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CE 383T
Plasticity in Structural Concrete

Class Notes
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Practical Design of Structural Concrete

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Practical Design of Structural Concrete

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"Practical Design of Structural Concrete"

Scope

- update of FIP Rec. 1984; the main intentions have been kept to:
  - -- to be used by practising engineers,
  - -- to treat in more detail the practical aspects of prestressing as in MC 90.
- changes mainly based on MC 90. Model code CEB-FIP
- fully based on member design with strut-and-tie models (STM), which necessitated a regrouping of the chapters.
- references to MC 90 like in previous edition.

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Scope

(1) These Recommendations apply to structural concrete using normalweight aggregates for all types of structures such as buildings and bridges, Structural concrete comprises all concrete used for structural purposes from plain concrete through applications with non-prestressed reinforcements, pretensioned or post-tensioned tendons or their combinations.

(2) The principles of these Recommendations also apply to the assessment of existing structures.

(3) The recommendations are intended for use by practising engineers to enable them to design according to modern concepts. The rules are given in a concise form suitable for qualified engineers with adequate experience in design and detailing.

(4) The rules are based on the CEB/FIP Model Code 1990 (MC 90) to which frequent reference is made in the right-hand margins of the present document. In some cases, explained by footnotes, deviations from MC 90 have been made in order to simplify the calculations and, where necessary, additional information has been introduced.

MC 90, 1.6

1 Principles

1.1 General

(1) The design of structures should involve the following steps:

- (a) check of the ultimate limit state (ULS)
- (b) check of the serviceability limit state (SLS)
- (c) compliance with detailing practice
- (d) compliance with technological requirements
- (e) compliance with durability requirements.

(2) Depending on the type of structure or the construction method employed, either the ULS or the SLS can be taken as the primary design criterion. In many cases only one of these checks will be needed if, according to experience, there is no doubt that the other one is respected a priori. In any case, detailing practice and technological requirements should be carefully observed, because they are as important to the serviceability and durability of concrete structures as checks by calculation. Especially the overall structural integrity is heavily dependent on the proper dimensioning and detailing especially at geometrical or load discontinuity regions (D-regions).

(3) The primary focus of structural design should be directed towards careful consideration of overall or global structural behaviour, and towards efficiently directing the flow of the forces throughout the structure. Thereby also possible damaging restraints and agressive environmental factors, should be considered.

(4) In general, the various load-carrying members of a structure should be interlinked so as to ensure a satisfactory overall performance with regard to structural stability and robustness. In particular, it should be ensured that the structure cannot be subjected to progressive collapse as a consequence of localized damage due to abnormal use or accident.

1.2 Ultimate limit state (ULS)

MC 90, 1.6

(1) The ULS verifications should be based on transparent and realistic models of structural behaviour and the correct failure mechanisms under ultimate loads. For these calculatins, the theory of plasticity (PT) provides a simple and efficient approach in many cases. Nonlinear analysis methods of a more general character may also be used, particularly in cases of instability.

(2) In all cases, it must be ensured that the structure is sufficiently ductile to be able to reach the assumed ultimate limit state without premature brittle failure, and to allow for restraint effects. In a sufficiently ductile structure the effects due to temperature, creep, shrinkage and foundation settlements have in general an insignificant influence on the ultimate load-carrying capacity.

(3) The ULS condition is satisfied if the following symbolic equation is respected for all relevant combinations of actions

$$F_{act,\,d}\,(\gamma_g\,G\,;\ \gamma_q\,\Sigma Q) \quad \leq \quad F_{res,d}\,(f_{ck}\,/\,\gamma_c\,;\ f_{sk}\,\,/\,\gamma_s) \qquad (1.1)$$

design value of load or actions ≤ design value of load carrying capacity

where the notation is as follows, with associated partial safety coefficients in parenthesis

G permanent actions (mean value (γ_g)

Q variable actions (nominal values) (γ_q)

f_{ck} characteristic concrete strength (γ_c)

f_{sk} characteristic strengths of reinforcing and prestressing steel (γ_s)

(4) Since it is very unlikely that the maximum values of all variable actions Q occur at the same time, representative values, as defined later on, may be introduced, using, in the case of two or more variable actions, combination values characterized by the coefficient ψ_o (see sect. 6.2.2).

(5) The effect of prestressing may either be considered on the action side by simulating it by forces due to the prestress being applied to the concrete, or by considering the high-strength prestressing steel as part of the member thus contributing to its resistance.

(6) The above general criterion (3) is satisfied, if all significant sections of the structure fulfil the following condition

$$S_d\,(\gamma_g G;\ \gamma_q \Sigma Q\,) \leq R_d\,(f_{ck}/\gamma_c\ ;\ f_{sk}/\gamma_s) \qquad (1.2)$$

design value of critical combination of action effects ≤ design value of resistant action effects

However, the overall structural integrity should be ensured by checking the layout and the anchorages of the reinforcements, especially for discontinuity regions.

(7) The distribution of internal forces in the structure shall satisfy the conditions of equilibrium. This is always the case for elastic distributions. Thus, if the structure has already been designed for serviceability conditions (SLS), the same distribution multiplied by a proper load factor can often been used for the check of capacity in ULS..

(8) For structures in which equilibrium is affected by the deformations of the members, the equilibrium condition should be formulated on the deformed structure. However, the equ. (1.2) may be used if the second order effects are included in the term S_d. It should be noted, that not always the ultimate capacity of the section is attained.

(9) Fatigue problems are normally not critical in reinforced and prestressed concrete, as long as serious cracking under the appropriate actions (γ_q =1) is prevented (see sect. 6.8).

1.3 Serviceability limit state (SLS)

MC 90, 1.6.6

(1) The SLS verifications should be based on transparent and realistic models of structural behaviour including, where relevant, cracking and time - dependent effects.

(2) Normally the SLS calculations are based on the theory of elasticity (ET). In certain cases, non-linear analysis methods may be used.

(3) Depending on the particular case, the SLS check should be done by one or more of the following three methods:

(a) by limiting stresses $\sigma_d < \sigma_{lim}$

(b) by limiting deformations (deflections or angles) $a_d < a_{lim}$

(c) by limiting crack widths $w_d < w_{lim}$

The limit values should be established on the basis of the functional requirements of the structure. Other requirements such as water tightness, tolerances, vibrations should also be checked in appropriate cases.

(4) For the SLS calculations, the effect of prestressing may either be considered on the action side, or in the material characteristics as an imposed deformation; however, normally it is considered as an external action. The prestressing force should be considered with its mean value.

(5) The check by calculation can sometimes be ommited by respecting minimum reinforcement or detailing regulations.

1.4 Design by testing

MC 90, Appendix C

(1) In special cases the design of structures or structural elements may be based on testing. The test should consider all possible unfavourable conditions for the real structure including any possible reduction of the concrete tensile strength.

(2) The following rules should be applied:

(a) The test results have to be interpreted by means of realistic analytical models from which the influence of the principal parameters involved may be estimated.

(b) The basic principles of the present Recommendations, notably the criteria of all limit states, have to be applied to this experimentally derived model.

(c) The partial safety coefficients have to be chosen conservatively according to adequate statistical and probabilistic considerations and the level of quality control.

(d) Major deviations from accepted principles or design rules (e.g. a bearing capacity that depends considerably on the concrete tensile strength) have to be justified, either by increasing the safety margins, or by a test series that is large enough to allow the estimation of the representative load carrying capacity based on the 5 % fractile strength of the materials used.

(e) The undertaking of alternative procedures to those outlined in the present Recommendations must be subjected to the control and agreement of an appropriate authority.

2 Material characteristics

MC 90, 2

2.1 Concrete

MC 90, 2.1

2.1.1 Concrete strength grades

(1) The following clauses apply to concrete with normal weight aggregates. For structural concrete containing a normal amount of reinforcement a density can usually be assumed of 25 kN/m^3.

(2) The design should be based on a concrete strength class defined by the characteristic compressice strength for a cylinder f_{ck}. If the strength is determined by testing cubes ($f_{ck,cube}$), the corresponding cylinder strength can be obtained by appropriate conversion factors given in table 2.1.1 of MC 90, 2.1.3.2. The testing conditions shall be in accordance with ISO 2736.

(3) The preferred concrete strength classes are given in table 2.1, and the main mechanical properties are defined for each class.

Table 2.1 Preferred concrete strength classes and mechanical properties

	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
f_{ck}	20	25	30	35	40	45	50	60	70	80
f_{ctm}	2.2	2.6	2.9	3.2	3.5	3,8	4.1	4.6	5.1	5.6
E_{cm}	29	30.5	32	33.5	35	36	37	39	41	43

with: f_{ck} = characteristic cylinder strength
f_{ctm} = mean tensile strength ; $f_{ctk,0.05} = 0.70\ f_{ctm}$; $f_{ctk,0.95} = 1.30\ f_{ctm}$
E_{cm} = secant modulus of elasticity at $\sigma_c = 0.40\ f_{ck}$

2.1.2 Design compressive strength

(1) The uniaxial design strength of concrete is

$$f_{1cd} = \alpha \ f_{ck} / \gamma_c \tag{2.1}$$

with: α = reduction factor considering reduction of f_{1cd} versus strength of control specimen and duration of loading:
$\alpha = 1.00$ for SLS ; $\alpha = 0.85$ for ULS
(other values may apply for other stress rates)

γ_c = partial safety factor
$\gamma_c = 1.00$ for SLS ; $\gamma_c = 1.50$ for ULS

(2) The variation of concrete strength with age depends on many parameters (e.g. curing conditions and cement type) and a universally applicable relationship cannot be given. In absence of more accurate data the strength increase may be estimated from Fig. 2.1, which is valid for two types of portland cement concrete.

2.1.3 Stress - strain diagram for concrete in compression

2.1.3.1 Stress - strain diagram for the analysis and for SLS

(1) The modulus of elasticity for linear-elastic analyses is given in table 2.1. The range of variation may extend from 0.7 E_{cm} to 1.0 E_{cm}.

(2) For more refined analyses the stress - strain diagram given in sect. 2.1.4.4.1 of MC 90 may be applied.

2.1.3.2 Stress - strain diagram for ULS

(1) For dimensioning cross-sections the parabola-rectangle stress - strain distribution of Fig. 2.2 is preferably be applied.

The maximum strain is defined as follows:

$$\varepsilon_{cu} = -0.0035 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad (2.2\text{ a})$$

$$\varepsilon_{cu} = -0.0035\,(50 / f_{ck}) \quad \text{for } f_{ck} > 50 \text{ MPa} \qquad (2.2\text{ b})$$

(2) Other equivalent diagrams may be used like the bi-linear diagram shown in Fig 2.3. A further simplified diagram is the uniform stress diagram given in sect. 5.2.2.2.

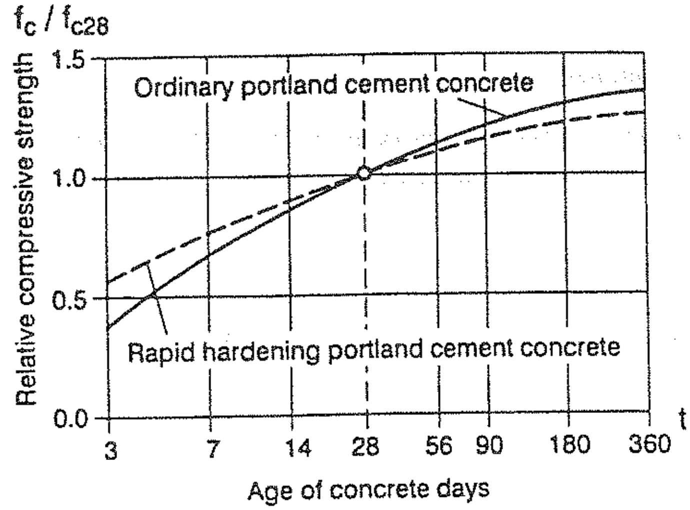


Fig. 2.1 Variation of concrete strength with age

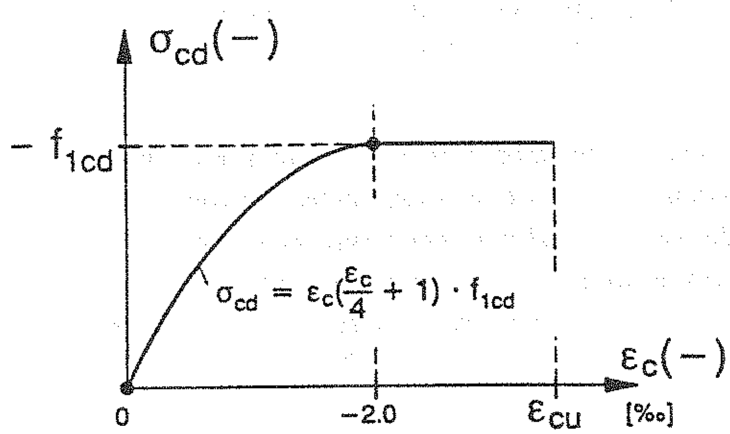


Fig. 2.2 Preferable stress-strain distribution in concrete compression zones

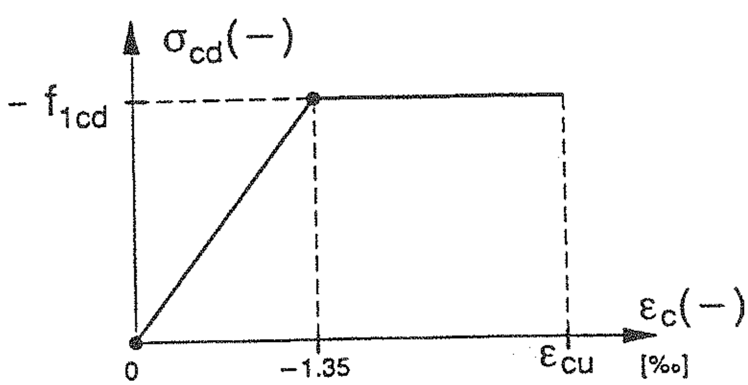


Fig. 2.3 Simplified bi-linear stress-strain distribution in concrete compression zones

2.1.4 Tensile resistance and cracking of concrete

(1) The basic reference value for assessing the strength of concrete ties is the uniaxial tensile strength, and its average value given in table 2.1 is:

$$f_{ctm} = 0.30\ f_{ck}^{2/3} \qquad (2.3\ a)$$

The lower and upper characteristic values given in Fig. 2.4 are:

$$f_{ctk,0.05} = 0.70\ f_{ctm} \quad \text{and} \quad f_{ctk,0.95} = 1.30\ f_{ctm} \qquad (2.3\ b)$$

(2) The design value is

$$f_{1ct,d} = f_{ct} / \gamma_{ct} \qquad (2.4)$$

with: f_{ct} = relevant value from above (1)
γ_{ct} = 1.7 for ULS (or 1.0 if more unfavourable)
γ_{ct} = 1.0 for SLS (or 1.3 in certain specific cases)

(3) The assessment of the crack formation requires to consider the realistic behaviour of the fracture process in tension zones. An empirically derived practical rule considering the size effect is to average the tensile stress over a representative depth c in the stress diagram and compare it with the relevant value of the axial tensile strength. This representative depth c may be taken as MC 90, 2.1.5.3

$$c = 3\ d_g \leq 50\ \text{mm} \qquad (2.5)$$

with: d_g = the maximum aggregate size

(4) As a practical application of the above rule, the maximum tensile stress at cracking (flexural tensile strength) may be derived for a rectangular section subjected to the cracking moment M_r combined with an axial force as follows (Fig. 2.5):

$$\sigma_{t,max} = \left(1 - \frac{c}{h}\nu_T\right) / \left(1 - \frac{c}{h}\right) f_{1ct,d} \leq 2\ f_{1ct,d} \qquad (2.6)$$

with: $\nu_T = \sigma_N / f_{1ct,d} = N_{Sd} / b\,h\,f_{1ct,d}$ (+ for tension)

$f_{1ct,d}$ = relevant design value for the axial tensile strength acc. to equ. (2.4)

The maximum tensile stress for a section subjected to pure bending is shown in Fig. 2.6 as a function of the depth of the member.

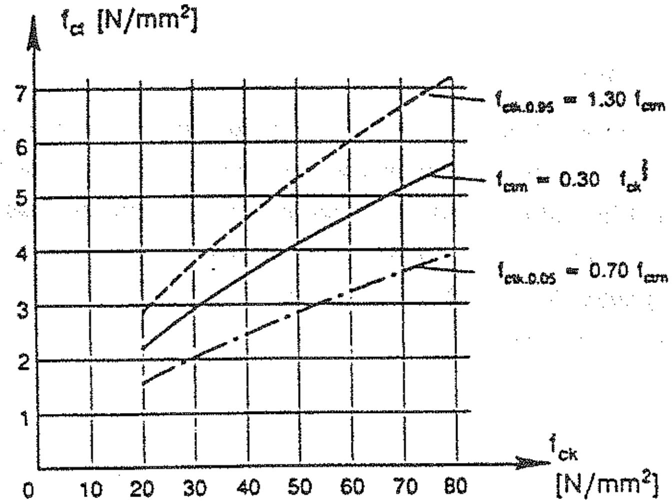


Fig. 2.4 Design values for the concrete tensile strength

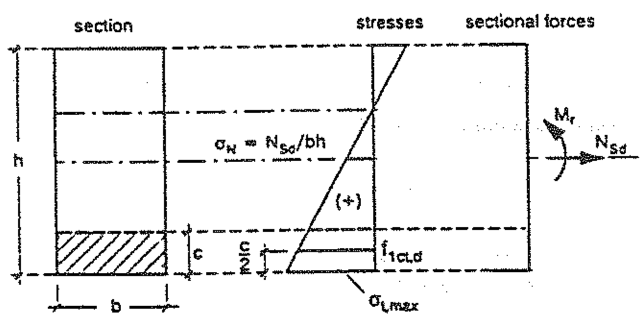


Fig. 2.5 Flexural tensile strength of a rectangular section subjected to combined bending moment and axial force

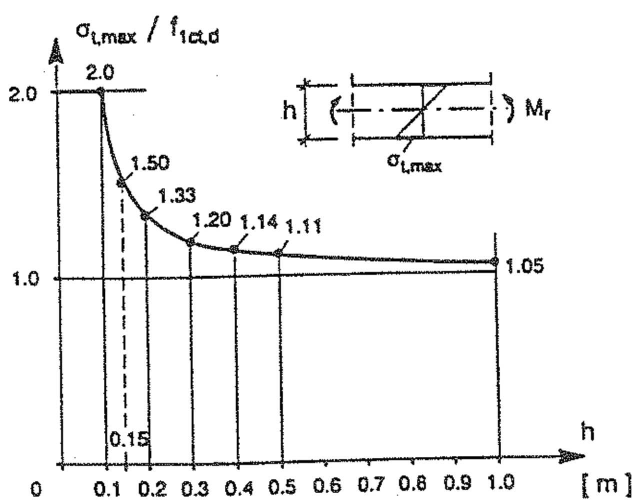


Fig. 2.6 Bending tensile strength as a function of the depth of the member (size effect) for c = 50 mm

2.1.5 Shrinkage and creep

MC 90, 2.1.6.4.3; 2.1.6.4.4

(1) The deformations of concrete due to shrinkage and creep may vary considerably with the type of cement and aggregates, with the climate (temperature and humidity), with the member size and time of loading. The final values in table 2.2 and table 2.3 are mean values, and apply to concrete of grades 20 to 50 MPa subjected to a stress not exceeding $0.4\ f_{c,\tau_0}$ at age τ_0 of loading.

Table 2.2 Final value for shrinkage strain ε_{cs} $[10^{-3}]$

Atmospheric condition	effective member size $2\ A_c\ /\ u$ [mm] 50	150	600
50 % (dry; inside)	- 0.53	- 0.51	- 0.36
80 % (humid; outside)	- 0.30	- 0.29	- 0.20

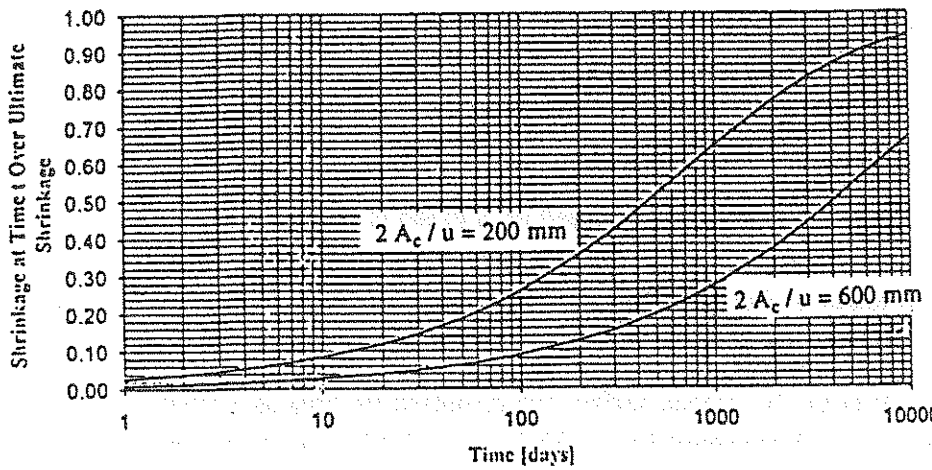
Table 2.3 Final value for the coefficient ϕ of the creep deformations for concrete grades $\leq$ C50 *

Age at loading τ_0 [days]	Atmospheric conditions: dry (indoors) (RH = 50 %)			humid (outdoors) (RH = 80 %)		
	effective member size $2\ A_c\ /\ u$ [mm]					
	50	150	600	50	150	600
1	5.6	4.6	3.7	3.7	3.3	2.8
7	3.9	3.2	2.6	2.6	2.3	2.0
28	3.0	2.5	2.0	2.0	1.8	1.5
90	2.4	2.0	1.6	1.6	1.4	1.2
365	1.8	1.5	1.2	1.2	1.1	1.0

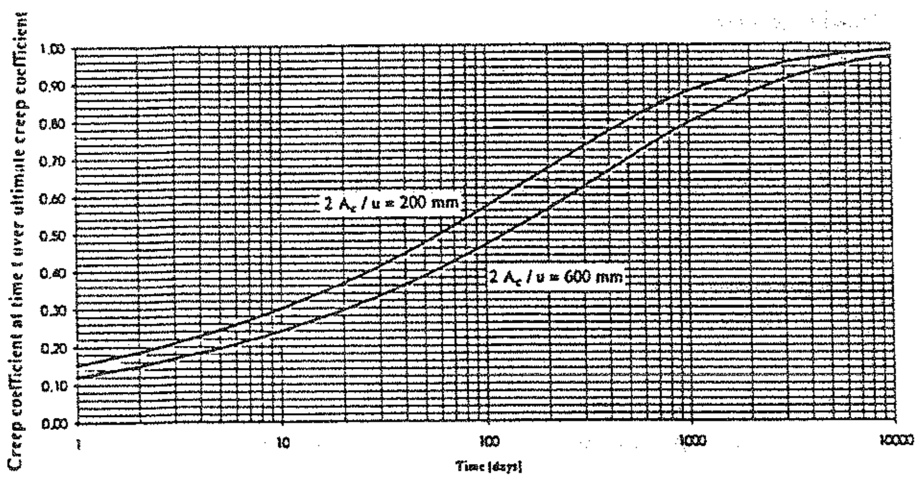
* The values of the creep coefficient f must be used in conjunction with the modulus of elasticity defined in table 2.1. For creep sensitive structures the characteristic values for the creep and shrinkage coefficients should be considered (see MC 90).

(2) The development of the shrinkage strain and of the creep coefficient ϕ_t with age may be estimated from Fig. 2.7.

(3) For special structures and other conditions more detailed information is required (see MC 90) or own tests have to be carried out.



a) development of shrinkage strain with time



b) average development of ϕ_t with time for all environmental conditions

Fig. 2.7 Development of shrinkage strain and of creep coefficient ϕ_t with time

2.1.6 Coefficient of thermal expansion

MC 90, 2.1.8.3

The coefficient of thermal expansion may vary between 6×10^{-6} and 15×10^{-6} depending on the type of aggregates and the moisture of concrete. For the structural analysis a value of 10×10^{-6} may be taken.

2.1.7 Fatigue strength

The properties of concrete in fatigue exhibit a large scatter, and especially the tensile strength should be used with caution. For ordinary buildings and bridges fatigue is rarely critical.

2.2 Reinforcing steel

2.2.1 Steel grades

(1) The design may normally be based on a grade of ribbed steel selected from the values S400 or S500, where the numbers denote the characteristic strength f_{yk} (N/mm^2) defined in section 2.2.2. Other values, according to national practice, may be chosen.

(2) As a criterium for ribbed surface the projected rib area as defined by the European Prestandard ENV 10080 (April 1995) may be chosen.

2.2.2 Tensile strength

MC 90, 2.2.4.1

(1) The characteristic strength f_{yk} is defined as the 5 % fractile of the yield strength f_y or the 0,2 % - proof stress (denoted as $f_{0,2}$).

(2) If the steel supplier guarantees a minimum value for f_y or $f_{0,2}$, that value may be taken as the characteristic strength.

(3) The design strength is

$$f_{yd} = f_{yk} / \gamma_s \qquad (2.7)$$

with: γ_s = partial safety factor

$\gamma_s = 1.00$ for SLS ; $\gamma_s = 1.50$ for ULS

2.2.3 Compressive strength

If the reinforcing steel is used in compression, normally the value f_{yk} respectively f_{yd} applies, unless the actual value in compression (f_{yck}) is smaller than in tension (f_{ytk}) (reference should be made to the approval document).

2.2.4 Modulus of elasticity and idealized stress-strain diagrams

(1) Due to the diversity and evolution of the manufacturing processes for bars and wires, various stress-strain diagrams may be encountered.

(2) As a simplification, actual stress-strain diagrams for all reinforcements of structural concrete may be replaced by an idealized characteristic diagram according to Fig. 2.8. Thereby the modulus of elasticity E_s may be taken as equal to 200 GPa.

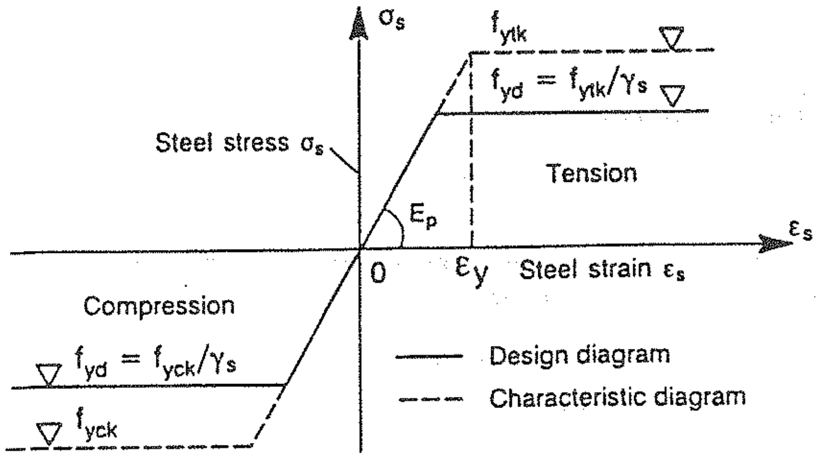


Fig. 2.8 Idealized stress-strain diagram for reinforcing steel

2.2.5 High bond reinforcement

Bars and wires may be considered to be high bond reinforcement if the projected rib area f_R (defined by ENV 10080) complies with the following values:

$f_R \geq 0.039$ for bar diameters $5 \leq Ø \leq 6$ mm
$f_R \geq 0.045$ for bar diameters $6.5 \leq Ø \leq 8.5$ mm
$f_R \geq 0.052$ for bar diameters $9 \leq Ø \leq 10.5$ mm
$f_R \geq 0.056$ for bar diameters $11 \leq Ø \leq 40$ mm

2.2.6 Ductility

MC 90, 2.2.4.4

(1) Adequate ductility is necessary whether or not moment redistribution is taken into account in the calculations. For design purpose, this may be defined by minimum specified values for the characteristic value of the ratio f_t / f_y and the characteristic elongation ε_{uk} at maximum load as follows:

$$(f_t / f_y) \geq 1,08 \quad \text{and} \quad \varepsilon_{uk} \geq 5\ \% \qquad (2.8)$$

The characteristic value of the ratio f_t / f_y corresponds to the 5 %-fractile of the relation between actual tensile strength and actual yield stress.

(2) If the above values are not respected, refer to MC 90.

2.2.7 Coefficient of thermal expansion

The coefficient of thermal expansion may be taken as 10×10^{-6} per degree centigrade.

2.2.8 Fatigue strength

MC 90, 2.2.4.5

(1) The characteristic fatigue strength $\Delta\sigma_{Rsk}$ is defined by S-N-curves.

