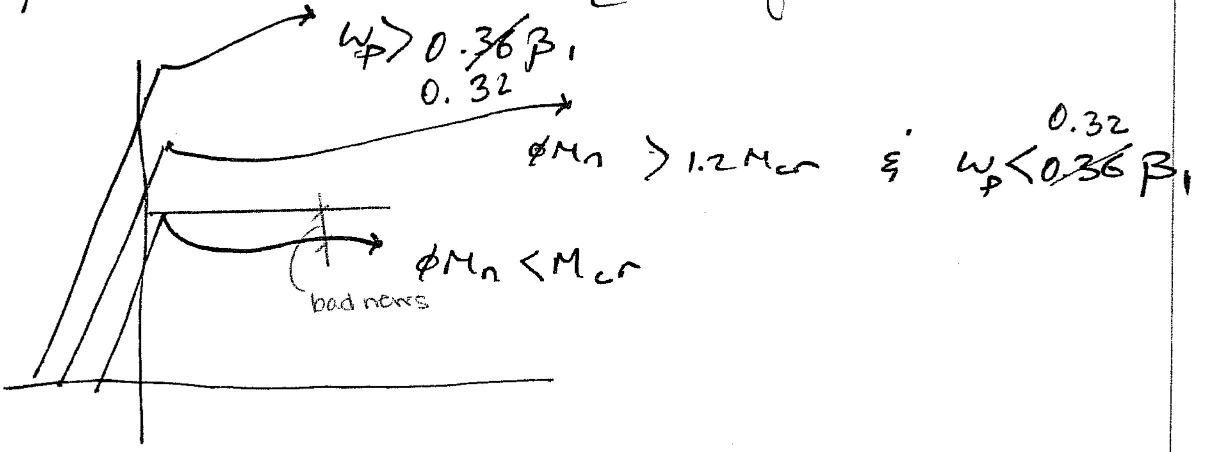
$$\left(\begin{array}{l} \text{or in nondimensional terms} \\ R_n = w_p (1 - 0.59 w_p) \\ R_n = \dfrac{M_n}{b d_p^2 f_c'} \end{array} \right)$$

Section 18.8.$\overset{2}{\cancel{3}}$ requires the total amount of prestressed and non prestressed reinforcement of flexural members to be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength.

$$\phi M_n \geqslant 1.2 M_{cr}$$

M_{cr} is computed by elastic theory using a modulus of rupture equal to $7.5\sqrt{f_c'}$

§18.8.$\overset{2}{\cancel{3}}$ is analogous to § 10.5 for R/C



§ 18.8.1 → the reinforcement index, $\omega_p = \dfrac{A_{ps} f_{ps}}{b\, d_p f_c'}$

$$\omega_p < 0.3\overset{2}{\cancel{6}}\beta_1$$

To ensure "yield" of the reinforcement

→ For a member w/ P/s bars and tension reinforcing bars

$$\omega_p + (\omega - \omega')\frac{d}{d_p} \leqslant 0.3\underset{2}{\cancel{6}}\beta_1$$

→ For a flanged section

$$\omega_{pw} + (\omega_w - \omega'_w)\frac{d}{d_p} \geq 0.36\beta_{2}$$

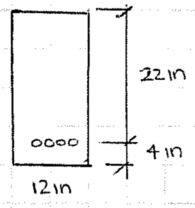
ω_{pw}, ω_w, ω' are reinforcement indices as computed for ω_p, ω, and ω' with

$b =$ web width

and the reinf. area is the area required to develop the compressive strength of the web only

FLEXURAL MEMBERS

Example



6 - 1/2 in low lax strands

$f'_c = 5\text{ ksi}$

$f_{pu} = 270\text{ ksi}$

what is M_n?

calculate f_{ps}:

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f'_c}\right]$$

$\gamma_p = 0.28$ (low lax strands)

$\beta_1 = 0.80$ ($f'_c = 5\text{ ksi}$)

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{6(0.153\text{ in}^2)}{(12\text{ in})(22\text{ in})} = 0.00348$$

$$f_{ps} = (270\text{ ksi})\left[1 - \frac{0.28}{0.80}(0.00348)\frac{270\text{ ksi}}{5\text{ ksi}}\right] = 252\text{ ksi} \longleftarrow \text{strand rupture does not control}$$

$$a = \frac{A_{ps} f_{pc}}{0.85 f'_c b} = \frac{6(0.153\text{ in}^2)(252\text{ ksi})}{0.85(5\text{ ksi})(12\text{ in})} = 4.54\text{ in}$$

$$M_n = A_{ps} f_{ps}(d_p - a/2) = 6(0.153\text{ in}^2)(252\text{ ksi})\left[22\text{ in} - \tfrac{1}{2}(4.54\text{ in})\right] = 380\text{ ft}\cdot\text{K}$$

$$\phi M_n = 0.9(380\text{ ft}\cdot\text{K}) = \underline{\underline{342\text{ ft}\cdot\text{K}}}$$

calculate ourselves to prove = 0.90

DESIGN PROCESS

Big decisions

- cross-section type → • structural system / construction sequence
- material properties
- tendon profile
- P/S force
- area of reinforcement

• span/depth ratios
• magnitude of loading
• location considerations

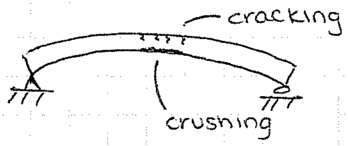
P/S force → stresses in tendons are limited by ACI (ch18) at jacking and transfer

Stress checks

Initial stage

- highest P/S force, P_i
- lowest concrete strength, f'_{ci}

Check both tension and compression limits



Final stage

- lowest P/S force, P_f
- concrete is strong, $>f'_c$
- moment is high, M_{max}

tensile stress limits generally control (more critical); use to determine P, e

Kern points

location of P/S to cause $f_{ct}=0$

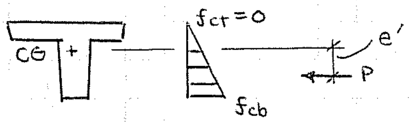
$K_b = \frac{S_t}{A}$ from centroid

$$f_{ct} = \frac{M\bar{y}_t}{I} - \frac{P}{A} = 0$$

$$\frac{Pe'}{S_t} - \frac{P}{A} = 0$$

S = section modulus

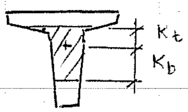
goal: no concrete in tension



$K_t = \frac{S_b}{A}$

Now you have a region from k_b to k_t in which there will be no tension

$f_c \leq 0$



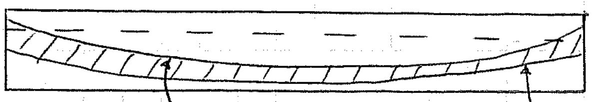
DESIGN PROCESS

Prestressing force

$$P_i \le \frac{M_{min}}{e-k_b}, \quad M_{min} = M_{DL}$$

$$P_f \ge \frac{M_{max}}{e+k_t}, \quad M_{max} \text{ includes all loads expected}$$

No tension in member



$$e_{min} = \frac{M_{max}}{P_f} - k_t \qquad e_{max} = k_b + \frac{M_{min}}{P_i}$$

in this range, no tension!

handout provides $e + k_t$ values

Design approach:

- at midspan, choose largest e possible
- from P_f equation (above), select smallest P_f possible
- detail.

$$P_f\,(k_t + k_b) \ge M_{max} - \frac{P_f}{P_i} M_{min}$$

Tension stress permitted

$$f_b \le f_t' \qquad \frac{P_i \cdot e}{S_t} - \frac{P_i}{A} - \frac{M_{min}}{S_t} \le f_t'$$

↖ or $0.5f_t'$ to be conservative

$$e \le k_b + \frac{M_{min} + S_t f_t}{P_i}$$

$$P_f \ge \frac{M_{max} - S_b f_b}{e + k_t}$$

$$e_{min} = \frac{M_{max} - S_b f_b}{P_f} - k_t$$

$$e_{max} = k_b + \frac{M_{min} + S_t f_t}{P_i}$$

- less abrupt harping in beams (easier to fabricate)
- more room for strand placement

6.8 THE DESIGN PROCESS

Desiger must decide on:
- cross section type
- material properties
- tendon profile
- prestressing force
- area of reinforement

Designer must also be mindful of:
- stress limits
- deflection limits / cracking
- load capacity

We'll look at all aspects of these.

6.8.1 CROSS-SECTION TYPE

- dictated by
  - structural system / construction sequence
  - span / depth ratios
  - magnitude of loading
- Figure 6-15 is useful guideline
- once prelim cross-section has been selected, can then calculate dead load of member and proceed to investigate design limits.

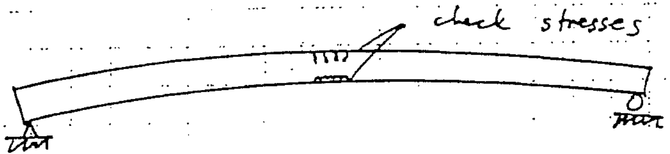
6.8.2 SATISFYING STRESS LIMITS

- Maximum stresses in tendons are controlled
  - at jacking
  - at transfer

  } see Ch. 18 ACI 318
- Maximum stresses in concrete are controlled as well
  - see Ch. 18
- In order to satisfy above, it is necessary to investigate conditions at various stage in life of beam
- At very least, must consider:

i) Initial Stage:
- here, prestress force has its highest value, P_i
- concrete still young, has low strength f_{ci}
- at midspan, the moment is at lowest value M_{min}
- at this stage, concerned with limiting:

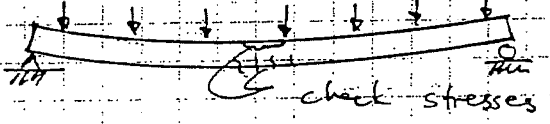
i) tensile stress on top face of beam
ii) compressive stress on bottom face of beam



ii) Final Stage:

- here, prestress force is at lowest value P_f
- concrete has reached compressive strength $\geq f'_c$
- moment is at its highest value, M_{max}

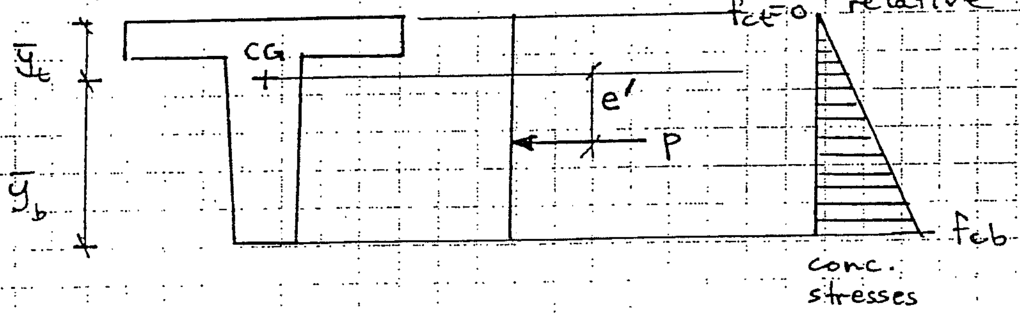
- at this stage, concerned about limiting tensile stress on bottom face & compressive stress on top face



In choosing : prestressing force, P ; tendon profile, e } tensile stress limit are usually more critical

KERN POINTS:

Consider a cross section subjected to section loads P, M where $M = P \cdot e'$ (e' is eccentricity of P relative to CG)



As e' is increased, top fibre stress starts as compressive, goes to zero, and then becomes tensile

At some point $f_{ct} = 0 \Rightarrow e' = k_b$

The distance k_b is known as the "bottom kern poin

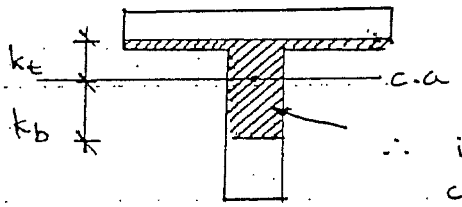
$$f_{ct} = \frac{M \cdot \bar{y}_t}{I} - \frac{P}{A} \Rightarrow \frac{P \cdot e'}{S_t} - \frac{P}{A} = 0 \Rightarrow e' = \frac{S_t}{A}$$

$$\therefore \boxed{k_b = \frac{S_t}{A}} \text{ from centroid}$$

- Similarly, to produce zero tensile stress on bottom fibre, line of action of P located distance k_t above centroid

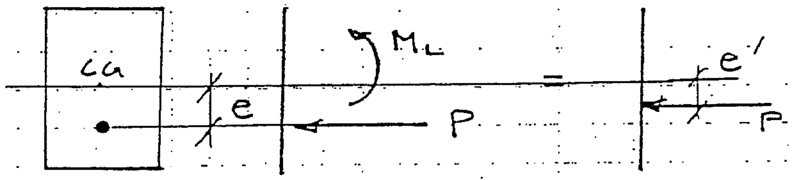
k_t = "top Kern point"

$$\boxed{k_t = \frac{S_b}{A}}$$



∴ if line of action of P falls in this region can be assured that $f_c \leq 0$ everywhere

- Now, total moment on cross section is combo of external moment (M_L) plus moment due to eccentricity of the tendon ($P \cdot e$)



$$Pe' = Pe - M_L$$

$$e' = e - \frac{M_L}{P}$$

- in order for no tension of top face

$$e' < k_b \Rightarrow e - \frac{M_L}{P} < k_b$$

$$\Rightarrow P \leq \frac{M_L}{e - k_b}$$

- similarly, for no tension on bottom face,

$$P \geq \frac{M_L}{e + k_t}$$

where e is eccentricity of <u>tendon</u>

- For initial condition: $M_L = M_{min}$ & we're concerned with tension on top

$$\therefore \quad \boxed{P_i \leq \frac{M_{min}}{e - k_b}} \quad * \quad (1)$$

Mmin = moment due to SW – no way moment can be smaller than this

- For final condition: $P = P_f$, $M = M_{max}$, and we're concerned with tension on bottom

$$\therefore \quad P_f \ge \frac{M_{max}}{e + k_t} \qquad (2)$$

e is tendon profile (+ve down from CG) ?

- Figure 6-23 gives approximate values of $(e + k_t)$ for typical prestressed sections ; these are useful for prelim design
- The preceding two equations can be combined to give:

$$P_f\,(k_t + k_b) \ge M_{max} - \frac{P_f}{P_i}\,M_{min} \qquad (3)$$

The term $(k_t + k_b)$ is the distance the compressive force can move without producing tension in concrete

Typical values of $(k_t + k_b)$ given in Fig. 6-21

- Design approach : i) at midspan, choose largest e possible
  ii) from (2), select smallest P_f possible
  detail accordingly

IF TENSILE STRESSES PERMITTED:

- ie. partially prestressed : $f_b > 0$ $(\le 0.5\, f'_c)$
- Initial Conditions : $P = P_i$; $M = M_{min}$
  let f_t be tensile stress permitted on top face

$$\frac{P_i \cdot e}{S_t} - \frac{P_i}{A} - \frac{M_{min}}{S_t} \le f_t$$

$$\Rightarrow \quad e \le k_b + \frac{M_{min} + S_t f_t}{P_i}$$

Final Conditions: $P = P_f$, $M = M_{max}$

let f_b be tensile stress permitted on bottom face

$$\frac{M_{max}}{S_b} - \frac{P_f}{A} - \frac{P_f \cdot e}{S_b} < f_b$$

$$\Rightarrow \quad P_f \geq \frac{M_{max} - S_b \cdot f_b}{e + k_t}$$

- Fig 6-23 illustrates the range of tendon eccentricities corresponding to above discussions

- Apparent why parabolic tendon profiles used; straight tendons usually won't meet stress criteria

- Remember to check compressive stresses too!

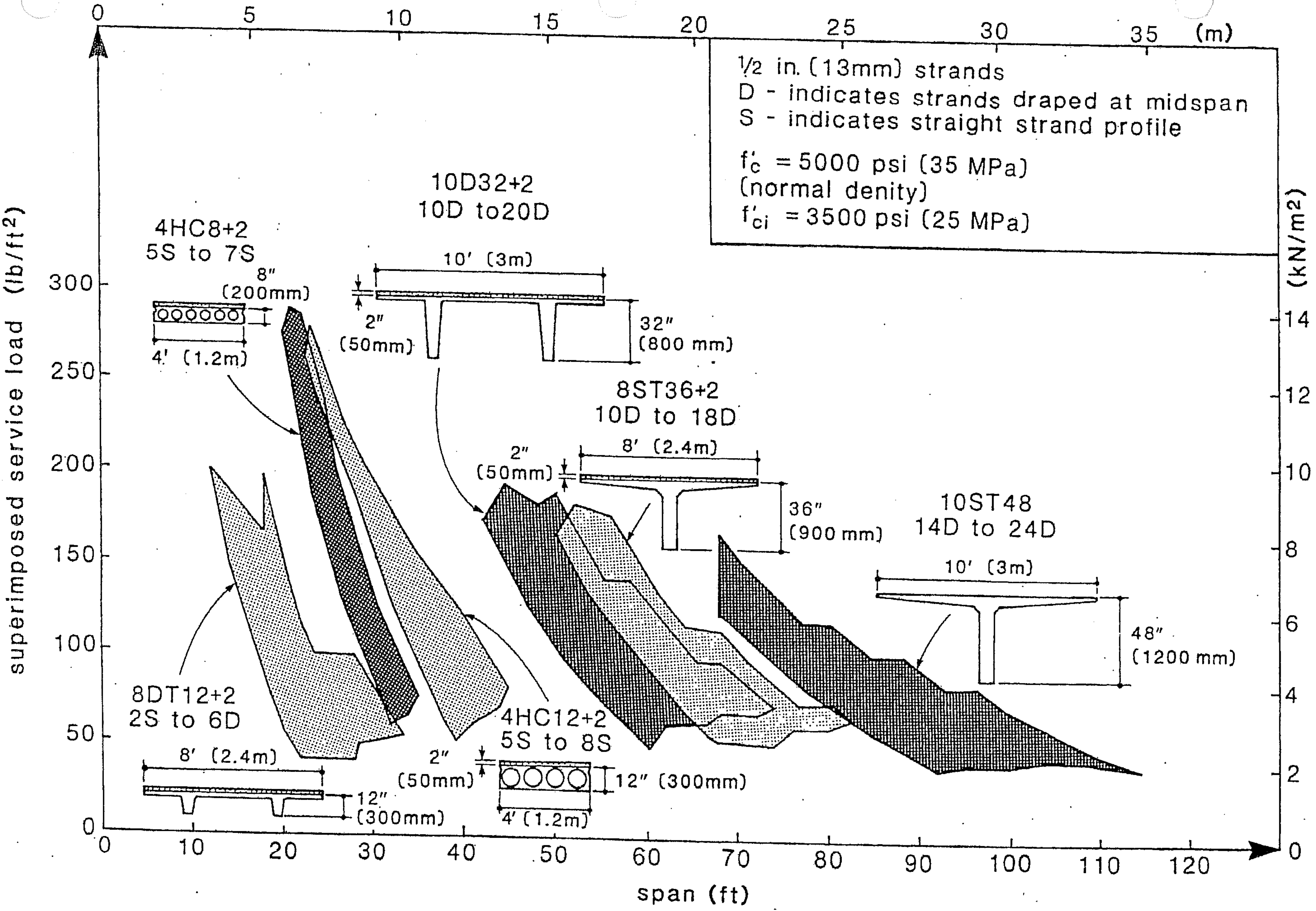


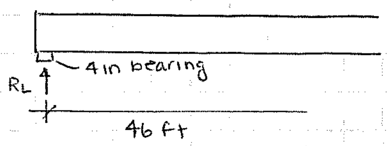
Figure 6-15 Span and load ranges for standard deck elements.

DESIGN PROCESS

Example:

Untopped double T	loads:	strands:
span = 46 ft	LL = 50 psf	$f_p = 0.74 f_{pu} \sim 200\text{k}$
$f'_{ci} = 3.5$ ksi	SDL = 15 psf	1/2" low lax strands
$f'_c = 5$ ksi	normal weight	
width = 8 ft	concrete	

Design:



Step 1: choose depth of section

span/depth = 20 to 30

Select PCI 8DT24 ← 24 in deep

$s/d = 23$

Step 2: choose prestressing

need Mmax, Mu

— from PCI book (and handout), SW = 52 psf = 418 plf

— superimposed dead load = (15 psf)(8 ft) = 120 plf

— live load = (50 psf)(8 ft) = 400 plf

$$M_{max} = \frac{WL^2}{8} = \frac{(46\text{ ft})^2}{8}(418\text{ plf} + 120\text{ plf} + 400\text{ plf}) = 248\text{ k}\cdot\text{ft}$$

$$M_u = \frac{(46\text{ ft})^2}{8}\left[1.2(418\text{ plf} + 120\text{ plf}) + 1.6(400\text{ plf})\right] = 340\text{ k}\cdot\text{ft}$$

tendon profile

from handout,

$$e + k_t = 0.7h = 0.7(24\text{ in}) = 16.8\text{ in}$$

$$e + k_b = 0.43h = 0.43(24\text{ in}) =$$

to satisfy bottom fiber stress limit,

$$7.5\sqrt{f'_c} = 530\text{ psi}$$

$$P_f \geq \frac{M_{max} - S_b f_b}{\underset{\nearrow \text{assume}}{e + k_t}} = \frac{248\text{ k}\cdot\text{ft} - (1224\text{ in}^3)(0.530\text{ ksi})}{16.8\text{ in}}$$

$P_f \geq 138.5\text{k}$, no cracking

$$\boxed{\begin{array}{c} f_{pi} = 200\text{ ksi}, \text{ elastic shortening} \rightarrow 187\text{ ksi} \\ \text{creep, etc} \rightarrow 157\text{ ksi} \end{array}}$$

So, $P_f = f_{pf} A_p = (157\text{ ksi})A_p = 138.5\text{k}$

$A_p = 0.88\text{ in}^2$ or greater

DESIGN PROCESS

Example, cont'd

Step 2, cont'd

If the design is controled by strength requirements

$\phi M_n = M_u \sim 0.77h \; A_{ps} f_{pu}$

similar to 0.9d for RC structures (assumed)

$(0.9)\, 340\,k\cdot ft = 0.77(24\,in)\, A_{ps}\, (270\,ksi)$

$A_{ps} = 0.82\,in^2$

$6 - \tfrac{1}{2}'' \phi$ strands $\rightarrow$ $0.92\,in^2$

satisfies both calculations ✓

Six strands, 3 in each leg

Step 3: choose tendon profile

e_{max} at midspan, ends

reduce eccentricity to minimize top fibre stresses

$6\sqrt{f'_{ci}} = 355$ psi at ends

$= 178$ psi at middle

$$e \le k_b + \frac{M_{min} + S_t f_t}{P_i}$$

$$k_b = \frac{S_t}{A} = \frac{3063\,in^3}{401\,in^2} = 7.64\,in$$

M_{min} = member self-weight moment at $l_d = 25$ in from end

only 23 in from bearing ℄

$$= (0.418\,klf)\frac{(46\,ft)}{2}(23\,in) - (0.418\,klf)\frac{(23\,in)^2}{2} = \frac{wL}{2}x - \frac{wx^2}{2}$$

$= 212$ k·in

$P_i = 6(0.153\,in^2)(187\,ksi) = 172\,k$

$$e \le 7.64\,in + \frac{(212\,k\cdot in) + (3063\,in^3)(0.355\,ksi)}{172\,k}$$

$e \le 15.2$ in — that's pretty big; tension at end does not control (likely)

can/should now check at midspan

choose a strand pattern similar to 8DT24 88D1, except use 6 strands, not 8

$e_e = 9.15$ in

$e_c = 14.65$ in

/ we can choose other eccentricities as needed; these slightly vary from design handbook values

DESIGN PROCESS

Example (cont'd)

Step 4: check concrete stresses

At support, 0.4L, midspan, initial and final checks

Stress limits

	initial	final
comp.	-2100 psi	-2250 -3000 psi
tension	177 355 psi	530 psi

sustained

member ends

Initial case

50d_b	0.4L	midspan
$f = {}^{-P_i}/_A +/- {}^{P_i e}/_S +/- {}^{M_{min}}/_S$		
f_t = 43 psi ✓	f_t = -84 psi ✓	f_t = -40 psi ✓
f_b = -1623 psi ✓	f_b = -1294 psi ✓	f_b = -1404 psi ✓
M_{min} = 212 k·in	M_{min} = 1273 k·in	M_{min} = 1327 k·in

all meet limits easily

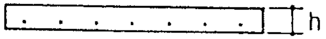
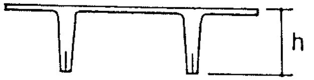
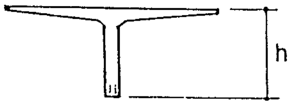
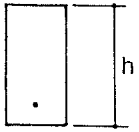
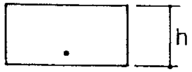
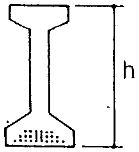
Final calculations

50d_b	0.4L	0.5L
$f = {}^{-P_f}/_A +/- {}^{P_f e}/_S +/- {}^{M_{max}}/_S$		
f_t = 13 psi	f_t = -655 psi	f_t = -642 psi
f_b = -1290 psi	f_b = 381 psi	f_b = 349 psi
M_{max} = 248 k·in	M_{max} = 2858 k·in	M_{max} = 2977 k·in

again, all are acceptable

look at stresses – 0.4L has larger values than midspan. M decreases by square, eccentricity linearly.