(2) In the absence of test results the values adopted for $\Delta\sigma_{Rsk}$ at 10^6 cycles may be $\Delta\sigma_{Rsk} = 195$ N/mm² for straight bars.

For bent bars with a mandrel diameter $d_b < 25\ Ø$, these values should be reduced by applying the following coefficient:

$$\xi = 0.35 + 0.026\ d_b / Ø \qquad (2.9)$$

(3) For welded bars including tack welding and for butt joints the value at 10^7 cycles may be taken as $\Delta\sigma_{Rsk} = 60$ N/mm² .

(4) For couplers, the manufacturer must justify the strength by test results by means of technical approval documents.

2.3 Prestressing steel

2.3.1 Steel grades

The grade of a prestressing steel shall be specified by its characteristic strength $f_{0,1k}$ (defined in section 2.3.2) and its characteristic tensile strength f_{ptk} as S ($f_{0,1k}$ / f_{ptk}).

2.3.2 Tensile strength

(1) The characteristic tensile strength of prestressing steel is $f_{0,1k}$.

(2) The design strength is defined as a simplification by

$$f_{ptd} = 0.90\ f_{ptk} / \gamma_s \qquad (2.10)$$

with γ_s = partial safety factor as defined in sect. 2.2.2.

2.3.3 Relaxation

The relaxation values to be taken into account for the final prestress can be obtained

(a) from data given in the approval documents, or

(b) by using approximate values, or

(c) from the results of reliable relaxation tests.

In the absence of more accurate information, as the final value of relaxation a 6 % loss may be adopted for $\sigma_{p0} = 0.7\ f_{ptk}$ for low relaxation steels and 12 % for other steels and the same initial stress.

2.3.4 Modulus of elasticity and idealized stress-strain diagram

(1) In the absence of more accurate information, the stress-strain-diagram in Fig. 2.9 may be used.

Unless more precise information is available, the modulus of elasticity of prestressing steel may be taken as:

- E_p = 205 kN/mm^2 for wires,
- E_p = 200 kN/mm^2 for bars,
- E_p = 195 kN/mm^2 for strands.

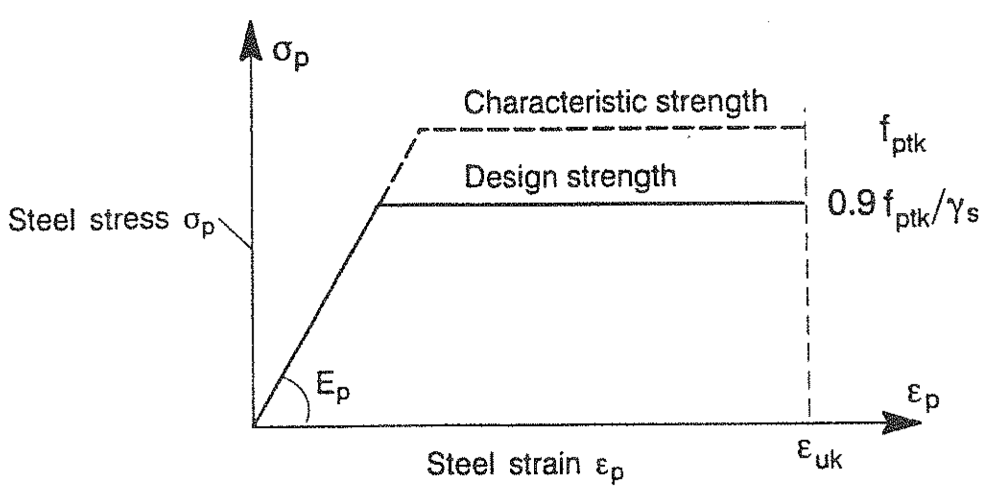


Fig. 2.9 Stress-strain diagram for prestressing steel

2.3.5 Bond properties of prestressing reinforcement

(1) The bond properties of prestressing reinforcement may be regarded equivalent to that of high bond reinforcing bars, if the criteria in sect. 2.2.6 are satisfied.

(2) The bond properties of smooth wires should be determined, either on basis of technical approval documents, or by means of tests corresponding to the conditions of use. Approximate values for the transmission and anchorage lengths are given in sect. 2.4.2.

2.3.6 Ductility

MC 90, 2.3.4.4

It should be shown that the ductility of the steel is adequate for its use in the event of a redistribution of stress. The unit elongation ε_{uk} at maximum load shall be at least equal to $\varepsilon_{uk} = 0.035$.

2.3.7 Coefficient of thermal expansion

MC 90, 2.3.5.3

The coefficient of thermal expansion may be taken as 10×10^{-6} per degree Centigrade.

2.3.8 Fatigue strength

(MC 90, 2.3.4.6)

(1) The characteristic fatigue strength $\Delta\sigma_{Rsk}$ is defined as the 10 %-fractile by S-N-curves .

(2) In the absence of test results, the values $\Delta\sigma_{sk}$ given in Table 2.4 may be adopted for steel embedded in concrete.

Table 2.4 Fatigue strength $\Delta\sigma_{Rsk}$ for steel embedded in concrete:

number of cycles	$N = 10^6$	$N = 2 \times 10^6$	$N = 100 \times 10^6$	
Pretensioning steel (straight):	185	175	120	N/mm^2
Posttensioning steel:				
- single layer of strand in plastic ducts (straight or curved)	185	175	120	N/mm^2
- curved tendons in plastic ducts	160	150	95	N/mm^2
- straight tendons in steel ducts	160	150	95	N/mm^2
- curved tendons in steel ducts	120	110	65	N/mm^2
- couplers	80	70	30	N/mm^2

2.4 Bond between concrete and reinforcement

MC 90, 6.9.3
6.9.4
6.9.5

2.4.1 High bond reinforcement

(1) Bars and wires may be considered to be high bond reinforcement if the relative rib area f_R satisfies the conditions given in sect. 2.2.6.

(2) The bond stress τ_{bd} may be assumed to be constant over the anchorage length l_b of a straight bar with the diameter Ø:

$$l_b = \text{Ø}\ f_{yd} / (4\ f_{bd}) \tag{2.11}$$

with: $f_{bd} = 1.05\ f_{ctm}$ (2.12)
limiting design value for bond stress τ_{bd} for good bond conditions (includes material safety factor)

(3) The values for f_{bd} and l_b / Ø for good bond conditions are given in Table 2.5 for the different concrete classes.

(4) The bond conditions are considered to be good for the cases shown in Fig. 2.10. In all other cases the bond conditions are considered poor and the limiting value f_{bd} should be multiplied by 0.70.

Table 2.5 Limiting values for the design bond stress f_{bd} and anchorage length l_b for the recommended concrete classes (for f_{yd} = 435 MPa)

	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
f_{bd}	2.3	2.7	3.0	3.4	3.7	4.0	4.3	4.8	5.4	5.9
l_b /Ø	47.3	40.3	36.2	32.0	29.4	27.2	25.3	22.6	20.1	19.4

(5) The limiting value f_{bd} may be increased in the presence of transverse pressure, so that the anchorage length l_b may be reduced by the factor:

- 2 /3 in case of an in-plane transverse pressure like e.g. at an end-anchorage over a support (C-C-T-node, see sect. 5.5.1).
- 0.50 in case of a similar pressures transversely to the possible planes of splitting .

(6) For bar diameters Ø > 32 mm the limiting value f_{bd} should be multiplied by [(132 - Ø) / 100].

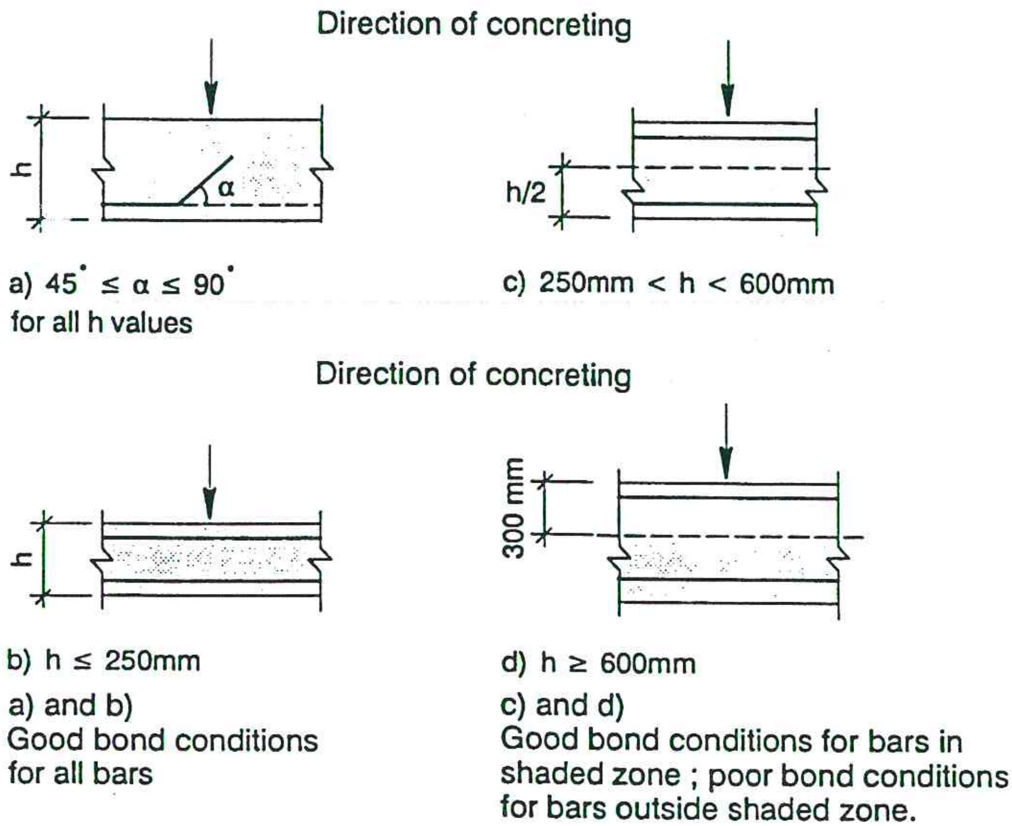


Fig. 2.10 Conditions for good bond of the reinforcement

2.4.2 Bond of posttensioning reinforcement

The bond properties of ribbed posttensioning reinforcement in grouted tendons may be regarded equivalent to that of high bond reinforcing bars, if the criteria in sect. 2.2.6 are satisfied.

2.4.3 Bond of pretensioning reinforcement

2.4.3.1 Bond strength

(1) The design value of the bond strength f_{bpd}, including the material safety factor, can be taken from Table 2.6. These values apply to good bond conditions as defined in Figure 2.11; in poor bond conditions the values should be multiplied by the factor 0.7.

Table 2.6 Design values of bond strength f_{bpd} for pretensioning tendons

Type of tendons	C20	C25	C30	C35	C40	C45	C50	C60	C70	C80
7-wire strands	1.2	1.4	1.6	1.8	1.9	2.1	2.2	2.5	2.8	3.0
Indented or crimped wires	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.9	3.2	3.5

2.4.3.2 Transfer of prestress

(1) At release of tendons, the prestress can be assumed to develop linearly from zero to its full value over the following transmission length l_{bpt}:

$$l_{bpt} = \alpha_1 \ \alpha_2 \ \alpha_3 \ \varnothing \ \sigma_{pi} \ / \ (4 \ f_{bpd}(t)) \qquad (2.13)$$

where: α_1 = 1.00 for gradual release
= 1.25 for sudden release

α_2 = 0.50 for verification at release of tendons
1.00 for verifications in ULS

$\varnothing$ = nominal diameter of tendon

σ_{pi} = prestress after release and before time-dependent losses

$f_{bpd}(t)$ = design value of bond strength according to Table 2.6 for concrete strength at time of release

2.4.3.3 Anchorage in ULS

The anchorage length in the ultimate limit state can be calculated as:

$$l_{bpd} = l_{bpt} + \alpha_4 \ \varnothing \ (\sigma_{pd} - \sigma_{pcs}) \ / \ (4 \ f_{bpd}) \qquad (2.14)$$

where l_{bpt} = transmission length acc. to equ. (2.13)
$\varnothing$ = nominal diameter of tendon,
σ_{pd} = stress to be anchored
σ_{pcs} = pretress after all losses
f_{bpd} = design value of bond strength acc. to Table 2.6

3 Prestressing

3.1 Types and Definiton of Prestressing

3.1.1 Definition of prestress

(1) The prestress is applied by a construction controlled process (prestressing) by stressing tendons (prestressing reinforcement) relatively to the concrete member. Generally, the prestress is defined by the relative deformation between the prestressing steel and the concrete member.

(2) Other means of prestressing are not considered in this document.

3.1.2 Types of prestress

MC 90, 4.1

(1) The prestress considered in these Recommendations is exerted by tendons made of high-strength steel (wires, strands or bars).

Tendons may be used

(a) internal to the concrete, and
- (a1) pretensioned, or
- (a2) post-tensioned; in this case they may be bonded by grouting, or provisionally or permanently unbonded.

(b) external to the concrete; they may then be
- (b1) totally within the external outline of the structure, or
- (b2) partially or totally outside (except in anchorage points) the outline of the structure. However, fatigue of such structures requires special considerations and is not covered in this document.

Further reference is made to the FIP-Recommendations "Acceptance of post-tensioning systems" (1993).

(2) The prestress may be
- non-detachable and non-adjustable (which is always the case for pretensioning and internal bonded tendons),
- non-detachable but adjustable,
- detachable and adjustable.

(3) Anchorages may be active or passive or coupling.

3.2 Initial prestress

MC 90, 4.2

3.2.1 Prestressing steel

The tensile stress in the tendons should not exceed the following values:

a) at the moment of tensioning: $\sigma_{pi} = 0.80\ f_{ptk}$

b) after transfer of prestress: $\sigma_{po} = 0.75\ f_{ptk}$

3.2.2 Time of prestressing

Where particular rules are not given, the time when prestressing takes place should be fixed with due regard to the following factors:

a) Conditions for the deformation of the component
b) Safety with respect to the actual compressive strength of concrete
c) Safety with respect to local stresses
d) Safety with respect to the anchorages of the tendons
e) Advantage of applying some prestress at an early stage
f) Early creep deformation in anchorage zones.

3.3 The decrease of prestressing force, the "losses of prestress"

MC 90, 4.3.2

The decrease of prestressing force, so-called "losses of prestress", should be determined thoroughly, because the prestressing force is to be considered with its mean value only (see sect. 3.4.2).

3.3.1 Losses before releasing the tendons (pretensioning)

The following losses should be considered in design:

a) losses due to friction at the deflectors; and losses due to movement in the anchoring devices (at the abutments) when anchoring on a prestressing bed,

b) losses due to relaxation of the pretensioned tendons during the period that elapses between the tensioning of the tendons and the prestressing of the concrete.

3.3.2 Immediate losses

(1) The following influences should be considered in design:

a) the instantaneous concrete deformation,
b) friction between tendon and sheathing,
c) draw-in of the anchorage,
d) steam curing, etc..

(2) The losses due to friction may be estimated as follows MC 90, 4.3.3.2

$$\Delta\sigma_{pi} = \sigma_{pi}\ [1 - \exp\{-\mu(\alpha + k\,x)\}] \qquad (3.1)$$

where:

$\Delta\sigma_{pi}$ = loss of stress in tendon at a distance x from anchorage,
σ_{pi} = stress in tendon at the anchorage,
μ = coefficient of friction,
α = sum of the angular displacements along x,
k = unintentional angular displacement (per unit length).

(3) If more accurate information is not available, the following μ- values can be accepted as being representative for unlubricated tendons. The coefficient k depends essentially on the accuracy with which the theoretical shape of the tendons is attained in practice; the following are approximate values for friction and wobble effects.

The following values μ may be adopted:

a) metal sheathing:

- for strands:	μ = 0.18 - 0.20
- for smooth wires:	μ = 0.18 - 0.20
- for wires that are not smooth:	μ = 0.30
- for bars:	μ = 0.30

b) other sheathing:

- for plastic ducts:	μ = 0.14
- for unbonded mono-strands	μ = 0.05 - 0.07

The following k-values may be assumed:

- in general:	$k = 0.005 - 0.010\ m^{-1}$
- in segmental construction:	$k = 0.010 - 0.020\ m^{-1}$

(4) With external prestressing, the friction is concentrated at deviation devices.

3.3.3 Time dependent losses

(1) The evaluation of the time dependent losses due to shrinkage and creep of the concrete and relaxation of the steel should take into account the interdependence of these phenomena.

(2) The time-dependent losses are calculated by considering the following two reductions of stress:

(a) the reduction of stress, due to the reduction of strain, caused by the deformation of concrete due to creep and shrinkage, under quasi-permanent actions:

(a1) for bonded tendons, the local deformation at the level of the tendons has to be considered;

(a2) for unbonded tendons, the deformation of the whole structure between the constraints of the tendons has to be taken into account;

(b) the reduction of stress within the steel due to the relaxation of this material under tension.

(3) The relaxation of steel depends on the reduction of strain due to creep and shrinkage of concrete. This interaction may be taken into account in a simplified manner by reducing the value of relaxation at constant length by 20%.

(4) An assessment of the total loss of prestress due to shrinkage, creep and relaxation can be carried out by means of the following formula:

$$\Delta\sigma_p = \frac{\alpha\phi(t,t_0)(\sigma_{cg} + \sigma_{cp0}) + E_p\varepsilon_{cs} + 0.8\Delta\sigma_{pr}}{1 + \alpha\frac{A_p}{A_c}\left(1 + \frac{A_c y_p^2}{I_c}\right)(1 + \chi\phi(t,t_0))} \qquad (3.2)$$

where:

all compressive strains and stresses, as well as prestressing losses are considered as negative.

α = E_s / E_c,

$\phi(t, t_o)$ = creep coefficient

t_o = age of concrete at prestressing

t = age of concrete at considered time

σ_{cg} = stress in the concrete at the level of the tendons due to permanent actions (excluding prestressing),

σ_{cp0} = initial stress in the concrete at the level of the tendons due to the prestressing alone

$\Delta\sigma_{pr}$ = loss of stress in the tendon (negative) due to relaxation (at constant length) acting alone,

χ = aging coefficient, which may be taken as 0.8 for long-time calculations,

3.4 Design considerations

3.4.1 Design value of the prestress and requirements

MC 90, 4.6.3
4.6.4

(1) In general, the effect of prestress has to be considered as descriped in 1.3 (4) für SLS and in 1.2 (5) for ULS.

(2) The design value for prestress in general is equal to the mean value in SLS and ULS.

(3) Normally it is not necessary to consider different fractiles for the prestressing force, but only the mean value

$$P_m = P_i - \Delta P \qquad (3.3)$$

where: P_m mean value at time t for section at distance x from origin,
P_i initial prestress at origin
ΔP immediate and time dependent losses.

(4) For most cases it is sufficient to consider the values of prestress at two different periods:

a) Initial prestress (t = 0) after transfer of prestress:

$$P_{m0} = P_i - \Delta P_0 \qquad (3.4)$$

b) Long-term prestress ($t = \infty$):

$$P_{m\infty} = P_i - \Delta P_0 - \Delta P_\infty \qquad (3.5)$$

In general, P_{m0} is critical for combination with the effects of permanent actions at transfer, whereas $P_{m\infty}$ is to be considered with the total actions.

(5) At transfer of prestress, Serviceability Limit State should be verified for P_{mo} in combination with permanent actions. Restraining effects during transfer shall be duly considere, such as caused by deformations of the scaffolding or by longitudinal restraints, etc..

(6) The tendon anchorages shall satisfy the requirements of the "FIP Recommendations for Acceptance of Post-Tensioning Systems" for the load transfer from the anchorage into the structure (local zone around anchorage).

(7) The transfer of the tendon force from the local zone around the anchorage into the D-region of the member or structure can be verified according to sect. 6.5.7. At Serviceability Limit State the initial prestressing force P_i shall be used as the applied tendon force. At Ultimate Limit State the characteristic tendon force ($A_p\ f_{ptk}$) shall be used as design value of the tendon force.

(8) Due consideration shall be paid to the forces in local zones caused by deviations of tendons; the verification can be carried out according to sect. 6.5.7.

3.4.2 Design of prestress

3.4.2.1 Definitions

(1) The following definitions are not intended as a classification of structures with regtard to prestressing, but rather as an indication concerning the design criteria to be applied. As a matter of principle, the whole range from full prestress to no prestress (reinforced concrete) is allowed,and it is up to the designer to choose the most appropriate degree of prestressing for a given structure.

(2) For the structural behaviour of prestressed structures the degree of prestress may be defined in either of the following ways:

a) The mechanical degree of prestressing pertains to the ULS, and it is defined as:

$$\lambda = A_p\, 0.9\, f_{ptk} \ / \ [\ A_s\ f_{yk} + A_p\, 0.9\, f_{ptk}\ ] \qquad (3.6)$$

where

A_p = area of prestressing steel in critical sections,
A_s = area of reinforcing steel in critical sections,
f_{ptk} = characteristic strength of prestressing steel,
f_{yk} = characteristic strength of reinforcing steel.

The mechanical degree of prestressing is, amongst others, particularly helpful in comparing designs based on different loading regulations and in appreciating test results.

b) The degree of load balancing pertains primarily to the SLS, and it is convenient for shallow members where deflection control for a given level of SLS loading is an important consideration. It is defined as:

$$\kappa = S_p / S_g \quad \text{or} \quad M_p / M_g \quad \text{or} \quad p / g \qquad (3.7)$$

(general) (normal) (special)

where:

S_p = (total) action effect due to prestress,
S_g = action effect due to permanent load,
M_p = bending moment due to prestress,
M_g = bending moment due to permanent load,
p = equivalent load due to prestress,
g = permanent load.

3.4.2.2 Design criteria for prestress

(1) The design criteria shall be established in order to combine the advantages of reinforced and prestressed concrete behaviour. The prestress mainly effects favourably the structural behaviour under service load conditions. The cracking load is increased and the steel stress after cracking are lower resulting in smaller crack widths. Furthermore, deflections for permanent loads can always be controlled by prestressing.

(2) The design criteria for prestressing may be based on the deflection control (common for slabs) or by limiting tensile stresses in the concrete to avoid cracking (common for beams) under frequent or permanent loads.

(3) Typical amounts of prestress in structures vary considerably with span, loading conditions, local codes, etc.. The amount of prestress can be expressed in different ways, and the following values can be often found and may serve as a starting point in a conceptual design.

	Average prestress 1) [N /mm²]	Load balancing 2) [%]	ratio per surface 3) [kg/m²]
Office floors	1.0 - 2.0	60 - 100	4 - 8
Raft foundations	0.75 - 1.50	60 - 100	10 - 20
Precast beam bridge (spans 20 - 35 m)	---	---	15 - 25
in-situ box girder bridge (spans 35 - 100 m)	---	---	30 - 40
precast segmental box girder bridge (spans 35 - 100 m)	---	---	35 - 45

1) Total effective prestress divided by total concrete section

2) Total effective deviation force of prestress divided by total permanent loads

3) Total weight of prestressing steel divided by surface of structure

4 Technological Details and Durability Requirements

MC 90, 1.5.2

4.1 Exposure classes

Environmental conditions mean those chemical and physical actions to which the concrete is exposed and which result in effects that are not considered as loads or action effects in structural design. In the absence of a more specific study, these environmental conditions may be classified in the exposure classes given in table 4.1.

Table 4.1: Exposure classes related to environmental conditions

Exposure class	Environmental conditions
1. Dry environment*	e.g. interior of buildings for normal habitation or offices
2. Humid environment	
(a) without frost	e.g. - interior of buildings where humidity is high (e.g. in commercial laundries) - exterior components - components in non-aggressive soil and/or water
(b) with frost	e.g. - exterior components exposed to frost - components in non-aggressive soil and/or water and exposed to frost - interior components when the humidity is high and exposed to frost
3. Humid environment with frost and de-icing agents	e.g. interior and exterior components exposed to frost and de-icing agents
4. Sea-water environment	
(a) without frost	e.g. – components partially immersed in sea-water or in the splash zone - components in saturated salt air (coastal area)
(b) with frost	e.g. - components partially immersed in sea- water or in the splash zone and exposed to frost - components in saturated salt air and exposed to frost
5. Aggressive chemical environment**	refer to MC 90, ENV 206, ISO.......

* This exposure class is valid only as long as during construction the structure or some of its components is not exposed to more severe conditions over a period of several months.

** May occur alone or in combination with classes 1 - 4.

4.2 Durability Design Criteria

MC 90, 8.4.1

(1) In order to satisfy the durability requirements the following criteria should be used:

a) An appropriate structural form should be selected at an early stage of the project, in order to avoid disproportionately sensitive structural arrangements and to secure adequate access to all critical parts of the structure for inspection and maintenance.

b) An appropriate quality of concrete in the outer layer ("skin") of the structural elements shall be secured. A dense, well compacted and well cured, strong and low permeability concrete is needed, which should not exhibit map cracking. Besides, an adequate thickness of cocnrete cover should be provided.

c) Adequate detailing of all structural concrete elements should ensure the integrity of critical surfaces or corners and edges in order to avoid any unforeseen concentration of aggressive influences.

d) Under specified environmental conditions and/or for small diameter reinforcing bars or prestressing single wires, nominal crack widths should be controlled under specified load conditions to avoid depassivation during the specified design life.

e) Under strongly aggressive environmental conditions, protective surface coatings may be needed.

(2) All exposed concrete surfaces should be adequately drained. Only preplanned ponding may take place. MC 90, 8.4.2

(3) Drainage of water over concrete should be limited as much as possible, and drainage over joints and seals should be avoided.

(4) In the selection of structural form adequate care should be taken to provide robustness against deleterious liquid or gaseous substances penetrating into the structure.

(5) The geometry of exposed structural components and the form, type and placing of joints, including construction joints, connections, and supports should be chosen such as to minimize the risks of local concentrations of deleterious substances. These concentrations may develop on the surface of the structure as well as within the concrete when these substances enter the concrete by permeation, diffusion, capillary action or similar.

(6) Care should be taken in the detailing of facades of buildings and structures in order to allow easy drainage of water and to facilitate clean-washing.

(7) Surface areas subjected to wetting splashing or water accumulation should be kept as small as possible.

ENV 10080 (1994)

4.3 Preferred nominal diameters for reinforcing bars

(1) The range of preferred nominal diameters Ø (mm) for bars is as follows: 6, 8, 10, 12,14, 16, 20, 25, 32, 40.

(2) Preferred diameters for wires used in welded mesh lie in the range 5 to 12 mm in steps of 0,5 mm, plus Ø14 and Ø16.

4.4 Minimum cover of reinforcements

MC 90, 8.4.3

(1) The minimum distance between any concrete surface and the nearest reinforcement bar, respectively of the nearest prestressing tendon or the sheathing for such tendons, shall be obtained from Table 4.2. The values are absolute minimum values with no downward tolerances allowed.

For exposure classes 2 to 4 the minimum values may be reduced by 5 mm, if the concrete strength class is at least C40.

(2) The nominal values, c_{nom}, are equal to the minimum values plus tolerance according to the rule

MC 90, 10.4

$$c_{nom} = c_{min} + \text{tolerance} \qquad (4.1)$$

Tolerance should be taken as 10 mm unless in the individual case it can be demonstrated that a lower value is obtainable (e.g. in the case of intensified quality control). The tolerance should not be less than 5 mm. The relevant values are given in Table 4.2.

Table 4.2 Minimum cover c_{min}

Exposure class	reinforcing bars		prestressing reinforcement	
	c_{min}	c_{nom}	c_{min}	c_{nom}
	[mm]		[mm]	
1	10	15 or 20	20	25 or 30
2	25	30 or 35	35	40 or 45
3, 4	40	45 or 50	50	55 or 60
5	*	*	*	*

* Depends on the individual type of environment encountered.

(3) To ensure that bond forces are safely transmitted and to prevent spalling of the concrete, the minimum cover of any bar, tendon or sheathing of diameter ∅ should be at least equal to ∅ (sect. 6.9.7.1).

(4) When anchoring is made by means of bends, hoops or loops, it is recommended that in the anchorage zone, the thickness of the cover should be equal to 3 ∅ .

(5) Where fire resistance is needed, other limits may apply.

4.5 Clear bar distances in the horizontal and vertical direction

MC 90, 9.1.3

4.5.1 Generally

(1) The bars in the various horizontal layers should be arranged in vertical planes, leaving sufficient space between them to allow for internal vibration. However, bundling is allowed, see sect. 5.8.

(2) The intermediate horizont or vertical free space between parallel single bars or horizontal layers of parallel bars, should be at least equal to the larges bar diameter but not less than 20 mm.

(3) The maximum size of the aggregate should be chosen to facilitate concreting and adequate compaction of the concrete surrounding the bars.

4.5.2 Members with post-tensioned prestressing reinforcement

MC 90, 9.1.7

(1) The sheathing shall be located so that

- the concrete can be safely placed without damaging the sheathings.
- the concrete can resist the forces from the sheathings in the curved parts under and after tensioning.
- no grout will leak into other sheathing during the grouting process.

(2) The minimum horizontal and vertical free spacing for the tendons are given in Fig. 4.1.

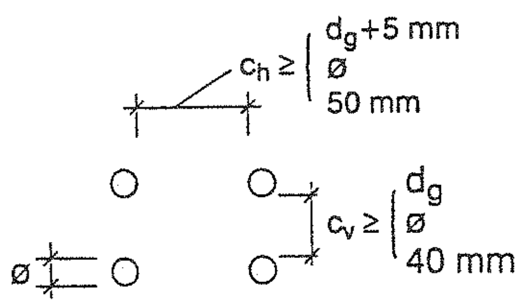


Fig. 4.1 Minimum clear spacing for sheathings (where d_g is the maximum aggregate size)

4.5.3 Members with pretensioned prestressing reinforcement

MC 90, 9.1.7

The minimum horizontal and vertical free spacing of tendons are given in Fig.4.2. Rules for bundling are given in sect. 5.8.

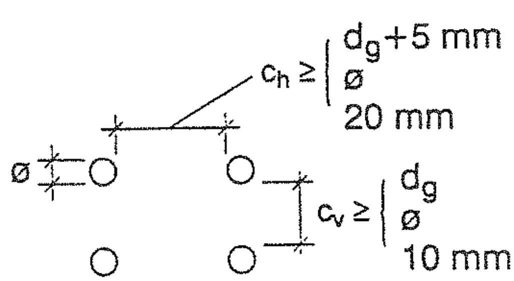
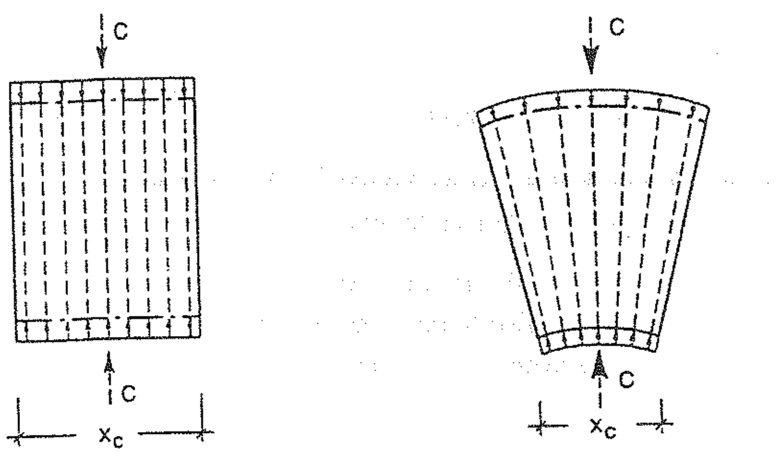


Fig. 4.2 Minimum clear spacing for pretensioning tendons (where d_g is the maximum aggregate size)

5 Strengths of ties, struts and nodes of strut-and-tie models

5.1 General

The elements of a strut-and-tie model are ties, struts and nodes. A tie normally is the resultant of a layer of reinforcing bars or prestressing reinforcement. A strut may either represent the resultant of a parallel respectively a prismatic compression stress-field (e.g .a compression chord or inclined struts in webs) or a fan-shaped compression stress-field (Fig. 5.1). A node is a confined volume of concrete, where either struts intersect or struts are deviated by ties anchored in the node. Nodes also follow from the reinforcement layout by deviating and splicing the reinforcements.



a) parallel or prismatic stress-field b) fan-shaped stress-field

Fig. 5.1 Typical compression stress-fields for struts

5.2 Strength of steel ties MC 90, 6.2.4

(1) In the ultimate limit state normally the tension reinforcements yields, so that the stress in the tie is (see sect. 2.3 and 2.3):

- for reinforcing steel: $\sigma_{sd} = f_{yd}$ (5.1)
- for bonded prestressing steel: $\sigma_{pd} = f_{pd}$ (5.2)

In the latter case it is considered, that the steel has been prestrained to such an amount, that the additional stresses due to loads lead to yielding.

(2) The resisting force of a tie is:

$$F_{RT} = A_s \ f_{yd} + A_p \ f_{pd} \qquad (5.3)$$

(3) In cases the prestress is applied as an external load in the analysis only the reserve capacity beyond the stress $\sigma_{p,Po}$ at decompression due prestressing force can be utilized:

$$\Delta\sigma_{pd} = f_{pd} - \sigma_{p,Po} \qquad (5.4)$$

(4) The stress - strain diagrams are given in sections 2.2.4 and 2.3.4.

5.3 Strength of Struts

5.3.1 Concrete strut in uniaxial compression

MC 90, 6.2.2.2

(1) The basis for any design strength is the uniaxial design strength f_{1cd} of concrete in compression as defined by equ. (2.1).

(2) The capacity of a strut may be determined from the compatibility of strains by using realistic stress-distributions respectively constitutive laws. Normally the parabola-rectangle stress diagram (Fig. 2.2) is recommended, but as a simplification also the bi-linear diagram (Fig. 2.3) may be used.

5.3.2 Capacity of a parallel compression field or prismatic strut

(1) The capacity of a prismatic strut or a parallel compression field of a strut-and-tie model is reduced to an effective strength $f_{cd,\,eff}$ out of different reasons. This strength depends on the state of stress and strain, as well as on the crack widths and geometrical disturbances, and it may be expressed in terms of the two alternative reduction-factors ν_1 and ν_2 :

$$f_{cd,\,eff} = \nu_1 \ f_{1cd} \quad \textbf{or} \quad f_{cd,\,eff} = \nu_2 \ f_{1cd} \qquad (5.5)$$

The resisting force of a strut is :

$$F_{Rcd} = A_c \ f_{cd,\,eff} \qquad (5.6)$$

with $A_c = x_c \, b$ = area of compression strut (Fig. 5.1)

(2) The reduction-factor ν_1 applies to an uncracked strut, if a rectangle stress-block is used instead of a realistic stress-distribution. In case of a compression chord of a beam with a linear strain distribution, the resultant force may be calculated with an average stress $f_{cd,\,eff}$ over the full depth of the compression zone (Fig. 5.2) with:

$$\nu_1 = (1 - f_{ck} / 250) \qquad (5.7)$$

The corresponding maximum strain at the extreme concrete fibre is:

$$\varepsilon_{cu} = -0.004 + 0.002\,(f_{ck} / 100) \qquad (5.8)$$

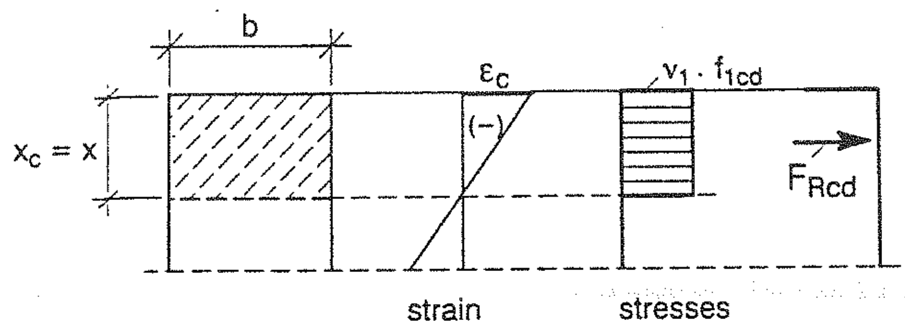


Fig. 5.2 Rectangle stress-distribution (stress-block)

(3) The factor ν_2 covers several influencing effects:

a) $\nu_2 = 0.80$ (5.9 a)
for struts with cracks parallel to the strut with bonded transverse reinforcement; the reduction is due to the transverse tension and to the disturbances by the reinforcement and the irregular crack surfaces.

b) $\nu_2 = 0.60$ (5.9 b)
for struts transferring compression across cracks with normal crack widths, e.g. in webs of beams.

c) $\nu_2 = 0.45$ (5.9 c)
for struts transferring compression over large cracks, like e.g. in members with axial tension or in tension - flanges.

(4) Alternatively for the cases b) and c) ($\nu_2 = 0.60$ or 0.45) the effective strength of struts with a transfer of forces over cracks may also be assessed by means of the constitutive laws for friction, given in sect. 5.4. Thereby due consideration has to be paid to realistically assess the spacing of cracks and the strain conditon.

5.3.3 Reinforced struts

MC 90, 6.2.5

(1) Reinforcing steel bars shall only be considered effective in compression struts if they are placed parallel to the strut, like in a compression chord or in a column.

(2) The bars must sufficiently be secured against buckling by transverse reinforcement, the amount of which is given in sect. 8...

(3) The resisting force of a reinforced strut is:

$$F_{Rc} = A_c \ f_{cd,\,eff} + A_{sc} \ \sigma_{scd} \qquad (5.10)$$

with: A_c = area of the compression strut
$f_{cd,\,eff} = \nu_1 \ f_{1cd}$ in chords of beams and columns
A_{sc} = area of compression reinforcement
$\sigma_{scd} = \varepsilon_c \ E_c \leq f_{ycd}$

5.3.4 Confined concrete struts

MC 90, 3.5.2

(1) The capacity of a strut may be increased by means of an appropriate amount of transverse reinforcement confining the concrete core A_{cc}. The increase in strength may be assessed by the following relationship shown in Fig. 5.3:

$$f_{ccd} \,/\, f_{1cd} = 1 + 1.6 \ \alpha \, \omega_w \qquad (5.11)$$

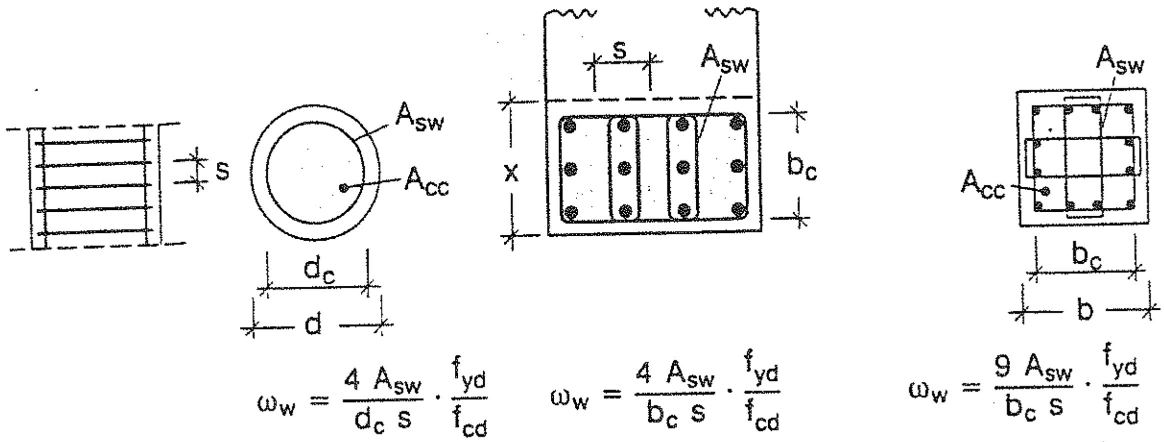
with α and ω_w as defined in Fig. 5.3 for typical cases.

(2) The resisting force of a strut is:

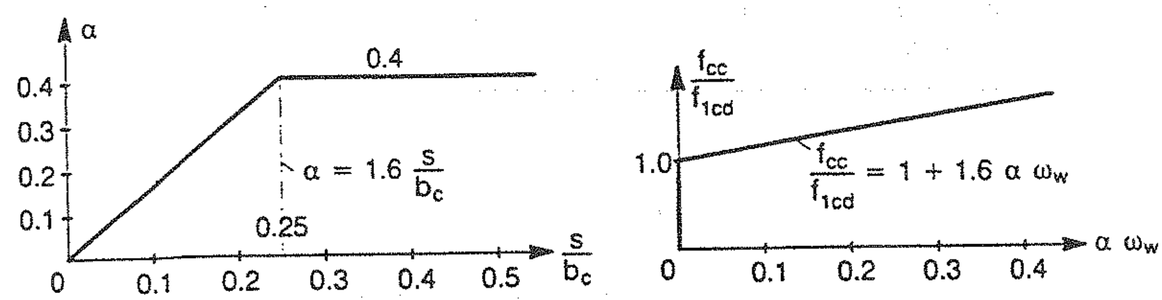
$$F_{Rc} = A_{cc} \ f_{ccd} \qquad (5.12)$$

with: A_{cc} = the concrete core encompassed by the confining transverse reinforcement

(3) The increase in ductility is important for the behaviour of members under reversed loading and earthquakes, and it may be assessed by the relationships given in MC 90, 3.5.2.1 (Fig. 3.5.8).



a) definition of ω_w



b) coefficient α c) capacity of confined concrete

Fig. 5.~~2~~3 Capacity of confined concrete struts

5.3.5 Struts crossed by bars or ducts

(1) If a strut is crossed by bars or ducts with the sum of diameters greater than b / 6 (b = width of strut), the compressive stresses shall be calculated on the basis of a reduced width:

$$b_{red} = b - h\ \Sigma\ Ø \qquad (5.13)$$

with: $\Sigma\ Ø$ = sum of the diameters of bars or ducts at most unfavourable level

η = coefficient depending on the stiffness of bars or ducts

η = 0.5 for bonded bars or grouted ducts,
η = 1.2 for unbonded tendons and ungrouted ducts

(2) The capacity of the strut is given by:

$$F_{Rc} = (x_c \; b_{red}) \; f_{cd,\,eff} \qquad (5.14)$$

with: x_c = depth of the strut (Fig. 5.1)

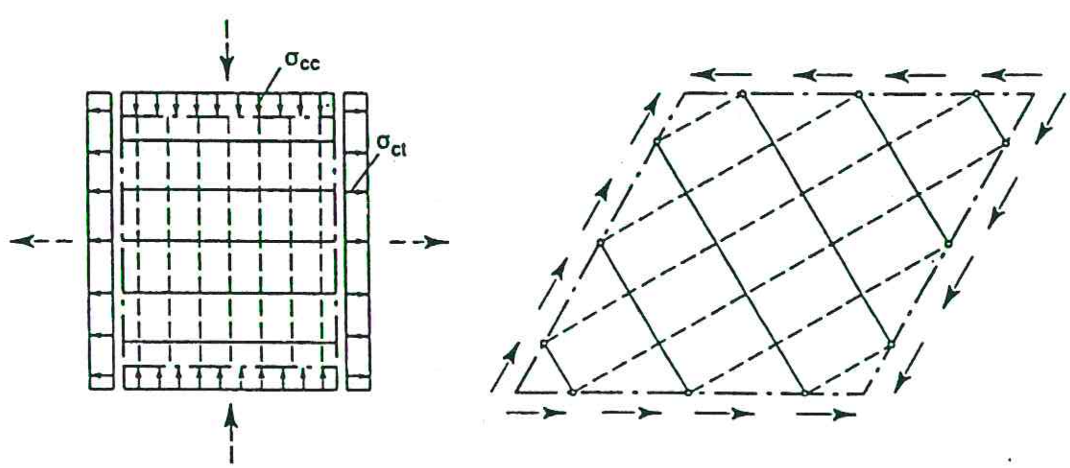
$f_{cd,\,eff}$ = effective strength acc. to equ. (5.5)

(3) Due consideration shall be given to the provision of transverse reinforcement.

5.4 Strength of Concrete Ties

(1) Although normally reinforcement is provided for taking the major tension forces, the ultimate capacity of a member often relies on the tensile resistance of concrete, like in case of members without transverse reinforcement or for bond and anchorages. The axial tensile strength defined in sect. 2.1.4 is the basic reference value for assessing the strength of concrete ties or cracking loads.

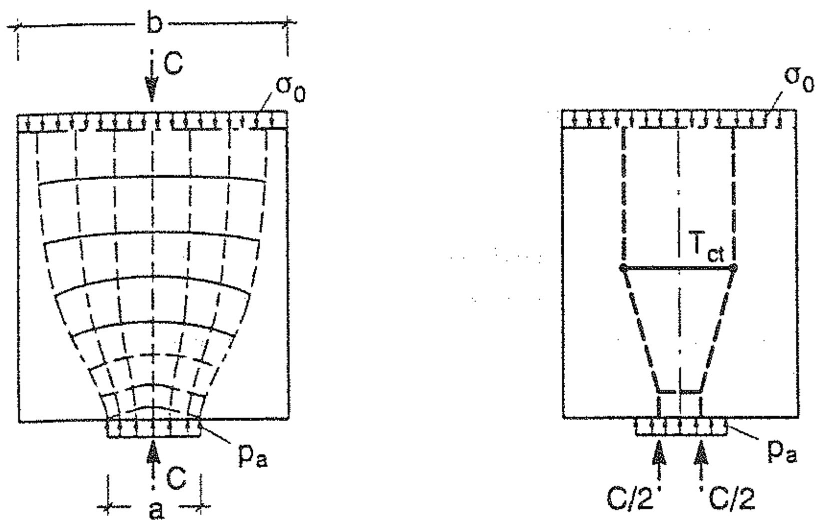
(2) For modelling uncracked regions in members the parallel biaxial stress-field in Fig. 5.4 may be used. It may also represent the behaviour of the concrete between cracks, like in case of webs without transverse reinforcement (slabs without shear reinforcement (sect. 6.5.1), whereby the concrete tensile strength cannot fully be utilized.



a) rectangular plate-element b) skew plate-element for webs

Fig. 5.4 Parallel biaxial tension-compression field in the concrete

(3) The biaxial bottle-shaped tension-compression stress-field in Fig. 5.5 may be used for modelling uncracked D-regions. Its capacity relies on transverse tensile stresses in the concrete, and these depend on the ratio of the width a of the loading plate to the total width b. For a ratio of about $a / b = 0.50$ the lowest value for the cracking load is attained for the pressure $p_a = 0.60\ f_{1cd}$.



a) stress-trajectories b) strut-and-tie model

Fig. 5.5 Bottle-shaped tension-compression field in the concrete for determining cracking loads of D-regions [Schlaich / Schäfer (1993)]

5.5 Transfer of forces by friction across interfaces

5.5.1 General

(1) The capacity for the transfer of compressive forces across an interface by means of concrete-to-concrete friction depends on the condtions of the interface and the material characteristics of the of the adjacent members. The capacity may generally be assessed by a shear-friction law:

$$\tau_{fd} = \beta \, f_{ctm} + \mu \, \sigma_f \qquad (5.15)$$

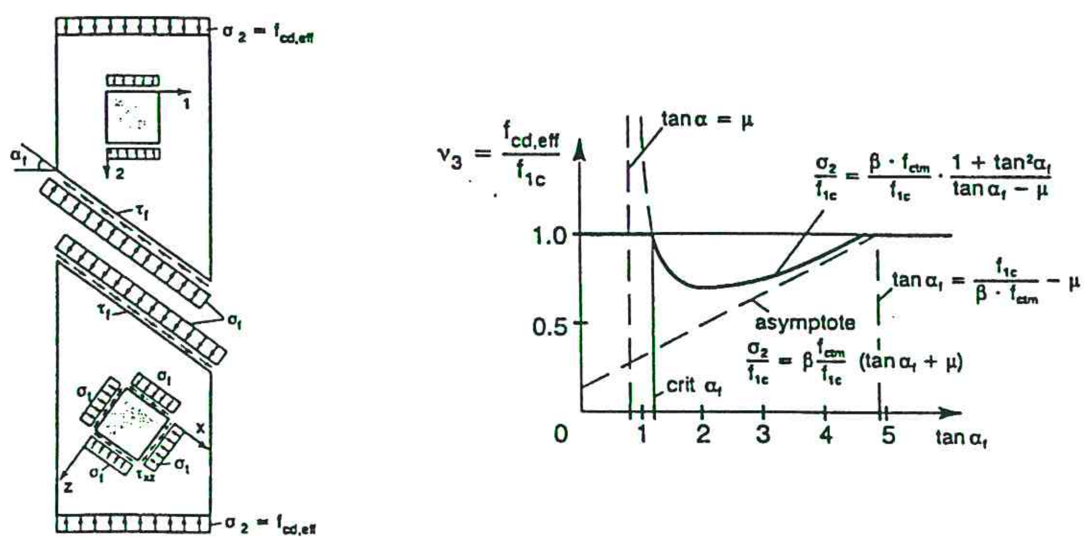
with: β = coefficient for the strength without normal stresses
σ_f = normal stress on interface
μ = friction coefficient for the roughness of the interface
f_{ctm} = average concrete tensile strength (Table 2.1)

(2) The strength of a strut transferring a parallel or prismatic compression field across an interface or a joint inclined at an angle α_f (Fig. 5.6) may be derived from equ. (5.15) and be expressed in terms of an effective strength:

$$f_{cd,\,eff} = \nu_3 \, f_{1cd} \qquad (5.16\ a)$$

$$\text{with: } \nu_3 = \beta \frac{f_{ctm}}{f_{1cd}} \; \frac{1 + \tan^2 \alpha_f}{\tan \alpha_f - \mu} \leq 1.0 \qquad (5.16\ b)$$

α_f = angle between direction of strut and joint



a) geometry and stresses b) factor ν_3 for the effective strength

Fig. 5.6 Transfer of compressive stresses of a strut across an interface or a joint by friction

5.5.2 Transfer of strut across joints

(1) The following values in table 5.1 may be assumed for the coefficients β and μ in equ. (5.15) according to the relevant interface condition.

(2) The maximum shear stress to be transferred shall be:

$$\tau_{fd} = 0.25 \ f_{1cd} \tag{5.17}$$

Table 5.1 Coefficients β and μ for the friction resistance of joints

Interface condition	β	μ
very smooth e.g. cast against steel or smooth wooden moulds	0.1	0.6
smooth e.g. slipformed or extruded, or left without further treatment after vibration	0.2	0.6
rough or toothed (indented) e.g. with exposed agregate, roughened by raking or brushing or provided with shear keys (indentations)	0.4	0.9

5.5.3 Transfer of strut over cracks (crack friction)

For crack widths not larger than 0.5 mm the capacity for the transfer of forces due to concrete-to-concrete friction over cracks may be assessed for concrete grades $f_{ck} < 50$ as follows:

$$\tau_{fd} = 0.50 \ f_{ctm} + 1.20 \ \sigma_f \tag{5.18}$$

5.6 Strength of Nodes and Anchorages

5.6.1 General

(1) The nodes shall be dimensioned and detailed so that all forces are balanced and any ties are anchored or spliced safely. The concrete is bi- or triaxially stressed, either only in compression in C-C-C-nodes (C = compressive force, strut) connecting struts or in compression and tension in C-C-T- or C-T-T-nodes (T = tension force, tie) if bonded reinforcement is anchored or spliced. The nodes must generally be verified by the following checks:

- verification of the anchorage of ties in the node.
- verification that the maximum compressive stress does not exceed the effective compressive strength.

(2) The anchorage length is defined by the begin and end of the deviations of the compression field by the reinforcement. Any anchorage of reinforcement requires transverse tension, which normally should be taken by reinforcement (additionally or existing), but often has to be taken by concrete tensile stresses. Therefore the transfer of the forces into the struts should be thoroughly investigated three-dimensionally, e.g. in the plane of load transfer and perpendicular to it.

(3) For C-C-T- or C-T-T-nodes the check of the compression stresses is often not decisive, because either the anchorage length or the limitations for the support pressure for the bearings govern the node dimensions. If in a general case such a check is required, a value of $\nu_2 = 0.85$ may be taken for the effective strength.

5.6.2 Compression nodes

(1) In nodes connecting only compression struts the bi- or triaxial hydrostatic compressive strength of the concrete may be utilized:

- for biaxial compression: $f_{2cd} = 1.20 \ f_{1cd}$ (5.19 a) ($\equiv 1.00 \ f_{cd}$)
- for triaxial compression: $f_{3cd} = 3.88 \ f_{1cd}$ (5.19 b) ($\equiv 3.30 \ f_{cd}$)

When utilizing such high strengths the magnitude of the transverse compression must be secured and shall be critically examined. The flow of the forces in the structure must be further followed up, because transverse tension may occur requiring a corresponding reinforcement.

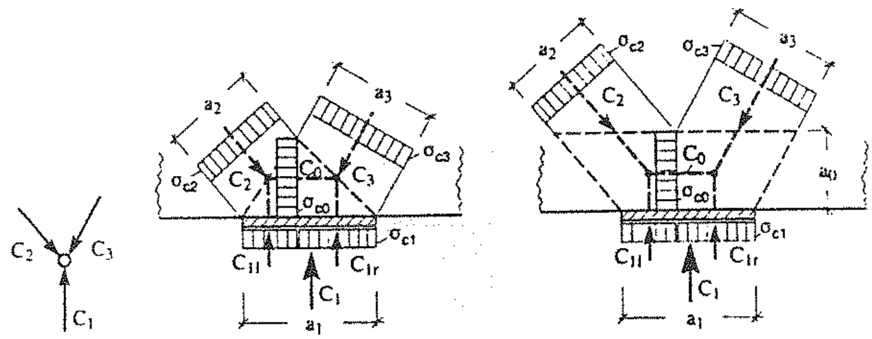
(2) Typical compression nodes are shown in Fig. 5.7. The capacity of the node may be checked by the local pressure σ_{c0} ($=\sigma_{c1}$ in Fig. 5.7) under the loaded area A_{c0} (see Fig. 5.8):

$$\sigma_{c0} = F_{Sd} / A_{c0} \leq f_{1cd} \sqrt{A_{c1} / A_{c0}} \leq f_{3cd} \qquad (5.20)$$

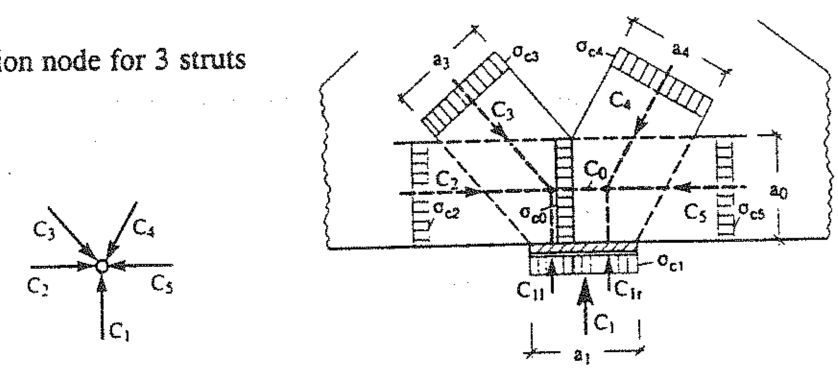
with: A_{c1} = the maximum area in A_c with the same centroid as the loaded area A_{c0}

Special considerations should be made in case of non-evenly distributed pressures and additionally applied horizontal forces.

(3) In special cases like prestressing anchorages locally higher strengths may be utilized by confining reinforcement, if relevant approval documents are provided.



a) compression node for 3 struts



b) compression node for 5 struts, e.g. at intermediate supports

Fig. 5.7 Typical compression nodes

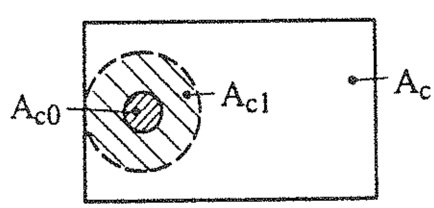


Fig. 5.8 Definition of areas for local pressures

5.6.3 Bends of bars and minimum radii of curvature of tendons

5.6.3.1 Bends of bars

(1) A typical node with a bend of a bar is shown in Fig. 5.9. If the angle θ is not equal to 45°, a part of the strut force is anchored at the bend. The average compressive stress at the node may be assessed by:

$$\sigma_c = C \;/\; b\, d_b \cos\theta \leq \nu_2\, f_{1cd} = 0.80\, f_{1cd} \qquad (5.21)$$

with: b = width of the strut at the node
d_b = diameter of bend

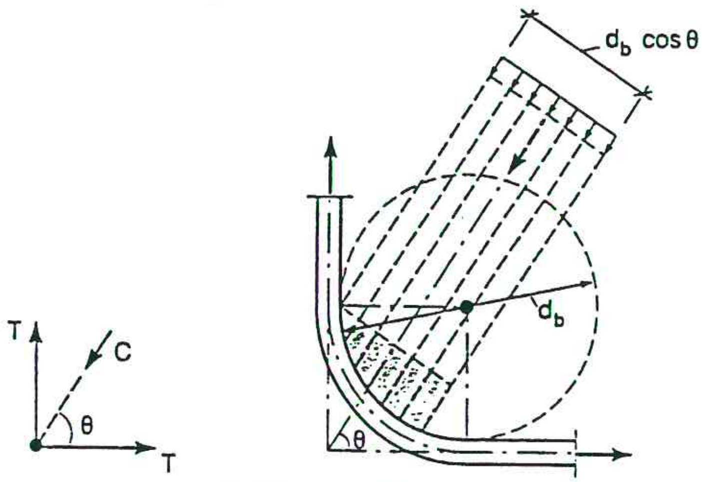


Fig. 5.9 C-T-T- node with a bend of a bar

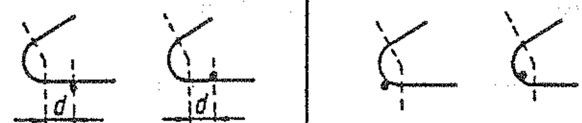
(2) The diameter of mandrel should be such to avoid crushing or splitting of the concrete under the effect of the bearing pressure inside the bend and to avoid cracks in the steel bar due to the bending. MC 90, 9.11.2.