Step 5: deflection calculations

Type of element	Live load kN/m2	Span/depth , ℓ/h ratio
	<dead load	40
	2.4 4.8	40-50 32-42
	2.4 4.8	20-30 18-28
	2.4 4.8	23-32 19-24
	<dead load	20
	<dead load	30
	highway loading	18

ure 6-19 Typical Span-to-Depth Ratios for Simply Supported Prestressed Concrete Mem-
s.

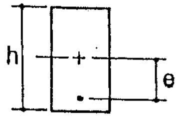
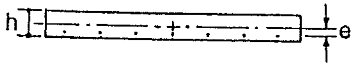
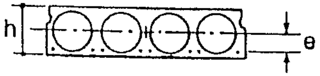
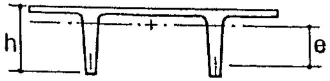
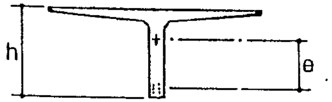
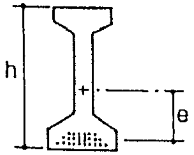
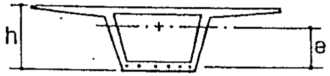
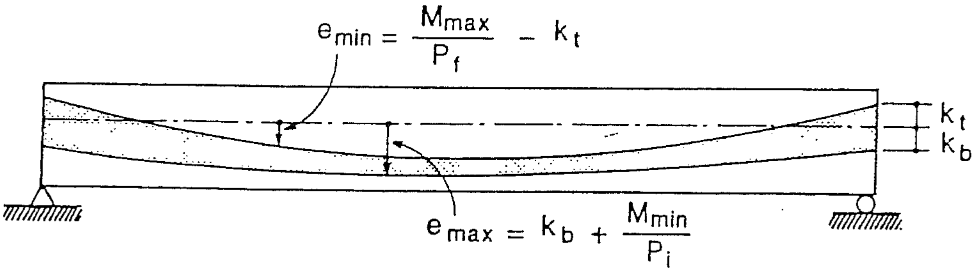
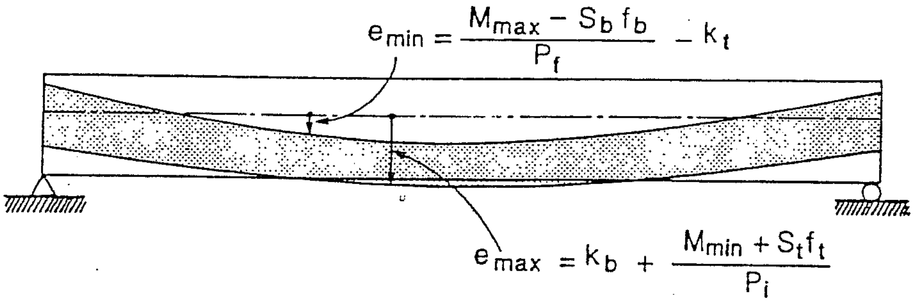
Cross-section shape	$e + k_t$	$k_t + k_b$
	0.50 h	0.33 h
	0.47 h	0.33 h
	0.58 h	0.49 h
	0.70 h	0.43 h
	0.76 h	0.48 h
	0.64 h	0.51 h
	0.82 h	0.56 h

Figure 6-21 Approximate Values of Flexural Lever Arms for Preliminary Service Load De-



(a) No tension permitted



(b) Tensile stresses permitted

Figure 6-23 Range of Tendon Eccentricities Corresponding to Stress Limits.

Strand Pattern Designation

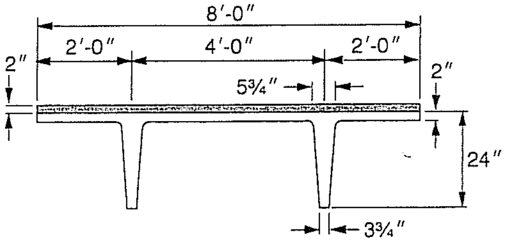
No. of strand (10)
S = straight D = depressed
108-D1
No. of depression points
Diameter of strand in 16ths

Safe loads shown include dead load of 10 psf for untopped members and 15 psf for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

Key
173 — Safe superimposed service load, psf
0.5 — Estimated camber at erection, in.
0.7 — Estimated long-time camber, in.

DOUBLE TEE

8′-0″ x 24″
Normal Weight Concrete



f'_c = 5,000 psi
f_{pu} = 270,000 psi

Section Properties

		Untopped		Topped	
A	=	401	in²	—	
I	=	20,985	in⁴	27,720	in⁴
y_b	=	17.15	in.	19.27	in.
y_t	=	6.85	in.	6.73	in.
S_b	=	1,224	in³	1,438	in³
S_t	=	3,063	in³	4,119	in³
wt	=	418	plf	618	plf
		52	psf	77	psf
V/S	=	1.41	in.		

8DT24

Table of safe superimposed service load (psf) and cambers (in.)

No Topping

Strand Pattern	e_e, in. e_c, in.	Span, ft																						
		30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74
68-S	11.15 11.15	173 0.5 0.7	147 0.6 0.8	126 0.6 0.8	108 0.7 0.8	92 0.7 0.8	79 0.7 0.8	68 0.7 0.8	58 0.7 0.8	50 0.7 0.7	43 0.6 0.6	36 0.6 0.4	30 0.5 0.2											
88-S	9.15 9.15		180 0.7 0.9	155 0.7 0.9	134 0.8 1.0	116 0.8 1.0	100 0.8 1.0	87 0.8 1.0	76 0.9 1.0	66 0.8 0.9	57 0.8 0.8	49 0.8 0.7	43 0.7 0.5	36 0.6 0.3	31 0.5 0.1									
88-D1	9.15 14.40				190 1.1 1.5	166 1.2 1.5	146 1.3 1.6	129 1.4 1.7	114 1.5 1.8	100 1.5 1.8	89 1.6 1.8	79 1.6 1.8	70 1.6 1.7	62 1.6 1.6	54 1.6 1.5	48 1.5 1.4	42 1.4 1.2	37 1.3 0.9	32 1.2 0.5					
108-D1	7.15 14.15								145 1.7 2.2	129 1.8 2.2	116 1.9 2.3	103 2.0 2.3	92 2.0 2.3	83 2.1 2.3	74 2.1 2.2	66 2.1 2.1	59 2.1 2.0	53 2.0 1.8	47 2.0 1.6	42 1.8 1.3	37 1.7 1.0	32 1.5 0.5		
128-D1	5.48 13.90															83 2.5 2.7	75 2.5 2.7	68 2.5 2.5	61 2.5 2.3	55 2.5 2.1	49 2.4 1.8	44 2.3 1.5	40 2.1 1.1	35 1.9 0.6
148-D1	4.29 13.65																				61 2.9 2.6	55 2.9 2.3	50 2.8 1.9	45 2.6 1.5

8DT24+2

Table of safe superimposed service load (psf) and cambers (in.)

2″ Normal Weight Topping

Strand Pattern	e_e, in. e_c, in.	Span, ft																			
		26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64
48-S	14.15 14.15	183 0.4 0.4	149 0.4 0.4	122 0.4 0.4	100 0.5 0.4	82 0.5 0.4	66 0.5 0.3	53 0.5 0.3	42 0.5 0.2	33 0.5 0.0											
68-S	11.15 11.15			175 0.5 0.5	147 0.6 0.6	123 0.6 0.6	103 0.7 0.6	86 0.7 0.5	72 0.7 0.4	60 0.7 0.3	49 0.7 0.2	39 0.7 0.0									
68-D1	11.15 14.65					184 0.7 0.8	156 0.8 0.8	133 0.9 0.8	113 0.9 0.8	96 1.0 0.7	81 1.0 0.7	69 1.0 0.6	58 1.0 0.5	48 1.0 0.3	39 1.0 0.1						
88-D1	9.15 14.40							190 1.1 1.1	165 1.2 1.1	143 1.3 1.2	124 1.4 1.2	107 1.5 1.1	93 1.5 1.1	80 1.6 0.9	69 1.6 0.8	59 1.6 0.6	51 1.6 0.4	43 1.6 0.1			
108-D1	7.15 14.15										142 1.7 1.5	124 1.8 1.5	109 1.9 1.4	96 2.0 1.3	84 2.0 1.2	74 2.1 1.0	64 2.1 0.7	56 2.1 0.5	48 2.1 0.1		
128-D1	5.48 13.90																	74 2.5 1.0	65 2.5 0.7	57 2.5 0.3	49 2.5 0.1

Strength based on strain compatibility; bottom tension limited to $12\sqrt{f'_c}$; see pages 2-2–2-6 for explanation.
Shaded values require release strengths higher than 3500 psi.

DESIGN PROCESS

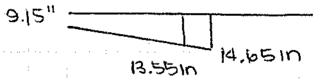
Example (cont'd)

Step 5: deflection calculations / flexural strength

Mu at midspan
0.96Mu at 0.4L

L = 46 ft

$M_u = 340\ k\cdot ft$, $0.96M_u = 326\ k\cdot ft$



at 0.4L:

$d_p = 13.55\ in + 6.85 in = 20.4 in$
$\uparrow y_t$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$

$$= (270\ ksi)\left[1 - \frac{0.28}{0.8}\cdot\frac{0.92\ in^2}{(96 in)(20.4 in)}\cdot\frac{270\ ksi}{5\ ksi}\right] = 268\ ksi$$

$$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{(0.92 in^2)(268\ ksi)}{0.85(96 in)(5\ ksi)} = 0.60 in$$

$c = \frac{a}{\beta_1}$ = small — large strain in strands, use $\phi = 0.9$

check: $\omega_p = \frac{A_{ps}f_{ps}}{bd_pf'_c} = 0.025 < 0.32\beta_1 = 0.256$

$$\phi M_n = 0.9\,(0.92 in^2)(268\ ksi)(20.4 in - \tfrac{1}{2}(0.60 in)) = 372\ k\cdot ft$$

$$\frac{\phi M_n}{M_u} = \frac{372\ k\cdot ft}{\underset{326}{\cancel{340}}\ k\cdot ft} = 1.14$$

check: $d - a/2 = 20.4 in - 0.3 in = 20.1 in = 0.84h$

we assumed $M_n \sim 0.77h$...
conservative assumption

At midspan,

$\phi M_n = 392\ k\cdot ft$

$\frac{392\ k\cdot ft}{340\ k\cdot ft} = 1.15$ ← about the same reserve strength at 0.4L and at midspan

DESIGN PROCESS

Example (cont'd)

Step 6: check reserve strength after cracking

$f_r = 7.5\sqrt{f_c'} = 530\text{psi}$

f_b at midspan, long-term = 349 psi

$f_r - f_b = 181\text{psi}$

↑ additional stress required to crack the bottom fiber

$\Delta f \cdot S_b = M_{add}$ for cracking

$= (181\text{ psi})(1224\text{ in}^3) = 222\text{ k}\cdot\text{in}$

$M_{cr} = M_{add} + M_{\text{for } 349\text{ psi}} = 2977\text{ k}\cdot\text{in} + 222\text{ k}\cdot\text{in}$
$= 3199\text{ k}\cdot\text{in}$

$\frac{\phi M_n}{M_{cr}} = \frac{392\text{ k}\cdot\text{ft}}{3199\text{ k}\cdot\text{in}} = 1.47 > 1.20$ — enough reserve strength after cracking

Step 7: check deflections

(see earlier notes)

* don't submit Δ calcs in HW 8

SHEAR DESIGN

ACI Provisions

$V_c + V_s \le 10\sqrt{f'_c}\, b_w d$

$= 0.25 f'_c\, b_w d$ AASHTO

consider $f'_c = 10000$ psi

$V_{n_{ACI}} \le 1000$ psi

$V_{n_{AASHTO}} \le 2500$ psi

AASHTO lets you "buy" strength with stirrups. Lots of steel → lots of large shear cracks. Steel does not engage until concrete cracks.

Alejandro's research is maxing out at 0.16 roots

Modified compression Field Theory – tricky and dangerous. Question yourself, and the calcs.

introduction

$V_u \le \phi V_n$, $V_n = V_c + V_s$

$V_u \le \phi V_c + \phi V_s$

the strength of concrete is limited in shear calculations

$\sqrt{f'_c} \le 300$ psi, $f'_c \le 10$ ksi

SHEAR DESIGN

Introduction

$\sqrt{f'_c} \leq 100$ psi, cannot take advantage of higher strength of concrete

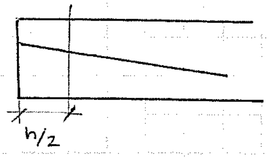
UNLESS: minimum shear reinforcement is provided

Maximum Factored Shear

located $d/2$ from support for RC concrete

$h/2$ for P/S concrete

↑ eccentricity changes so much at the end of a beam



capacity of concrete

$$V_c = \underbrace{\left[0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u} \right]}_{[2\sqrt{f'_c} - 5\sqrt{f'_c}]\ \text{limits}} b_w d$$

$b_w d$: shear area

$\frac{d}{a}$, inverse of shear span-to-depth ratio – shear strength of concrete increases with decreasing a/d

Simplified Method:

– effective P/S force > $0.4 f_{pu} A_p$

$f_p \sim 100$ ksi

Examples: PT with losses

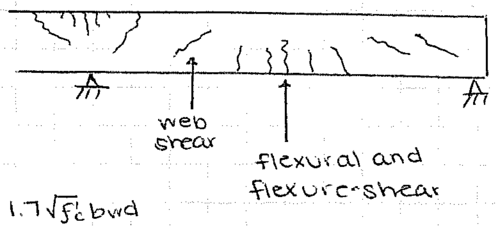
transfer length region

$V_c \leq V_{cw}$ from more complicated method

Detailed Method

considers flexure-shear cracking

web-shear cracking



Flexure-shear cracking

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d + \frac{\overset{V_u - V_d}{V_i}\, M_{cr}}{\underset{M_u - M_d}{M_{max}}} + V_d \geq 1.7\sqrt{f'_c}\, b_w d$$

dead load contributions removed from factored values

M_{cr}: $f_r = \frac{My}{I} + \frac{P}{A}$ possibly done for calibration reasons

V_d : shear force from unfactored dead load (noncomposite)

: . . . unfactored SW and superimposed DL (composite)

V_i : The factored shear force resulting from externally applied loads that cause M_{max}

– Flexure-shear cracks need flexural cracks first, which come from large moments

$V_i = V_u - V_d$

SHEAR DESIGN

Capacity of concrete

Detailed method (cont'd)

Web shear concrete

PS adds to V_n

$$V_{cw} = \left[3.5\sqrt{f'_c} + 0.3 f_{pc}\right] b_w d + V_p$$

principal diagonal tension in the web exceeds the tensile strength of concrete

V_p: vertical component of the P/S force

V_{cw} usually governs for heavily P/S beams with thin webs, near simple supports

Alternate method

- compute shear force from DL + LL
- find principal stress of $4\sqrt{f'_c}$ at the centroidal axis

7. PRESTRESSED CONCRETE - SHEAR (ACI-318-05)

- The design of prestressed concrete members for shear is essentially the same as for reinforced concrete members.

- Only the computation of nominal shear strength provided by Concrete V_c and several details of design differ from the procedures used for Reinforced Concrete.

$$V_u \leqslant \phi V_n \qquad \text{——} \qquad \text{Eq'n } (11\text{-}1)$$

$$V_n = V_c + V_s \qquad \text{Eq'n } (11\text{-}2)$$

concrete contribution $= 2\sqrt{f'_c}\,bd$

steel contribution $= \dfrac{A_s f_y d}{s}$

A_s = steel for shear, A_{sv}, not all (or longitudinal) steel

Therefore

$$V_u \leqslant \phi V_c + \phi V_s$$

The nominal shear strength provided by concrete, V_c, is assumed to be equal to the shear causing inclined cracking in concrete.

7.1 Concrete Strength

Section 11.1.2 restricts the concrete strength that can be used in computing the concrete contribution because of a lack of shear test data for high strength Concrete. The limit does not allow $\sqrt{f'_c}$ to be greater than 100 psi

Hence $f'_c \leqslant 10\,000$ psi or 10 ksi

(- The limit can be exceeded if a multiple of the minimum shear reinforcement is provided as specified in § 11.1.2.1)

7.2 Location for Computing Maximum Factored Shear

§ 11.1.3 allows V_u to be computed at a distance from the face of the support, provided that

(i) the support reaction, in the direction of the applied shear, introduces compression into the end regions of the member.

(ii) loads are applied at or near the top of the member

(iii) no concentrated load occurs between the face of the support and the critical section

For P/s concrete members the critical section for computing V_u is located $\boxed{h/2}$ from the face of the support Except for inverted tees — shear at support

7.3 Shear strength provided by concrete for prestressed members

Section 11.4 provides two approaches to determining the nominal shear strength provided by concrete. V_c.

7.3.1 Simplified Method

the use of this simplified method is limited to prestressed members with an effective prestress force not less than 40% $f_{pu} A_p$ (i.e. > 40% of the tensile strength of the prestressing reinforcement).

- Originally proposed by MacGregor & Hanson (1969)

$$V_c = \left(0.6 \sqrt{f'_c} + 700 \frac{V_u d}{M_u}\right) b_w d \qquad \text{Eq } (11.9)$$

$$2 \sqrt{f'_c}\, b_w d \leq V_c \leq 5 \sqrt{f'_c}\, b_w d$$

and

$$V_c \leq V_{cw} \; (\S 11.4.2.2)$$

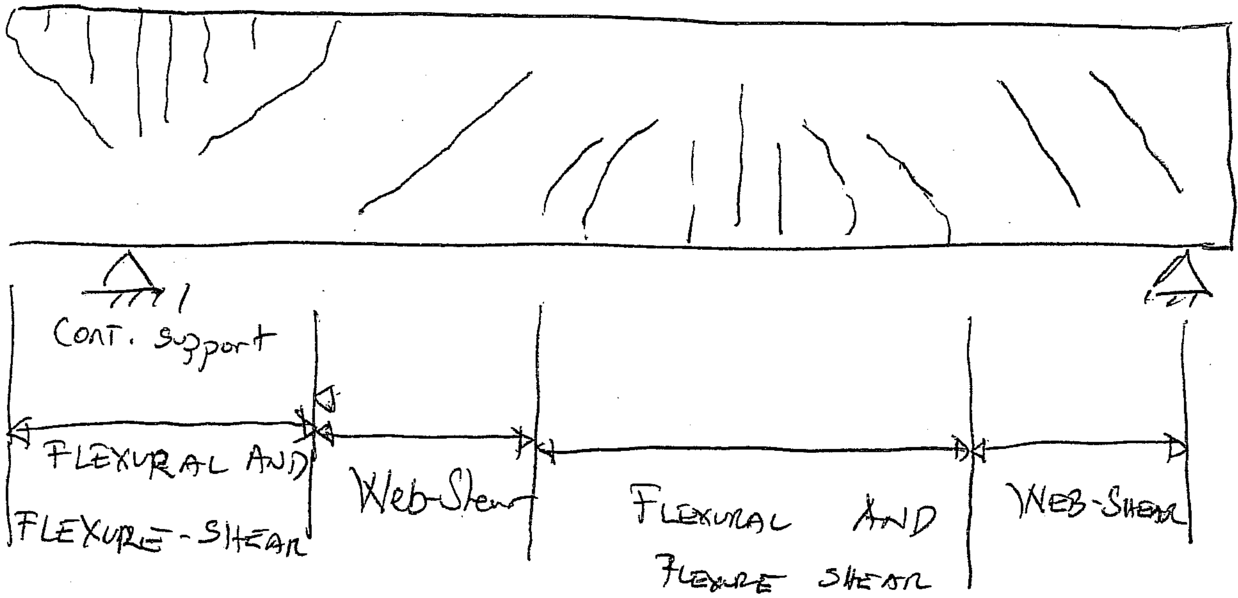
$\downarrow$

computed considering the effects of transfer length and debonding (§11.4.4) which apply in regions near the ends of pretensioned members.

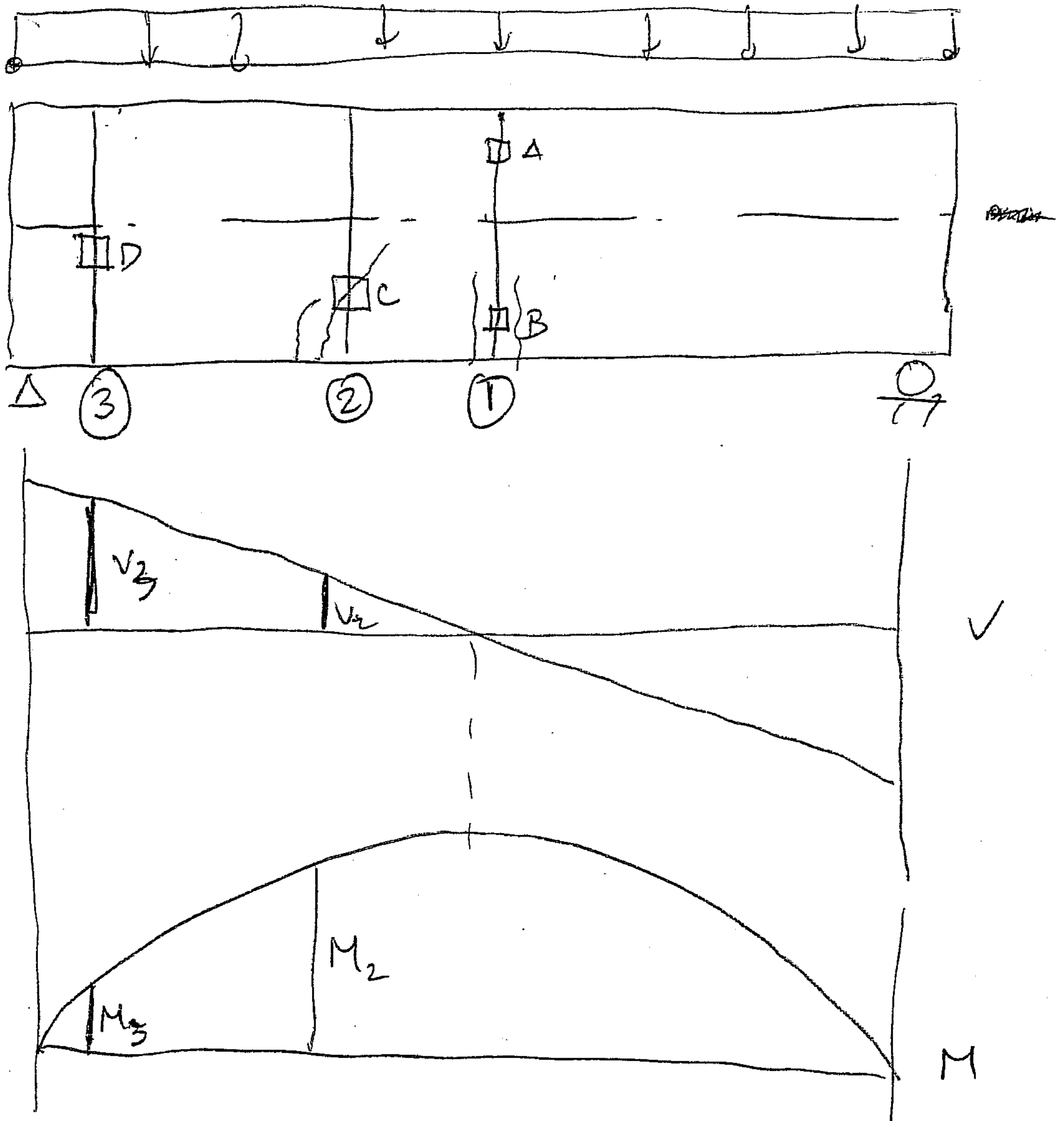
7.3.2 DETAILED METHOD

Two types of cracks have been observed in P/s concrete members:
- flexure - shear cracking
- web - shear cracking.

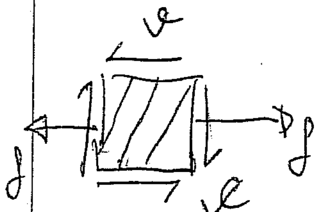
Since the nominal shear strength from concrete is assumed to be equal to the shear causing inclined cracking of concrete, the detailed method provides equations to determine the nominal shear strength for both types of cracking.



-The nominal shear strength provided by concrete, V_c, is taken as the lesser shear causing the two types of cracking



Sec ②

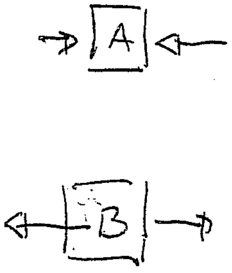


$f = \frac{My}{I} =$ flexure

$\tau = v = \frac{VQ}{It} =$ shear

flexure & shear somewhat balanced

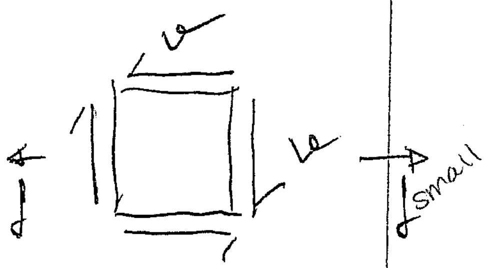
Sect ①



No Shear Stresses

flexural stresses only

Sec ③



- Significant Shear Stresses
- Minimal flexural stress

∴ Failure is a function of V

7.3.2.1 Flexure Shear Cracking

Flexure-shear cracking occurs when flexural cracks, which are initially vertical, become inclined under the influence of shear.

$$V_{ci} = 0.6\sqrt{f_c'}\, b_w d + \frac{\overset{(V_u - V_d)}{V_i}\, M_{cr}}{\underset{(M_u - M_d)}{M_{max}}} + V_d$$

$$V_{ci} \geq 1.7\sqrt{f_c'}\, b_w d \quad \text{---Eq}(11.10)$$

$M_{cr} \rightarrow$ calculate using $f = \frac{My}{I} + \frac{P}{A}$

$\downarrow$

= modulus of rupture

or alternatively can use the code expression

no explanation why not $7.5\sqrt{f_c'}$

$$M_{cr} = \left(\frac{I}{\underset{\uparrow\ \text{tension}\ \equiv y_b}{y_{tens.}}}\right)\left(6\sqrt{f_c'} + \underset{\downarrow}{f_{pe}} - f_d\right)$$

due to effective prestress after losses

V_{ci} typically governs for member subject to uniform loading

V_{ci} = Sum of three parts

Three Parts of Equation

1. The shear force required to transform a flexural crack into an inclined crack - $0.6\sqrt{f_c'}\, b_w d$
2. The unfactored dead load shear force - V_d
3. The portion of the remaining factored shear force that will cause a flexural crack to initially occur - $V_i M_{cr}/M_{max}$

For noncomposite members

V_d = shear force caused by the unfactored dead load

For composite members

V_d = unfactored selfweight + unfactored superimp. dead load

-The load combination used to determine V_i and M_{max} is the one that causes maximum moment at the section under consideration.

V_i = the factored shear force resulting from externally applied loads occurring simultaneously with M_{max}

$$V_i = V_u - V_d$$

* For composite members V_i, may be determined by subtracting V_d from the shear force resulting from the total factored loads, V_u.

Similarly $M_{max} = M_u - M_d$

* When calculating the cracking moment M_{cr}, the load used to determine f_d is the same unfactored load used to compute V_d.

7.3.2.2 Web-Shear Cracking, V_{cw}

* Web-Shear cracking occurs when the principal diagonal tension in the web exceeds the tensile strength of the concrete.

$$V_{cw} = \left(3.5\sqrt{f_c'} + 0.3 f_{pc}\right) b_w d + V_p$$

f_{pc} is the compressive stress in concrete (after allowance for all prestress losses) at the centroid of the cross-section. (+'ve quantity)

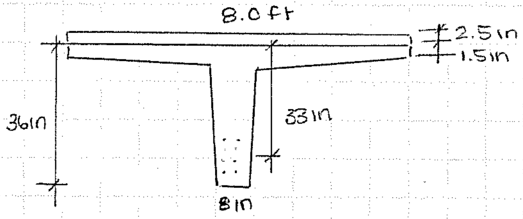
V_p = is the vertical component of the effective prestress force, (which is only present when strands are draped or harped)

V_{cw} usually governs for ~~[illegible]~~ heavily prestressed beams with thin webs, especially when the beam is subject to large concentrated loads near simple supports.

→ An alternate method for determining the web shear strength V_{cw} is to compute the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f'_c}$ at the centroidal axis of the member.

SHEAR DESIGN

Example



$f'_c = 5$ ksi ~~Na~~ lightweight, $w_c = 120$ pcf

$f'_{c_{top}} = 4$ ksi ($w_c = 150$ pcf)

12 - ½" ϕ strands, 270 ksi
single depression point

L = 60 ft

dead load = 725 plf (includes topping)

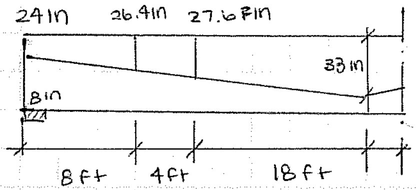
1.2DL + 1.6LL = 2.24 klf

$f_{pf} = 150$ ksi

$A = 570$ in^2
$I = 68{,}917$ in^4
$y_b = 26.01$ in
$y_t = 9.99$ in

} precast single-T

composite $y'_b = 29.27$ in



V(x=0) = 67.2 k
V(x=8in) = 62.1 k
+ h/2

- §11.0: d need not be taken as less than 0.8h for shear strength calcs.

  0.8(38.5 in) = 30.8 in or 2.57 ft

- check if we can use Eq. 11.9

  $f_{sc} = 150$ ksi $> 0.40(270$ ksi$) = 108$ ksi

  ∴ we can use the simplified expression

- typical calculations using Eq. 11.9 for a section 8 ft from the support

$$V_u = \left[\frac{60\text{ ft}}{2} - 8\text{ ft}\right] \times 2.24\text{ klf} = 49.3\text{ k}$$

$$M_u = (30\text{ft})(2.24\text{klf})(8\text{ft}) - (2.24\text{klf})\cdot\tfrac{1}{2}(8\text{ft})^2 = 466\text{ k}\cdot\text{ft}$$

For the composite section,

$$d = 26.40\text{ in} + 2.5\text{ in} = 28.9\text{ in} < 0.8h = 30.8\text{ in}$$

$$V_c = \left[0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right] b_w d \qquad [\text{Eq. } 11.9]$$

but not less than $2\sqrt{f'_c}\, b_w d$

nor greater than $5\sqrt{f'_c}\, b_w d$

SHEAR DESIGN

Example, cont'd

concrete contribution

$$V_c = \left[0.6\,(\underset{\substack{\uparrow\\ \text{factor to account}\\ \text{for lightweight concrete}}}{0.85})\sqrt{5000\,\text{psi}} + 700\,\frac{(49.3\,\text{K})(\underset{\substack{\uparrow\\ \text{d comp.}}}{28.9\,\text{in}})}{466\,\text{K}\cdot\text{ft}}\right](8\,\text{in})(\underset{\substack{\uparrow\\ 0.8h}}{30.8\,\text{in}})$$

$$V_c = 52.8\,\text{K}$$

$$\left.\begin{array}{l} 2\,\text{roots} = 29.6\,\text{K} \\ 5\,\text{roots} = 74.8\,\text{K} \end{array}\right\rangle \text{both bound } V_c\text{, so calculation is ok}$$

$\uparrow$ use $0.85\sqrt{f'_c}\,b_w d$, with $d = 0.8h$

Comments:

1. $V_u = 49.3\,\text{K}$
   $\phi V_c = 39.6\,\text{K}$ – need to add stirrups
2. For simply supported members subject to uneven loads, $\frac{V_u d}{M_u}$ term in Eq. 11.9 becomes a simple function of d/ℓ, where ℓ is the simple span length.

$$V_c = \left[0.6\sqrt{f'_c} + 700d\,\frac{\ell - 2x}{x(\ell - x)}\right] b_w d$$

where x = distance to the point in question

Ex. at $x = 8\,\text{ft}$:

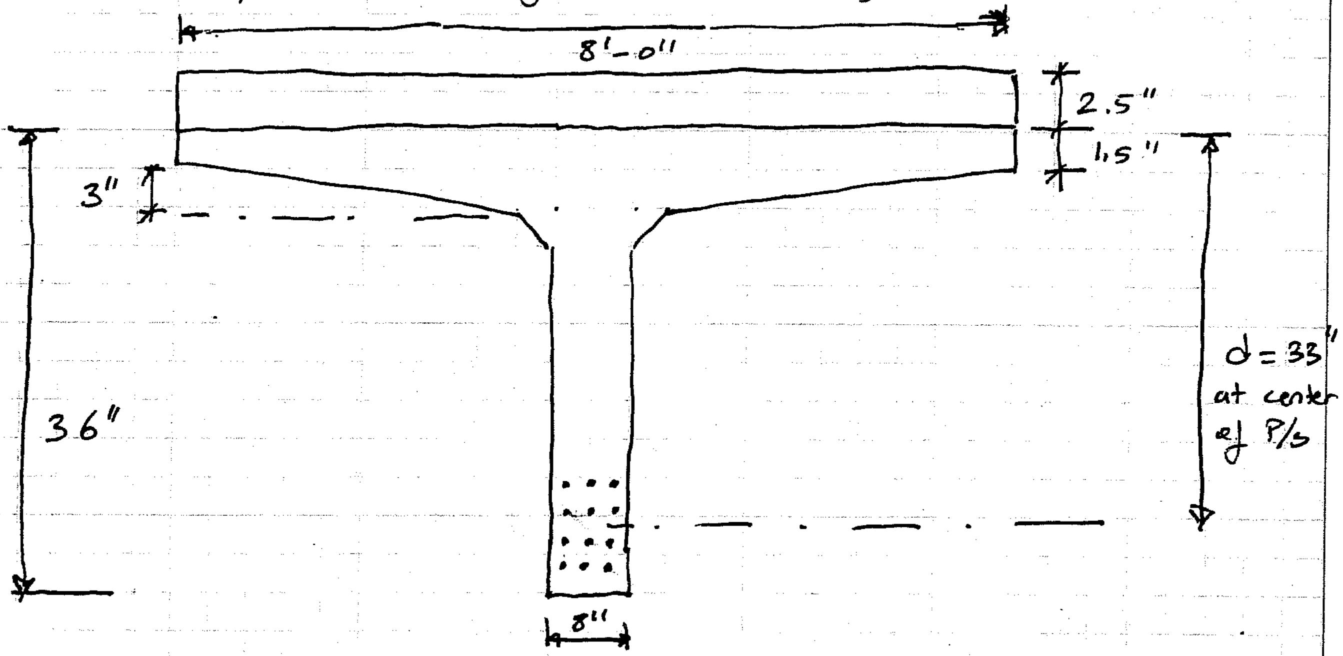
$$V_c = \left[0.6(0.85)\sqrt{f'_c} + 700\,(28.9\,\text{in})\,\frac{(60\,\text{ft} - 16\,\text{ft})}{(8\,\text{ft})(60\,\text{ft} - 8\,\text{ft})}\right](8\,\text{in})(30.8\,\text{in})$$

$$= 52.8\,\text{K} \quad \text{(same value)}$$

much easier to compute directly!

DESIGN FOR SHEAR : EXAMPLE

For the prestressed single tee shown, determine shear requirements using V_c (Eq 11.9)



Precast concrete: $f'_c = 5000$ psi (sand lightweight, $w_c = 120$ pcf)

Topping concrete: $f'_c = 4000$ psi (normal weight, $w_c = 150$ pcf)

Prestressing Steel ; - Twelve ½" dia. 270 ksi strands

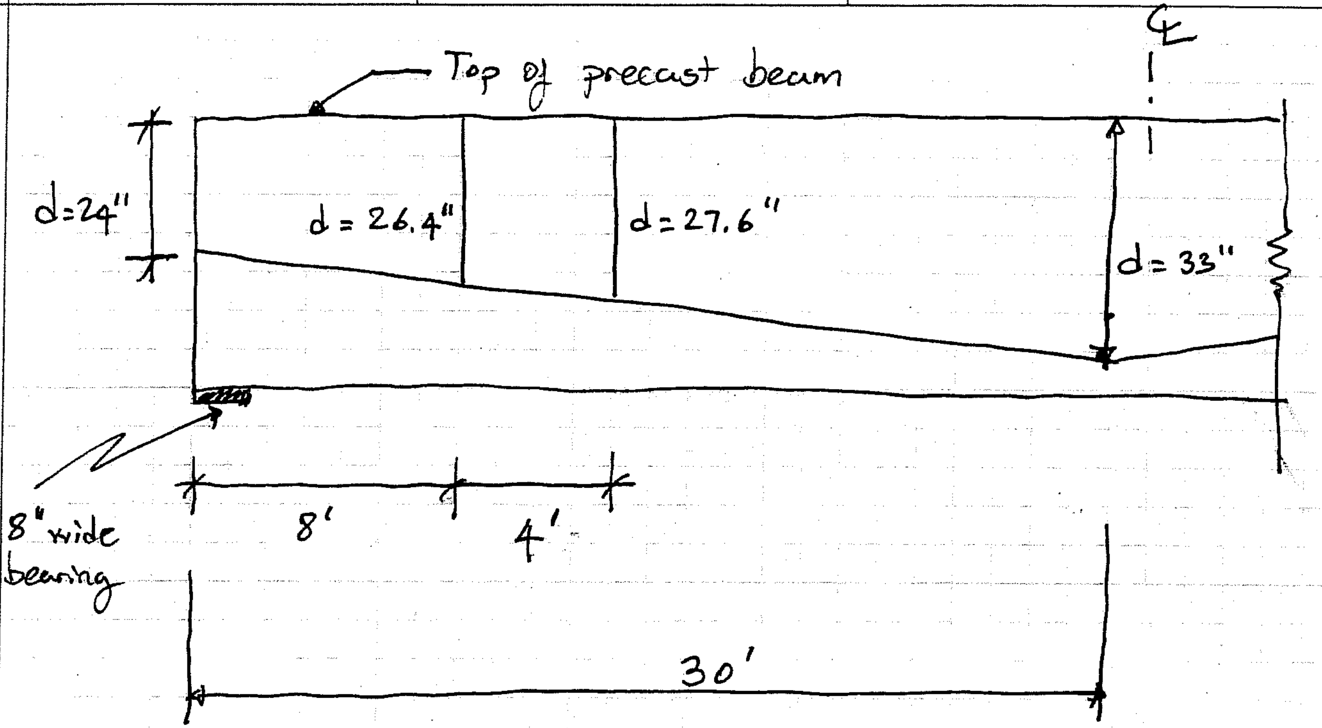
- Single depression at mid-span.

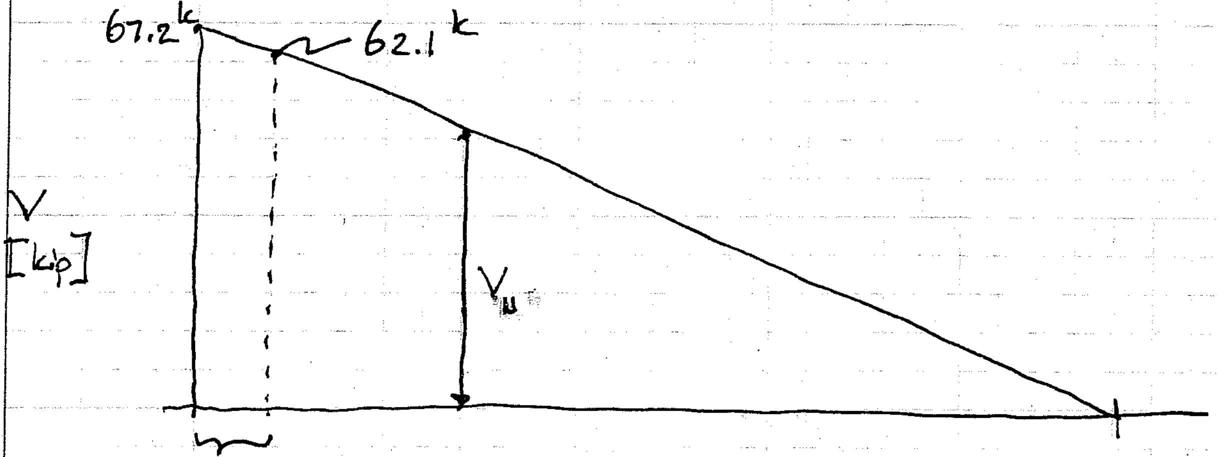
Span = 60 ft (simple)

D.L. = 725 lb/ft (includes topping) $W_U = 1.2D + 1.6L = 2.24$ k/ft

$f_{pf} = f_{se} = 150$ ksi (after all losses)

Precast Section:		Composite Section:
	$A = 570\ in^2$	
	$I = 68917\ in^4$	
	$y_b = 26.01$ in	$y_{bc} = 29.27$ in.
	$y_t = 9.99$ in.	





$$\text{support width} + h/2 = 8" + \underbrace{\frac{(36+2.5)}{2}}_{19.25"} = 27.25"$$

$$V_{x=27.25"} = 67.2 - \frac{1}{12}(27.25) * 2.24 = 62.1^{k}$$

§ 11.0 → "d" need not be taken less than 0.8h for shear strength calculations

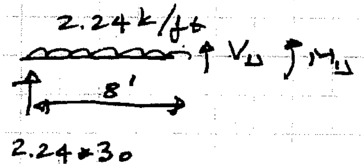
$$0.8\left[38.5"\right] = 30.8" = 2.57'$$

Check if we can use Eq. 11.9.

$$f_{se} = 150 \text{ ksi} > \underbrace{0.40 * 270}_{40\% \text{ of } f_{pu}} = 108 \text{ ksi}$$

→ Typical Calculations using Eq. 11.9 for a section 8' from support

$$V_u = \left(\frac{60}{2} - 8\right) 2.24 = 49.3^k$$



$$M_u = 30 * 2.24 * 8 - 2.24 * 8 * 4 = 466 \text{ ft-kips.}$$

For composite section

$$d = 26.40 + 2.5 = 28.9'' < 0.8h = 30.8'' \qquad \text{use } \underline{\underline{d = 30.8''}}$$

$$V_c = \left(0.6\sqrt{f_c'} + 700 \frac{V_u d}{M_u}\right) b_w d$$

but not less than $2\sqrt{f_c'}\, b_w d$

nor greater than $5\sqrt{f_c'}\, b_w d$

* Since the precast section utilizes sand lightweight concrete, all $\sqrt{f_c'}$ terms must be multiplied by 0.85.

* Total effective depth ⇒ $d = 26.4 + 2.5 = 28.9$, must be used in $V_u d / M_u$ term rather than $0.8h$ which is used elsewhere.

$$V_c = \left[0.6 * 0.85\sqrt{5000} + \frac{700 * 49.3 * 28.90}{466 * 12} \right] * 8 * 30.8$$

$$= 52.8^k$$

$$2\sqrt{f_c'}\, b_w d = 2 * 0.85\sqrt{5000} * 8 * 30.8 = 29.6^k < V_c = 52.8^k \checkmark$$

$$5\sqrt{f_c'}\, b_w d = 5 * 0.85\sqrt{5000} * 8 * 30.8 = 74.0^k > V_c = 52.8^k \checkmark$$

$$\phi V_c = 0.75 * 52.8 = 39.6^k$$

Note: 1. $V_u = 49.3^k$ $\phi V_c = 39.6^k \rightarrow$ design for stirrups!

2. For simply supported members subject to uniform loads "$V_u d / M_u$" term in Eq. (11.9) becomes a simple function of d/l where l is the span length.

$$V_c = \left[0.6\sqrt{f_c'} + 700\, d\, \frac{l - 2x}{x(l-x)} \right] b_w d \quad \text{---- (Eq 11.9 simp.)}$$

where x is the distance from the support to the section being designed.

At $x = 8'$

$$V_c = \left[0.6 * 0.85\sqrt{5000} + 700 * 28.90 * \frac{(60-16)}{8(60-8)12} \right] 8 * 30.8$$

$$= 52.8^k$$

- In the end regions of pretensioned members, the shear strength provided by concrete V_c may be limited by the provisions of § 11.4.3.
- For this design § 11.4.3 does not apply because the section at $h/2$ is farther out into the span than the bond transfer length.
- the following will, however, illustrate typical calculations to satisfy § 11.4.3.

→ Compute V_c at 10" from the end of the member.

Bond transfer length for ½" diameter strand = 50 * ½ = 25"

Prestress force at 10" location.

$$F_{p,eff} = \left(\frac{10}{25}\right) 150 * 0.153 * 12 = 110.2^k$$

Vertical Component of prestress at 10" location.

$$\text{Slope} = \frac{d_{\mathbb{C}} - d_{end}}{\ell/2} = \frac{33-24}{30*12} = 0.025$$

$$V_p = P * \text{slope} = (110.2)(0.025) = 2.8^k$$

For composite section $d = 28.9"$ → use $0.8h = 30.8"$

M_d (unfactored weight of precast unit + topping)

= 214.4 k-in.

f_{pc} = comp. stress in concrete = 130 psi (comp.)