These requirements are covered if the minimum diameter of mandrel of bars complies with the values given in Table 5.2.

Table 5.2 Minimum values for the diameter of mandrel

hooks and loops (Fig.5.11)		bent-up bars and curved bars		
bar diameter Ø		concrete cover perpendicular to the plane of the bend		
< 20 mm	≥ 20 mm	100 mm and 7Ø	> 50 mm and 3Ø	≤ 50 mm and ≤ 3Ø
4 Ø	7 Ø	10 Ø	15 Ø	20 Ø

(3) The minimum diameter of mandrel of welded mesh fabric should comply with Fig. 5.10.



welds outside bend	welds inside bend
$d < 4\,Ø$: $20\,Ø$ $d \geq 4\,Ø$: see table 5.1	$20\,Ø$

Fig 5.10 Minimum diameter of bend for welded mesh fabric

5.6.3.2 Minimum radii of curvature for tendons

(1) Unless otherwise stated in technical approval documents, the following approximate values may be taken from an empirical formula for the minimum radii of curvature:

a) internal, bonded tendons in corrugated ducts:

$$R_{min}\ [m] = 3\ \sqrt{f_{ptk}\, A_p\ [MN]} \geq 2.5\ m \qquad (5.22\ a)$$

b) external, unbonded multistrand tendons in smooth pipe:

$$R_{min}\ [m] = 1.5\ \sqrt{f_{ptk}\, A_p\ [MN]} \geq 2.0\ m \qquad (5.22\ b)$$

c) internal, unbonded monostrand tendon (0.6 in diam.):

$$R_{min} = 2.5\ [m] \qquad (5.22\ c)$$

5.6.4 Nodes with anchorages of reinforcing bars

(1) The anchorage length $l_{b,net}$ depends on the type of anchorage as well as on the actual stress in the reinforcement, and it can be calculated from the basic value l_b as follows:

$$l_{b,net} = \alpha_a\ l_b\ (A_{s,requ} / A_{s,prov}) \qquad (5.23)$$

with: l_b = basic anchorage length acc.to equ.(2.11)
$A_{s,requ}$ = required area of reinforcement
$A_{s,prov}$ = provided area of reinforcement
α_a = coefficient for type of anchorage, see Fig. 5.11

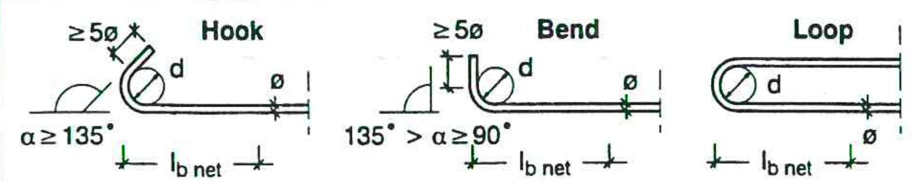
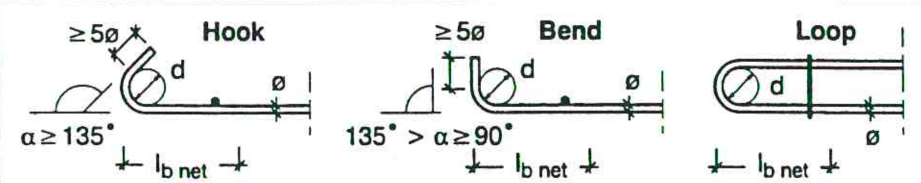
	Anchorage element	Coefficient α_a for anchorage in: Tension	Compression
1		1·0	1·0
2		0·7	1·0
3		0·7	0·7
4		0·5	0·7

Fig. 5.11 Coefficient α_a for the type of anchorage

(2) A minimum anchorage length should be provided of

- for anchorages in tension: $l_{b,min} = 0.3\ l_b > 10\ \varnothing > 100$ mm

- for anchorages in compression: $l_{b,min} = 0.6\ l_b > 10\ \varnothing > 100$ mm

(3) A transverse reinforcement with an area of 25 % of that of the main reinforcement should be provided for all anchorages, unless a sufficient transverse compression exists. The transverse reinforcement should be evenly distributed over the anchorage length, with at least one bar placed near the hook, bend or loop.

(4) Bars with diameters $\varnothing > 32$ mm should be anchored by bond of straight bars or by means of mechanical devices.

(5) The transverse reinforcement may be anchored in the chords using one of the anchor elements shown in Fig. 5.12. Bars with diameters greater than Ø16 mm should not be used as transverse reinforcement. Inside hooks or bends a longitudinal bar should be provided. The diameter of bend for hooks and loops must comply with sect. 5.6.3.

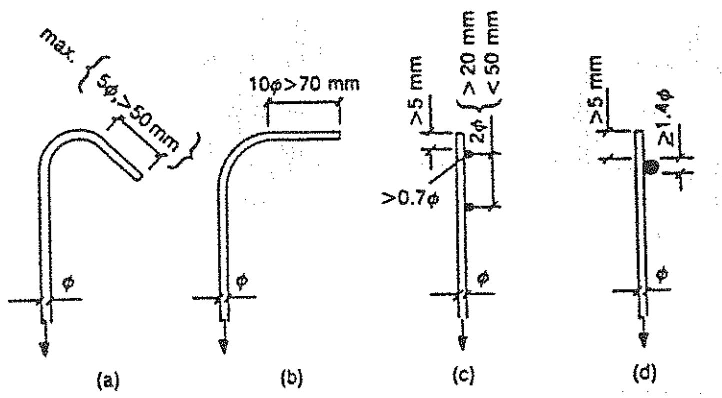


Fig. 5.12 Types of anchoragees for the transverse reinforcement

(6) Typical cases for a C-C-T- node with end-anchorages of reinforcing bars are shown in Fig. 5.13. The concrete stresses σ_{c2} in the inclined strut depend on the depth u of the node. Fig. 5.12 a shows the extreme case with u = 0 due to no anchorage length behind the anchor plate, whereas Fig. 5.12 c applies to nodes where several layers of bars are anchored. In the latter case the height u should be restricted to a value $u \leq 1.5\ a_1$, and in case of very high forces T, the transfer of the strut force C_2 across the top interface by friction should be checked acc. to sect. 5.5.
For the limiting values for the concrete stresses the value $\nu_2 = 0.85$ applies, see sect. 5.6.1 (3).

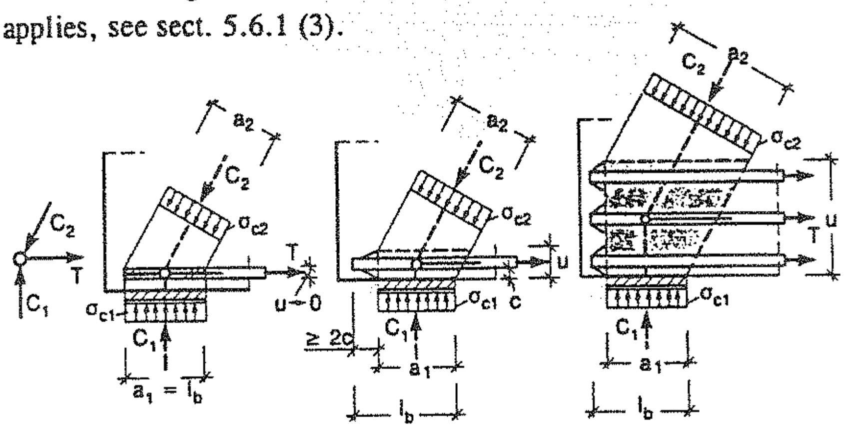


Fig. 5.13 T-C-C- node with an end-anchorage of reinforcement

(7) The end-anchorage of hooks in a C-C-T- node in Fig. 5.14 demonstrates, that all three directions of a node with an anchorage should be looked at. The anchor plate should not be placed less than about 2 c (with c = concrete cover) from the edge in order to avoid any spalling off of the bottom corner.

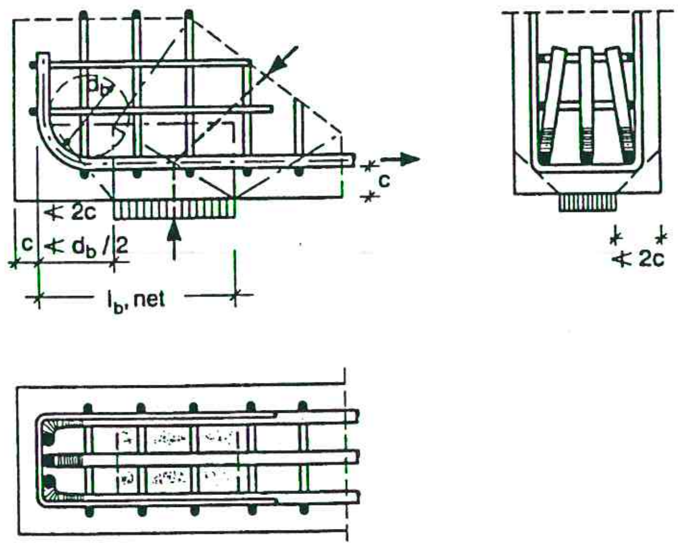
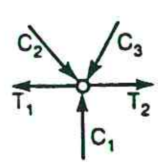


Fig. 5.14 End-anchorage of bars with hooks

(8) At intermediate supports of less slender beams and deep beams the reinforcement may have to be anchored in the node shown in Fig. 5.15, which combines the two types of a C-C-C - node with a C-C-T- node.



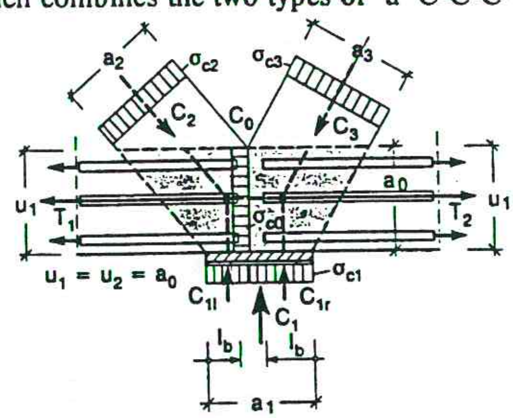


Fig. 5.15 Combination of different node types at intermediate supports of deep beams and non-slender beams

(9) The anchorage lengths of pretensioning reinforcement is dealt with in sect. 2.4.3.

5.6.5 Nodes with anchoring devices

(1) In case of short anchorage lengths anchor-plates or anchoring devices should be provided. These should not be placed in the tension zones of members

(2) If an anchor- plate is used, then the load transfer from the tie to the struts may be regarded like a compression node (Fig. 5.16). The anchor-plate must be dimensioned for the relevant stress distribution in the node face.

(3) The use of anchoring devices like studs, button heads or bolts requires appropriate technical approval documents.

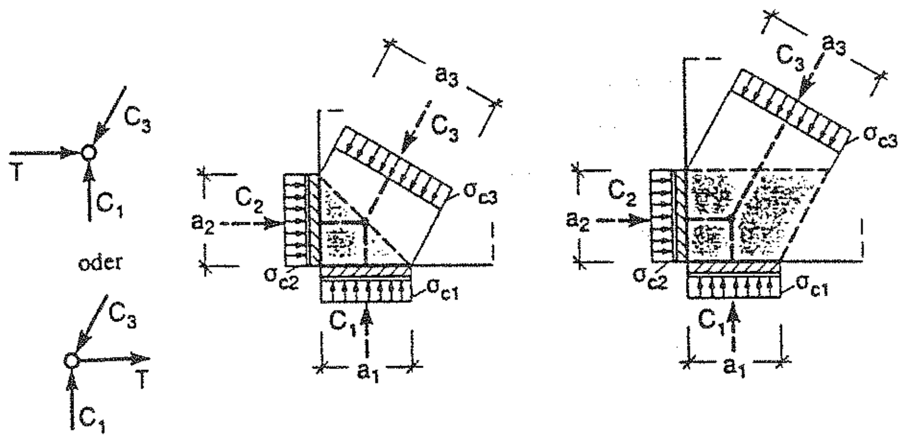


Fig. 5.16 Anchor-plates for the end-anchorage of reinforcing bars result basically in a C-C-C - node

5.7 Splices of reinforcements

5.7.1 General requirements

(1) Forces may be transmitted from one bar to another by
- lapping without or with hooks, bends or loops.
- welding.
- mechanical devices.

(2) Any splice requires transverse tension in the plane of load transfer and perpendicular to it, which should be taken by appropriate reinforcements.

5.7.2 Splices by overlapping of bars

MC 90, 6.9.1

5.7.2.1 General requirements

9.1.2

(1) Laps between bars should be detailed such that the forces are fully transmitted from one bar to the other without causing spalling of the concrete cover or excessive cracking. Laps should not be located at sections, where the stress in the reinforcement is high, e.g $>0.80\ f_{yd}$. Laps should be placed symmetrically and parallel to the outer faces of the member.

(2) The laps between bars should be detailed and staggered in accordance with Fig. 5.17.

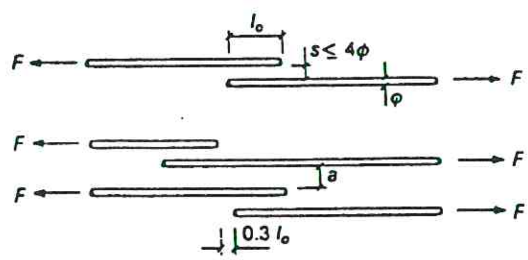


Fig. 5.17 Staggering of lapped bars

5.7.2.2 Lap length

MC 90, 6.9.6

(1) The required lap length is given by

$$l_0 = \alpha_s\ l_{b,net} \geq l_{0,min} \qquad (5.24)$$

with: $l_{b,net}$ = anchorage length acc. to equ.(5.23),

α_s = coefficient given in Table 5.3 depending on the percentage of bars lapped within $1.3\ l_0$ from the centre of the splice (see Fig. 5.18).

(2) The required lap length should be increased by the spacing between the spliced bars, if this spacing exceeds $4\ \varnothing$.

(3) The minimum lap length $l_{0,min}$ is

$$l_{0,min} > 15\ \varnothing \quad \text{or} \quad > 200\ \text{mm} \qquad (5.25)$$

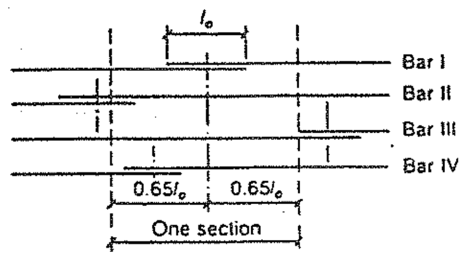


Fig. 5.18 Staggering of lapped splices

Table 5.3 Coefficient α_s for the lap length of lapped bars

Percentage of lapped bars relative to total area of steel	≤ 20 %	25 %	33%	50 %	> 50 %	bars in compression
coefficient α_s	1.2	1.4	1.6	1.8	2.0	1.0

5.7.2.3 Permissible percentage of lapped reinforcement

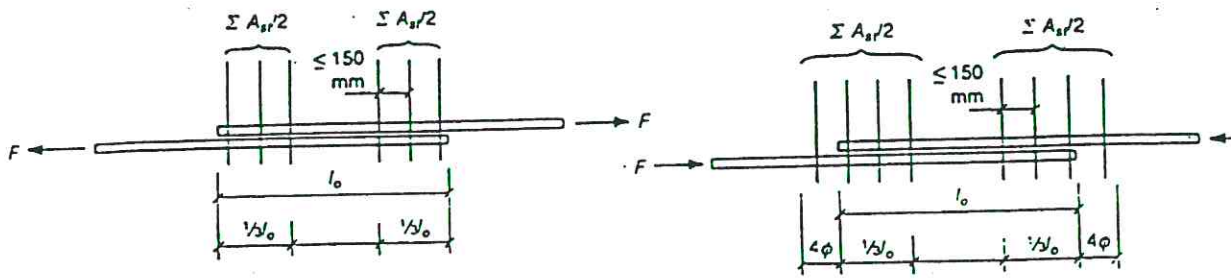
(1) For lapped bars in tension 100 % may be lapped in one section, if they are placed only in one layer. For bars in several layers only 50 % may be lapped in one section.

(2) For lapped bars in compression 100 % may be lapped in any section

5.7.2.4 Transverse reinforcement

(1) The transverse reinforcement available out of other reasons may be also be regarded to cover the transverse tensile forces of a lap, if bars of less than Ø16 mm and if less than 25 % of the total reinforcement are lapped. MC 90, 9.1.2.2.3

(2) If bars Ø > 16 mm or if more than 25 % of the total reinforcement are lapped, a special transverse reinforcement should be placed. The amount and distribution of which corresponds to the principles stated in sect. 5.7.4. These bars should be placed between the lapped bars and the concrete surface and the amount and distribution should comply with that given in Fig. 5.19. For a spacing of laps less than 10 Ø the transverse reinforcement should consist of stirrups.



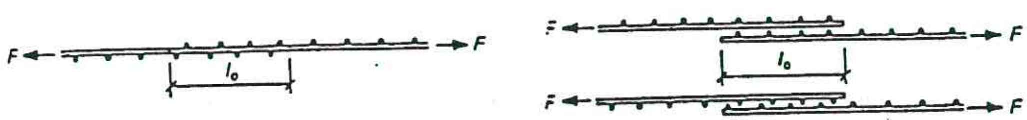
a) in tension b) in compression

Fig. 5.19 Detailing of lapped bar splices

5.7.3 Lapping of welded mesh fabrics

(1) Laps between bars should be detailed such that the forces are fully transmitted from one bar to the other without causing spalling of the concrete cover or excessive cracking. In general, laps should not be located at sections, where the stress in the reinforcement is high, (e.g $>0.80\ f_{yd}$). MC 90, 6.9.8

(2) Laps may be made either by intermeshing or layering of the sheets (Fig. 5.20), whereby intermeshing should be adopted for repeated loading.



a) intermeshed laps b) layered laps

Fig. 5.20 Types of laps for welded fabrics

(3) The lap length l_0 should be calculated as in section 5.7.2.2:

$$l_0 = \alpha_s\ l_{b,net} > l_{0,min} \qquad (5.26)$$

with:

$$\alpha_s = 0.50 + \frac{A_{s,requ}}{A_{s,prov}} \quad \text{and} \quad 1 < \alpha_s < 2$$

$l_{0,min}$ = max $\{0.75\ l_b\ ,\ 15\ \varnothing\ ,\ s_{tr}\ ,\ 200\ \text{mm}\}$

$l_{b,net}$ = value given by equ. (5.23)

$A_{s,prov}$ = value acc. to sect. 5.6.4

s_{tr} = spacing of cross wires

(4) Laps in several layers should be staggered by 1.3 l_b.

(5) The maximum percentage of the main reinforcement that may be lapped by layering at any one section is:

- 100 % if A_s /s < 1200 mm^2/m,
- 60 % if A_s /s > 1200 mm^2/m, and this mesh is an interior layer of a multiple layer.

For intermeshed fabrics the clauses of section 5.7.2.3 apply.

(6) All wires acting as secondary reinforcement and the cross wires of lapped wire meshes may be lapped in one section.

(7) No additional transverse reinforcement is required within the lap length.

5.7.4 Splices by mechanical devices

The use of mechanical devices requires appropriate technical approval documents. These should specify the following characteristics of the connection:

- charactersitic values of yield and rupture strength.
- deformation properties of the connection.
- fatigue characteristics.

5.8 Special rules for bundled bars and for bundled tendons

5.8.1 Bundled bars

(1) Bars of the same diameter may be bundled. The maximum number of bars in a bundle is limited to three, except for vertical bars in compression and laps, where 4 bars may be bundled.

(2) Arrangements of three or more bars in contact in one plane (horizontal or vertical) should not be used.

(3) For all design purposes, bundles of bars containing n bars having the same diameter should be replaced by a single notational bar having the same centre of gravity and an equivalent diameter of

$$\varnothing_n = \varnothing \sqrt{n} \leq 55 \text{ mm} \tag{5.27}$$

(4) The cover and the spacings of the bundles should be measured from the actual outer contour of the bundle.

(5) Bundles bars should only be provided with straight anchorages, whereby the anchorages of the single bars should be staggered. For bundles of 2, 3 or 4 bars the staggering should be respectively 1.2- or 1.3- or 1.4- times the anchorage length of the individual bar. The anchorage length for a complete bundle with $\varnothing_n < 32$ mm may be determined on basis of the equivalent diameter.

(6) Laps can only be made with one bar of a bundle at any one section. The laps should be staggered in accordance with clause (5).

5.8.2 Bundled tendons

(1) Up to two tendon ducts may be bundled transversally to the tendon curvature, or for straight tendons.

(2) Up to four monostrands may be bundled transversally to the tendon curvature.

(3) Tendon ducts may touch locally if they cross approximately perpendicular, or if they touch only over a small length longitudinally.

6 Ultimate Limit State Design

MC 90, 6

6.1 General Requirements and Definitions

MC 90, 6.1

(1) It shall be demonstrated that for the structure as a whole and for its members or for certain regions the probability of reaching an ultimate limit state (ULS) is small.

(2) It is generally of advantage to discern two typical regions in structures: the B- and D-regions. In a B-region the Bernoulli-hypothesis of plane sections remaining plane applies, and therefore standard dimensioning procedures may be developed (see sect. 6.4).

In regions with statical or geometrical discontinuities (D-regions) non-linear strain distributions occur. Such D-regions are e.g. regions with load applications or member connections like frame corners, regions with openings or abrupt changes of section.

(3) The determination of the resistance shall be based on physical models of the internal forces and the external reactions of the structure. The internal model shall represent a coherent system of internal forces, repectively of struts or compression stress-fields and of ties and nodes in equilibrium with the design loads and reactions.

(4) When determining the model compatibility should at least approximately considered. It is generally advantageous if the model is orientated by the stress fields determined from a linear-elastic analysis. The model may be modified to account for cracking and yielding of reinforcements.

(5) The assumed nodes and ties must comply with the detailing of the reinforcements. The reinforcements must extend to the extreme fibres of the nodes or of the deviated compression stress-fields. The axes of the reinforcements have to coincide with the axes of the corresponding ties in the model.

6.2 Actions and action effects

6.2.1 Definitions

(1) In general, permanent actions may be represented by a single (mean) value.

(2) The characteristic values of variable actions should be chosen along the lines of Appendix 1. For live loads nominal values can be used.

(3) The prestress is defined in sect. 3.4.1. The partial safety factor is 1.0.

(4) Unbonded /external tendons should be treated as seperate members in the analyis. The strain in the prestressing steel is equal to the strain corresponding to the forces defined in sect. 3.4.1, increased by the mean concrete strain between two successive points of anchorage or fixation due to load effects. This increase can be determined with a non-linear analysis of the entire structure, whereby, normally the tendons do not attain their yield strengths at ULS. For simplicity, a verification based on a linear-elastic analysis can be performed neglecting any increase in strain in the tendon.

see FIP Recomm. "Flat Slabs"

6.2.2 Combination of actions

MC 90, 1.6

(1) The applied loads or the acting internal forces should be determined with the partial safety coefficient shown in table 6.1.

Table 6.1 Partial safety coefficients for ULS

Actions	unfavourable effect	favourable effect
Permanent	$\gamma_g = 1.35$	$\gamma_g = 1.00$
Variable	$\gamma_q = 1.50$	---

(2) Thus, the following combinations of actions should be considered

a) in the case of an unfavourable effect of G:

$$1.35\ G + 1.50\ Q_1 + 1.5\ \Sigma\,\psi_o\,Q_2 \quad (6.1)$$

b) in the case of a favourable effect of G

$$1.00\ G + 1.50\ Q_1 + 1.5\ \Sigma\,\psi_o\,Q_2 \quad (6.2)$$

where Q_1 is the basic variable action and $\psi_o\,Q_2$ is the combination value of the other variable actions, see table 6.2.

Table 6.2 Combination values for ULS actions

	dwellings	Offices or retail stores	Parking areas	Highway bridges	Wind or snow
ψ_o	0.3	0.6	0.6	0.3	0.5

(3) Indirect actions (imposed or restrained deformations) need only to be considered if they are exceptionally large, to the extent of impairing the capacity for redistribution of internal forces, i.e. if a significant part of the plastic range in the M-k-diagram has already been used for the redistribution of the indirect action effects (Fig. 6.1). In this case, these effects are to be taken with their full value on the action side (without ψ_o).

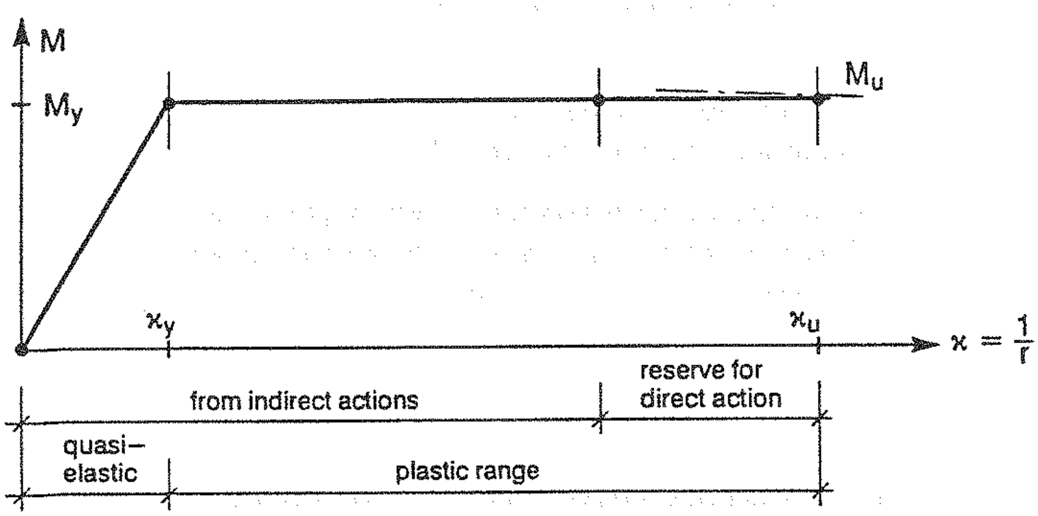


Fig. 6.1 Moment-curvature diagram

6.2.3 Resistant action effects

The resistant internal forces should be determined on basis of the resistances given in section 5.

6.3 Structural analysis

6.3.1 General requirements

(1) The ULS check should be carried out according to the theory of plasticity (PT) or an appropriate non-linear method.

(2) Variable actions should be considered for the worst loading case.

(3) As effective widths of flanges the values defined in sect. 7.3.2 may be used, unless proof is given for other values.

6.3.2 Static method of the theory of plasticity (PT)

(1) The use of the static method of PT is recommended whenever possible, since it yields a lower bound for the ultimate load of the structure. To this end, a plausible distribution of the internal forces is chosen, and the cross-sections or elements of the structure are dimensioned accordingly. The assumed distribution of internal forces has to satisfy the conditions of equilibrium, and should in general not differ too much from the elastic one. Otherwise, it is necessary to verify that the ductility of the structure is sufficient to allow the assumed plastic redistribution of internal forces.

(2) Linear-elastic analyses with or without redistribution are possible applications of the static method of PT.

(3) In principle, the hyperstatical effects of prestressing have no influence on the bearing capacity of the structure; however, they often give a good indication of the suitability of the assumed distribution of internal forces.

6.3.3 Kinematic method of the theory of plasticity

The kinematic method of the theory of plasticity may be used for determining the resistance of a structure, like e.g the yield line theory for slabs. The designer has to make sure by experience or trial and error, that the selected mechanism does not give an overestimated upper bound for the ultimate load.

6.3.4 Plastic rotation capacity and check of ductility

(1) In general, the required plastic rotation capacity has to be estimated considering the non-linear behaviour of the structure (e.g. cracking and yielding).