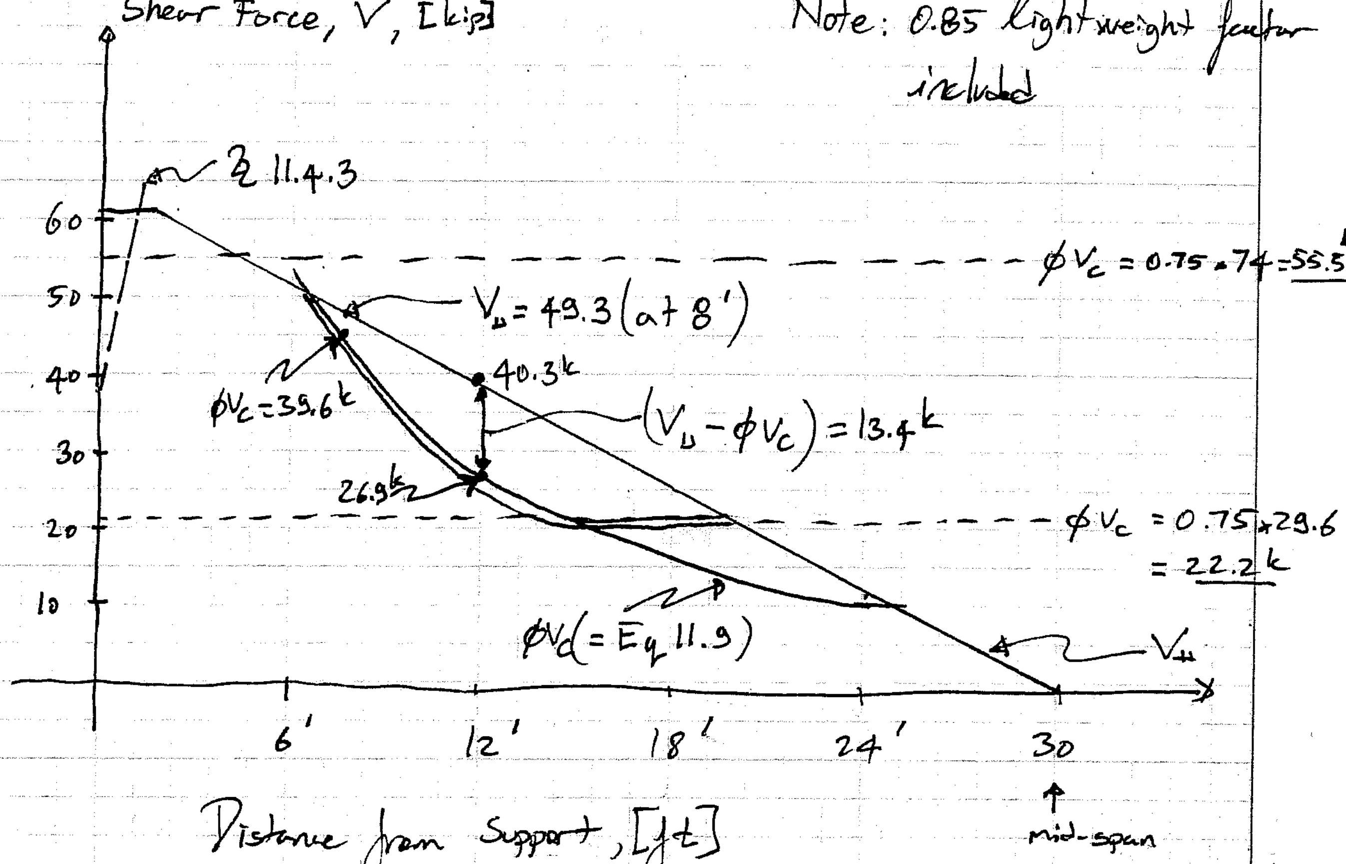
$$V_{cw} = \left(3.5\sqrt{f_c'} + 0.3 f_{pc}\right) b_w d + V_p \qquad (\text{Eq } 11.12)$$

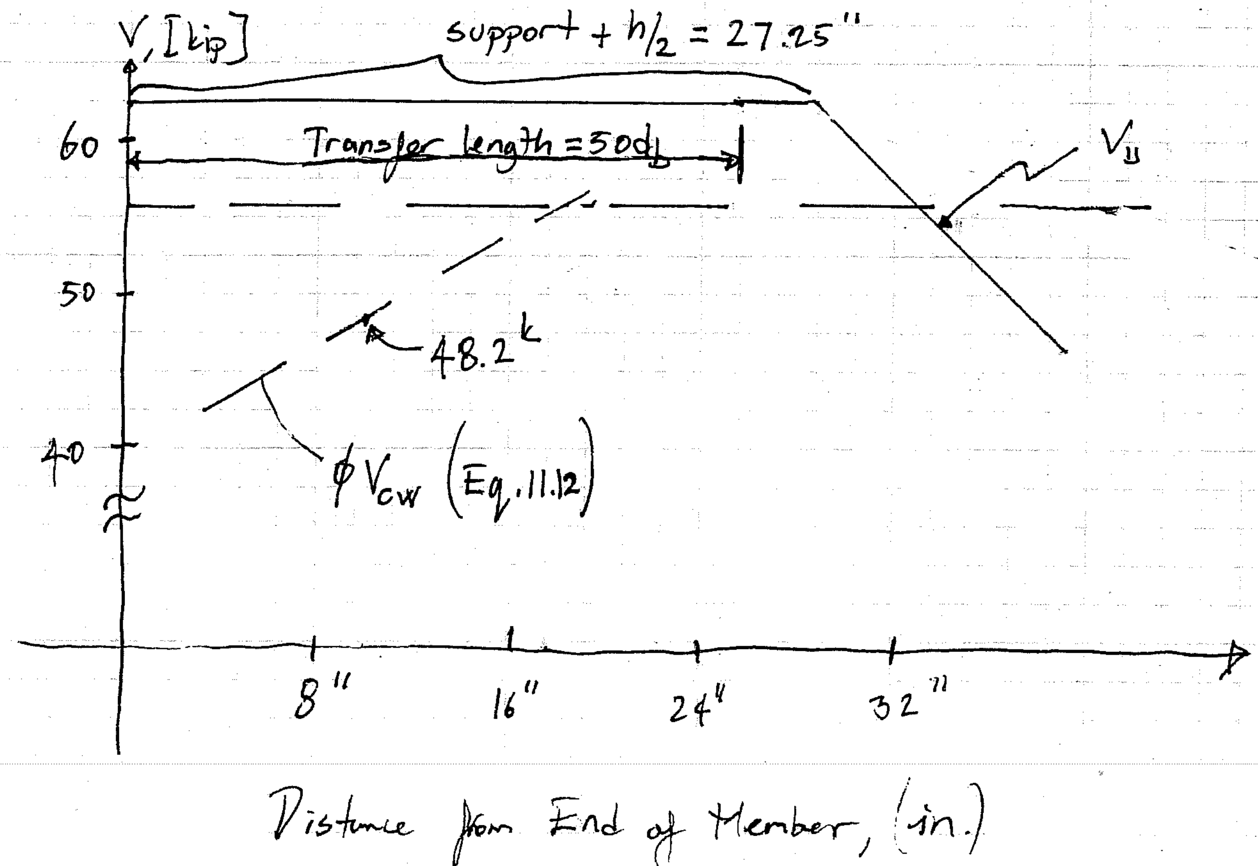
$$= \left(3.5 * 0.85 * \sqrt{5000} + 0.3 * (130)\right) 8 * 30.8 + 2.8$$

$$= 64.24^k$$

$$\phi V_{cw} = 0.75 * 64.2^k = 48.2^k$$

Summary of Results





- Compare V_u with ϕV_c
- Where $V_u > \phi V_c \Rightarrow$ shear reinforcement must be provided to carry excess shear.
- Minimum shear reinf. requirements should also be checked.

→ Shear Reinforcement required at 12' from support.

$d = 30.6''$ → use in $V_u d / M_u$ term.

$$M_u = 30 * 2.24 * 12 - 2.24 * 12 * 6 = 645 \text{ k-ft}$$

$$V_u = \left[\left(\frac{60}{2}\right) - 12\right] * 2.24 = 40.3^k$$

$$V_c = \left(0.6\sqrt{f_c'} + 700\,\frac{V_u d}{M_u}\right) b_w d$$

$$= \left(0.6 * 0.85 * \sqrt{5000} + 700 * \frac{40.3 * 30.6}{645 * 12}\right) * 8 * 30.8$$

$$= 35.9^k$$

$$\phi V_c = 26.9^k$$

$$A_v = \frac{(V_u - \phi V_c)\,s}{\phi f_y d} = \frac{(40.3 - 26.9) * 12}{0.75 * 60 * 30.8} = 0.11\,\text{in}^2/\text{ft}$$

$$A_v(\text{min}) = \frac{50\, b_w s}{f_y} = 50\,\frac{8 * 12}{60\,000} = 0.08\,\text{in}^2/\text{ft}$$

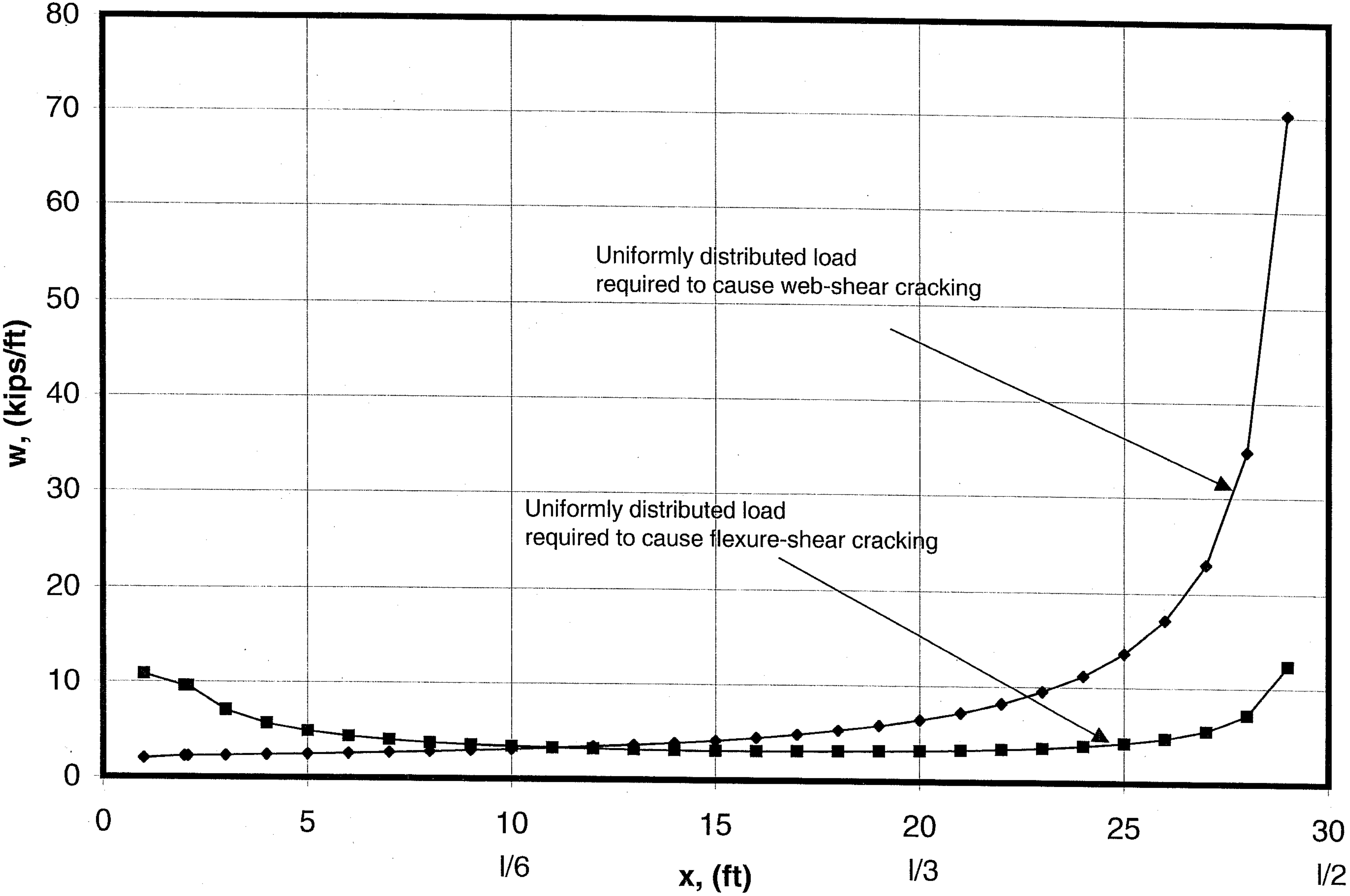
$$0.75\sqrt{f_c'}\,\frac{b_w s}{\;} = 0.07\,\text{in}^2/\text{ft}$$

$$A_v(\min) = \frac{A_{ps}}{80}\,\frac{f_{pu}}{f_y}\,\frac{s}{d}\sqrt{\frac{d}{b_w}} \quad (\text{Eq } 11.14)$$

$$= \frac{1.84}{80} * \frac{270}{60} * \frac{12}{30.8}\sqrt{\frac{30.8}{8}} = 0.08 \text{ in}^2/\text{ft}$$

Use $A_v = 0.11 \text{ in}^2/\text{ft}$ ← governs

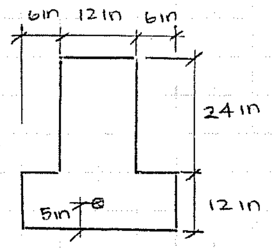
Use No 3 Stirrups @ 18 in $(A_v = 0.15 \text{ in}^2/\text{ft})$


Uniformly distributed load
required to cause web-shear cracking
Uniformly distributed load
required to cause flexure-shear cracking
w, (kips/ft)
0
10
20
30
40
50
60
70
80
0
5
10
15
20
25
30
l/6
l/3
l/2
x, (ft)

SHEAR CALCS

Example

For the simple span pretensioned ledger beam shown, determine V_c (by 11.10 and 11.12)(V_{ci} and V_{cw})



$A = 576 in^2$
$I = 63,936 in^4$
$S_b = 4262 in^3$
$y_b = 15 in$
$f'_c = 6000 psi$
$L = 24 ft$ simplespan

16 - 1/2 in 270 KSI strand
$P_{eff} = 396.6 k$
$f_{peff} = 162 ksi$
$e_{end} = e_{mid} = 10 in$

Loads:

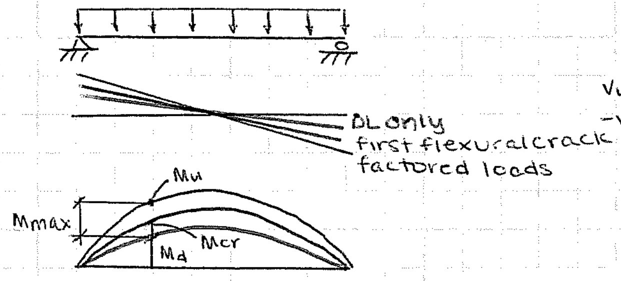
$\omega_D = 5.49 k/ft$
$\omega_L = 5.0 k/ft$

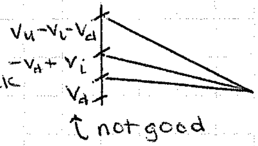
Find:

V_c at 0.3L, at support

Solution:

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d + V_d + \underbrace{\frac{V_i M_{cr}}{M_{max}}}_{\text{shear at the time of flexural cracking}}$$

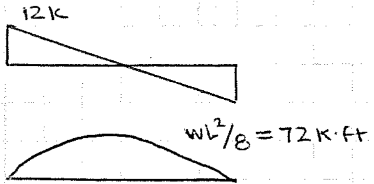




$V_1 = V_u - V_d$

$M_{max} = M_u - M_d$

consider a unit load on beam (1k/ft)



V_c at 0.3L:

$0.3L = 0.3(24 ft) = 7.2 ft$

$V = 4.8 k$

$M = (12 \overset{k}{\cancel{ft}})(7.2 ft) - (7.2 k)(3.6 ft)$

$= 60.48 k \cdot ft$

$W_d = 5.49 k/ft$
$W_L = 5.0 k/ft$
$W_u = 1.2 W_D + 1.6 W_L = 14.59 k/ft$

scale → ; ← from unit load

$V_d = (5.49 k/ft)(4.8) = 26.3 k$

$V_u = (14.59 k/ft)(4.8 k/k/ft) = 70.03 k$

$V_i = V_u - V_d = 43.7 k$

SHEAR CALCS

Example (cont'd)

More calcs:

$$M_{max} = M_u - M_d = \left[14.59\,k/ft - 5.49\,k/ft\right](60.48\,k\cdot ft/k/ft) = 6604\,k\cdot in$$
$$= 550.4\,k\cdot ft$$

$$M_{cr}: \frac{-P}{A} - \frac{Pe}{S_b} + \frac{M_d}{S_b} + \frac{M_{cr}}{S_b} = 6\sqrt{f'_c}$$

code recommends 6, although 7.5 makes more sense

$$M_{cr}\ \frac{-396.6\,k}{576\,in^2} - \frac{(396.6\,k)(10\,in) + (5.49\,k/ft)(60.48\,k\cdot ft/k/ft)}{4262\,in^3} + \frac{M_{cr}}{4262\,in^3} = 6\sqrt{6000\,psi}$$

$$M_{cr} = 4897\,k\cdot in$$

$$V_{ci} = 0.6\sqrt{f'_c}\,b_w d + V_d + \frac{V_i M_{cr}}{M_{max}}$$

$$V_{ci} = \underline{0.6\sqrt{6000\,psi}\,(12\,in)(31\,in)} + 26.3\,k + \underline{\frac{(43.7\,k)(4897\,k\cdot in)}{6604\,k\cdot in}}$$
$$= 17.3\,k + 26.3\,k + 32.4\,k$$
$$= 76.0\,k$$

causes flexural crack

needed to change flexural crack to shear crack

$$\phi V_{ci} = 0.75(76.0\,k) = 57\,k$$
$$V_u = 70\,k$$
$$V_s = V_n - V_c = \frac{V_u}{\phi} - V_c = \frac{70\,k}{0.75} - 76.0\,k = 17.3\,k$$

$$V_s = A_v f_y \frac{d}{s} > 17.3\,k$$

(capacity needed in stirrups)

$$A_v = \frac{V_s \cdot s}{f_y \cdot d} = \frac{17.3\,k\,(12\,in)}{(60\,ksi)(31\,in)} = 0.11\,in^2/ft$$

need 0.11 in² through 12 in longitudinal

Minimum A_v requirements

$$A_v = \underbrace{0.75\sqrt{f'_c}}_{>50,\ \text{does not govern}}\,b_w\frac{s}{f_y} = 0.75\sqrt{6\,ksi}\,(12\,in)\frac{12\,in}{60\,ksi} = 0.11\,in^2 \text{ per } 12\,in$$

$f'_c > 4444$ psi

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y \cdot d}\sqrt{\frac{d}{b_w}} = \frac{(2.488\,in^2)(270\,ksi)(12\,in)}{80(60\,ksi)(31\,in)}\sqrt{\frac{31\,in}{12\,in}} = 0.09\,in^2/ft$$

Maximum spacing = 3/4 d, = 23.25 in

$$\text{Use \#3 stirrups at 18 in, } A_v = \frac{0.11\,in^2 \times 2}{1.5\,ft}$$
$$= 0.15\,in^2/ft$$

SHEAR CALCS

Example (cont'd)

calculate shear reinforcing required at support (not $h/2$, as is inverted tee)

at support, $P = 0$

$$V_{cw} = \left[3.5\sqrt{f'_c} + 0.3 f_{pc}\right] b_w d + V_p = 3.5\sqrt{6\,\text{ksi}}\,(12\,\text{in})(31\,\text{in}) = 100.9\,\text{K}$$

$$V_s = V_n - V_c = \frac{V_u}{\phi} - V_c = \frac{175.1\,\text{K}}{0.75} - 100.9\,\text{K} = 132.4\,\text{K}$$

$$V_u = (12\,\text{ft})(14.59\,\text{K/ft}) = 175.1\,\text{K}$$

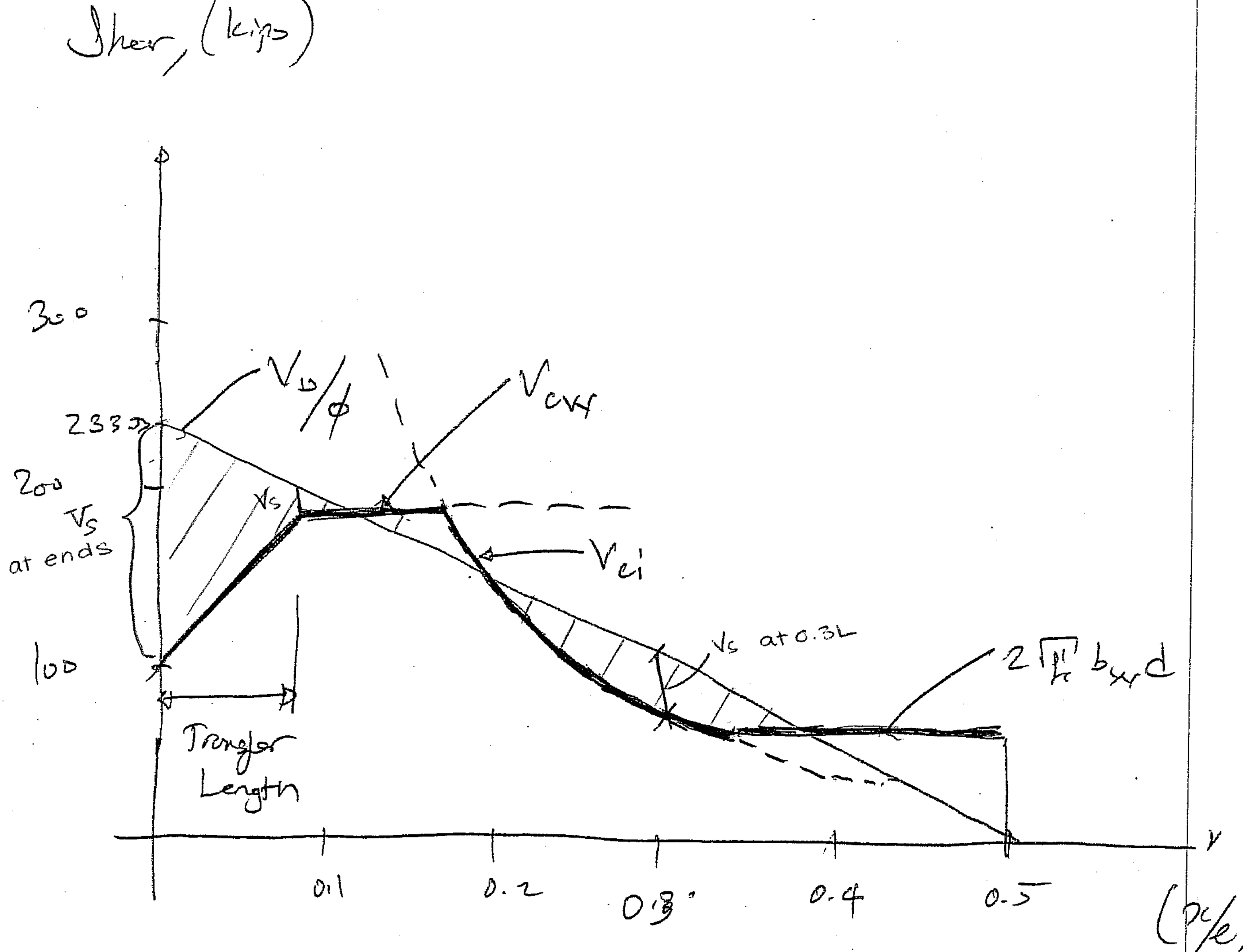
$$A_v = \frac{V_s \cdot s}{f_y\, d} = \frac{(132.4\,\text{K})(12\,\text{in})}{(60\,\text{ksi})(31\,\text{in})} = 0.85\,\text{in}^2$$

options: Twin #3 at 4 in, $A_v = 1.33\,\text{in}^2/\text{ft}$

#4 at 4 in, $A_v = 1.2\,\text{in}^2/\text{ft}$

#3 at 3 in, $A_v = 0.88\,\text{in}^2/\text{ft}$

considering #3s are used at midspan and 3 in spacing is mighty small, maybe this is the best. or, even $s = 6$ in

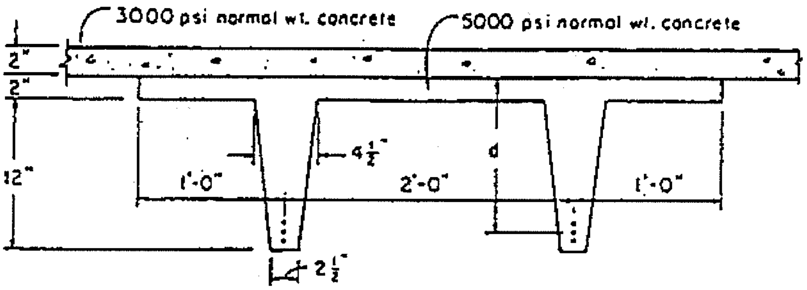

Shear, (kips)
300
233
200
100
Vs
at ends
V_u/ϕ
V_{cw}
Vs
V_{ci}
Vs at 0.3L
$2\sqrt{f'_c}\, b_w d$
Transfer
Length
0.1
0.2
0.3
0.4
0.5
(x/L

Problem 1:

For the prestressed double tee shown, determine shear requirements using V_c Equation 11.9 of ACI 318-05 and provide a sketch summarizing your shear design.