(2) The plastic rotation capacity of a reinforced concrete flexural member may be estimated from the equation (3.7.2) in MC 90.

(3) Sufficient ductility may be presumed to exist in flexural members with a depth of the compression zone of $x < 0.3\ d$.

(4) The rotation capacity can be increased by

a) increasing ε_{cu}, e.g. by closely spaced stirrups confining the compression chord (see sect. 5.2.4).

b) decreasing x / d (e.g. by compression reinforcement).

(5) In cases of high reinforcing ratios or high normal forces (due to actions), more detailed checks have to be carried out. Special attention should be given to cases where high strength concrete or steel with a small unit elongation is used.

6.4 Design of B-regions

MC 90, 6.3.1

6.4.1 Basic assumptions

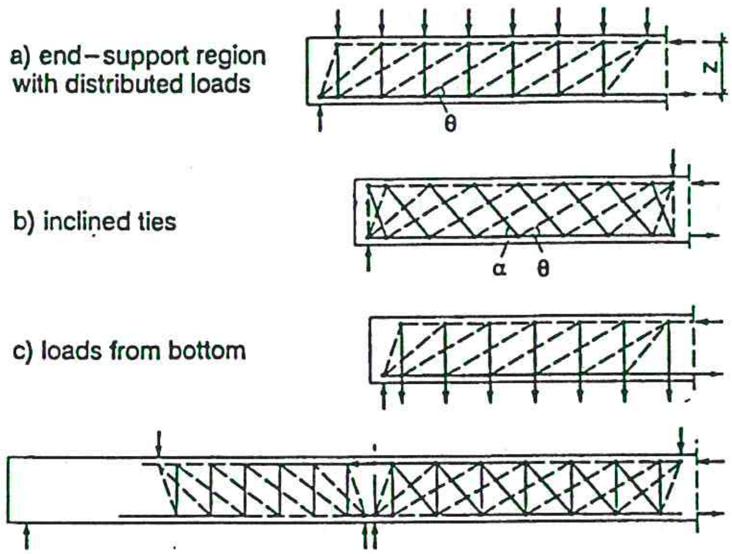
(1) The design model for B-regions of linear members with rectangular cross-section subjected to bending moments combined with an axial force and a shear force is a truss with longitudinal chords and a web (Fig. 6.2). The web consists of inclined concrete struts representing a uniaxial compression stress-field, and ties representing normally the distributed transverse reinforcement (Fig. 6.3). For members with low amounts of transverse reinforcement and members without transverse reinforcement the ties may also represent a tension stress-field in the concrete (Fig. 6.4).

(2) The geometry of the truss model is determined by the inner lever arm z between the chords and the angle θ of the inclined struts or compression stress-field in the web. Thereby the angle θ is measured against the tension chord.

The inner lever arm follows from the flexural design of section 6.4.2 for the sections with maximum moments. It may be assumed constant throughout the region, in which the bending moments retains the same sign.

The angle θ of the inclined struts follow from the shear design of section 6.4.3, and it varies with the magnitude of the axial force or prestress. It may be assumed constant throughout the region, in which the shear force retains the same sign.

3) A linear member with addititonal torsion and more complex sections, like box-beams or T-beams, can be subdivided into several walls representing webs and flanges. These walls can then be designed for their individual action effects using the model described before.



d) continuous girder with point loads

Fig.: 6.2 Truss models for structural concrete members

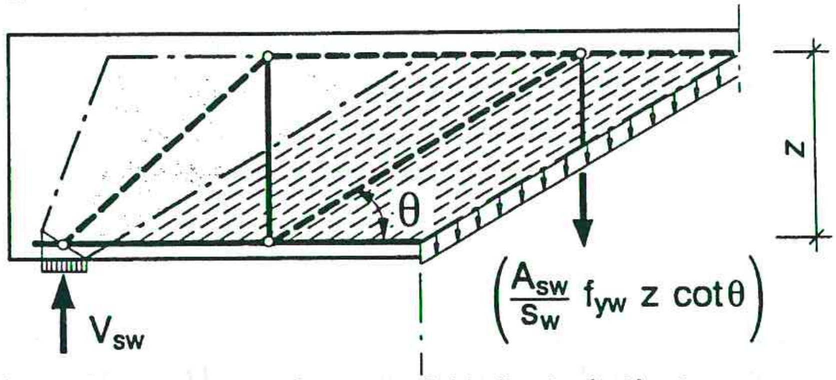


Fig. 6.3 Compression stress-fields for the inclined struts

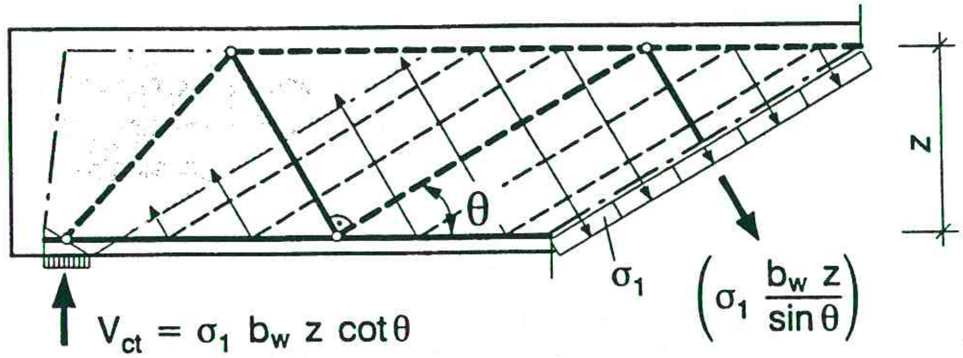


Fig. 6.4 Model with concrete ties respectively with an inclined biaxial tension - compression field in the web for members with low and without transverse reinforcement

6.4.2 Flexural design and inner lever arm of the truss

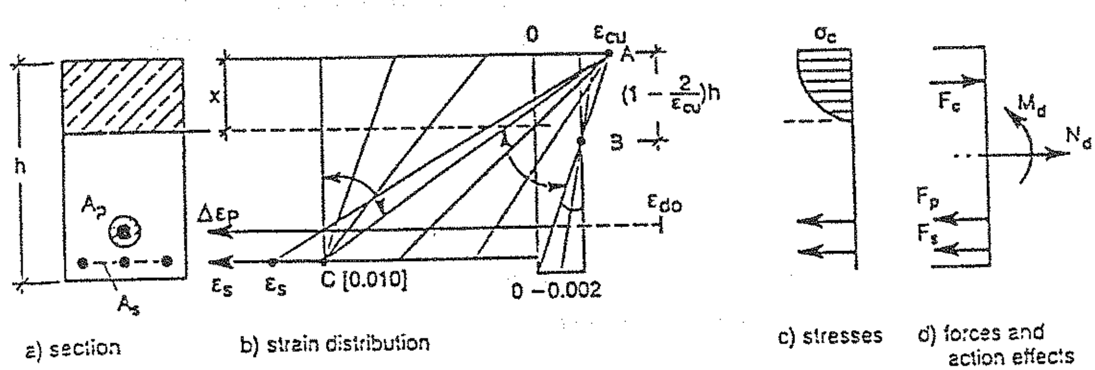
MC 90, 6.3.2

(1) The design moment and axial force are resisted by the chords in a distance z, and the forces in the chords should be derived on the basis of the following assumptions :

a) the distribution of the longitudinal strains is linear over the depth of the section.
b) the section is situated at a crack and the tensile stresses in the concrete are neglected.
c) bonded reinforcement is subjected to the same variations in strain as the adjacent concrete.
d) the total strain of bonded prestressing reinforcements considers the prestrain ε_{d0} corresponding to the prestressing force after creep and shrinkage.

(2) The strain diagram (Fig. 6.5) shall pass either through point A, defined by the maximum compressive strain ε_{cu} , or through point B in case of uncracked members.

In cases where the steel strain may have to be limited, e.g. for low-ductility steel, the strain diagram is also restricted through point C, which is defined by a steel strain of ε_{su} = 0.010; other values may apply if the ductility of the steel is known.



ε_c = compressive strain

ε_{cu} = maximum compressive strain acc. to equ. (2.2) for parabola-rectangle-diagram or equ.(5.8) for a rectangle distribution

ε_s = strain in reinforcing steel

$\Delta\varepsilon_p$ = increase of strain in bonded prestressing reinforcement due to action effects

ε_{d0} = prestrain corresponding to the prestressing force after creep and shrinkage

Fig. 6.5 Strain distribution over the depth of the member

(3) The design diagrams for steel are defined in the Figures 2.8 and Fig. 2.9. The design diagrams for the concrete are defined in Fig. 2.2 or Fig. 2.3, respectively in case of a rectangle distribution in Fig. 5.2.

(4) For any combination of action effects the resistances of the chords shall not exceed the values given in sections 5.2 and 5.3.

(5) An increase in strength of the compression chord due to confinement may be taken into account according to sect. 5.3.4. The area of the compression zone is only that of the confined concrete.

6.4.3 Shear design and angle θ of the inclined struts

MC 90, 6.

6.4.3.1 General requirements

(1) The truss models shown in Fig. 6.2 and Fig. 6.3 are only valid as long as the transverse reinforcement is so closely spaced, that the inclined compression stress-fields can develop. Therefore, the spacing of stirrups in the longitudinal direction shall not exceed the lesser of $s_w = z / 5$ or 200 mm.

(2) If larger spacings are used than the above given, then the compression field is not uniformly supported by the stirrups. In this case, the strength of the inclined strut (see equ.(5.6)) is limited to the following values for ν_2:

- $\nu_2 = 0.60$ for $0.20 < s_w / z \leq 0.40$
- $\nu_2 = 0.45$ for $0.40 < s_w / z \leq 0.60$

(3) In any case shall the maximum spacing of the transverse reinforcement not exceed the lesser of the following values:

- in the longitudinal direction: $s_{w,max} = 0.6\,z$ or 400 mm.
- in the transverse direction: $s_{w,max} = 0.6\,z$ or 400 mm.

(4) The angle α of any transverse reinforcement (see Fig. 6.2 b) shall not be smaller than 45°.

(5) The stirrups shall be adequately anchored to the chords, see section 5.6.4 (5).

(6) Bent-up bars are not recommended.

6.4.3.2 Design of the transverse reinforcement

(1) The design of the stirrups in the web starts from the fundamental free-body diagram in Fig. 6.6. The basic equation for any shear design follows directly from the vertical equilibrium:

$$V_{Rd} \geq V_{Sd} \tag{6.3}$$

The resistance follows from equilibrium at the inclined crack:

$$V_{Rd} = V_{sw} + V_f + V_P \tag{6.4 a}$$

with: V_{Rd} = acting design shear force in a distance ($z \cot\theta$) from the face of the support (see sect. 6.5.2.1)

V_{sw} = shear force carried by the stirrups over the cracks

V_f = vertical component of friction forces at crack (Fig. 6.6 b)

V_P = vertical component of force in prestressing tendon.

The equation simplifies for a structural concrete member without inclined prestressing tendons to:

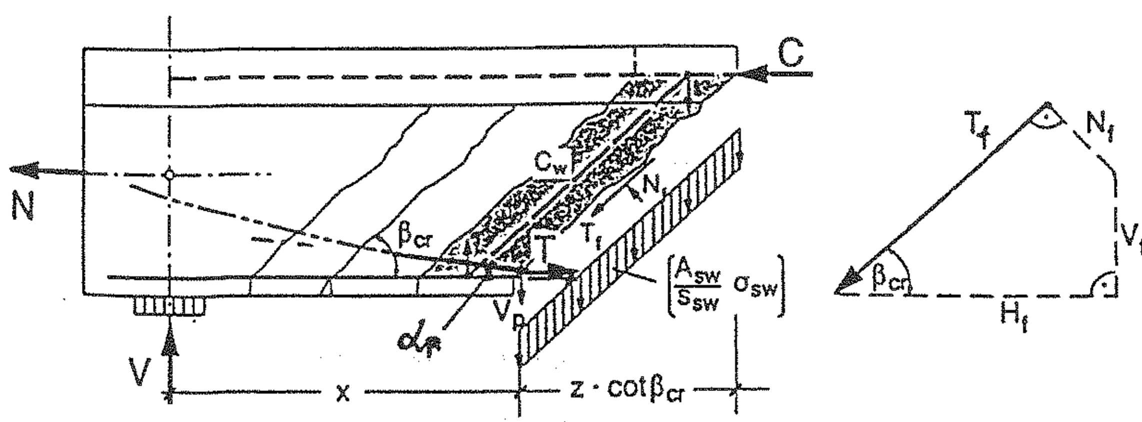
$$V_{Rd} = V_{sw} + V_f \tag{6.4 b}$$

(2) The vertical component of the force in prestressing tendon may be assumed as

$$V_P = P_0 \sin\alpha_p \tag{6.5}$$

with: $P_0 = \sigma_{p0}\ A_p$ = mean value of prestressing force (sect. 3.4.1)

α_p = angle of tendon at considered section



a) free-body diagram b) friction forces at crack

Fig. 6.6 Free-body diagram for the end-support of am beam with all forces at the failure surface in the B-region

(3) The shear force component V_{sw} carried by all the stirrups across the crack is:

$$V_{sw} = (A_{sw} / s_w)\ f_{yw}\ z\ \cot\beta_{cr} \qquad (6.6)$$

with: A_{sw} = area of transverse reinforcement
s_w = stirrup spacing in the longitudinal direction
f_{yw} = yield strength of transverse reinforcement
z = inner lever arm
β_{cr} = the crack angle

(4) The crack angle and the shear force component V_f due to friction depend on the axial force as well as on the strains and crack widths in the web. As an approximation the following values may be assumed:

- in case of axial compression:

$$\cot\beta_{cr} = 1.20 - 0.2\ \sigma_{xd} / f_{ctm} \qquad (6.10\ a)$$

$$V_f = 0.10\ (1 - \cot\beta_{cr} / 4)\ (b_w\ z\ f_{cwd}) \geq 0 \qquad (6.11\ a)$$

- in case of axial tension:

$$\cot\beta_{cr} = 1.20 - 0.9\ \sigma_{xd} / f_{ctm} \qquad (6.10\ b)$$

$$V_f = 0.10\ (1 - 0.36 / \cot\beta_{cr})\ (b_w\ z\ f_{cwd}) \geq 0 \qquad (6.11\ b)$$

with: $\sigma_{xd} = N_{sd} / A_c$ = axial stress [(-) in compression]

(5) For a reinforced concrete member without an acial force both equ. (6.10) and (6.11) lead to

$$V_f = 0.070\ (b_w\ z\ f_{cwd}) \qquad (6.12\ a)$$

and the resistant shear force is:

$$V_{Rd} = (A_{sw} / s_w)\ f_{yw}\ z\ 1.20 + 0.070\ (b_w\ z\ f_{cwd}) \qquad (6.12\ b)$$

6.4.3.3 Determination of the angle θ of the inclined struts

The angle θ of the inclined struts is required for evaluating the force in the tension chord and it may be determined from equ.(6.4) and equ. (6.12) as follows:.

$$\cot\theta = V_{Rd} / V_{sw} \qquad (6.13)$$

6.4.3.4 Upper limit of resistant shear force

(1) The capacity of the web to transfer any shear force is limited by the compressive stresses in the strut between the cracks attaining the limiting stress $f_{cwd} = \nu_2 \; f_{1cd}$. The inclined compressive stresses are given by:

$$\sigma_{cw} = V_{Sd} \; / \; b_w \, z \quad \sin\theta \; \cos\theta \tag{6.14 a}$$

(2) The upper limit of the resistant shear force follows from that:

$$V_{Rd,max} = b_w \; z \; f_{cwd} \quad \sin\theta \; \cos\theta \tag{6.15}$$

For $\theta = 45°$ and $\nu_{2,max} = 0.80$ the highest value is reached with

$$V_{Rd,max} = 0.5 \; b_w \; z \; f_{cwd} = 0.4 \; b_w \; z \; f_{1cd} \tag{6.16}$$

6.4.4 Forces in the chords of the B-region

(1) The chord forces may be derived from the truss model and are as follows:

- tension chord:

$$F_{St} = \frac{|M_{Sd}|}{z} + N_{Sd}\frac{(z - z_s)}{z} + \frac{V_{Sd}}{2}(\cot\theta - \cot\alpha) \tag{6.17}$$

- compression chord:

$$F_{Sc} = \frac{|M_{Sd}|}{z} - N_{Sd}\frac{z_s}{z} - \frac{V_{Sd}}{2}(\cot\theta - \cot\alpha) \tag{6.18}$$

(2) At sections of maximum moments the shear force is zero and highest values are attained for the forces, which then are compared with the resisting forces.

6.4.5 Design of flanges of chords

(1) The flanges of sections act as chords for members over their effective width. The force transfer from the web into the flange may be determined by means of a truss model.

(2) In the analysis and design the same effective width b_{eff} may be assumed as stated in sect. 7.3.2. It may be kept constant over the length of the member. Larger values may be assumed if the reinforcement is correspondingly designed and detailed.

(3) The flange and the web are connected by the shear force v_{fl} per unit length

$$v_{fl} = \Delta F \;/\; \Delta x \tag{6.19}$$

with: ΔF_{fl} = change of chord force in flange over Δx
Δx = length under consideration

In the B-region the following value applies:

$$v_{fl} = V_{Sd} / z \tag{6.20}$$

(4) The transverse reinforcement per unit length in the flange follows from the truss model (Fig. 6.7):

$$n_{sf} = A_{sf}\, f_{yd} = v_{fl} \;/\; \cot\theta_{fl} \tag{6.21}$$

(5) The angle θ_{fl} respectively the transverse reinforcement may be determined for the above shear force from the rules in sect. 6.4.3.2.

Alternatively and for simplicity also the following values may be assumed for the angle θ_f of the struts in the flanges:

- $\cot\theta_{fl} = 2.00$ for compression flanges,
- $\cot\theta_{fl} = 1.25$ for tension flanges.

The reinforcement may be distributed over the length a_{fl} (Fig. 6.7).

(6) The compressive stress in the inclined struts is given by:

$$\sigma_{cf} = v_{fl} \;/\; h_{fl} \;\sin\theta_{fl}\;\cos\theta_{fl} \tag{6.22}$$

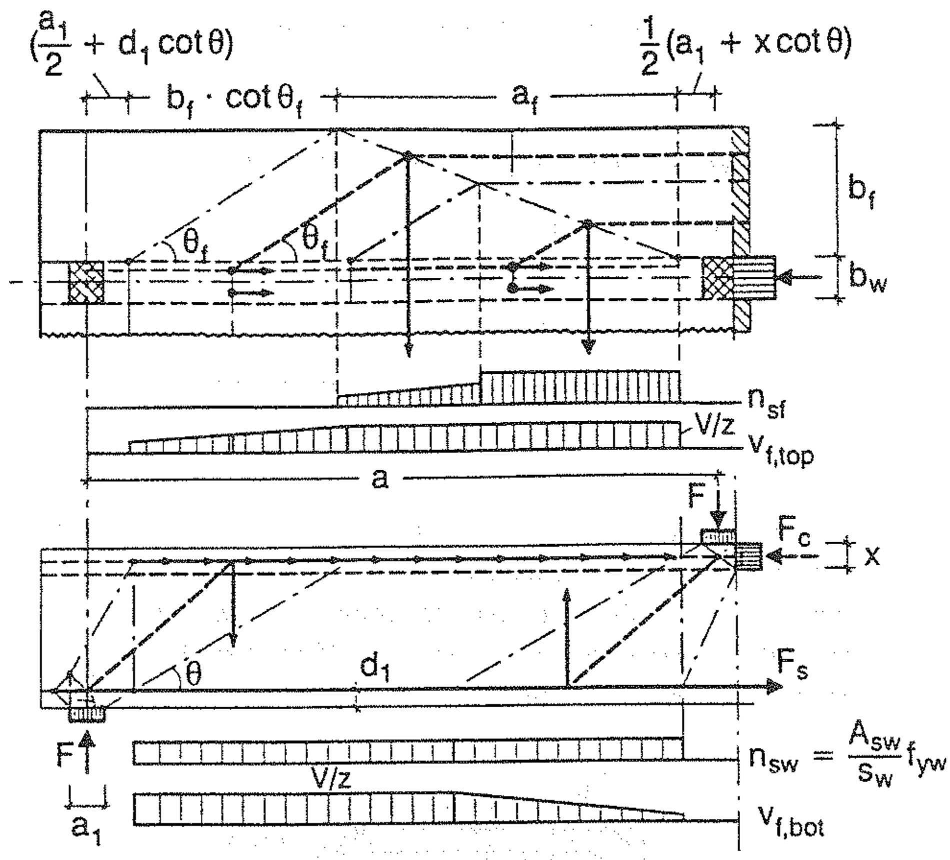


Fig. 6.7 Truss model and stress fields for the transverse reinforcement of a compression flange

6.4.6 B-regions of members with torsion

(1) This section applies to B-regions of members where torsional resistance is required for equilibrium combined with bending and shear. Torsion due to compatibility may usually be neglected in the design, but appropriate minimum reinforcements should be provided. Warping torsion, which may be predominant for members with open cross-sections, is not treated here.

(2) In torsion the equilibrium is maintained by a closed flow of tangential forces (circulatory torsion), which combine with the tangential forces due to the shear force in the web. For calculating the resistance an equivalent hollow section with thin walls (real or notational for members with solid cross-sections) is considered (see Fig. 6.8).

(3) The shear flow due to the torsional moment T_{Sd} alone is:

$$\tau_{t,i}\ t_{ef,i} = T_{Sd} \,/\, (2\ A_{ef}) \tag{6.23}$$

and the tangential force $V_{Sd,i}$ in a wall is (Fig. 6.9):

$$V_{Sd,i} = \tau_{t,i}\ t_{ef,i}\ z_i \tag{6.24}$$

with: A_{ef} = the area enclosed by the centre lines of the walls

z_i = the distance between the intersections of adjacent walls

$t_{ef,i}$ = the effective thickness of the wall i

The centre lines of the wall are defined by the axes of the longitudinal bars in the corners, and the effective thickness t_{ef} ($=t_{ef,i}$) of a wall then is twice the distance from the centre line to the external face of the wall. In case of hollow sections the effective thickness shall not exceed the actual wall thickness.

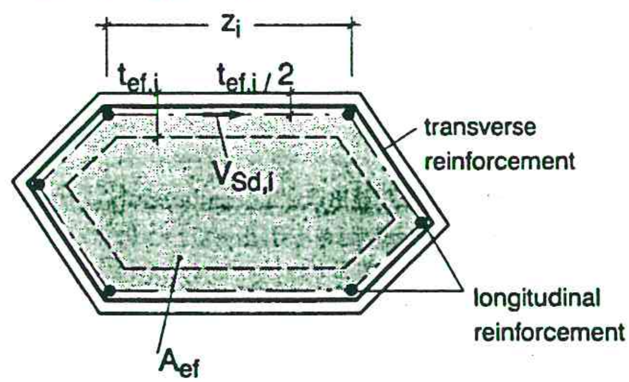
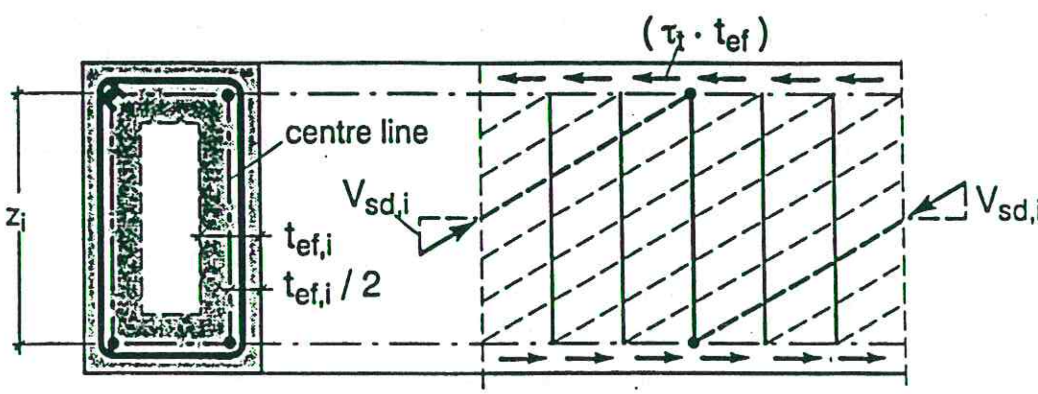


Fig. 6.8 Notations for circulatory torsion



a) equivalent hollow section b) truss model in a wall

Fig. 6.9 Definition of hollow section and model for torsion

(4) The transverse reinforcement in the wall should be designed for the combined shear flow due to torsion and shear, i.e. for the shear force

$$V_{Sdi} = V_{Sdi,T} + V_{Sdi,V} \tag{6.25}$$

The design may be carried out for this shear force according to the rules in sect. 6.4.3.2, whereby the normal force of each wall may be considered.

(5) The longitudinal reinforcement should be designed for the following distributed force over the perimeter u_{ef} of the area A_{ef} :

$$n_{sl} = \frac{\Sigma A_{sl} f_{yl}}{u_{ef}} = \frac{T_{Sd}}{2 A_{ef}} \cot\theta \tag{6.26}$$

In chords these forces may be reduced by corresponding compressive forces. This reinforcement should be distributed over the perimeter, but at least one bar should be placed in each corner of the stirrups.

(6) The upper limit of the resistance is determined by the inclined compression in the shear walls and may be regarded as given, if the following condition is satisfied:

$$\frac{T_{Sd}}{T_{Rd}} + \frac{V_{Sd}}{V_{Rd}} \leq 1 \tag{6.27}$$

with: V_{Rd} = upper limit of shear force acc. to equ. (6.15).

The upper limit of torsional moment is determined for the same angle θ as used for V_{Rd} and is:

$$T_{Rd} = 0.50\ f_{cwd}\ A_{ef}\ t_{ef}\ \sin\theta\ \cos\theta \tag{6.28}$$

6.4.7 Shear in joints

(1) The capacity for the transfer of shear forces across an interface or a joint, such as a construction joint or a joint between in-situ concrete and a precast member, depends on the capacity of the concrete-to-concrete friction limiting the transfer of the inclined compressive forces across the interface (see sect. 5.5). Based on that, the resistance to shear forces across a joint is given by:

$$\tau_{fRd} = \beta\ f_{ctd} + \mu\ \sigma_{cd} + (\mu \sin\alpha_j + \cos\alpha_j)\ \rho\ f_{yd} \qquad (6.29)$$

with:
- τ_{fRd} = design value for shear transfer by concrete friction
- β = coefficient acc. to Table 5.1
- μ = coefficient acc. to Table 5.1
- f_{ctd} = design value of concrete tensile strength acc.sect. 2.1.4
- ρ = A_s / A_j = reinforcing ratio
- A_s = area of reinforcement crossing joint
- A_j = area of joint
- f_{yd} = yield strength of reinforcement
- σ_{cd} = normal stress on interface due to loading only (+ in compression)
- α_j = angle of reinforcement, see Fig. 6.10

(2) The maximum shear stress to be transferred follows from the upper limit for the inclined struts acc. to sect. 6.4.3.4. Such a check is not required if

$$\tau_{fRd} \leq 0.25\ f_{1cd} \qquad (6.30)$$

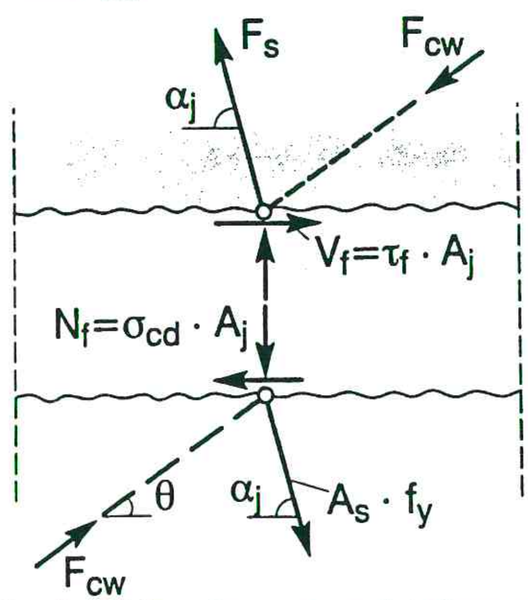


Fig. 6.10 Transfer of shear forces across interfaces or joints

6.5 Design of Discontinuity Regions (D-regions)

MC 90, 6.8

6.5.1 Requirements and general criteria for modelling

(1) The determination of resistance for discontinuity regions (D-regions) shall be based on physical models according to the requirements given in section 6.1. The model for a D-region must comply with that of the adjacent B-region(s), if any.

(2) It is to be verified, that under the action of the design loads the stresses in the struts and ties do not exceed the strength criteria given in sections 5.1 to 5.4, and that the nodes and anchorages comply with sections 5.5 to 5.7.

6.5.2 Statical discontinuities: beam supports and corbels

6.5.2.1 Direct supports of beams

MC 90, 6.8.2.2.1

(1) At "direct supports" the support force is applied by compressive stresses at the bottom face of the member. The support force $A = V_A$ is transferred into the member by an inclined strut representing a fan-shaped compression field (Fig. 6.5.1). The geometry of the fan is defined by the flattest angle θ, which is the angle θ of the compression field in the B-region intersecting the axis of the tension chord.

(2) At an end support the following force F_{sA} in the tension chord has to be anchored in the node over the support plate:

$$F_{sA} = V_A \cot\theta_A + N(1 - z_{s1}/z) \qquad (6.5.1)$$

whereby N (+) for tension.

The angle θ_A for the resultant of the fan-shaped compression field follows from the geometry of the fan:

$$\cot\theta_A = [0.5\ a_1/z + (d_1/z + 0.5)\cot\theta] \qquad (6.5.2)$$

This normally results in a value of $\cot\theta_A = 1.20$ or $\theta_A = 40°$.

(3) The distributed loads q over the fan are directly carried to the support, so that the transverse reinforcement near the support may be designed for the following force:

$$n_{sw,d} = A_{sw}\, f_{wwd} / s_w = V_A - q\,[\,0.5\, a_1 + (d_1 + z) \cot\theta\,] \quad (6.5.3)$$

(4) At intermediate supports the design model in the web is like a combination of two end-supports for the relevant shear forces (see Fig. 6.5.2).

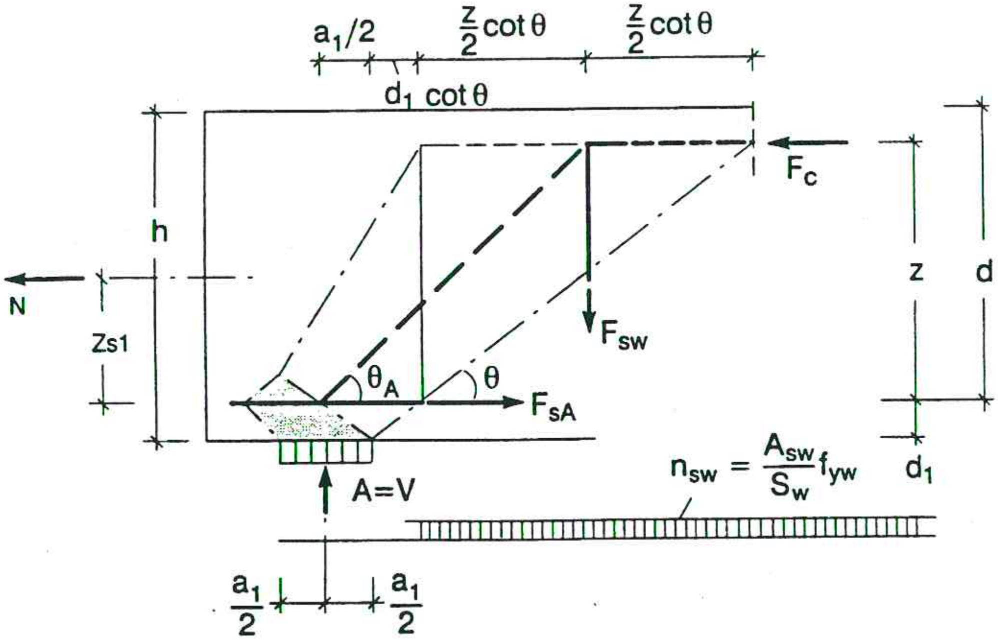


Fig. 6.5.1 Strut-and-tie model for a "direct" end-support

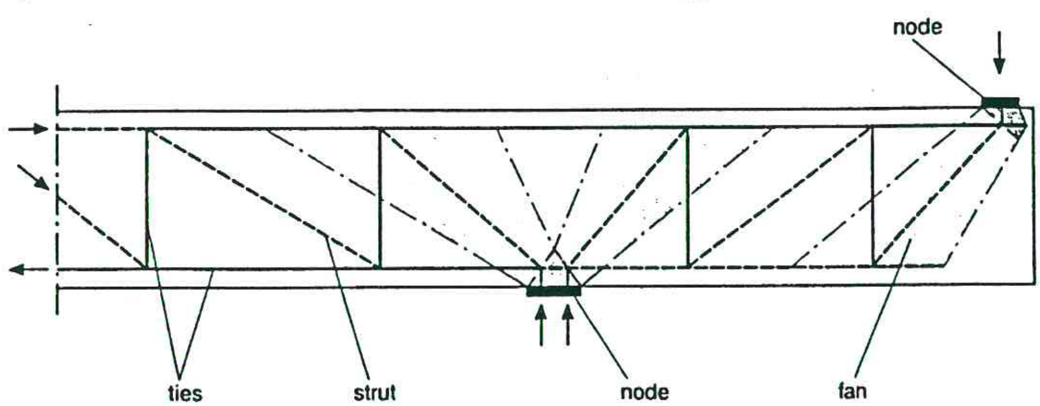


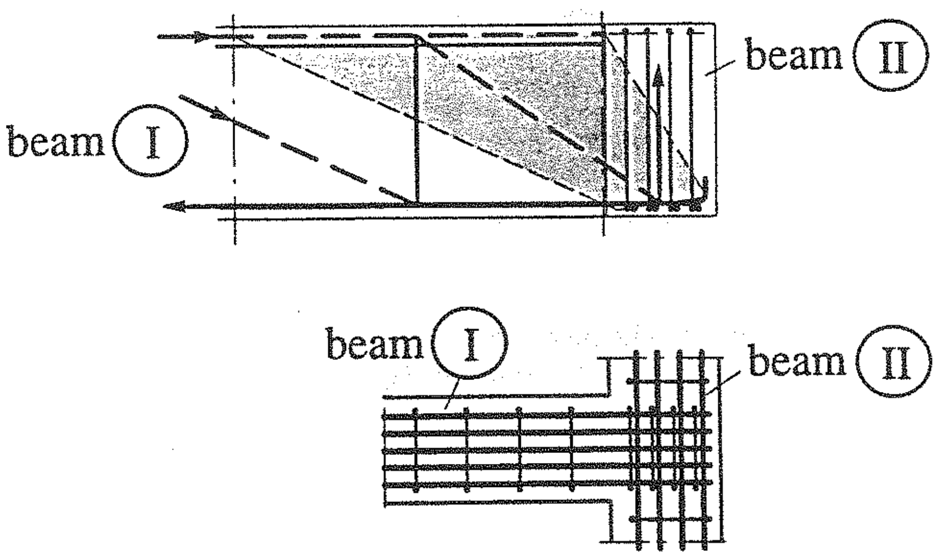
Fig. 6.5.2 Strut-and-tie model for a "direct" intermediate support

6.5.2.2 Indirect supports

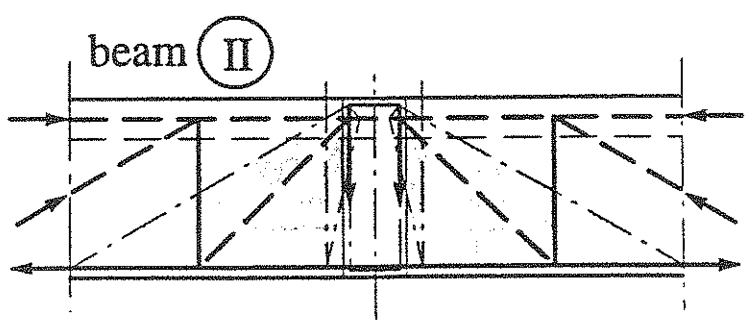
(1) At "indirect supports" of intersecting members the support is provided by tensile stresses over the depth of the member. The total support force has to be transferred to the top of the member by means of "hanging-up" reinforcement within the width of the web (Fig. 6.5.3).

(2) The load transfer into the webs of the intersecting members and the web design may be assumed like for a direct support

(3) Careful consideration should be paid to the anchorage of the main reinforcement, because there is no benefit of transverse compression for the anchorage length acc. to sect. 2.4.1 (5). The begin of the anchorage length is defined by the first stirrup at the inner face of the supporting beam, which defines the deviation of the compression field.



a) indirect end-support and arrangement of hanging-up reinforcement



b) strut-and-tie model in supporting beam

Fig. 6.5.3 Strut-and-tie model for an "indirect" end-support

6.5.2.3 Point load near an support and corbels

(1) A loads near a support of a beam (Fig. 6.5.4) or a load on the cantilevering part of a corbel (Fig. 6.5.5) may directly be transferred to the support by means of an inclined strut. The transverse reinforcement may be designed for the following part of the load: MC 90, Fig. 6.8.4

$$F_1 / F = (2a / z - 1) / 3 \qquad (6.5.4)$$

for : $z / 2 \leq a \leq 2z$

The transverse reinforcement shall be distributed over the length a_w shown in Fig. 6.5.4. As an estimate the value $a_w = (0.85\,a - z/4)$ may be taken.

(2) Unless more refined considerations are made, the strength of the inclined strut $\sigma_{cw} < \nu_2 f_{1cd} = 0.60 f_{1cd}$ is given, if the width of the loading plate fulfils the condition

$$a_F > \frac{x}{\sin\theta_2}\left[\frac{\nu_1}{0.60\cos\theta_2} - \cos\theta_2\right] \qquad (6.5.5)$$

with the angle θ_2 determined from $\cot\theta_2 = a / z$.

(3) Members with point loads near ($a < z / 2$) or over an support require horizontal reinforcement (Fig. 6.5.6) . Unless refined considerations are made it should be designed for

$$T_3 = 0.20 F \qquad (6.5.6)$$

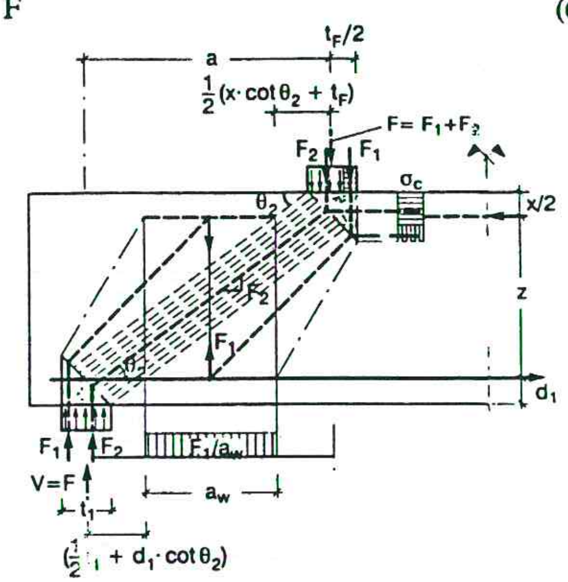
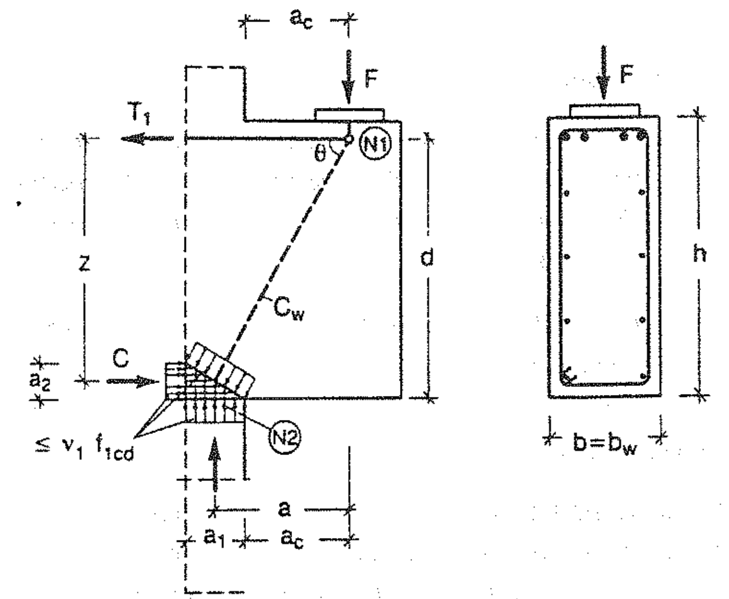


Fig. 6.5.4 Strut-and-tie model for a point load near an end-support



Step 1 : $a_1 = F/b\ \nu_1\ f_{1cd}$ with $\nu_1 = (1 - \frac{f_{ck}}{250})$ $\Rightarrow$ $a = a_c + \frac{a_1}{2}$

Step 2 : $a_2 = d - \sqrt{d^2 - 2a\ a_1}$ $\Rightarrow$ $z = d - \frac{a_2}{2}$

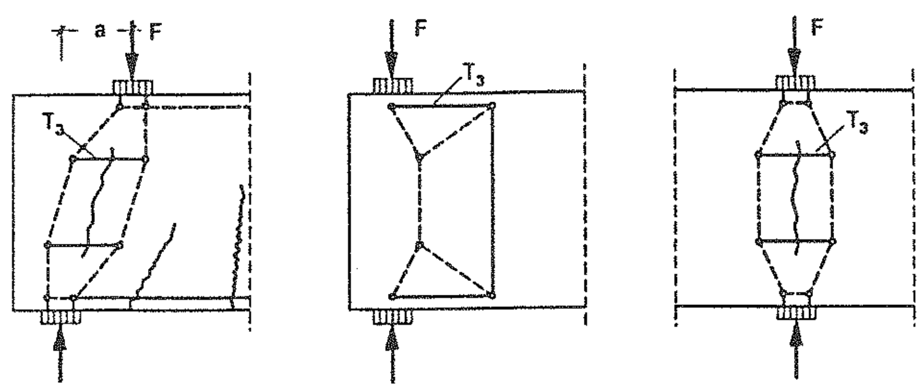
Step 3 : $\cot\theta = \frac{a_2}{a_1} = \frac{a}{z}$

Step 4 : $T_1 = F \cot\theta$ $\Rightarrow$ $A_{s1} = T_1/f_{syd}$

Step 5 : check of node : a) anchorage of tie T_1
b) loading pressure $< 0.80\ f_{1cd}$

Step 6 : check of strut C_w : not required if horizontal reinforcement acc. to Fig 6.9.7a is provided

Fig. 6.5.5 Strut-and-tie model for a corbel (with $a < z / 2$)



a) case $a < z$ b) case $a = 0$ c) point load on a strip

Fig. 6.5.6 Models for the horizontal reinforcements for loads near or over a support

6.5.3 Deep beams

Mc 90, 9.2.5

(1) Deep beams may be designed with the strut-and-tie models. Special attention shall be paid to the anchorage in the nodes at the supports. Minimum reinforcements of 0.1 % in each direction should be placed on each face.

(2) The strut-and-tie model and the distribution of the reinforcement for a deep beam on two supports is shown in Fig. 6.5.7. The inner lever arm may be assumed at about $z = 0.6\ l$, so that the force in the tension chord is about $F_s = 0.2\ q\ l = 0.4\ A$.

(3) The support zones of continuous deep beams may be designed with the model shown in Fig. 6.5.8. Unless more refined considerations are made, the reinforcement over the support should be designed for the force $F_s = 0.2\ q\ l$ and should be distributed over $0.6\ l$. The force in the tension chords should be assumed $F_s = 0.16\ q\ l$ in an end-span (Fig. 6.5.8 a) and $F_s = 0.09\ q\ l$ in intermediate spans (Fig. 6.5.8 b) .

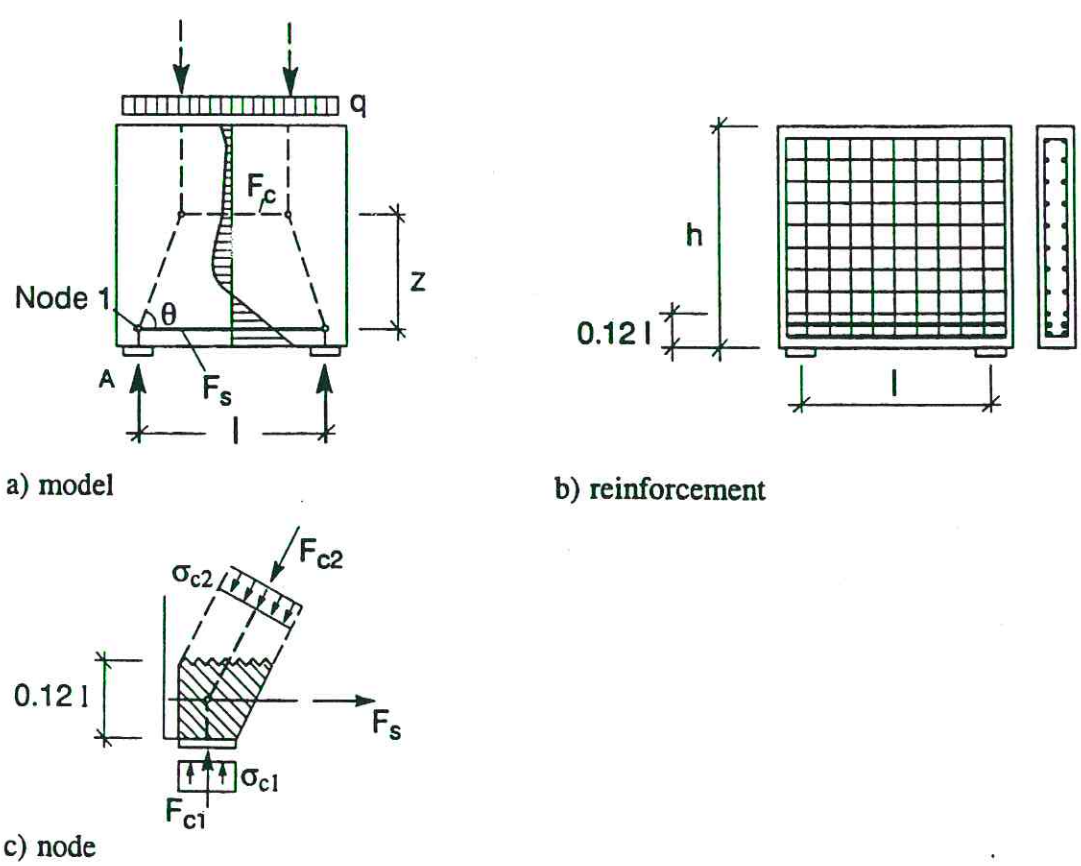
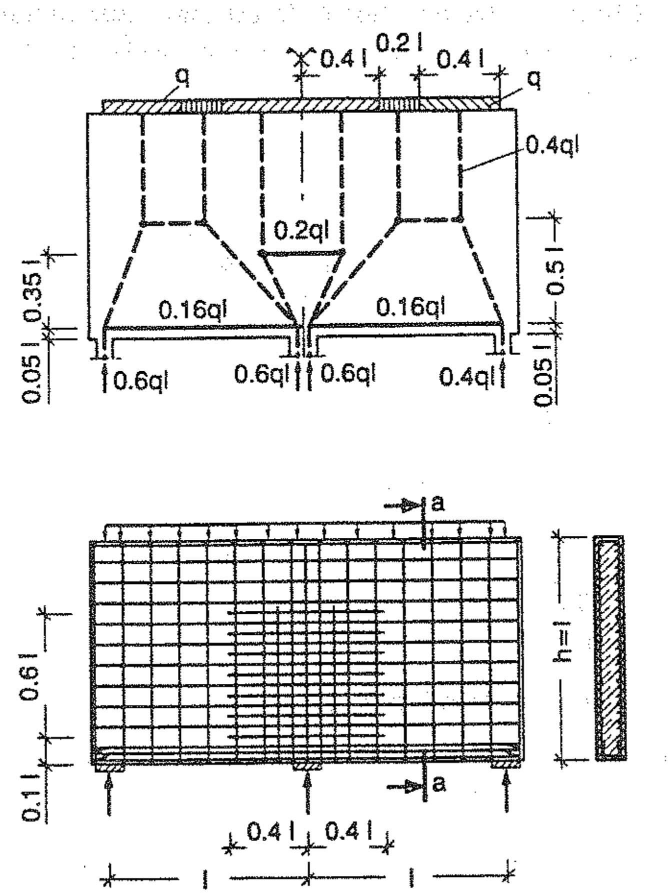
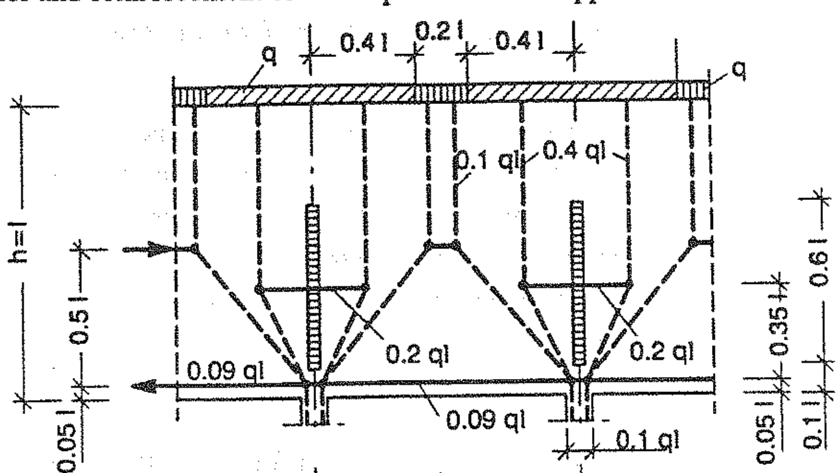


Fig. 6.5.7 Strut-and-tie model and distribution of reinforcement for a deep beam on two supports



a) model and reinforcement for a deep beam on 3 supports



b) model for an intermediate span of a continuous deep beam

Fig. 6.5.8 Strut-and-tie model for continuous deep beams and distribution of reinforcement

6.5.4 Deviation of forces

(1) Changes of the direction of forces may result in transverse tensile stresses or "bursting forces" (Fig. 6.5.9), which should be resisted by suitably anchored reinforcement.

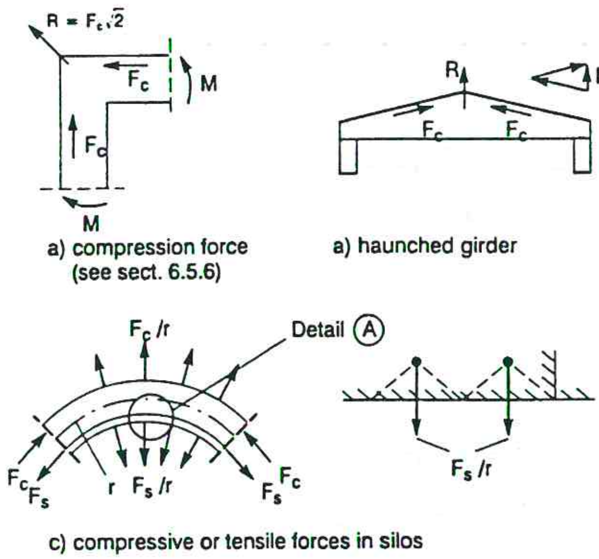


Fig. 6.5.9 Examples for transverse tension due to deviation of forces

6.5.5 Frame corners and beam-column-connections

6.5.5.1 Frame corner with negative (closing) moment

(1) The basic strut-and-tie model for frame corners with negative (closing) moments is shown in Fig. 6.5.10. The critical sections 1 - 1 and 2 - 2 for determining the maximum chord forces lie within the beam-column connection in a distance $x_1/2$ respectively $x_2/2$ from the beam-column interfaces. Compression reinforcement in the chords should not be provided, because it cannot be anchored in the node N1; however, the bi- or triaxial strength may be utilized for the node N1.

(2) The strength of the strut C3 in Fig. 6.5.10 is normally determined by the dimensions of the node N2 at the bend of the main bars (see sect. 5.5.3.1 and Fig. 5.8). It should not exceed the capacity of the bottle-shaped strut of sect. 5.3 (3), unless the connection is reinforced in both directions.

(3) The basic strut-and-tie model of Fig. 6.5.10 is only valid for members of about equal inner lever arms z_1 and z_2. For values $z_1 < z_2$ horizontal reinforcement is required in the connection (Fig. 6.5.11). This horizontal reinforcement may either be determined from the model given in sect. 6.5.2.3 (Fig. 6.5.11 a) or from the force $\Sigma T_h = T_2 - T_1$ according to the model in Fig. 6.5.11 b.

(4) Any splicing of the chord reinforcements requires additional transverse reinforcements. The side cover of the main bars should be secured by stirrups, and longitudinal bars should be provided perpendicular to the plane of the bend.

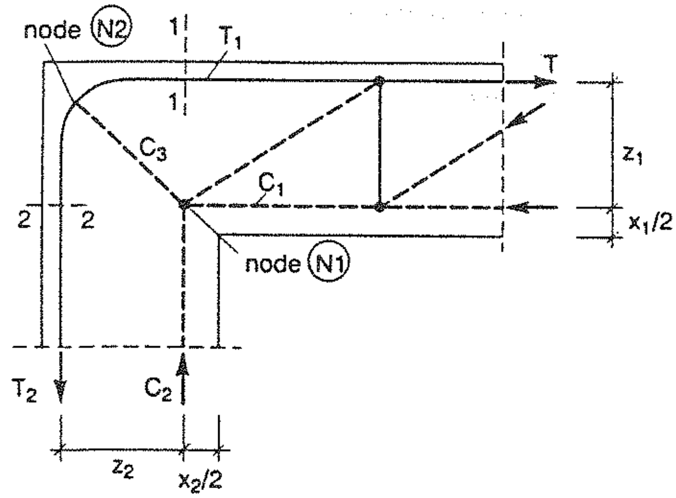
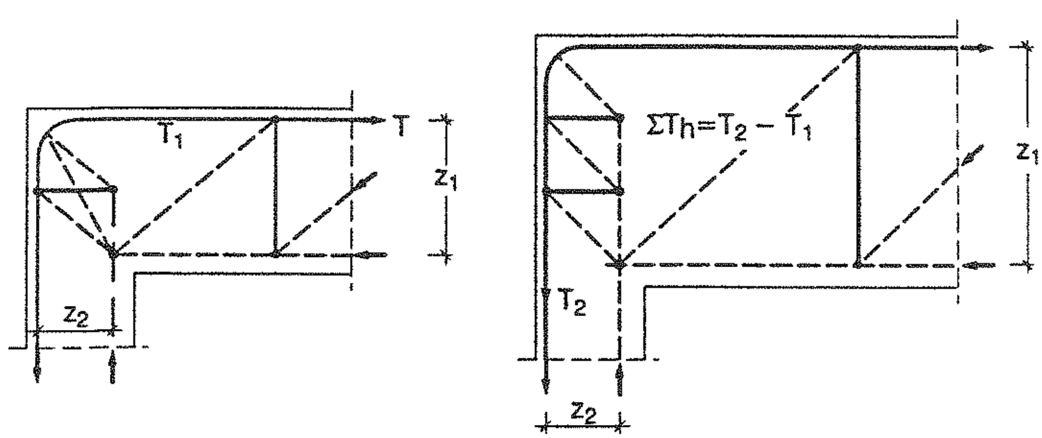


Fig. 6.5.10 Basic strut-and-tie model for frame corners with negative moments



a) model for case $z_1 > z_2$ b) model for case $z_1 >> z_2$

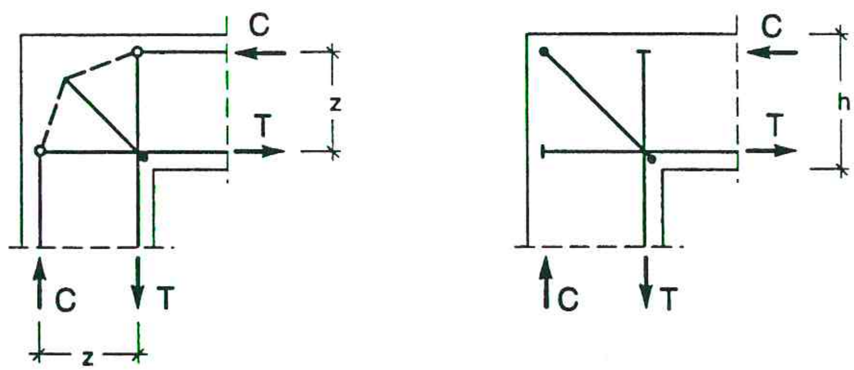
Fig. 6.5.11 Strut-and-tie model for frame corners with negative moments and differing depths of the members

6.5.5.2 Frame corners with positive ("opening") moments

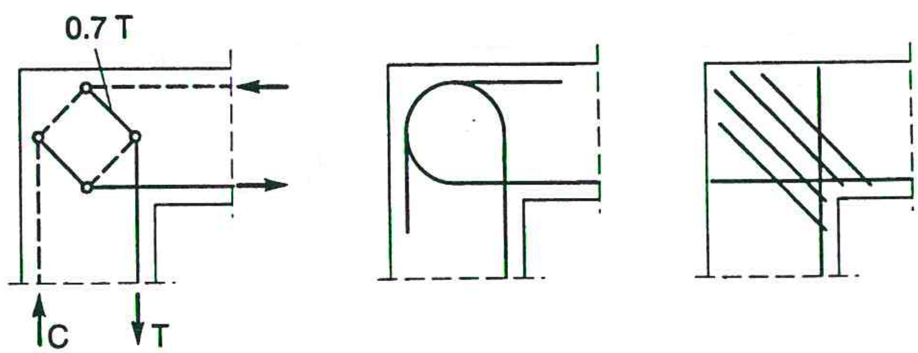
(1) In frame corners with positive ("opening") moments the corner may spall off (see Fig. 6.5.9) and must be secured by appropriate reinforcement. A basic strut-and-tie model (a) and a more refined one (b) are shown in Fig. 6.5.12 along with possible reinforcement lay-outs.