Precast concrete : $f_c' = 5{,}000$ psi
Topping concrete : $f_c' = 3{,}000$ psi
Prestressing steel : Six ½ in. diameter 270 ksi strands.
Span : 30 ft (out-to-out)
Dead Load : 288 lb/ft (beam + topping)
Live Load : 280 lb/ft
F_{se} (after all losses) : 157 ksi
Precast section only :
- d = 8.5 in. at the end
- d = 11.5 in. at the center
- Single point depression at midspan

Note that the beam is simply supported and supports (bearing pads) are 8" wide.



Properties	Area, in^2	Weight, psf	I, in^4	Y_b, in.
Precast	180	47	2,864	10.0
Composite			4,203*	11.45*

* *Corrected for difference in concrete strengths.*

Problem 2:

Using the detailed method of the ACI 318-05, calculate the uniformly distributed load that would cause web-shear cracking 4 ft. away from the support. Also calculate the uniformly distributed load that would cause flexure-shear cracking. Comment on your results.

For 2 webs : $b_w = 2\,(3.5) = 7''$

At 15" = bearing + h/2 (approx)

Eqn (11-12) : $V_{cw} = (3.5\sqrt{f'_c} + 0.3\ f_{pc})\ b_w\ d + V_p$

$= (3.5\sqrt{5000} + 0.3\,(304))\,(2 \times 3.5)\,(12.8) + 1442$

$= 31{,}788\ lb = 31.8\ kip$

$\phi\ V_{cw} = 0.75\ (31.8) = 23.9\ kip > V_u = 10.9\ kip$

Max limit for Vc using Eqn (11-9) :

$V_c = \left(0.6\sqrt{f'_c} + 700\ \frac{V_u\ d}{M_u}\right)\ b_w\ d$

$= \left(0.6\sqrt{5000} + 700\ \frac{(10.9)\,(11.2)}{(14.3 \times 12)}\right)(2 \times 3.5)\left(\overset{0.8h}{\boxed{12.8}}\right)$

$= 48.4\ k > V_{cw} = 31.8\ k$ ok!

At 3 ft :

Eqn (11-9) : $V_c = \left(0.6\sqrt{5000} + 700\ \frac{(9.5)\,(11.2)}{(32.1 \times 12)}\right)(2 \times 3.5)\,(12.8)$

$= 21.1\ kip$

$\phi\ V_c = 0.75\ (21.1) = 15.8\ k > V_u = 9.5\ kip$

At 8 ft :

Eqn (11.9) : $V_c = \left(0.6\sqrt{5000} + 700\ \frac{(5.6)\,(11.2)}{(70 \times 12)}\right)(2 \times 3.5)\,(12.8)$

$= 8.5\ kip$

$\phi\ V_c = 0.75\ (8.5) = 6.4\ kip > V_u = 5.6\ kip$

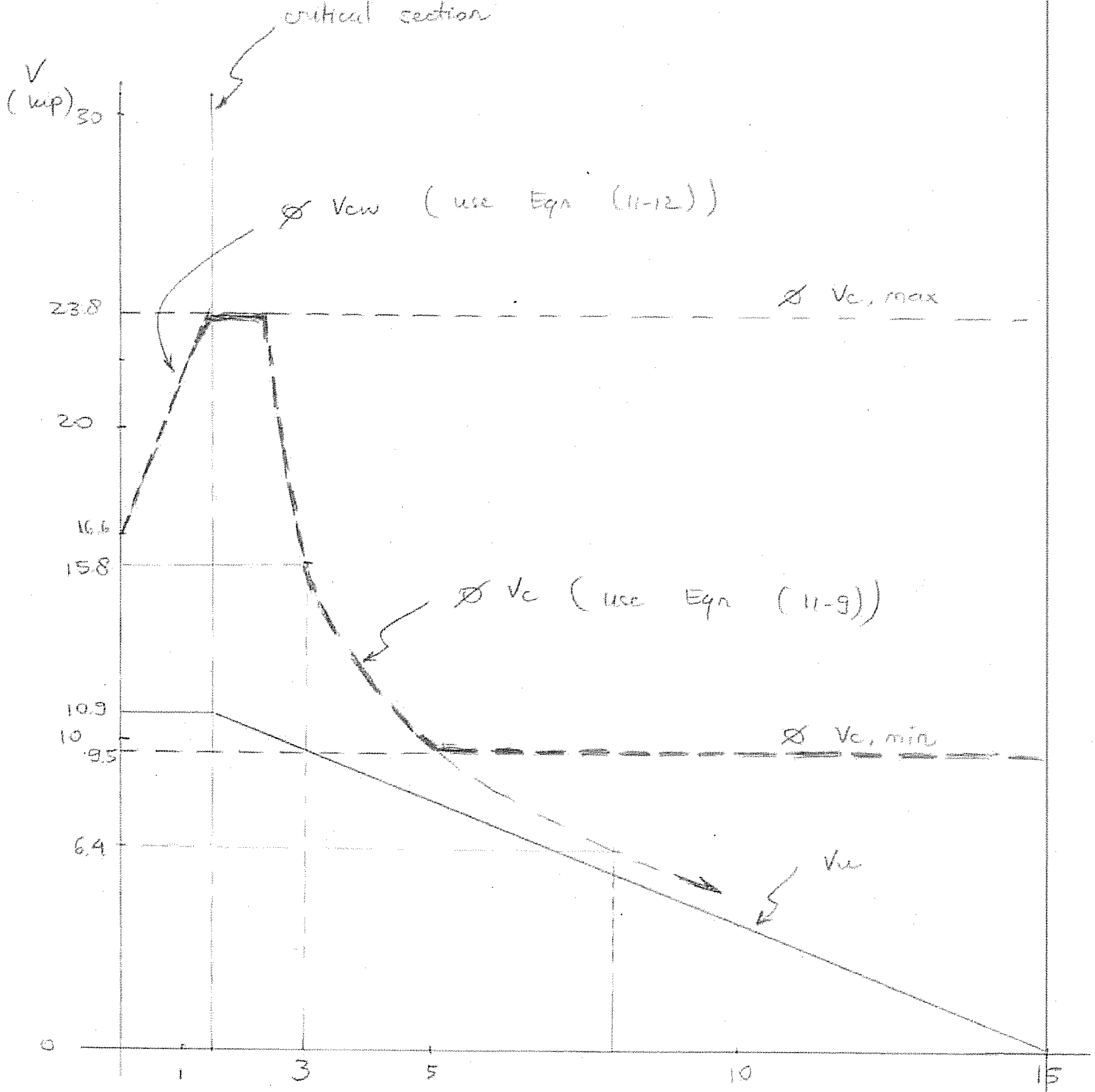
Minimum and maximum Vc :

$V_{c,\,min} = 2\sqrt{f'_c}\ b_w\ d = 2\sqrt{5000}\ (2 \times 3.5)\,(12.8) = 12.7\ kip$

$V_{c,\,max} = 5\sqrt{f'_c}\ b_w\ d = 5\sqrt{5000}\ (2 \times 3.5)\,(12.8) = 31.7\ kip$

$\phi\ V_{c,\,min} = (0.75)\,(12.7) = 9.5\ kip$

$\phi\ V_{c,\,max} = (0.75)\,(31.7) = 23.8\ kip$



Note At $x = 0 \Rightarrow P = 0,\ M = 0$, Eqn. (11-12) reduces to

$$V_{cw} = 3.5\sqrt{f_c'}\ b_w\ d = \frac{3.5\sqrt{5000}}{1000}\ (2\times3.5)(12.8)$$

$$= 22.2\ \text{kip}$$

$$\phi\ V_{cw} = (0.75)(22.2) = 16.6\ \text{kip}$$

$$A_{v,min} = \frac{A_{ps}\ f_{pu}\ s}{80\ f_y\ d}\sqrt{\frac{d}{b_w}} = \frac{(6\times0.153)(270)(12)}{80\ (60)\ (12.8)}\sqrt{\frac{12.8}{2\times3.5}} = 0.0655\ \text{in}^2/$$

Use #3 (single leg) @ 10" per web !

* Web shear cracking : At 4' away from the support :

Eqn. (11-12) : $V_{cw} = (3.5\sqrt{f'_c} + 0.3\, f_{pc})\, b_w\, d + V_p$

At 4' : $f_{pc} = 505$ psi

$\Rightarrow V_{cw} = (3.5\sqrt{5000} + 0.3\,(505))\,(2\times3.5)\,(12.8) + 2400$

$= 38.1$ k

$V @ 4' = w\left(\frac{L}{2} - x\right) = w\,(15-4) = 38.1 \Rightarrow \boxed{w = 3.46\ k/ft}$

* Flexural shear cracking :

Eqn. (11-10) : $V_{ci} = \underbrace{0.6\sqrt{f'_c}\, b_w\, d}_{V_1} + \underbrace{V_d}_{V_2} + \underbrace{\frac{V_i\, M_{cr}}{M_{max}}}_{V_3}$

where : $V_1 = 0.6\sqrt{5000}\,(2\times3.5)\,(12.8) = 3.8$ kip

$V_2 = 3.17$ kip

$V_3 = \frac{V_i\, M_{cr}}{M_{max}}$, $M_{cr} = 1198.3$ k "

Find V_i : $V_i = V_u - V_D$

At the end : $V_u = \frac{w_u\,(30)}{2}$

$V_D = 4.32$ kip

$\Big\}\ V_i = 15\, w_u - 4.32$

At 4 ft : $V_{factor} = 0.7333$

$V_i = 0.7333\ (V_{i,\,end})$

$= 0.7333\ (15\, w_u - 4.32)$

Find M_{max} : At midspan : $M_u = \frac{w_u\,(30)^2}{8} = 112.5\, w_u$

$M_D = 32.4$ k.ft

$M_{max} = M_u - M_D = 112.5\, w_u - 32.4$

At 4ft : $M_{factor} = 0.4622$

$M_{max} = 0.4622\ (M_{max,\ midspan})$

$= 0.4622\ (112.5\, w_u - 32.4)$

Therefore: $V_3 = \frac{V_i \cdot M_{cr}}{M_{max}} = \frac{0.7333\,(15\,w_u - 4.32) \cdot 1198.3/12}{0.4622\,(112.5\,w_u - 32.4)}$

And $V_{ci} = V_1 + V_2 + V_3$

$= 3.8 + 3.17 + \frac{0.7333\,(15\,w_u - 4.32)\ 1198.3/12}{0.4622\,(112.5\,w_u - 32.4)}$

$V_{ci} = w_u \left(\frac{L}{2} - x\right) = 11\ w_u$

Set these two equations equal to each other:

$11\ w_u = 3.8 + 3.17 + \frac{0.7333\,(15\,w_u - 4.32) \cdot 1198.3/12}{0.4622\,(112.5\,w_u - 32.4)}$

$\boxed{w_u = 2.55\ ^{k}/_{ft}}$ #

∴ Flexural shear cracking will occur first

SHEAR CALCS

or, COMPOSITE CONSTRUCTION

Introduction

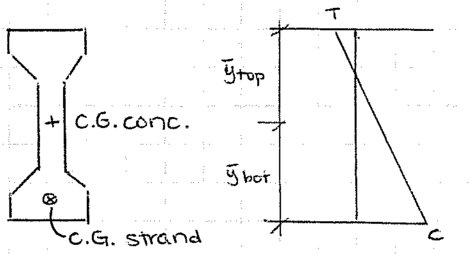
Three important stress checks

Stage 1 – initial conditions at transfer

Stage 2 – conditions at time of wet concrete placement (all load, no stiffness)

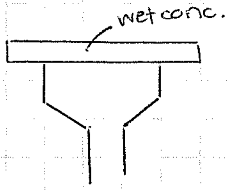
Stage 3 – final conditions

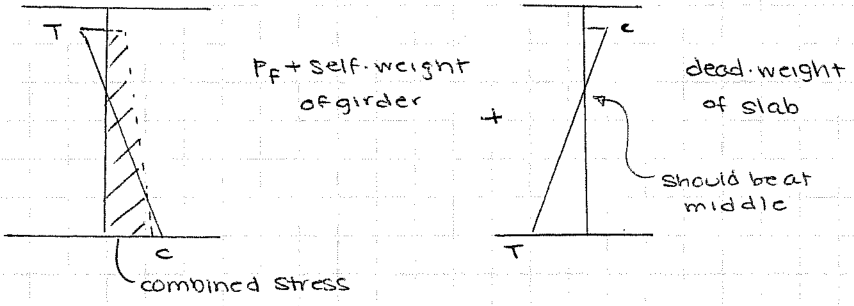
Stage 1:



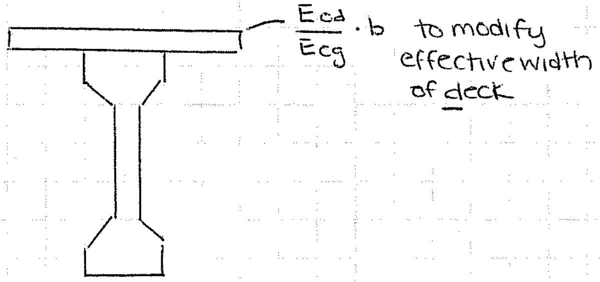
P_i plus dead weight of girder

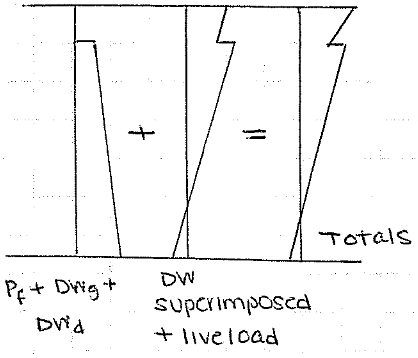
Stage 2:





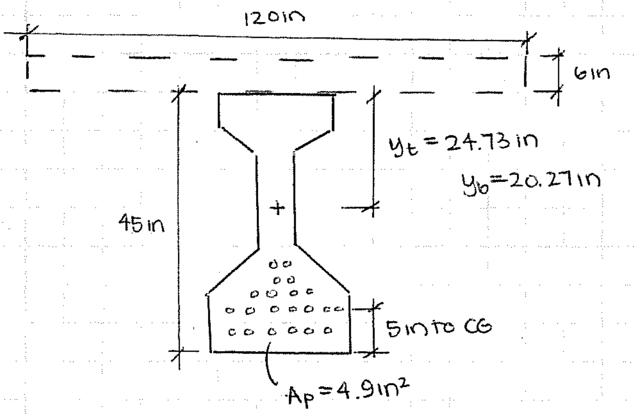
Stage 3:





COMPOSITE CONSTRUCTION

Example



$A = 560 \text{ in}^2$
$I = 125{,}930 \text{ in}^4$
$y_b = 20.27 \text{ in}$
$y_t = 24.73 \text{ in}$
$e = 15.27 \text{ in}$
$P_i = 800 \text{ k}$
$P_f = 672 \text{ k}$
$\phi = 2.5$, 5% relax
$L = 90 \text{ ft}$
$f'_{ci} = 4500 \text{ psi}$
$f'_c = 6000 \text{ psi}$
$f'_{c\,deck} = 4000 \text{ psi}$

Check stresses in the composite beam.

- unshored during deck pour
- check only at midspan
  (for this example!)
- loads:
  - DL_{beam}: $(560 \text{ in}^2)(150 \text{ lb/ft}^3) = 0.583 \text{ k/ft}$
  - DL_{slab}: $(6 \text{ in})(120 \text{ in})(150 \text{ lb/ft}^3) = 0.75 \text{ k/ft}$
  - LL: $(50 \text{ psf})(10 \text{ ft}) = 0.500 \text{ k/ft}$
    none of the live load is sustained
- moments:

$$M_{DW\,beam} = \frac{wL^2}{8} = \frac{(0.583 \text{ k/ft})(90 \text{ ft})^2}{8} = 590.3 \text{ k}\cdot\text{ft}$$

$$M_{DW\,deck} = 759.4 \text{ k}\cdot\text{ft}$$

$$M_{LL} = 506.3 \text{ k}\cdot\text{ft}$$

Stress calculations

Stage 1:

$$f_{cb} = \frac{-800\text{k}}{560 \text{ in}^2} + \frac{(-800\text{k})(15.27 \text{ in})(20.27 \text{ in})}{125{,}930 \text{ in}^4} + \frac{(590.3 \text{ k}\cdot\text{ft})(20.27 \text{ in})}{125{,}930 \text{ in}^4}$$

$$f_{cb} = -2.25 \text{ ksi} < 0.6 f'_{ci} = -2.7 \text{ ksi}$$

$$f_{ct} = \frac{-800\text{k}}{560 \text{ in}^2} - \frac{(-800\text{k})(15.27 \text{ in})(24.73 \text{ in})}{125{,}930 \text{ in}^4} - \frac{(590.3 \text{ k}\cdot\text{ft})(24.73 \text{ in})}{125{,}930 \text{ in}^4}$$

$$f_{ct} = -0.42 \text{ ksi (compression)}$$

Stage 2:

$$f_{cb} = \frac{\overset{\text{conservative}}{-672\text{k}}}{560 \text{ in}^2} + \frac{(-672\text{k})(15.27 \text{ in})(20.27 \text{ in})}{125{,}930 \text{ in}^4} + \frac{(590.3 \text{ k}\cdot\text{ft})(20.27 \text{ in})}{125{,}930 \text{ in}^4}$$

$$+ \frac{(759.4 \text{ k}\cdot\text{ft})(20.27 \text{ in})}{125{,}930 \text{ in}^4} = -0.245 \text{ ksi} \quad \checkmark$$

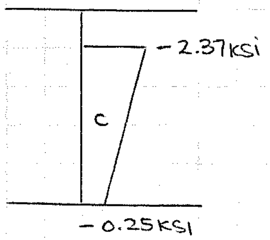
f_{ct} = change signs, y_{bot} to y_{top}, $f_{ct} = -2.37 \text{ ksi}$ ✓

COMPOSITE CONSTRUCTION

Example (cont'd)

Stress calculations

Stage 2 stress profile



all compression, woo!

up next:

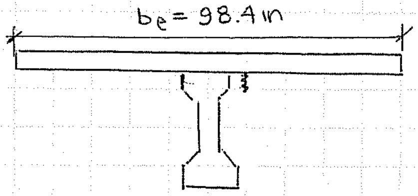
- new I, new location of C.G.
- live loads do next to nothing

COMPOSITE CONSTRUCTION

Example, cont'd

Section Properties

	Net	S.T. Transformed
E_c		
E_p		
A_{trans}	560 in²	587 in²
I_{trans}	125,930 in⁴	132,134 in⁴
$y_{b\,trans}$	20.27 in	19.5 in
$y_{t\,trans}$	24.73 in	25.5 in
n		6.57



$E_{c\,slab} = 3605$ ksi

$E_{c\,beam} = 4415$ ksi

$$\frac{E_{cs}}{E_{cb}} = 0.82 \qquad b_e = 120\text{ in}(0.82) = 98.4\text{ in}$$

$$A_{comp} = 587\text{ in}^2 + (98.4\text{ in})(6\text{ in}) = 1177\text{ in}^2$$

$$\bar{y}_{t\,comp} = \frac{(6\text{ in})(98.4\text{ in})(3\text{ in}) + (587\text{ in}^2)(31.5\text{ in})}{1177.4\text{ in}^2} = 17.21\text{ in}$$

$$\bar{y}_b = 51\text{ in} - 17.21\text{ in} = 33.79\text{ in}$$

$$I_{comp} = 132{,}134\text{ in}^4 + (587\text{ in}^2)(31.5\text{ in} - 17.21\text{ in})^2 + \tfrac{1}{12}(98.4\text{ in})(6\text{ in})^3 + (98.4\text{ in})(6\text{ in})(3\text{ in} - 17.21\text{ in})^2 = 372{,}989\text{ in}^4$$

Stage 3: live load application $\boxed{(\Delta f)}$

$$f_{t_{slab}} = \frac{-(506.3\text{k}\cdot\text{ft})(17.21\text{ in})}{372{,}989\text{ in}^4}(0.82) = -0.23\text{ ksi}$$

↑ returns it to deck concrete

$$f_{b_{slab}} = \frac{-(506.3\text{k}\cdot\text{ft})(11.21\text{ in})}{372{,}989\text{ in}^4}(0.82) = -0.15\text{ ksi}$$

$$f_{t_{beam}} = \frac{-(506.3\text{k}\cdot\text{ft})(11.21\text{ in})}{372{,}989\text{ in}^4} = -0.18\text{ ksi}$$

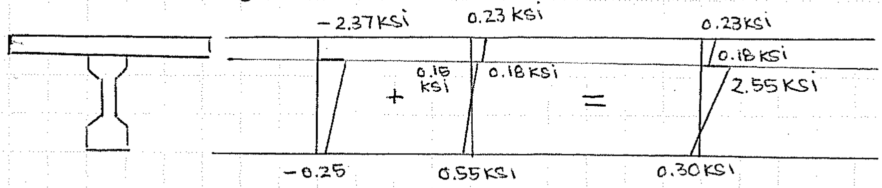
– strains are compatible, Es are different

$$f_{b_{beam}} = \frac{(506.3\text{k}\cdot\text{ft})(33.79\text{ in})}{372{,}989\text{ in}^4} = 0.55\text{ ksi}$$

COMPOSITE CONSTRUCTION

Example (cont'd)

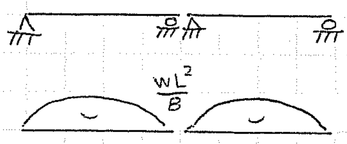
Stress Additions – stage 3

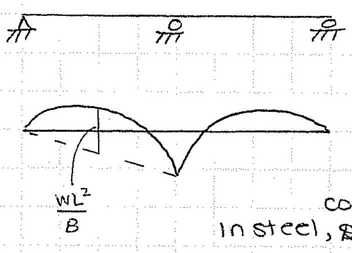


- All compressive stresses are less than 2.7 ksi
- All tensile stresses are less than $7.5\sqrt{f'_c} = 581$ psi

STATICALLY INDETERMINATE SYSTEMS

Introduction





In steel, ~~SS~~ continuous uses a smaller, cheaper section than SS.