(2) The nodes with the anchorages of the main ties of the tension chords shall be thoroughly investigated. The capacity of the frame corner may be considerably be reduced with increasing reinforcement ratios due to the anchorages.

(3) Inclined bars in the corner improve the capacity and the serviceability of an opening frame corner with larger reinforcing ratios, and Fig. 6.5.13 gives two possible solutions.

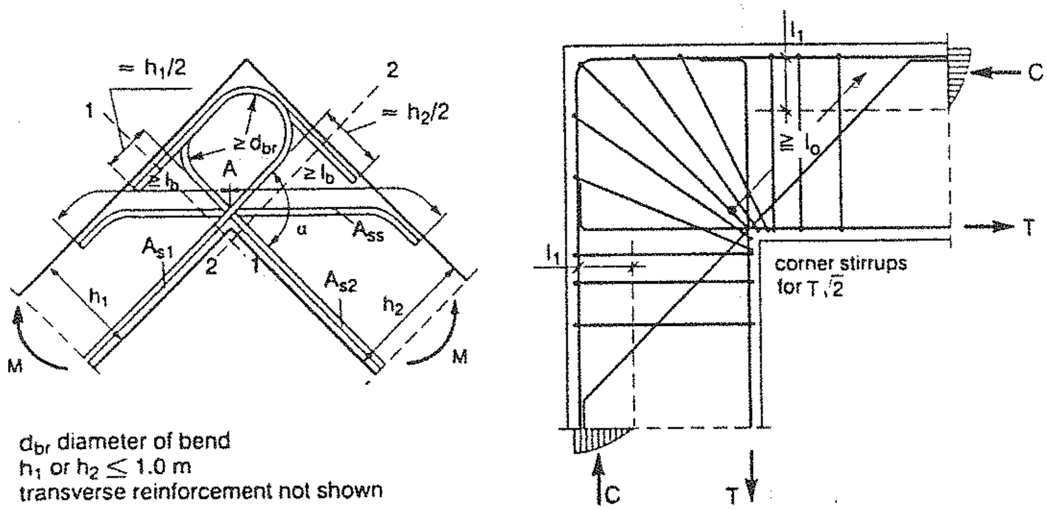


a) basic model and reinforcement for low moments



b) model and reinforcement for moderate moments

Fig. 6.5.12 Models for frame corners with low and moderate moments



a) looped reinforcement and inclined bars

b) cogged bars in combination with inclined bars

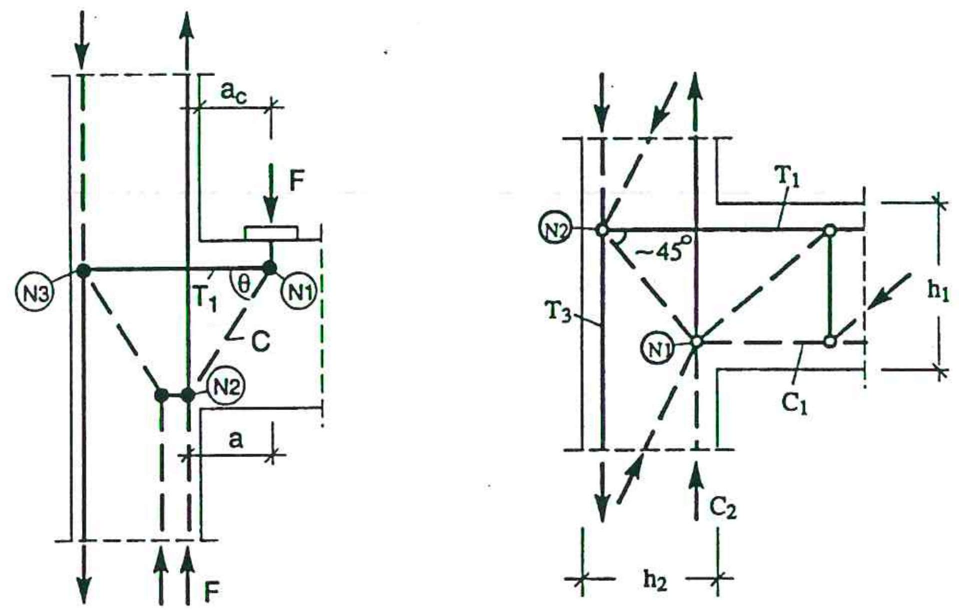
Fig. 6.5.13 Possible reinforcements for frame corners with large opening moments

6.5.5.3 Beam-column connection for an external column

(1) Simple models for a beam-column connection of members with equal depths are shown in Fig. 6.5.14. The forces for dimensioning the connection have to be determined in the sections through the node N1, as explained in 6.5.6.1 (1).

(2) The capacity of the connection depends highly on the requirements for the diameter of bent and the anchorage at the node N2. In case of insufficient anchorage lengths at node N2 additional ties for the force of about $\Delta T = T_1 - T_3 = 0.3\ T_1$ are required over and below the reinforcement for T_1, as shown in Fig. 6.5.15.

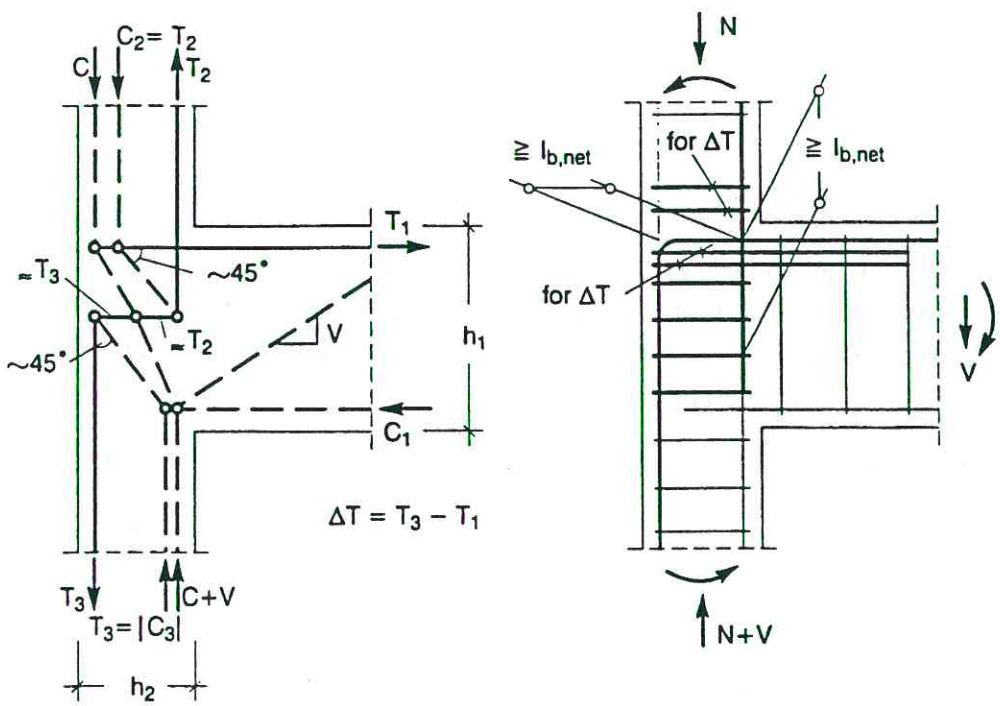
(3) For connections of members with differing depths $h_1 > h_2$ (Fig. 6.5.15) additional horizontal reinforcement is required in the connection (see also Fig. 6.5.11), which may be designed for the force T_3,



a) column - corbel connection

b) connection of beam and external column

Fig. 6.5.14 Basic strut-and-tie models for beam-column connections



a) model

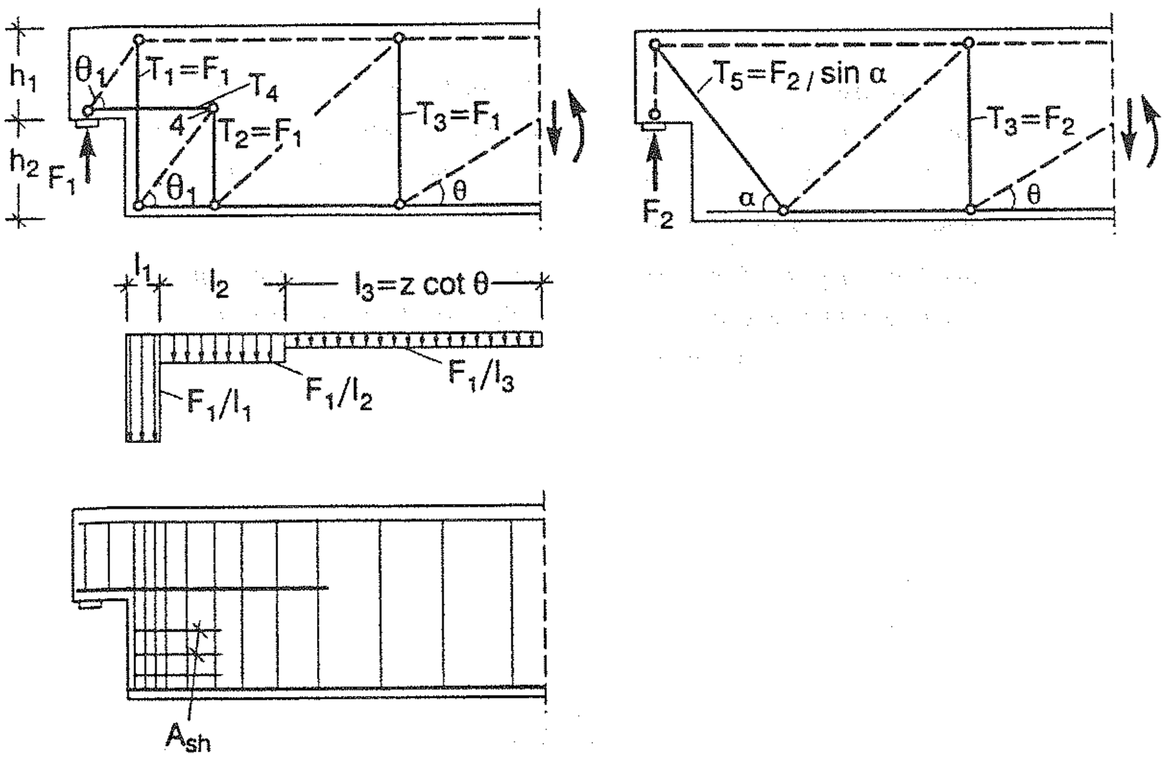
b) reinforcement

Fig. 6.5.15 Refined model for a beam-column connection of members with differing depths

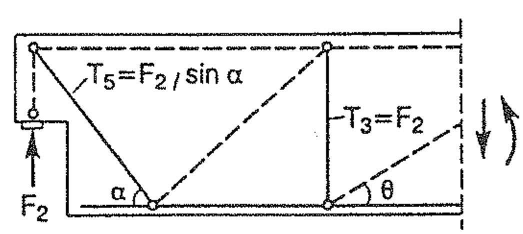
6.5.6 Half joints and steps in members

(1) Half joints or stepped-beam ends should be designed on basis of a combination of the two strut-and-tie models in Fig. 6.5.16 a and b. Due consideration should be paid to possible horizontal forces due to friction of the support.

(2) The model in Fig. 6.5.16 a requires transverse reinforcement for the force $(T_1+T_2) = 2\ F_1$ distributed as shown. The horizontal reinforcement for the tie T_4 must be extended beyond the node N4 by at least half of its anchorage length. Additional horizontal loops or hairpins should be provided in the bottom half of the step, if $h_2 > h_1$ or if $h_2 > 300$ mm.



a) model (1) with horizontal tie at the support and reinforcement



b) model (2) with inclined tie at the support

Fig. 6.5.16 Strut-and-tie models for half joints

6.5.7 Point loads in direction of member axis and anchorage zones of prestressing reinforcements

6.5.7.1 D-region at end-support of rectangular members

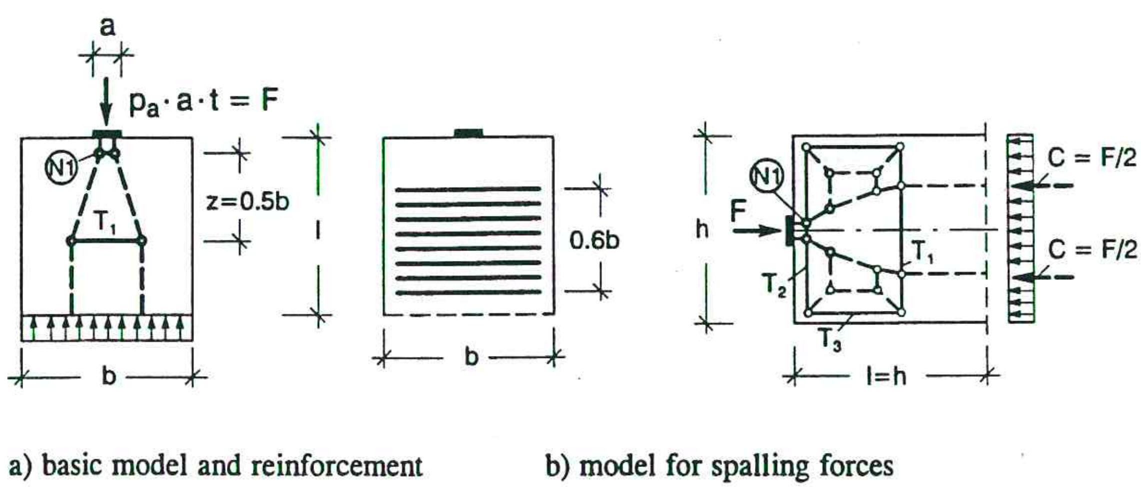
(1) The basic model in Fig. 6.5.17 a applies to the D-region of a point load on a wall or a prestressing anchor in a member with rectangular cross-section. The location of the tie T_1 was orientated by the stress distribution of a linear-elastic calculation. The force T_1 may be assumed at about $T_1 = 0.25\ (1 - a / b)\ F$.

(2) The refined model in Fig. 6.5.17 a assesses the tensile forces ("spalling forces") in the concrete due the compatibility of the inclined struts with the unstressed corners. The forces $T_{sp,1}$ and $T_{sp,2}$ may be assumed at 0.02 F and are normally covered by minimum reinforcements.

(3) The basic models for excentrically applied loads follow from equilibrating the applied load with the linear-elastic stress distribution at the border to the B-region at the opposite end of the D-region (modelling with load-path method).

(4) Similarly, the forces may be derived for an end-support with an anchorage of a tendon (Fig. 6.5.19). For simplicity the forces for the case given above may be taken.

(5) The check of the node N1 shall comply with sect. 5.5.2.



a) basic model and reinforcement b) model for spalling forces

Fig. 6.5.17 Basic model for a concentrated load in the member axis

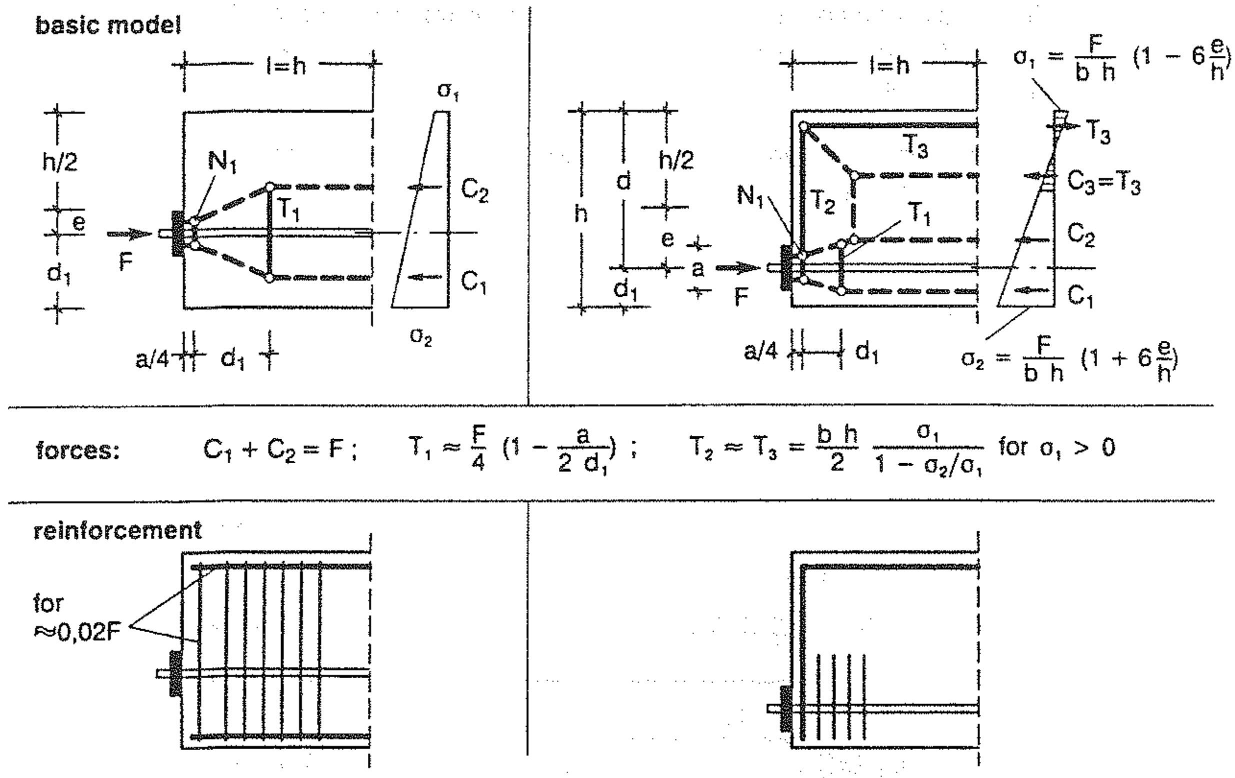
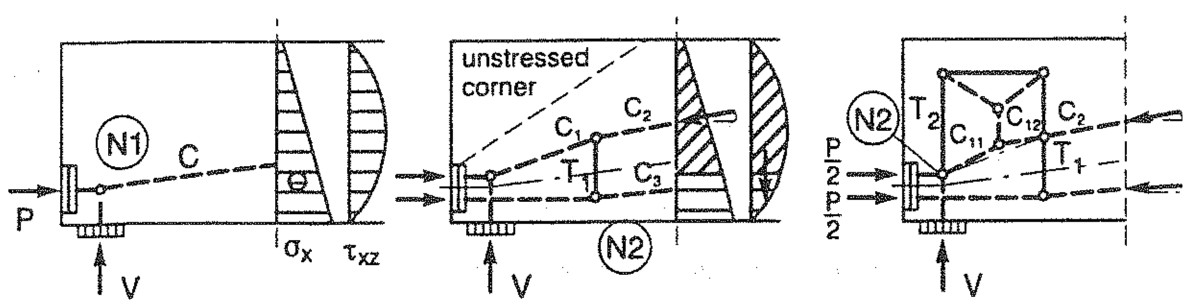


Fig. 6.5.18 Excentric point load in direction of the member axis



a) simple model with resultant C b) refined model for assessing T_1 c) further refined model

Fig. 6.5.19 Anchorage of a tendon at an end-support

(6) In case of the anchorage of a pretensioned prestressed reinforcement (Fig. 6.5.19) the length of the D-region may be assumed at ($l_{b,pt}$ + h), whereby $l_{b,pt}$ is the transmission length acc. to sect. 2.4.2. The modelling of the strut-and-tie model follows the rules given before (see. Fig. 6.5.18).

(7) The transverse force T_1 may be assumed to be taken by the concrete if the following conditions are fulfilled:

$$c / \varnothing > 2.5 \quad \text{and} \quad c_{eff} / \varnothing > 2.25 \qquad (6.5.6)$$

with: c = concrete cover
c_{eff} = $[2c + 1.5(n-1)\, s_n] / 2n$ = effective cover
n = number of tendons
s_n = clear distance between tendons

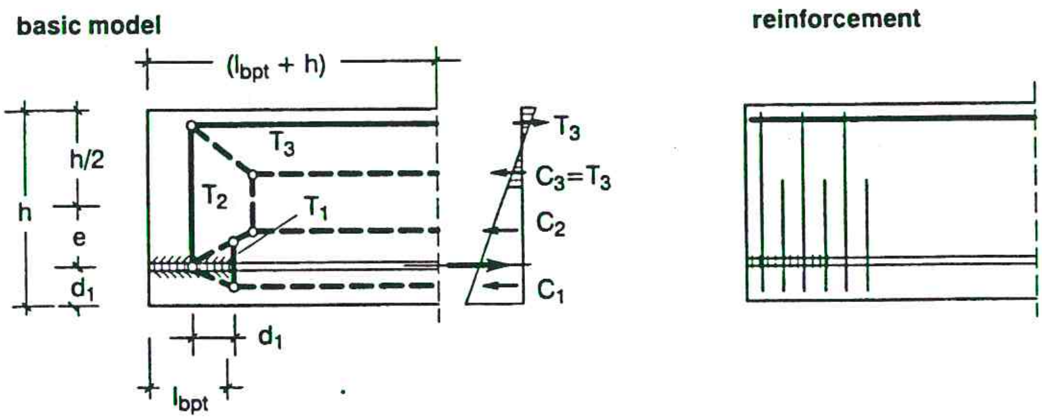


Fig. 6.5.20 End-anchorage of a pretensioned member

6.5.7.2 End support of a beam with flanges

(1) The dispersion of a concentrated load into a member with flanges (Fig. 6.5.21) requires transverse reinforcement in the flange additionally to the transverse web reinforcement.

(2) The dispersion of the prestress in a T-beam or a box-beam (Fig. 6.5.22) follows the same principles. The transverse tie T_1 in the web of the D-region covers the forces due to the combined action of the prestress and the support force respectively the shear force.

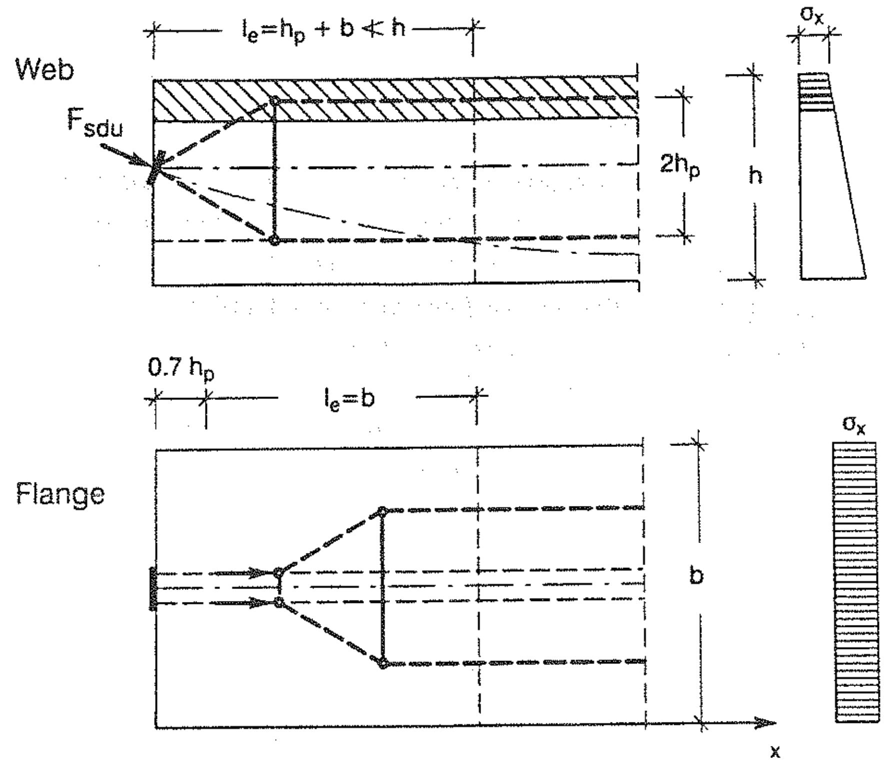


Fig. 6.5.21 Dispersion of the prestress into a T-beam

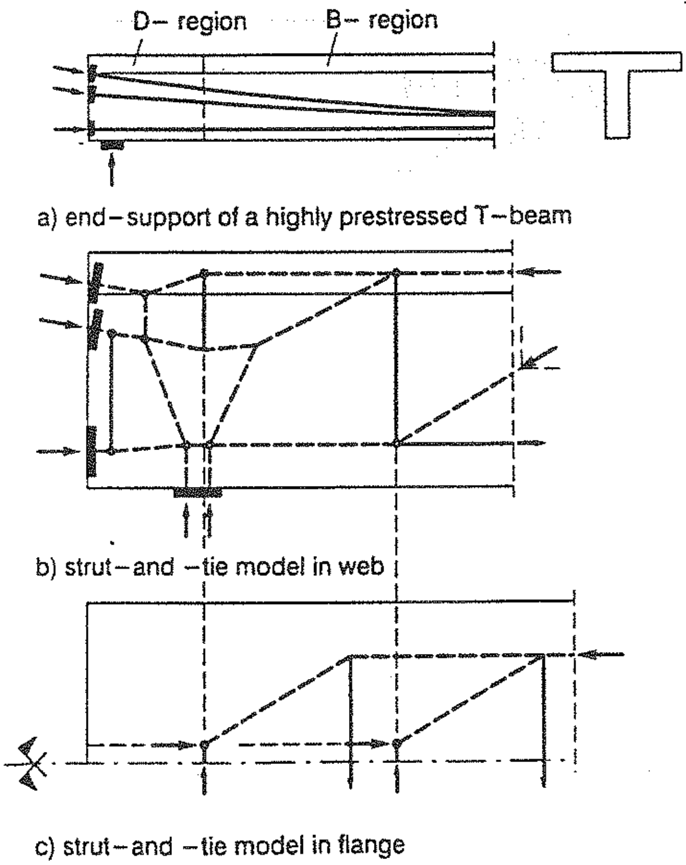


Fig. 6.5 22 Strut-and-tie model for an end-support for a T-beam with prestressing anchors

6.5.8.3 Interior anchor zones and construction joints with prestressing anchors

(1) If a load F is applied at an interior anchor (stressing pocket) of a structural concrete member, about 25 % of it should be tied back by reinforcement besides the anchor, as shown by the strut-and-tie model in Fig. 6.5.23. This tensile force may be reduced by a force of ($5\ A_1\ \sigma_c$) in case of compressive stresses σ_c behind the anchorage (with A_1 = loaded area of anchor). The transverse reinforcement may be designed for the forces given in Fig. 6.5.21, and due consideration should be paid to proper anchorage lenghts.

(2) The same principles apply to internal blisters and Fig. 6.5.22 shows the corresponding strut-and-tie model.