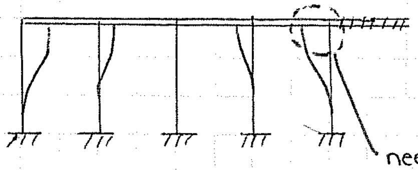
In concrete, maybe just have continuity through the ~~deck~~ deck (live load response)

Advantages & Applications

- Span to depth ratio is improved
    - for continuous flat slab plates: 45-55 as opposed to ~30 for simple span (thinner deck, less dead load moment)
    - for continuous box-girders: 25-35
- the elimination of anchorages and intermediate supports through continuous post-tensioning
- commonly used in flat-plates (one- or twoway PT)
- long-span P/S concrete bridges, particularly for cast-in-place OR L.L. continuous beams
    - ↳ two simple span beams made continuous
- cantilevered box-girder bridges
- cable-stayed bridges with PT decks

Disadvantages and/or concerns

1. Significantly higher frictional losses in PT, due to larger number of bends and tendons
2. Maximum moment and maximum shear occur at the same section. ⟶ diminishes capacity (considering crack formation; flexure-shear cracking expressions)
3.



add PT, then long-term effects — deck/beams want to move inwards

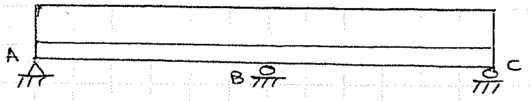
need to account for this, allow horizontal members to move, else get cracking

Excessive lateral loads or movement

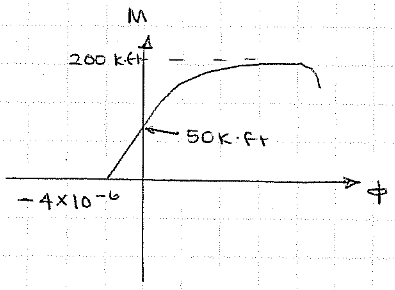
4. Higher secondary stresses due to shrinkage, creep, temperature effects
5. Possible need for negative reinforcement

STATIC INDETERMINACY

Example

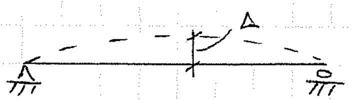


L = 60 ft (2 · 30 ft spans)



Given the M·φ relationship
for the P/S concrete beam
determine support reactions
at A/B/C

A possible solution:



Δ:

$\phi = -4\times10^{-6}$ at $M=0$

$$\Delta = (-4\times10^{-6})(30\text{ft})(15\text{ft}) = 0.26\text{in upward}$$

to bring it back down to the support ($\Delta = 0$),

$$\Delta_{load} = \Delta = \frac{PL^3}{48EI}$$

find EI using the slope of the M·φ curve

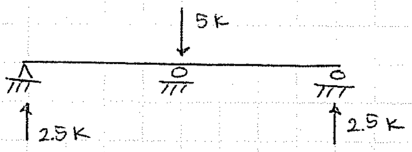
$$EI = \frac{50\text{ k}\cdot\text{ft}}{4\times10^{-6}} = 150\times10^{6}\text{ k}\cdot\text{in}^2$$

$$P = \frac{48EI}{L^3}\Delta = \frac{(0.26\text{ in})\,48\,(150\times10^{6}\text{ k}\cdot\text{in}^2)}{(60\text{ft})^3} = 5.02\text{ k}$$

→ only valid in linear·elastic range, so check that:

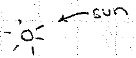
$$M = \frac{PL}{4} = (5.02\text{k})\frac{1}{4}(60\text{ft}) = 75.23\text{ k}\cdot\text{ft}\ ✓$$

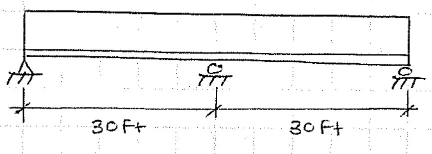
Reactions at A, C = ½ P = ½ R_B

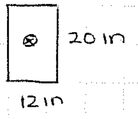


STATIC INDETERMINACY

Example

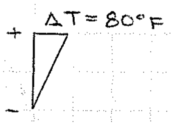






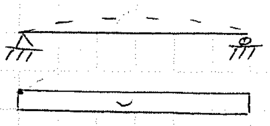
$f'_c = 5 \text{ ksi}$

$\alpha = 5.5 \times 10^{-6} / {}^\circ F$



what are support reactions?

Solution: remove support



$$\phi = \alpha \Delta T \frac{1}{d} = (5.5 \times 10^{-6}/{}^\circ F) \frac{80^\circ F}{20 \text{ in}} = 22 \times 10^{-6} \text{ rad/in}$$

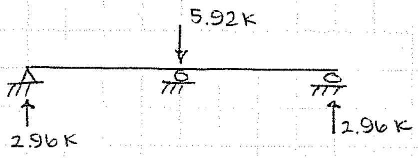
$$E_c = 57\sqrt{f'_c} = 4031 \text{ ksi}$$

$$I = 1/12\,(12 \text{ in})(20 \text{ in})^3 = 8000 \text{ in}^4$$

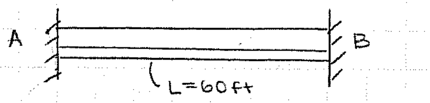
$$\Delta = (22 \times 10^{-6}/\text{in})(30 \text{ ft})(15 \text{ ft}) = 1.43 \text{ in}$$

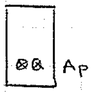
$$\Delta_{load} = \Delta = \frac{PL^3}{48EI}$$

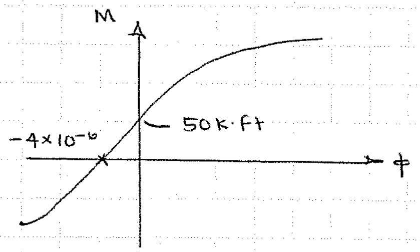
$$P = \frac{48\,(4031 \text{ ksi})(8000 \text{ in}^4)}{(60 \text{ ft})^3}\,(1.43 \text{ in}) = 5.92 \text{ K}$$



consider Fixity







Given the M·ϕ relationship for a typical section, determine the restraint moments

$$EI = \frac{50 \text{ k}\cdot\text{ft}}{4 \times 10^{-6}} = 150 \times 10^6 \text{ k}\cdot\text{in}^2$$

Release fixities:

what rotation occurs?

$$\theta = (4 \times 10^{-6})(30 \text{ ft}) = 0.00144 \text{ rad} = 0.1^\circ$$

STATIC INDETERMINACY

Example, cont'd

$$\theta = 0.1^\circ = \frac{M}{EI}(30\text{ ft})$$

$$M = \frac{0.1^\circ\ (150\times10^6\ \text{k}\cdot\text{in}^2)}{30\text{ ft}}$$ (use radians, not degrees)

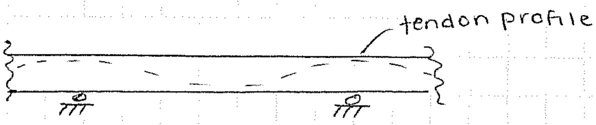
$M = 600\text{ k}\cdot\text{in} = 50\text{ k}\cdot\text{ft}$, the restraint moment

equals decompression moment ($\phi = 0$); moment needed to make the beam straight

Continuous Beams

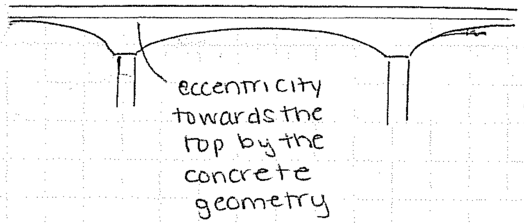
fully or partially continuous beams

Example A:



- curved tendons plus straight beam
- curved tendons are placed at the tension side of the beam (e.g. slabs)

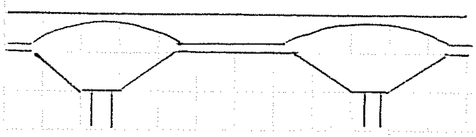
Example B:



very little loss in tendons – no bending, no friction losses

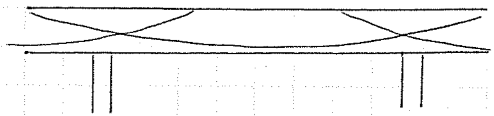
- straight tendons plus curved beam
- more efficient for longer spans and heavier loads
- segmental boxes often do this

Example C:



- combination of A and B
- permits optimum depth of beam, ideal position of steel

Example D:

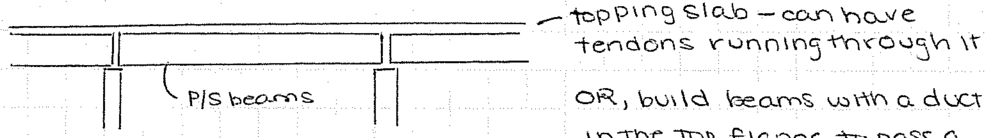


- overlapping tendons
- short tendons (small losses), variable number of strands, etc.
- P/S force can change along the length of the beam

CONTINUOUS BEAMS

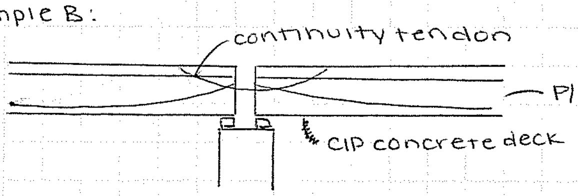
Partially continuous beams

Example A:

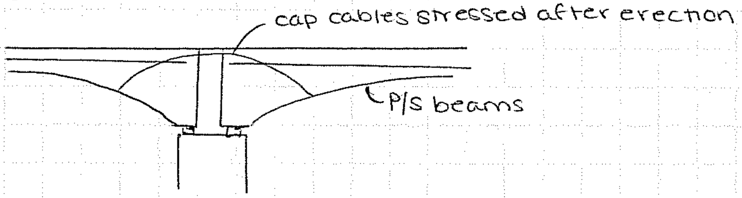


OR, build beams with a duct in the top flange to pass a tendon through

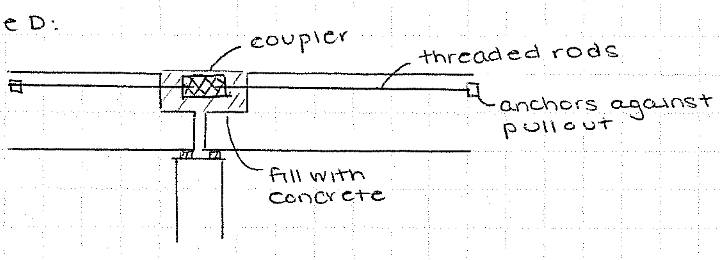
Example B:



Example C:



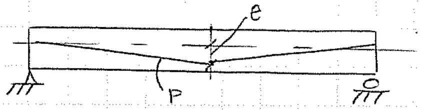
Example D:

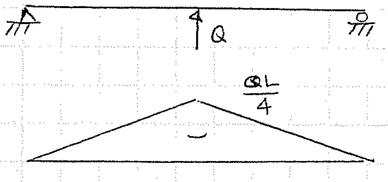


ANALYSIS OF BEAMS

Load-balancing

Example





can we match M from Q to M from prestressing?

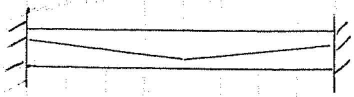
$$\frac{QL}{4} = Pe$$

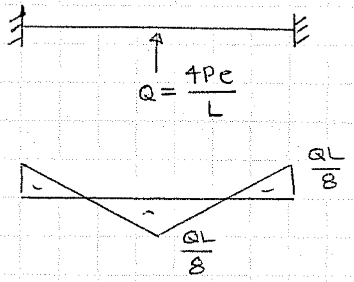
$$Q = \frac{4Pe}{L} \quad \text{ignoring } P/A$$

value of load balancing:

- add load on the beam
- determine how much is held by strands

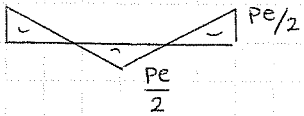
Structural analysis





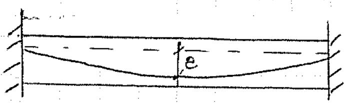
$$\frac{QL}{8} = \frac{4Pe}{L} \cdot \frac{L}{8} = \frac{Pe}{2}$$

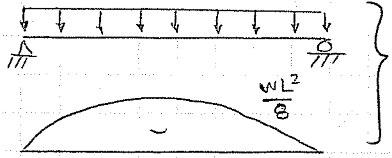
M_o + restraint



$$\text{curvature} = \frac{Pe}{2EI}$$

Example

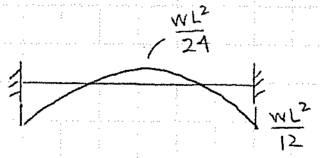




negative of what we need

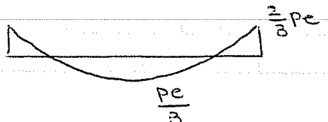
$$Pe = \frac{wL^2}{8}, \quad w = \frac{8Pe}{L^2}$$

Now, what are fixed end moments?



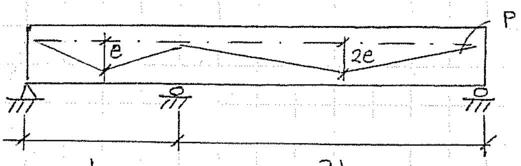
$$\frac{wL^2}{12} = \frac{8Pe}{12} = \frac{2}{3}Pe$$

M_o + restraint:



BEAM ANALYSIS

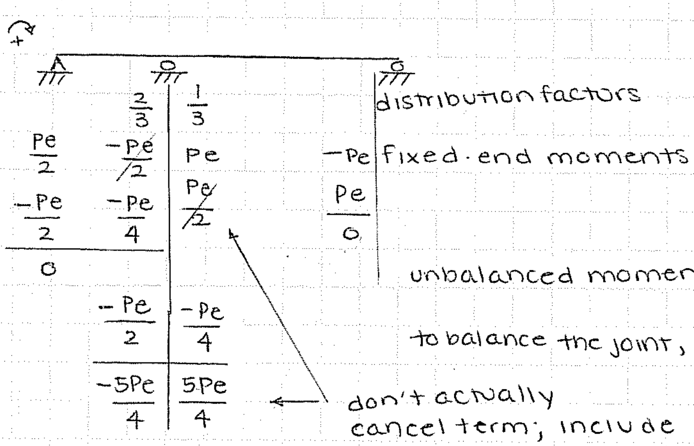
Example



derive M_o diagram

- two-span post-tensioned beam
- use Hardy cross theories

Moment distribution method

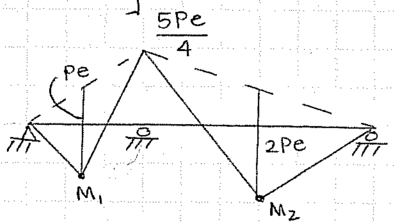


unbalanced moment: $Pe - \frac{Pe}{4} = \frac{3}{4}Pe$

to balance the joint, add $-\frac{3}{4}Pe$ to joint

don't actually cancel term; include in total (5/4 Pe)

[M_o + restraint]



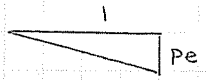
$M_1 = Pe - \frac{5Pe}{8} = \frac{3Pe}{8}$

$M_2 = 2Pe - \frac{5Pe}{8} = \frac{11Pe}{8}$

observation

Harped strands

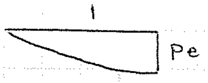
fixed-end moment = $\frac{Pe}{2}$



$A = \frac{1}{2}Pe$

Draped strands

$FEM = \frac{2}{3}Pe$



$A = \frac{2}{3}Pe$

Not a coincidence.

TWO-WAY SLAB SYSTEMS

Equations, etc. from handout

Punching shear

$$V_c = (\beta_p\sqrt{f'_c} + 0.3 f_{pc}) b_o d + V_p$$

V_p ← vertical component of P/S force

β_p ↑ 3.5 or $(\alpha_s d / b_o + 1.5)$

f_{pc} ↖ average value in two directions

$125\,psi \le f_{pc} \le 500\,psi$

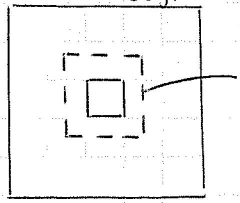
Shear strength

$$v_c = \beta_p\sqrt{f'_c} + 0.3 f_{pc} + \frac{V_p}{b_o d}$$ – same equation, for stresses instead of forces.

$$f'_c \le 5\,ksi$$

P/S strand spacing limits

- no more than $8 \times h_{slab}$ or 5 ft
- minimum of two through critical shear section over column



critical perimeter, b_o

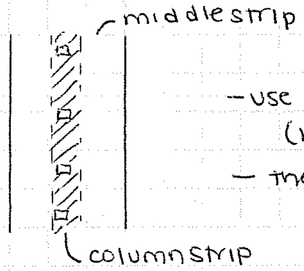
Bayrak says pass strand <u>through</u> column, too

Example notes (see handout)

Design Steps

1. Slab thickness
2. Equivalent frame
3. Load balancing

Distributing strands



- use as few strands in the middle strip as necessary (based on spacing requirements)
- then, group the rest in the column strip

07 December 07

26

Prestressed Slab Systems

UPDATE FOR THE '02 CODE

Section 18.3.3 defines two-way prestressed slab systems as Class U. This permits the calculation of concrete service load flexural stresses using an uncracked section (18.3.4), and results in an increase in allowable concrete flexural tensile stress from $6\sqrt{f'_c}$ to $7.5\sqrt{f'_c}$.

The wording of 18.9.3 was revised to indicate that the minimum bonded reinforcing steel requirements in this Section apply to all two-way flat slab systems, not just flat plates.

Section 18.10.4.1 permits redistribution of negative moments in continuous prestressed flexural members in accordance with new 8.4, provided bonded reinforcement in accordance with 18.9 is provided at supports. This is a significant departure from previous codes.

Load factors and strength reduction factors, ϕ, have been extensively revised in Chapter 9 and are discussed in other parts of this document. The new factors have been incorporated in Design Example 26.1.

INTRODUCTION

Four code sections are particularly significant with respect to analysis and design of prestressed slab systems:

Section 11.12.2—Shear strength of prestressed slabs.

Section 11.12.6—Shear strength of prestressed slabs with moment transfer.

Section 18.4.2—Permissible compressive stresses.

Section 18.7.2—Determination of f_{ps} for calculation of flexural strength.

Section 18.12—Prestressed slab systems.

Discussion of each of these code sections is presented below, followed by Example 26.1 of a post-tensioned flat plate. The design example illustrates application of the above code sections as well as general applicability of the code to analysis and design of post-tensioned flat plates.

11.12.2 Shear Strength

Section 11.12.2 contains specific provisions for calculation of shear strength in two-way prestressed concrete systems. At columns of two-way prestressed slabs (and footings) utilizing unbonded tendons and meeting the bonded reinforcement requirements of 18.9.3, the shear strength V_n must not be taken greater than the shear strength V_c computed in accordance with 11.12.2.1 or 11.12.2.2, unless shear reinforcement is provided in accordance with 11.12.3 or 11.12.4. Section 11.12.2.2 gives the following value of the shear strength V_c at columns of two-way prestressed slabs:

$$V_c = \left(\overset{\checkmark\ 3.5\sqrt{f'_c}}{\beta_p}\sqrt{f'_c} + 0.3f_{pc}\right) b_o d + V_p \qquad \textit{Eq. (11-36)}$$

Equation (11-36) includes the term β_p which is the smaller of 3.5 or $(\alpha_s d/b_o + 1.5)$. The term $\alpha_s d/b_o$ is to account for a decrease in shear strength affected by the perimeter area aspect ratio of the column, where α_s is to be taken as 40 for interior columns, 30 for edge columns, and 20 for corner columns. f_{pc} is the average value of f_{pc} for the two directions, and V_p is the vertical component of all effective prestress forces crossing the critical section. If the shear strength is computed by Eq. (11-36), the following must be satisfied; otherwise, 11.12.2.1 for nonprestressed slabs applies:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

In accordance with the above limitations, shear strength Eqs. (11-33), (11-34), and (11-35) for nonprestressed slabs are applicable to columns closer to the discontinuous edge than 4 times the slab thickness. The shear strength V_c is the lesser of the values given by these three equations. For usual design conditions (slab thicknesses and column sizes), the controlling shear strength at edge columns will be $4\sqrt{f'_c}b_o d$.

11.12.6 Shear Strength with Moment Transfer

For moment transfer calculations, the controlling shear stress at columns of two-way prestressed slabs with bonded reinforcement in accordance with 18.9.3 is governed by Eq. (11-36), which could be expressed as a shear stress for use in Eq. (11-40) as follows:

$$v_c = \beta_p\sqrt{f'_c} + 0.3f_{pc} + \frac{V_p}{b_o d} \qquad \textit{Eq. (11-36)}$$

If the permissible shear stress is computed by Eq. (11-36), the following must be satisfied:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

For edge columns under moment transfer conditions, the controlling shear stress will be the same as that permitted for nonprestressed slabs. For usual design conditions, the governing shear stress at edge columns will be $4\sqrt{f'_c}$.

18.4.2 Permissible Compressive Stresses

In 1995, Section 18.4.2 increased the permissible concrete service load flexural compressive stress under total load from $0.45f'_c$ to $0.60f'_c$, but imposed a new limit of $0.45f'_c$ for sustained load. This involves some judgment on the part of designers in determining the appropriate sustained load

18.7.2 f_{ps} for Unbonded Tendons

In prestressed elements with unbonded tendons having a span/depth ratio greater than 35, the stress in the prestressed reinforcement at nominal strength is given by:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \qquad \textit{Eq. (18-5)}$$

but not greater than f_{py}, nor (f_{se} + 30,000).

Nearly all prestressed one-way slabs and flat plates will have span/depth ratios greater than 35. Equation (18-5) provides values of f_{ps} which are generally 15,000 to 20,000 psi lower than the values of f_{ps} given by Eq. (18-4) which was derived primarily from results of beam tests. These lower values of f_{ps} are more compatible with values of f_{ps} obtained in more recent tests of prestressed one-way slabs and flat plates. Application of Eq. (18-5) is illustrated in Example 26.1.

18.12 SLAB SYSTEMS

Section 18.12 provides analysis and design procedures for two-way prestressed slab systems, including the following requirements:

taught in concrete II.

computer program

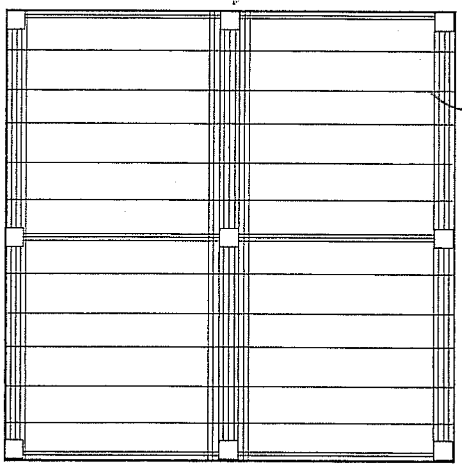
1. Use of the Equivalent Frame Method of 13.7 (excluding 13.7.7.4 and 13.7.7.5), or more detailed analysis procedures, is required for determination of factored moments and shears in prestressed slab systems. According to References 26.1 and 26.4, for two-way prestressed slabs, the equivalent frame slab-beam strips would not be divided into column and middle strips as for a typical nonprestressed two-way slab, but would be designed as a total beam strip.

2. Spacing of tendons or groups of tendons in one direction shall not exceed 8 times the slab thickness nor 5 ft. Spacing of tendons shall also provide a minimum average prestress, after allowance for all prestress losses, of 125 psi on the slab section tributary to the tendon or tendon group. Special consideration must be given to tendon spacing in slabs with concentrated loads.

3. A minimum of two tendons shall be provided in each direction through the critical shear section over columns. This provision, in conjunction with the limits on tendon spacing outlined in item 2 above, provides specific guidance for distributing tendons in prestressed flat plates in accordance with the "banded" pattern illustrated in Fig. 26-1. This method of tendon installation is widely used and greatly simplifies detailing and installation procedures.

Calculation of equivalent frame properties is illustrated in Example 26.1. Tendon distribution is also discussed in this example.

References 26.1 and 26.4 illustrate application of ACI 318 requirements for design of one-way and two-way post-tensioned slabs, including detailed design examples.



carry loads

minimum to control shrinkage, temperature, etc.

Figure 26-1 Banded Tendon Distribution

// Method / design is from FSEL research

REFERENCES

26.1 *Design of Post-Tensioned Slabs*, Post-Tensioning Institute, 2nd. ed., Phoenix, AZ, 1995.

26.2 *Continuity in Concrete Building Frames*, Portland Cement Association, Skokie, IL, 1959.

26.3 *Estimating Prestress Losses*, Zia, P., Preston, H. K., Scott, N. L., and Workman, E. B., Concrete International, :Design and Construction, V. 1, No. 6, June 1979, pp. 32-38.

26.4 *Design Fundamentals of Post-Tensioned Concrete Floors,* Aalami, B. O., and Bommer, A., Post-Tensioning Institute, Phoenix, AZ, 1999

26.5 Burns, Ned. (of UT-Austin)

Example 26.1—Two-Way Prestressed Slab System

Design a typical transverse equivalent frame strip of the prestressed flat plate with partial plan and section shown in Figure 26-2.

f'_c = 4000 psi; w = 150 pcf (slab and columns)

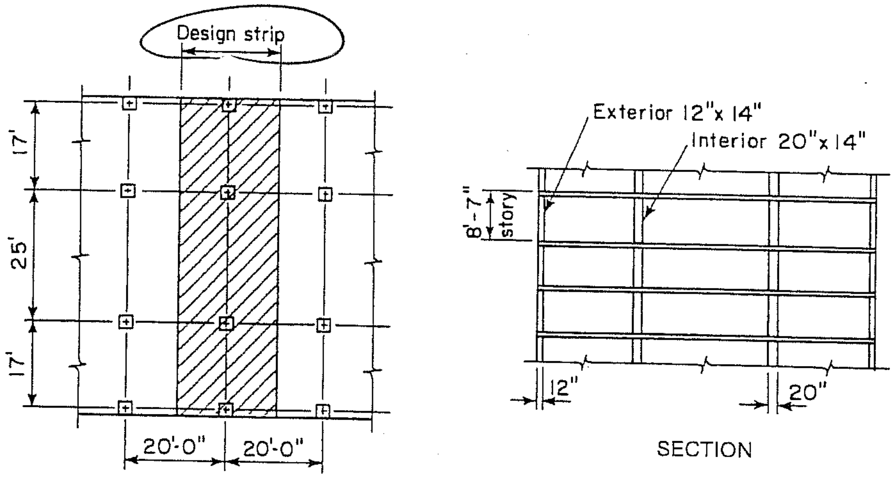
f_y = 60,000 psi

f_{pu} = 270,000 psi

Live load = 40 psf
Partition load = 15 psf

Reduce live load in accordance with general building code. For this example live load is reduced in accordance with IBC 2000, Section 1607.9.2.

Required minimum concrete cover to tendons 1.5 in. from the bottom of the slab in end spans, 0.75 in. top and bottom elsewhere.



PARTIAL PLAN

SECTION

Figure 26-2

Calculations and Discussion

Code Reference

1. Slab Thickness

For two-way prestressed slabs, a span/depth ratio of 45 typically results in overall economy and provides satisfactory structural performance.[26.1]

Slab thickness:

Longitudinal span: 20 × 12/45 = 5.3 in.
Transverse span: 25 × 12/45 = 6.7 in.
} rule of thumb, not code limit

Use 6-1/2 in. slab. thus, 6½ < 6.7 in. is okay

Slab weight = 81 psf
Partition load = 15 psf
Total dead load = 81 + 15 = 96 psf

Span 2:
Reduced live load (IBC 1607.9.2)
Live load = 40(1-0.08(500-150)/100) = 29 psf LL =
Factored dead load = 1.2 × 96 = 115 psf
Factored live load = 1.6 × 29 = 47 psf

Total load = 125 psf, unfactored
= 162 psf, factored

Spans 1 and 3:
Reduced live load (IBC 1607.9.2) 2L+16
Live load = 40(1-0.08(340-150)/100) = 34 psf
Factored dead load = 1.2 × 96 = 115 psf
Factored live load = 1.6 × 34 = 55 psf
Total load = 130 psf, unfactored
= 170 psf, factored

2. Design Procedure

Assume a set of loads to be balanced by parabolic tendons. Analyze an equivalent frame subjected to the net downward loads according to 13.7. Check flexural stresses at critical sections, and revise load balancing tendon forces as required to obtain permissible flexural stresses according to 18.3.3 and 18.4.

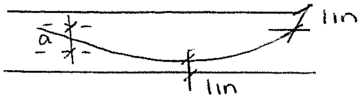
When final forces are determined, obtain frame moments for factored dead and live loads. Calculate secondary moments induced in the frame by post-tensioning forces, and combine with factored load moments to obtain design factored moments. Provide minimum bonded reinforcement in accordance with 18.9.

Check design flexural strength and increase nonprestressed reinforcement if required by strength criteria. Investigate shear strength, including shear due to vertical load and due to moment transfer, and compare total to permissible values calculated in accordance with 11.12.2.

3. Load Balancing

Arbitrarily assume the tendons will balance 80% of the slab weight (0.8 × 0.081 = 0.065 ksf) in the controlling span (Span 2), with a parabolic tendon profile of maximum permissible sag, for the initial estimate of the required prestress force F_e: 81 psf SW

Maximum tendon sag in Span 2 = 6.5-1-1 = 4.5 in.



t = 6.5 in
3/4 in clear cover
1/4 in = 1/2 of strand

$$F_e = \frac{w_{bal}L^2}{\underbrace{8a}_{e\ (\text{kinda})}} = \frac{\overbrace{0.8(0.081)}^{W_{bal}}\overbrace{(25)^2}^{L^2}(12)}{8(4.5)} = 13.5\ \text{kips/ft}$$

P/S force per 1 ft strip of slab

Assume 1/2 in. diameter (cross-sectional area = 0.153 in.^{2}), 270 ksi seven-wire low relaxation strand tendons with 14 ksi long-term losses (Reference 26.3). Effective force per tendon is 0.153[(0.7 × 270) - 14] = 26.8 kips, where the tensile stress in the tendons immediately after tendon anchorage = $0.70f_{pu}$.

18.5.1(c)

For a 20-ft bay, 20 × 13.5/26.8 = 10.1 tendons.

$$ft \cdot \frac{k}{ft} \cdot \frac{1}{k}$$

Use 10-1/2 in. diameter tendons/bay

$F_e = \overset{\#}{10} \times \overset{F}{26.8}/\overset{L}{20} = 13.4$ kips/ft

$f_{pc} = F_e/A = 13.4/(6.5 \times 12) = 0.172$ ksi > 125 psi , < 500 psi

Actual balanced load in Span 2:

$$w_{bal} = \frac{8F_e a}{L^2} = \frac{8(13.4)(4.5)}{12 \times 25^2} = 0.064 \text{ ksf}$$

Adjust tendon profile in Spans 1 and 3 to balance same load as in Span 2:

$$a = \frac{w_{bal}L^2}{8F_e} = \frac{\overset{W_{bal}}{0.064}(\overset{L}{17})^2(12)}{\underset{F}{8(13.4)}} = 2.1 \text{ in.} \quad a = \text{sag}$$

Midspan cgs = (3.25+5.5)/2-2.1 = 2.275 in say 2.25 in.
Actual sag in Spans 1 and 3 = (3.25+5.5)/2-2.25 = 2.125 in.
Actual balanced load in Spans 1 and 3 =

midspan cgs = 2.275in, use 2.25in

$$w_{bal} = \frac{8(13.4)(2.125)}{17^2(12)} = 0.066 \text{ ksf}$$

then check actual sag.

4. Tendon Profile

no moment, want tendons at center

not realistic; do not do this

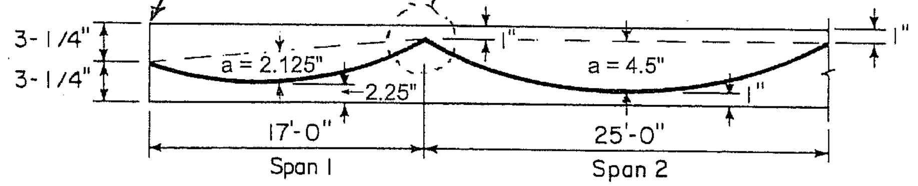


Figure 26-3

Net load causing bending:
Span 2:

$w_{net} = \overset{\text{unfactored load}}{0.125} - \overset{P/T}{0.064} = 0.061$ ksf – P/T has reduced loads on section

Spans 1 and 3:

$w_{net} = \underset{\text{unfactored load}}{0.130} - \overset{P/T}{0.066} = 0.064$ ksf

Now use these loads and design as normal (RC [illegible])

5. Equivalent Frame Properties 13.7

a. Column stiffness. 13.7.4

Column stiffness, including effects of "infinite" stiffness within the slab-column joint (rigid connection), may be calculated by classical methods or by simplified methods which are in close agreement. The following approximate stiffness K_c will give results within five percent of "exact" values.[26.1]

$K_c = 4EI/(\ell - 2h)$

where ℓ = center-to-center column height and h = slab thickness.

For exterior columns (14 × 12 in.):

$I = 14 \times 12^3/12 = 2016 \text{ in.}^4$

$E_{col}/E_{slab} = 1.0$

$K_c = (4 \times 1.0 \times 2016)/[103 - (2 \times 6.5)] = 90 \text{ in.}^3$

$\Sigma K_c = 2 \times 90 = 180 \text{ in.}^3$ (joint total)

Stiffness of torsional members is calculated as follows: 13.7.5

$C = (1 - 0.63\ x/y)\ x^3y/3$ 13.0

$= [1 - (0.63 \times 6.5/12)]\ (6.5^3 \times 12)/3 = 724 \text{ in.}^4$

$$K_t = \frac{9CE_{cs}}{\ell_2\,(1 - c_2/\ell_2)^3}$$ R13.7.5

$$= \frac{9 \times 724 \times 1.0}{(20 \times 12)\,(1 - 1.17/20)^3} = 32.5 \text{ in.}^3$$

$\Sigma K_t = 2 \times 32.5 = 65 \text{ in.}^3$ (joint total)

Exterior equivalent column stiffness (see ACI 318R-89, R13.7.4):

$1/K_{ec} = 1/\Sigma K_t + 1/\Sigma K_c$

$K_{ec} = (1/65 + 1/180)^{-1} = 48 \text{ in.}^3$

For interior columns (14 × 20 in.):

$I = 14 \times 20^3/12 = 9333 \text{ in.}^4$

$K_c = (4 \times 1.0 \times 9333)/[103 - (2 \times 6.5)] = 415 \text{ in.}^3$

$\Sigma K_c = 2 \times 415 = 830 \text{ in.}^3$ (joint total)

$C = [1 - (0.63 \times 6.5/20)]\,(6.5^3 \times 20)/3 = 1456 \text{ in.}^4$

$$K_t = \frac{9 \times 1456 \times 1.0}{240\,(1 - 1.17/20)^3} = 65 \text{ in.}^3$$

$\Sigma K_t = 2 \times 65 = 130 \text{ in.}^3$ (joint total)

$K_{ec} = (1/130 + 1/830)^{-1} = 112 \text{ in.}^3$

b. Slab-beam stiffness. *13.7.3*

Slab stiffness, including effects of infinite stiffness within slab-column joint, can be calculated by the following approximate expression.[26.1]

$K_s = 4EI/(\ell_1 - c_1/2)$

where ℓ_1 = length of span in direction of analysis measured center-to-center of supports and c_1 = column dimension in direction of ℓ_1.

At exterior column:

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 12/2] = 111 \text{ in.}^3$

At interior column (spans 1 & 3):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 20/2] = 113 \text{ in.}^3$

At interior column (span 2):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(25 \times 12) - 20/2] = 76 \text{ in.}^3$

c. Distribution factors for analysis by moment distribution.

Slab distribution factors:

At exterior joints = $111/(111 + 48) = 0.70$

At interior joints for spans 1 and 3 = $113/(113 + 76 + 112) = 0.37$

At interior joints for span 2 = $76/301 = 0.25$

6. Moment Distribution—Net Loads

Since the nonprismatic section causes only very small effects on fixed-end moments and carryover factors, fixed-end moments will be calculated from $FEM = wL^2/12$ and carryover factors will be taken as $COF = 1/2$.

For Spans 1 and 3, net load FEM $= 0.064 \times 17^2/12 = 1.54$ ft-kips

For Span 2 net load FEM $= 0.061 \times 25^2/12 = 3.18$ ft-kips

Note that since live load is less than three-quarters dead load, patterned or "skipped" live load is not required. Maximum factored moments are based upon full live load on all spans simultaneously. *(13.7.6.2)*

Table 26-1 Moment Distribution—Net Loads
(all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-1.54	-1.54	-3.18
Distribution	+1.08	-0.61	+0.41
Carry-over	+0.31	-0.54	-0.21
Distribution	-0.22	+0.12	-0.08
Final	-0.37	-2.57	-3.06

7. Check Net Stresses (tension positive, compression negative)

a. At interior face of interior column:

Moment at column face = centerline moment + $Vc_1/3$ (see Ref. 26.2):

$$-M_{max} = -3.06 + \frac{1}{3}\left(\frac{0.061 \times 25}{2}\right)\left(\frac{20}{12}\right)$$

$$= -2.64 \text{ ft-kips}$$

$S = bh^2/6 = 12 \times 6.5^2/6 = 84.5 \text{ in.}^3$

$$f_{t,b} = -f_{pc} \pm \frac{M_{net}}{S_{t,b}} = -0.172 \pm \frac{12x2.64}{84.5} = 0.172 \pm 0.375 = +0.203, -0.547 \text{ ksi}$$

Allowable Tension = $7.5\sqrt{4000} = 0.474$ ksi *18.3.3*
At top 0.203 ksi applied < 0.474 allowable OK

Allowable compression under total load = $0.60f'_c = 0.6 \times 4000 = 2.4$ ksi *18.4.2(b)*
At bottom 0.547 ksi applied < 2.4 ksi allowable OK

Allowable compression under sustained load = $0.45 \times 4000 = 1.8$ ksi *18.4.2(a)*
0.547 ksi applied under total load < 1.8 ksi allowable under sustained load OK
(regardless of value of sustained load).

b. At midspan of Span 2:

$$+M_{max} = (0.061 \times 25^2/8) - 3.18 = +1.59 \text{ ft-kips}$$

$$f_{t,b} = -f_{pc} \mp \frac{M_{net}}{S_{t,b}} = -0.172 \mp \frac{12x1.59}{84.5} = -0.172 \mp 0.226 = -0.398,\ +0.054 \text{ ksi}$$

Compression at top 0.398 < 1.8 ksi allowable sustained load < 2.4 ksi allowable total load O.K. Tension at bottom 0.054 ksi applied < 0.474 ksi allowable O.K.

When the tensile stress exceeds $2\sqrt{f'_c}$ in positive moment areas, the total tensile force N_c must be carried by bonded reinforcement. For this slab, 18.9.3.2
$2\sqrt{4000} = 0.126 \text{ ksi} > 0.054 \text{ ksi}$. Therefore, positive moment bonded reinforcement is not required. When it is, the calculation for the required amount of bonded reinforcement is done as follows (refer to Figure 26-4).

$$y = \frac{f_t}{f_t + f_c}(h) \text{ in.}$$

$$N_c = \frac{12(y)(f_t)}{2} \text{ kips/ft}$$

$$A_s = \frac{N_c}{0.5f_y} \text{ in.}^2\text{/ft}$$

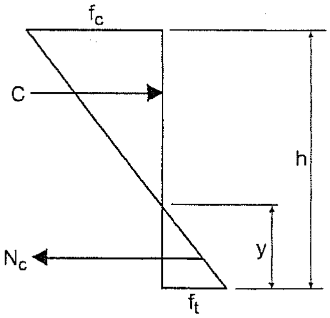


Figure 26-4

Determine minimum bar lengths for this reinforcement in accordance with *18.9.4* (Note that conformance to *Chapter 12* is also required.)

Calculate deflections under total loads using usual elastic methods and gross concrete section properties *(9.5.4)*. Limit ***computed*** deflections to those specified in Table *9.5(b)*.

This completes the service load portion of the design.

8. Flexural Strength

a. Calculation of design moments.

Design moments for statically indeterminate post-tensioned members are determined by combining frame moments due to factored dead and live loads with secondary moments induced into the frame by the tendons. The load balancing approach directly includes both primary and secondary effects, so that for service conditions only "net loads" need be considered.

At design flexural strength, the balanced load moments are used to determine secondary moments by subtracting the primary moment, which is simply $F_e \times e$, at each support. For multistory buildings where typical vertical load design is combined with varying moments due to lateral loading, an efficient design approach would be to analyze the equivalent frame under each case of dead, live, balanced, and lateral loads, and combine the cases for each design condition with appropriate load factors. For this example, the balanced load moments are determined by moment distribution as follows:

For spans 1 and 3, balanced load FEM $= 0.066 \times 17^2/12 = 1.59$ ft-kips

For span 2, balanced load FEM $= 0.064 \times 25^2/12 = 3.33$ ft-kips

Table 26-2 Moment Distribution—Balanced Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	+1.59	+1.59	+3.33
Distribution	-1.11	+0.64	-0.44
Carry-over	-0.32	+0.56	+0.22
Distribution	+0.22	-0.13	+0.09
Final	+0.38	+2.66	+3.20

Since the balanced load moment includes both primary (M_1) and secondary (M_2) moments, secondary moments can be found from the following relationship:

$M_{bal} = M_1 + M_2$, or $M_2 = M_{bal} - M_1$

The primary moment M_1 equals F_e x e at any point ("e" is the distance between the cgs and the cgc, the "eccentricity" of the prestress force).

Thus, the secondary moments are:

At an exterior column:

$M_2 = 0.38 - (13.4 \times 0/12) = 0.38$ ft-kips

At an interior column:

Spans 1 and 3,

$M_2 = 2.66 - 13.4\,(3.25 - 1.0)/12 = 0.15$ ft-kips

Span 2,

$M_2 = 3.20 - (13.4 \times 2.25)/12 = 0.69$ ft-kips

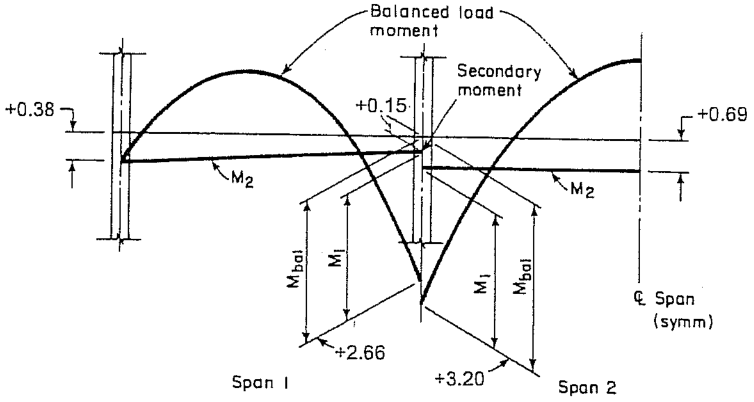


Figure 26-5

Factored load moments:
Spans 1 and 3: $w_u = 170$ psf
Span 2: $w_u = 162$ psf

For spans 1 and 3, factored load FEM $= 0.170 \times 17^2/12 = 4.09$ ft-kips

For span 2, factored load FEM $= 0.162 \times 25^2/12 = 8.44$ ft-kips

Table 26-3 Moment Distribution—Factored Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-4.09	-4.09	-8.44
Distribution	+2.86	-1.61	+1.09
Carry-over	+0.81	-1.43	-0.55
Distribution	-0.57	+0.33	-0.22
Final	-0.99	-6.80	-8.12

Combine the factored load and secondary moments to obtain the total negative design moments. The results are given in Table 26-4.

Table 26-4 Design Moments at Face of Column (all moments are in ft-kips)

	Span 1		Span 2
Factored load moments	-0.99	-6.80	-8.12
Secondary moments	+0.38	+0.15	+0.69
Moments at column centerline	-0.61	-6.65	-7.43
Moment reduction to face of column, $Vc_1/3$	+0.48	+0.80	+1.13
Design moments at face of column	-0.13	-5.85	-6.30

Calculate total positive design moments at interior of span:

For span 1,

$V_{ext} = (0.170 \times 17/2) - (6.65 - 0.61)/17$

$= 1.45 - 0.36 = 1.09$ kips/ft

$V_{int} = 1.45 + 0.36 = 1.81$ kips/ft

Distance x to location of zero shear and maximum positive moment from centerline of exterior column:

$x = 1.09/0.170 = 6.42$ ft

End span positive moment $= (0.5 \times 1.09 \times 6.42) - 0.61 = 2.89$ ft-kips/ft (including M_2)

For span 2,

$V = 0.162 \times 25/2 = 2.03$ kips/ft

Interior span positive moment $= -7.43 + (0.5 \times 2.03 \times 12.5) = 5.26$ ft-kips/ft (including M_2)

b. Calculation of flexural strength.

Check slab at interior support. Section 18.9.3.3 requires a minimum amount of bonded reinforcement in negative moment areas at column supports regardless of service load stress levels. More than the minimum may be required for flexural strength. The minimum amount is to help ensure flexural continuity and ductility, and to control cracking due to overload, temperature, or shrinkage.

$A_s = 0.00075A_{cf}$ *Eq. (18-8)*

where

A_{cf} = larger cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab.

$$A_s = 0.00075 \times 6.5 \times \left(\frac{17 + 25}{2}\right) \times 12 = 1.23 \text{ in.}^2$$

Try 6-No. 4 bars. Space bars at 6 in. on center, so that they are within the column width plus 1.5 times slab thickness on either side of column. *18.9.3.3*

Bar length $= [2 \times (25 - 20/12)/6] + 20/12 = 9$ ft-5 in. *18.9.4.2*

For average one-foot strip:

$A_s = 6 \times 0.20/20 = 0.06 \text{ in.}^2/\text{ft}$

Initial check of flexural strength will be made considering this reinforcement.

Calculate stress in tendons at nominal strength:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p}$$ *Eq. (18-5)*

With 10 tendons in 20 ft bay:

$\rho_p = A_{ps}/bd_p = 10 \times 0.153/(20 \times 12 \times 5.5) = 0.00116$

$f_{se} = (0.7 \times 270) - 14 = 175$ ksi *18.5.1, 18.6, Reference 3*

$f_{ps} = 175 + 10 + 4/(300 \text{x} 0.00116) = 175 + 10 + 12 = 197$ ksi

f_{ps} shall not be taken greater than $f_{py} = 0.85f_{pu} = 230$ ksi >197
or $f_{se} + 30 = 205$ ksi > 197 OK *18.7.2(c)*

$A_{ps}f_{ps} = 10 \times 0.153 \times 197/20 = 15.1$ kips/ft

$A_sf_y = 0.06 \times 60 = 3.6$ kips/ft

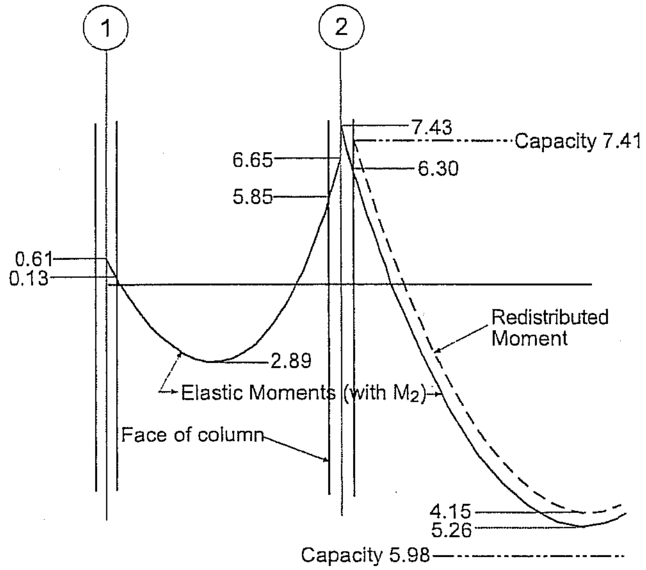


Figure 26-6 Moments in ft-kips

$$a = \frac{A_{ps}f_{ps} + A_sf_y}{0.85f'_cb} = \frac{15.1 + 3.6}{0.85 \times 4 \times 12} = 0.46 \text{ in.}$$

$$c = a/\beta_1 = 0.46/0.85 = 0.54 \text{ in.}$$

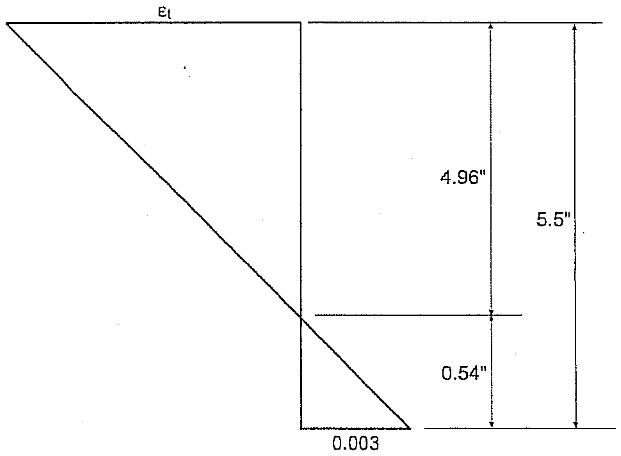


Figure 26-7 Strain Diagram at Interior Support

$\varepsilon_t = (5.5\text{-}0.54) \times 0.003/0.54 = 0.028$ therefore tension controlled$\phi = 0.9$ *9.3.2, 10.3.4*

Since the bars and tendons are in the same layer:

$$\left(d - \frac{a}{2}\right) = \left(5.5 - \frac{0.46}{2}\right)/12 = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times (15.1 + 3.6) \times 0.44 = 7.41$ ft-kips/ft > 6.30 ft-kips/ft OK. *9.3.2.1*

Since there is excess negative moment capacity available, use moment redistribution to increase the negative moment and minimize the positive moment demand in Span 2. Note that the actual inelastic moment redistribution occurs at the positive moment section of Span 2.

Permissible change in negative moment = $1000\varepsilon_t = 1000(0.028) = 28\% > 20\%$ max *18.10.4.1* *8.4*

Available increase in negative moment = $0.2 \times 6.30 = 1.26$ ft-kips/ft

Actual increase in negative moment = Minimum capacity – Elastic Negative Moment
= 7.41 – 6.30 = 1.11 ft-kips/ft < 1.26 available O.K.

Minimum design positive moment in Span 2 = 5.26 – 1.11 = 4.15 ft-kips/ft

Capacity at midspan of Span 2 (no bonded reinforcement required):

$A_{ps}f_{ps}$ = 15.1 kips/ft

$$a = \frac{15.1}{0.85 \times 4 \times 12} = 0.37 \text{ in.}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{5.5} = 0.079 < 0.375, \text{ therefore tension controlled.}$$ 9.3.2.2 10.3.4

$$\left(d - \frac{a}{2}\right) = \frac{5.5 - \frac{0.37}{2}}{12} = 0.44 \text{ ft}$$

At center of span,

ϕM_n = 0.9 × (15.1) × 0.44 = 5.98 ft-kips/ft > 4.15 OK at midspan

Check positive moment capacity in Span 1:

$$\left(d - \frac{a}{2}\right) = \frac{(6.5 - 2.25) - \frac{0.37}{2}}{12} = 0.39 \text{ ft}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{4.25} = 0.102 < 0.375, \text{ therefore, tension controlled}$$ 9.3.2.2 10.3.4

ϕM_n = 0.9 × (15.1) × 0.39 = 5.30 ft-kips/ft > 2.89 OK at midspan

Exterior columns:

A_s minimum = 0.00075 × 20 × 12 × 6.5 = 1.17 in^2 use 6-#4 bars

A_s = 6 × 0.2/20 = 0.06 in^2/ft

A_sf_y = 0.06 × 60 = 3.6 kips/ft

ρ_p = 10 × 0.153/(12 × 20 × 3.25) = 0.00196

f_{ps} = 175 + 10 + 4/(300 × 0.00196) = 192 ksi

$A_sf_{ps} = 10 \times 0.153 \times 192/20 = 14.7$ kips/ft

$$a = \frac{14.7+3.6}{0.85 \times 4 \times 12} = 0.45 \text{ in}$$

$\varepsilon_t = (5.5\text{-}0.53) \times 0.003/0.53 = 0.028$, therefore, tension controlled, $\phi = 0.9$ *9.3.2* *10.3.4*

Tendons:

$$\left(d - \frac{a}{2}\right) = \frac{(3.25) - \dfrac{0.45}{2}}{12} = 0.25 \text{ ft}$$

Rebar:

$$\left(d - \frac{a}{2}\right) = \frac{(5.5) - \dfrac{0.45}{2}}{12} = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times [(14.7 \times 0.25) + (3.6 \times 0.44)] = 4.73$ ft-kips/ft > 0.13 OK

This completes the design for flexural strength.

9. Shear and Moment Transfer Strength at Exterior Column *11.12.6* *13.5.3*

   a. Shear and moment transferred at exterior column.

   $V_u = (0.170 \times 17/2) - (6.65 - 0.61)/17 = 1.09$ kips/ft

   Assume building enclosure is masonry and glass, weighing 0.40 kips/ft.

   Total slab shear at exterior column:

   $V_u = [(1.2 \times 0.40) + 1.09] \quad 20 = 31.4$ kips

   Transfer moment = 20 (0.61) = 12.2 ft-kips
   (factored moment at exterior column centerline = 0.61 ft-kips/ft)

   b. Combined shear stress at inside face of critical transfer section.

   For shear strength equations, see Part 16.

R11.12.6.2

where (referring to Table 16-2: edge column-bending perpendicular to edge)

$d \approx 0.8 \times 6.5 = 5.2$ in.

$c_1 = 12$ in.

$c_2 = 14$ in.

$b_1 = c_1 + d/2 = 14.6$ in.

$b_2 = c_2 + d = 19.2$ in.

$$c = \frac{b_1^2}{(2b_1 + b_2)} = 4.40 \text{ in.}$$

$A_c = (2b_1 + b_2)\, d = 252$ in.^{2}

$J/c = [2b_1 d\,(b_1 + 2b_2) + d^3\,(2b_1 + b_2)/b_1]/6 = 1419$ in^3

$$\gamma_v = 1 - \gamma_f$$ *Eq. (11-39)*

$$= 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.37$$ *13.5.3.2*

$$v_u = \frac{31400}{252} + \frac{0.37 \times 12.2 \times 12000}{1419} = 163 \text{ psi}$$

c. Permissible shear stress (for members without shear reinforcement). *11.12.6.2*

$\phi v_n = \phi V_c/(b_o d)$ *Eq. (11-20)*
where V_c is defined in 11.12.2.1 or 11.12.2.2

For edge columns:

$$\phi v_n = \phi 4\sqrt{f_c'} = 0.85 \times 4\sqrt{4000} = 215 \text{ psi} > 163 \quad \text{O.K.}$$ *11.12.2.1*

d. Check moment transfer strength. *13.5.3*

Although the transfer moment is small, for illustrative purposes, check the moment strength of the effective slab width (width of column plus 1.5 times the slab thickness on each side) for moment transfer. Assume that of the 10 tendons required for the 20 ft bay width, 3 tendons are anchored within the column and are bundled together across *13.5.3.2* the building. This amount should be noted on the design drawings. Besides providing flexural strength, this prestress force will act directly on the critical section for shear and improve shear strength. As previously shown, a minimum amount of bonded reinforcement is required at all columns. For the exterior column, the required area is:

$A_s = 0.00075A_{cf} = 0.00075 \times 6.5 \times 20 \times 12 = 1.17 \text{ in}^2$ *Eq. (18-8)*

Use 6-No. 4 bars, 5 ft in length (including standard end hook).

Calculate stress in tendons:

Effective slab width = 14 + 2 (1.5 × 6.5) = 33.5 in.

$$\rho_p = \frac{3 \times 0.153}{33.5 \times 3.25} = 0.0042$$

$f_{ps} = 175 + 10 + 4/(300\text{x}0.0042) = 188.2$ ksi

Corresponding prestress force $= 3 \times 0.153 \times 188.2 = 86.4$ kips

$A_sf_y = 6 \times 0.20 \times 60 = 72.0$ kips

$A_{ps}f_{ps} + A_sf_y = 158.4$ kips

$a = 158.4/(0.85\text{x}4\text{x}33.5) = 1.39$ in.

tendon $(d_p\text{- } a/2) = (3.25 - 1.39/2)/12 = 0.21$ ft

rebar $(d - a/2) = (5.5 - 1.39/2)/12 = 0.40$ ft

$\phi M_n = 0.9\,[(86.4 \times 0.21) + (72 \times 0.40)] = 42.25$ ft-kips

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.63$$ *Eq. (13-1)*

$\gamma_f M_u = 0.63\,(12.2) = 7.69$ ft-kips << 42.25 ft-kips O.K.

10. Shear and Moment Transfer Strength at Interior Column *11.12.6* *13.5.3*

a. Shear and moment transferred at interior column.

Direct shear and moment to the left and right of interior columns is calculated in Step 8 above.

$V_u = (1.81 + 2.03)\,20 = 76.8$ kips

Transfer moment $= 20\,(7.43 - 6.65) = 15.6$ ft-kips

b. Combined shear stress at face of critical transfer section. For shear strength equations, see Part 16.

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$ *R11.12.6.2*

where (referring to Table 16-1: interior column)

$d \approx 0.8 \times 6.5 = 5.2$ in.

$c_1 = 20$ in.

$c_2 = 14$ in.

$b_1 = c_1 + d = 25.2$ in.

$b_2 = c_2 + d = 19.2$ in.

$A_c = 2\,(b_1 + b_2)\,d = 462$ in.^{2}

$J/c = [b_1 d\,(b_1 + 3b_2) + d^3]\,/3 = 3664$ in.^{3}

$\gamma_v = 1 - \gamma_f$ *Eq. (11-39)*

$$= 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.43$$ *13.5.3.2*

$$v_u = \frac{76{,}800}{462} + \frac{0.43 \times 15.6 \times 12{,}000}{3664} = 188 \text{ psi}$$

c. Permissible shear stress.

For interior columns, Eq. (11-36) applies: *11.12.2.2*

$$\phi v_c = \phi\left(\beta_p\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b_o d}\right)$$ *Eq. (11-36)*

where $\beta_p = \left(\dfrac{\alpha_s d}{b_o} + 1.5\right)$ but not greater than 3.5

$$b_o = 2\,[(20 + 5.2) + (14 + 5.2)] = 88.8 \text{ in.}$$

$\alpha_s = 40$ for interior columns

$$d = 5.2 \text{ in.}$$

$$\beta_p = \frac{40 \times 5.2}{88.8} + 1.5 = 3.8 > 3.5, \text{ use } 3.5$$

V_p is the shear carried through the critical transfer section by the tendons. For thin slabs, the V_p term must be carefully evaluated, as field placing practices can have a great effect on the profile of the tendons through the critical section. Conservatively, this term may be taken as zero. *R11.12.2.2*

$$\phi v_c = 0.85\left[3.5\sqrt{4000} + (0.3 \times 172)\right] = 232 \text{ psi} > 188 \text{ psi O.K.}$$

d. Check moment transfer strength. *13.5.3*

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.57$$ *Eq. (13-1)*

Moment transferred by flexure within width of column plus 1.5 times slab thickness on each side = 0.57 (15.6) = 8.89 ft-kips. *13.5.3.2*

Effective slab width = $14 + 2\,(1.5 \times 6.5) = 33.5$ in.

Say $A_{ps}f_{ps} = 86.4$ kips (same as exterior column)

$A_s = 0.00075A_{cf} = 0.00075 \times 6.5 \times (17 + 25)/2 \times 12 = 1.23$ in.^{2} *Eq. (18-8)*

Use 6-No. 4 bars ($A_s = 1.20$ in.^{2})

$A_sf_y = 1.20 \times 60 = 72.0$ kips

$A_{ps}f_{ps} + A_sf_y = 86.4 + 72.0 = 158.4$ kips

$$a = \frac{158.4}{0.85 \times 4 \times 33.5} = 1.39 \text{ in.}$$

$(d - a/2) = (5.5 - 1.39/2)/12 = 0.40$ ft

$\phi M_n = 0.9 (158.4 \times 0.40) = 57.0$ ft-kips >> 8.89 ft-kips O.K.

This completes the shear design.

11. Distribution of tendons.

In accordance with 18.12.4, the 10 tendons per 20 ft bay will be distributed in a group of 3 tendons directly through the column with the remaining 7 tendons spaced at 2 ft-6 in. on center (4.6 times slab thickness). Tendons in the perpendicular direction will be placed in a narrow band through and immediately adjacent to the columns.

TORSION

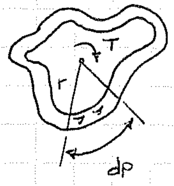
Introduction

- torsional cracks (goes all the way around)
- torsional strength

SEE HANDOUT

Thin-walled Tubes

q = shear flow = (shear stress) × (wall thickness)



$$dT = q\,r\,dp$$

integrate to get $T = q\int_{p} \overbrace{r\,dp}^{2\times \text{Area}}$

↑ perimeter (2π)

$T = q\,2A_o$, A_o = area enclosed by the centerline of the tube

$$\boxed{v = \text{shear stress from torsion} = \frac{T}{2A_o t}}$$

torsional stiffness

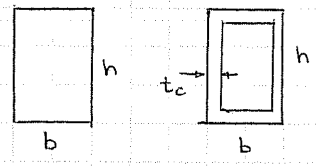
$$= \frac{\text{torsion}}{\text{twist}}, \quad GK = G\,\frac{4A_o^2 t}{P_o}$$

← perimeter of tube

Torsional Response

cracks occur when principal tensile stress ≥ $4\sqrt{f'_c}$, or the cracking stress from a direct tension test

Solid section – assumptions!



$t_c = \frac{3}{4}\frac{A_c}{P_c}$, $A_c = bh$

$P_c = 2(b+h)$

↑ empirical value

$A_o \sim \frac{2}{3}A_c$, $v = \frac{T\,P_c}{A_c^2}$

EQUIVALENT THIN-WALLED APPROACH

cracking torsion

$$\boxed{T = \frac{A_c^2}{P_c}\,4\sqrt{f'_c}\,\sqrt{1 + \frac{f_{pc}}{4\sqrt{f'_c}}}} \qquad f_{pc} = \frac{P_{ps}}{A_g}$$

↑ very large influence

TORSION

Major references

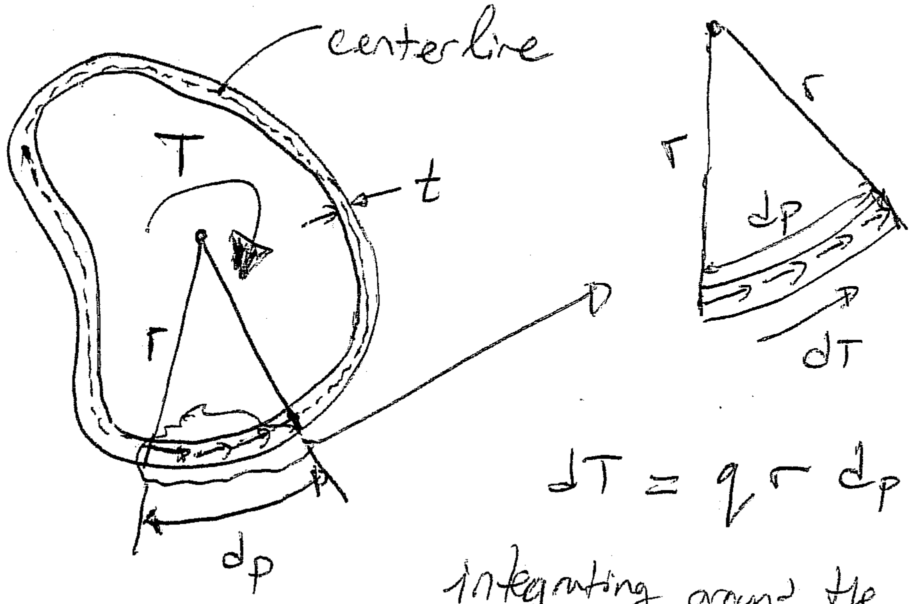
1. Collins & Mitchell
2. Hsu – University of Houston
3. ACI Torsion Document (ACI-ASCE Committee 445) under development

THIN-WALLED TUBES

Thin-walled → shear stresses are constant across the wall thickness Assumption (thin wall)

shear flow = (shear stress) * (wall thickness) = q

$q = \tau * t$



$$dT = q\,r\,d_p$$

integrating around the perimeter/section

$$T = q \int_p \underbrace{r\,d_p}$$

~twice the area of the

$$T = q\,2A_0$$

A_0 = area enclosed by the centerline of the tube
or// area enclosed by shear flow

∴ v = shear stress caused by torsion $= \dfrac{T}{2A_0 t}$

TORSIONAL STIFFNESS

$$= \frac{\text{torsion}}{\text{twist}}$$

$$GK = G\,\frac{4A_0^2\,t}{p_0}$$

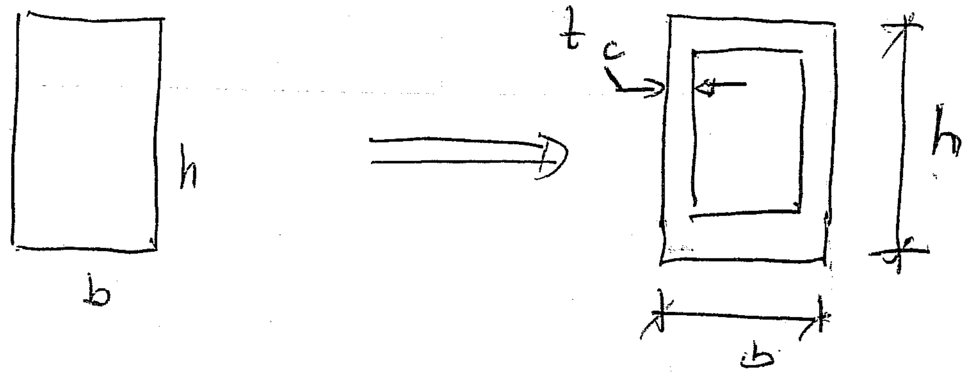
TORSIONAL RESPONSE : UNCRACKED, P/S, SOLID SECTION

Diagonal cracks will form when principal tensile stress $= 4\sqrt{f_c'} = f_{cr} =$ (direct tension test)

Thin-walled sections → can use the aforementioned equations to relate shear stress to torque

Solid Sections → need to make simplifying assumptions
need to be conservative

"Equivalent thin-walled tube" approach



* Same external dimensions

$$t_c = \frac{3}{4}\frac{A_c}{P_c}$$

$$A_c = b\,h$$

$$P_c = 2(b+h)$$

A_o = Area ~~of~~ enclosed by shear flow path

* can be calculated from external dimensions

⇒ OR * $A_o \approx \frac{2}{3} A_c$

Shear stress $v = \dfrac{T\,p_c}{A_c^2}$

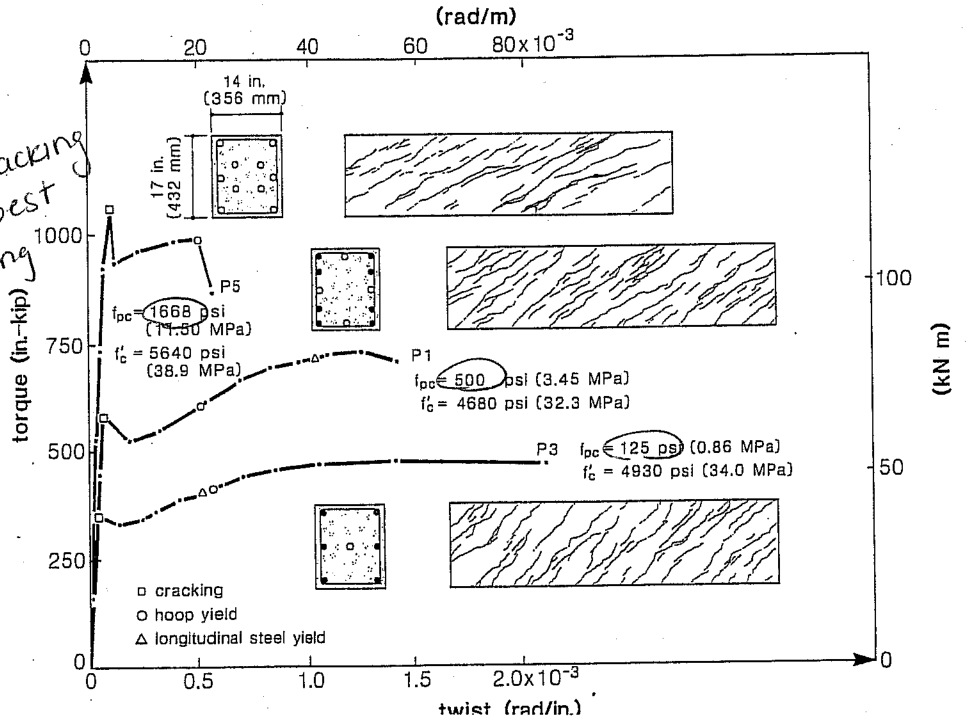
* Shear stresses calculated by the "EQUIVALENT THIN-WALLED TUBE" approach are / can be considered the average of elastic and plastic theories...

Elastic → Saint-Venant
Plastic → Nadai

CRACKING TORSION

$$T_{cr} = \frac{A_c^2}{P_c} 4\sqrt{f_c'} \sqrt{1 + \frac{f_{pc}}{4\sqrt{f_c'}}} \quad (psi)$$

Example: (3 experiments)



higher cracking torque is best (not limiting design)

* Major difference among the three specimens is the amount of P/S force.

T_{cr} for P_3

$$T_{cr} = \frac{\overset{A_c}{(14\times 17)^2}}{\underset{P_c}{2\,(14 \overset{+}{\cancel{\times}} 17)}}\ \overset{4\sqrt{f'c}}{\swarrow}\ 4\sqrt{4930}\ \sqrt{1+\frac{\overset{f_{pc}\ \swarrow}{125}}{\underset{4\sqrt{f'c}}{4\sqrt{4930}}}}$$

$= 308$ in-kips - good correlation

T_{cr} for P_1

$$T_{cr} = \frac{(14\times 17)^2}{2\,(14 \overset{+}{\cancel{\times}} 17)} \times 4\sqrt{4680}\ \sqrt{1+\frac{500}{4\sqrt{4680}}}$$

$= 420$ in-kips - less close; conservative

T_{cr} for P_5

$$T_{cr} = \frac{(14\times 17)^2}{2\,(14 \overset{+}{\cancel{\times}} 17)} \times 4\sqrt{5640}\ \sqrt{1+\frac{1668}{4\sqrt{\underset{5640}{\cancel{\text{[illegible]}}}}}}$$

$= 703$ in-kips - very conservative

Torsional Behavior after cracking

- Space Truss / strut-and-tie
- Skewed Bending (skewed bending)
- MCFT - Modified Compression Field Theory
  in AASHTO LRFD

→ Complicated / iterative

See Collins & Mitchell - big discussion in book on MCFT
OR Hsu

All cracking is bad, especially in bridges
equilibrium torsion - beam falls off support