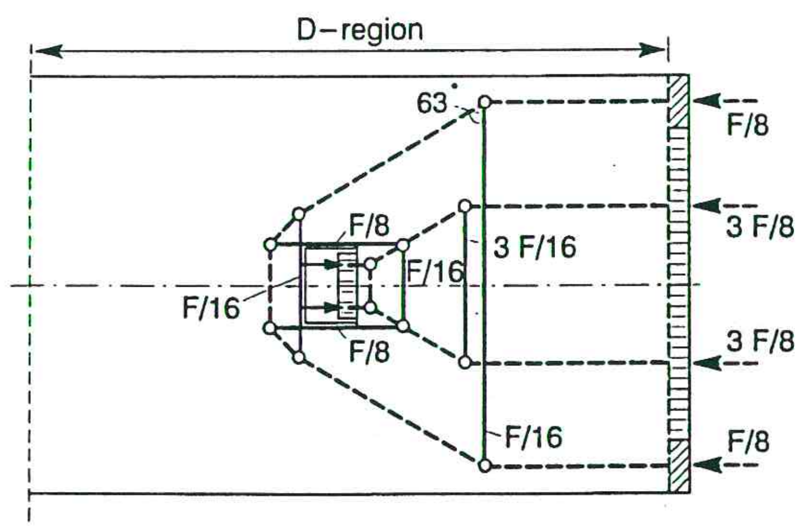
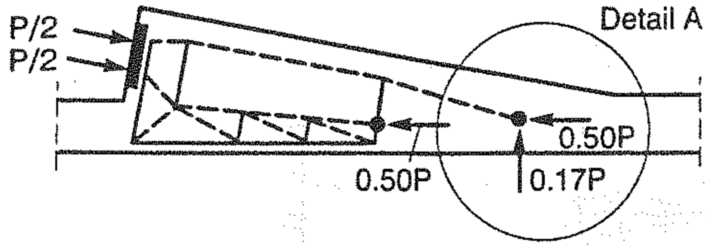


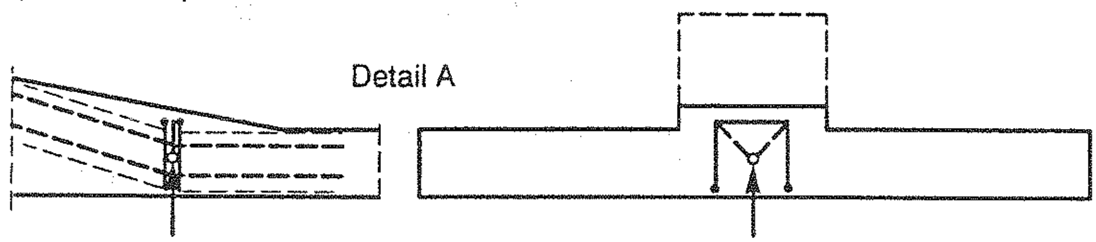
Fig. 6.5.23 Strut-and-tie model for an interior anchorage of a tendon

(2) In case of an internal blister the pretressing force causes transverse forces due to the dispersion of the pretress as well as due to local bending, as shown by the strut-and-tie model in Fig. 6.5.24 a and b.

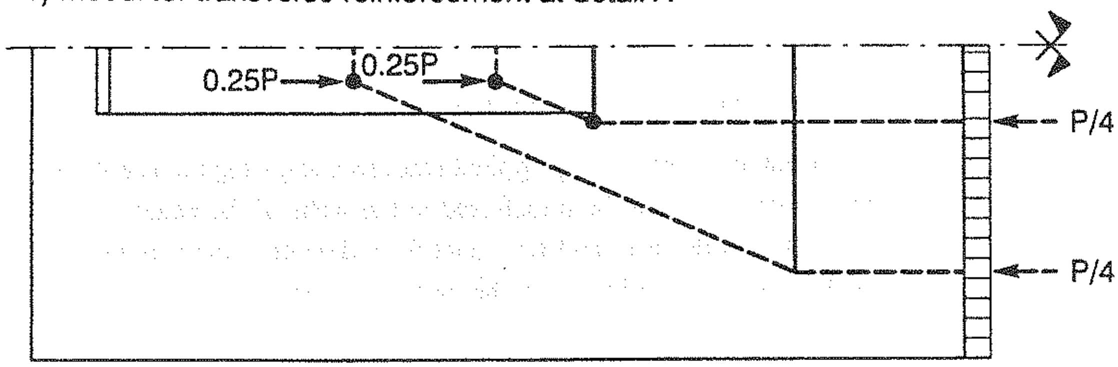
For the further dispersion of the pretress into the slab a simplified model is shown in Fig. 6.5.24 c; additionally longitudinal reinforcement should be provided for tying back some part of the prestressing force as shown in Fig. 6.5.23 for the interior anchorage.



a) transfer of prestress into slab



b) model for transverse reinforcement at detail A



c) simple model for load dispersion in slab

Fig. 6.5.24 Strut-and-tie model and reinforcement for a blister

(3) A construction joint with a coupling anchor represents a D-region, where the forces concentrate at the anchor, so that tension may occur in the section and at the edges (Fig. 6.5.25). Therefore, an appropriate amount of minimum longitudinal reinforcement should be provided, which crosses the joint. The problem is greatly reduced if only a part of the tendons is coupled and if the coupling anchors are distributed over the depth of the web.

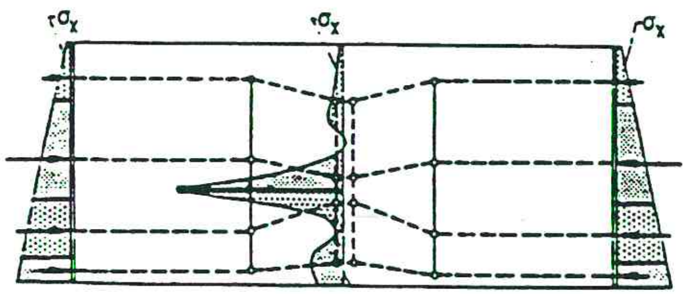


Fig. 6.5.25 Stress distributions and strut-and-tie model for the D-region at a construction joint with a coupling anchor

6.5.7.4 Deviator for external tendons

The deviation of externally applied tendons causes high concentrated forces, which have to be transferred to the webs of the beam. The Fig. 6.5.26 shows the strut-and-tie model for a deviator, and it demonstrates that special attention has to be paid to the transverse ties in the deviator.

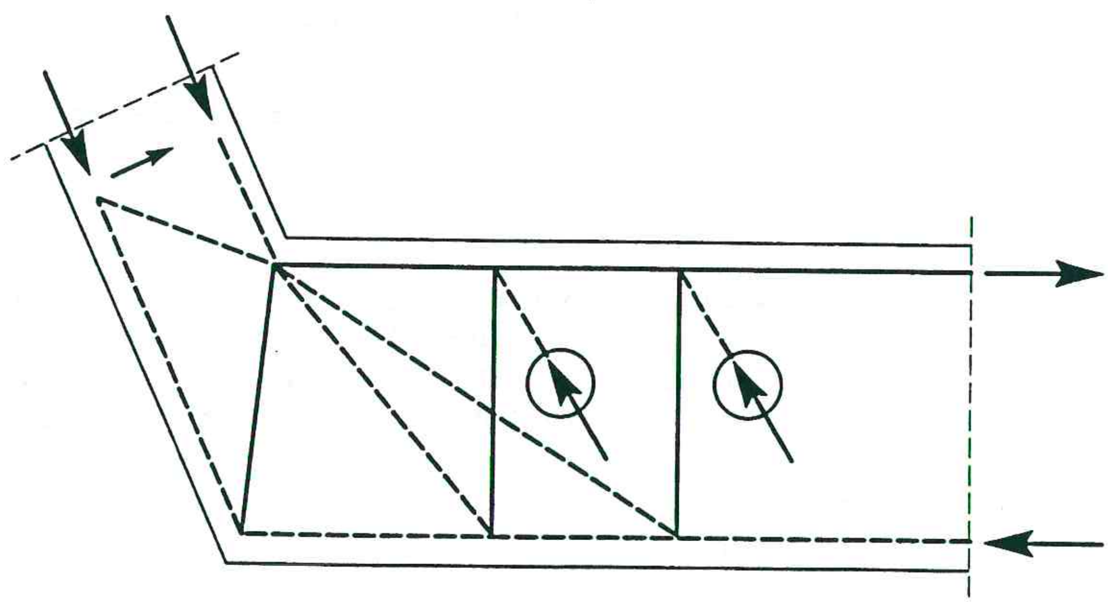


Fig. 6.5.26 Strut-and-tie model for the deviator of external tendons

6.6 Design of slender compressed members

6.6.1 General

(1) This section deals with the design of slender compressed members, e.g. columns, for which deformations may have a significant effect on bending moments. This so called "second order effect" should be calculated taking into account the non-linearity due to cracking, inelastic material properties and creep.

(2) Second order effects can be disregarded if the slenderness of the structure is sufficiently low, see sect. 6.6.2. The effect of creep on second order deformations can be disregarded in some cases (see sect. 6.6.3).

(3) The effect of geometrical imperfections should be considered (see sect. 6.6.4).

(4) Simplified analyses of structures can be made by conventional methods based on second order elastic theory, if member stiffnesses are reduced to reflect the influence of cracking, material non-linearity and creep (see sect. 6.6.5).

(5) For isolated members various simplified methods can be used. One method is described in section 6.6.6.

(6) An accurate non-linear calculation can be made by assuming a linear strain distribution with proper stress-strain curves for the concrete (e.g. acc. to 2.1.3.1 (2)) and the reinforcement, by satisfying the conditions of equilibrium and deformation compatibility in selected cross sections and by integrating the curvature to obtain the deflection. Creep can be included in the stress-strain curve of concrete. Tension stiffening can be considered, but its effect is usually small.

A simplification of this method is to consider only one cross section in a column and to assume a certain distribution of the curvature along the column.

6.6.2 Slenderness

(1) Second order effects can be neglected under certain circumstances, depending on slenderness, eccentricities and axial load. One simple citerion is that the slenderness ratio fulfils the criterion $\lambda \leq 25$.

$$\lambda = l_o \ / \ i \qquad (6.6.1)$$

where

l_o = buckling length (effective length) defined below

$i = \sqrt{I/A}$ = radius of gyration

I = moment of inertia, normally for uncracked section

A = area of cross-section, normally for uncracked section

(2) The buckling length is defined as the length of a pin-ended column with constant axial load, having the same cross-section and buckling load as the actual column. For isolated columns with constant axial force and cross section, the buckling length can be determined directly for certain boundary conditions, and some basic cases are given in Fig 6.6.1.

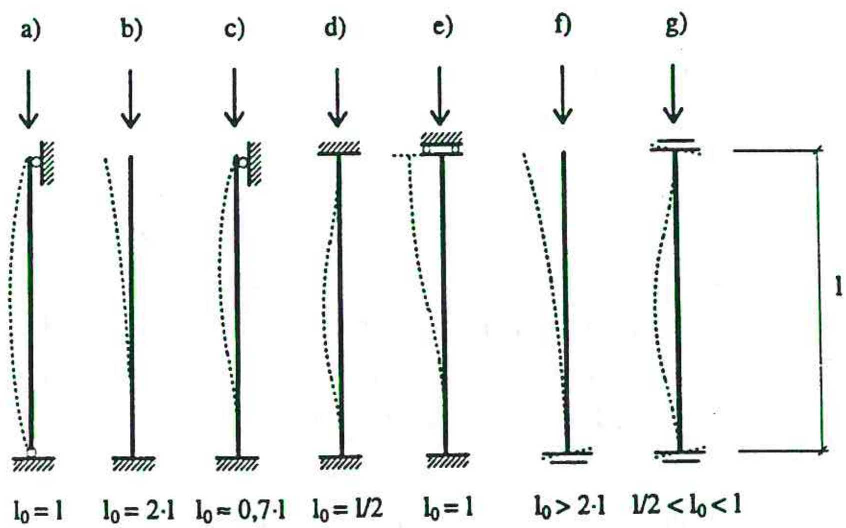


Fig. 6.6.1 Examples for the buckling length for isolated columns

(3) If the buckling load (instability load) of a structure is known, the following general definition of the buckling length can be used for individual columns:

6.6.3 Effects of creep

(1) The effects of creep can be taken into account by means of an "effective creep ratio":

$$\varphi_{ef} = \varphi \cdot M^0_{Sg} / M^0_{Sd} \qquad (6.6.3)$$

where: φ = basic creep ratio according to sect. 2.1.5

M^0_{Sg} = first order moment under (unfactored) quasi-permanent load (see sect. 7.2.2)

M^0_{Sd} = first order moment under design load

The first order moments M^0_{Sg} and M^0_{Sd} should include the effects of imperfections under quasi-permanent and design loads respectively (see sect. 6.6.4).

(2) The effects of creep may be neglected, i.e. $\varphi_{ef} = 0$, if at least <u>two</u> of the following conditions is fulfilled:

$$\lambda \leq 40 \qquad (6.6.4\ a)$$

$$e_0 / h \geq 2 \qquad (6.6.4\ b)$$

$$N_{Sg} / N_{Sd} \leq 0.15 \qquad (6.6.4\ c)$$

where: λ = slenderness ratio acc. to sect. 6.6.2

e_0 = first order eccentricity M^0_{Sd} / N_{Sd}

M^0_{Sd} = first order moment under design load (including imperfections)

N_{sd} = axial force under design load

N_{sg} = axial force under (unfactorred) quasi-permanent load (see sect. 7.2.2)

6.6.4 Geometrical imperfections

(1) The effect of geometrical imperfections should be included in the analysis (unless a specific method is used which in itself includes the effect of a relevant imperfection).

(2) The effect of imperfections can be based on an inclination α, with the follow design value α_a for one individual column:

$$\alpha_a = 0{,}01 / \sqrt{l} \tag{6.6.5}$$

where l = length of member in [m]

(3) To calculate the combined effect of imperfections from horizontally connected vertical elements, a mean value α_{am} according to eq. (6.6.6) can be used.

$$\alpha_{am} = \alpha_a \cdot \sqrt{0{,}5\,(1 + 1/m)} \tag{6.6.6}$$

where: m = number of vertical elements contributing to the combined effect

(4) In the design of isolated columns m is always equal to 1, i.e. $\alpha_{am} = \alpha_a$, and l is the actual length of the column, i.e. generally not the buckling length.

(5) In other cases than in (4) m and l depend on the case considered, see Fig. 6.6.2:

a) case a in Fig. 6.6.2 a for calculating the total effect of imperfections on a "bracing member" A. Thereby, l and m can be defined as follows:

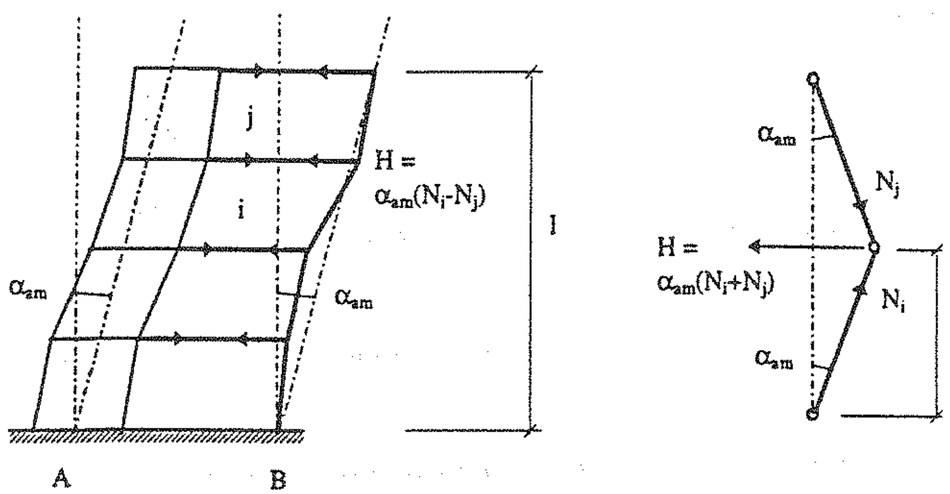
- - for braced columns which are continuous throughout the building: l is the height of the building and m is the number of continuous members (including the bracing ones).
- -- for columns consisting of storey high elements: l is the storey height and m is the number of individual members.

b) case b in Fig. 6.6.2 b for calculating the total effect on a "floor diaphragm" transferring the horizontal loads from braced to bracing members. In this case, l is the storey height and m the total number of columns in the two storeys which contribute to the total horizontal force on the floor.

(6) In the analysis of entire structures the effect of imperfections can be represented by horizontal forces, see Fig. 6.6.2.

(7) For isolated columns the effect of imperfections can be represented by horizontal forces according to Fig. 6.6.3 or, for non-sway columns, by an eccentricity e_a:

$$e_a = \alpha_a \cdot l / 2 \tag{6.6.7}$$



a) case a: effect of imperfections for bracing member b) case b: effect of imperfections for floor structure

Fig. 6.6.2 Representation of geometrical imperfections

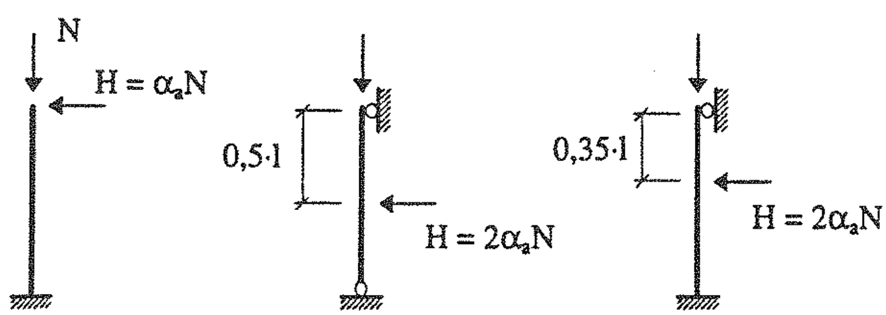


Fig. 6.6.3 Effect of imperfections on isolated columns, expressed in terms of equivalent horizontal forces H

6.6.5 Method based on estimation of secant stiffness

(1) With a reduced bending stiffness, i.e. a secant stiffness taking into account cracking, material non-linearity and creep, analysis can be made formally by second order elastic theory. In some cases amplification factors formulated on the basis of a fictive buckling load can be used, provided the buckling load is based on such a reduced stiffness (i.e. a stiffness reflecting the conditions of the structure in ULS, not the stiffness of the structure in a fictive state of buckling).

This is applicable for isolated members as well as for whole structures. The effect of imperfections according to 6.6.4 should be included in the analysis.

The cross-sections are designed in ULS for the normal forces and total moments resulting from the analysis.

(2) The general definition of the secant stiffness is:

$$EI = M \,/\, (1/r) \tag{6.6.8}$$

where $1/r$ = curvature at moment M and normal force N.

(3) A simple estimate of the secant stiffness in is given by eq (6.6.9). It is generally conservative if the total reinforcement ratio $\rho \geq 0{,}01$.

$$EI = 0.4 \cdot E_c I_c \;/\; (1 + \varphi_{ef}) \tag{6.6.9}$$

where: E_c = design value of the modulus of elasticity of concrete,
I_c = moment of inertia
φ_{ef} = effective creep ratio according to sect. 6.6.3

(4) If the reinforcement is known (or assumed) and if the reinforcement ratio is $\rho \geq 0.01$, then eq. (6.6.10) can be used as a better estimate:

$$EI = 0.2 \cdot E_c I_c \;/\; (1 + \varphi_{ef}) + E_s I_s \tag{6.6.10}$$

where: E_s = design value of modulus of elasticity of reinforcement,
I_s = moment of inertia of reinforcement (with respect to CG of cross section)

(5) If the cross section can be shown to be uncracked under design moment (including second order moment), then eq (6.6.11) may be used.

$$EI = 0.8 \cdot E_c I_c \,/\, (1 + \varphi_{ef}) \qquad (6.6.11)$$

(6) Other methods for calculating the secant stiffness may be used.

6.6.6 Simplified method for isolated columns

(1) The following method is based on a simple estimate of the curvature in the critical section, giving the deformation and hence the second order moment. This is added to the first order moment to give the total design moment for which, together with the axial force, the cross section is designed in ULS. The method should be limited to cases where the first order moment M^0_{Sd} corresponds to an eccentricity $e_0 \geq 0.1\,h$.

(2) The method is best suited for individual columns with constant axial load and boudary conditions acc. to Fig. 6.6.1 a) to e). It can also be used for other cases , provided the buckling lenght defined in sect. 6.7.2 (2) is determined with due consideration of elastic restraints and/or variation of axial force.

(3) The total design moment is

$$M_{Sd} = M^0_{Sd} + M_2 \qquad (6.6.12)$$

where: M^0_{Sd} = first order design moment, including the effect of imperfections

M_2 = second order moment

Differing first order end moments M^0_1 and M^0_2 in a non-sway column acc. to Fig. 6.6.1 a), c), d) and g) may be replaced by an equivalent value:

$$M^0_e = 0.6 \cdot M^0_1 + 0.4 \cdot M^0_2 \qquad (6.6.13)$$

M^0_1 and M^0_2 should be inserted with the same sign if they give tension on the same side of the column, otherwise with opposite signs. Further, $|M^0_2| \geq |M^0_1|$.

(4) The second order moment M_2 is determined as follows:

$$M_2 = N_{sd} \; e_2 \qquad (6.6.14\ a)$$

$$e_2 = (1/r) \cdot l_0^2 / \beta \qquad (6.6.14\ b)$$

where: N_{sd} = axial force under design load

e_2 = deformation (eccentricity)

$1/r$ = curvature, see (5) below

l_0 = buckling length, see sect. 6.6.2

β = factor depending on the curvature distribution

For the factor β in equ. (6.6.14 b) normally $\beta = 10$ ($\approx \pi^2$) can be used. If the first order moment is caused by a concentrated horizontal load near midheight, ot at the top of a cantilever column, then $\beta = 12$ can be used. If the reinforcement is curtailed according to the moment diagram, then $\beta = 8$ should always be used.

(5) For cross-sections with symmetrical reinforcement, the curvature can be estimated as follows:

$$1 / r = (1 + \varphi_{ef}/2) \cdot \alpha \cdot 2\, \varepsilon_{yd} / z_s \qquad (6.6.15)$$

where: φ_{ef} = effective creep ratio, see sect.6.6.3

e_{yd} = f_{yd} / E_s

z_s = distance between the centres of the reinforcements on each side of the cross-section

α = $(N_{ud} - N_{Sd}) / (N_{ud} - N_{bal}) \leq 1$

N_{ud} = axial capacity of cross-section $= (f_{1cd}\ A_c + f_{yd}\ A_s)$

N_{sd} = the actual design axial force

N_{bal} = the axial force which gives maximum moment capacity, normally $\approx 0.4 \cdot f_{1cd}\ A_c$

6.7 Design of slabs

6.7.1 General and design model

(1) The design model for the B-region of slabs consists of two chords or outer layers connected by the web or inner layer in between. In the general case of bi- or triaxially loaded slabs the chords are represented by biaxially loaded plate elements resisting the in-plane effects of the moments and axial forces and torsion, as well as the additional forces due to the transfer of shear and torsion in the web due to the truss action.

(2) In B-regions of slabs primarily subjected to moments parallel to the directions of the reinforcment the flexural design to determine the inner lever arm between the chords follows sect. 6.4.2 and the shear design follows sect. 6.7.2.

(3) The D-region design for punching of a column through a two-way slab is dealt with in sect. 6.7.3.

6.7.2 Shear design of one-way spanning slabs or members

(1) The structural behaviour of slender members without transverse reinforcement may be representred by the truss model with a biaxial tension-compression field in the web shown in Fig. 6.7.1 a. According to this model the required force in the tension chord is:

$$F_s = M / z + 0.58 V \qquad (6.7.1)$$

(2) The ultimate strength of this model is not determined by the tensile strength of the concrete ties, but by the shear transfer mechanisms across the cracks, i.e. mainly the friction across the crack surfaces and the dowel action (see Fig. 6.7.1 b). Based on constitutive laws for these mechanisms an explicit equation can be derived for the design capacity, which may be simplified as follows:

$$V_{Rd} = b_w\, d\ 0.12\ f_{ck}^{2/3}\ \beta_N \ / \ [1 + 0.007\ d / \rho] \qquad (6.7.2)$$

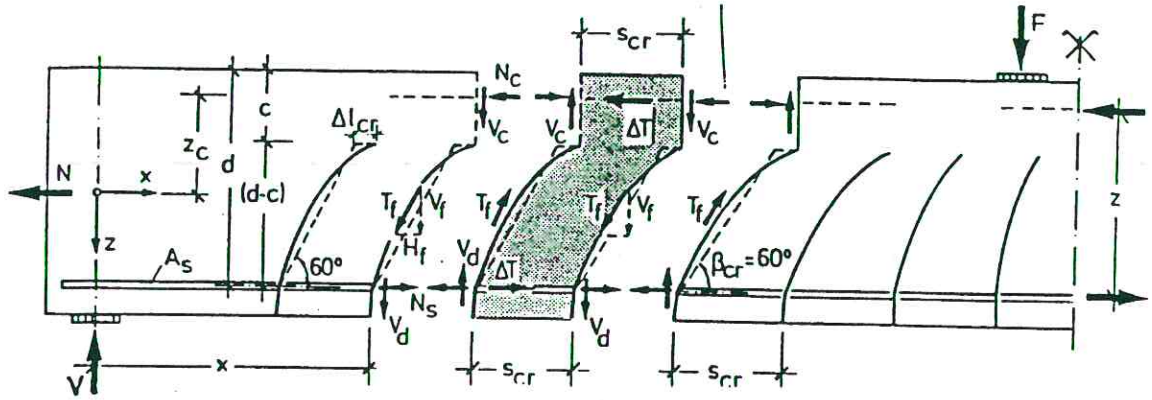
where: d [m] = effective depth

f_{ck} [MPa]

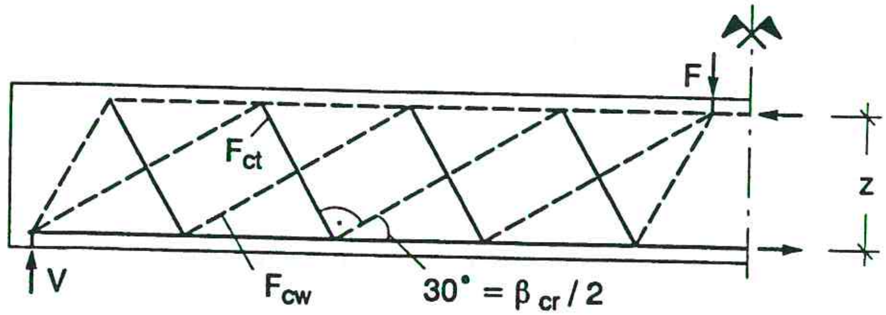
ρ [-] = $A_s / b\, d$ = reinforcing ratio of longitudinal reinforcement

$\beta_N = [1 - (\sigma_N / 400)\ (d / \rho)]$ = factor for influence of axial forces or of prestress

$\sigma_N = N / b\, d$ = axial stress [MPa]; (+ for tension)



a) cracked member and shear transfer mechanisms across cracks



b) truss model with biaxial tension - compression field in the concrete

Fig. 6.7.1 Shear transfer mechanisms and truss model for members without transverse reinforcement

(3) If V_{Sd} exceeds V_{Rd} acc. to equ. (6.5.1), then transverse reinforcement is required, which should be designed according to sect. 6.4.3.2. However, no minimum transverse reinforcement need to be provided in regions with $V_{Sd} < V_{Rd}$.

(4) If in two-way slabs the principal shear is not in the direction of the longitudinal reinforcement the design may be carried out acc. to sect. 6.4.2.5 of MC 90,

6.7.3 Punching

6.7.3.1 General

(1) The punching resistance to the transverse effects, i.e. the transfer of concentrated forces (loads or reactions) acting on slabs without shear reinforcement may be verified in terms of nominal shear stresses at control perimeters around the concentrated force. This empirical approach for assessing the punching resistance does not imply any physical meaning for the nominal shear stress on the defined section.

(2) The applied shear τ_{sd} can be determined at a critical section taken on a perimeter at a distance 2.0 d from the boundary of the loaded area, as shown in Fig. 6.7.2 and Fig. 6.7.3, and should satisfy the following condition:

$$\tau_{Sd} = P_{Sd} / u_1 d \leq \tau_{Rd} \qquad (6.7.3)$$

where P_{Sd} = punching load due to the applied external loads; this may include the vertical effect of prestress acting inside a perimeter at a distance $h / 2$ from the periphery of the loaded area.
u_1 = perimeter at a distance 2 d from the boundary of the loaded area (see Fig. 6.7.2 and Fig. 6.7.3)
d = effective depth

(3) The applied load P_{Sd} may be reduced by the loads within the control perimeter defined in (2), which especially is relevant for column bases.

6.7.3.2 Symmetric punching of slabs without shear reinforcement

(1) For the above defined critical section, the resistant punching force for symmetrically loaded (interior) columns is given by:

$$\tau_{Rd} = P_{Rd} / u_1 d = 0.12\ \xi\ (100\ \rho\ f_{ck})^{1/3} \qquad (6.7.4)$$

where: ξ = $(1 + 200/d)$ factor for size effect, with d [mm]

$\rho\,[-] = \sqrt{\rho_x\ \rho_y}$ = ratio of flexural reinforcement.
In each direction ρ should be calculated for a width equal to the dimension of the loaded area plus 3d on each side of it (or the slab edge if it is closer).

(2) For prestressed slabs the beneficial effects of the prestress may be considered as in the FIP Recommendations for Slabs and Foundations.

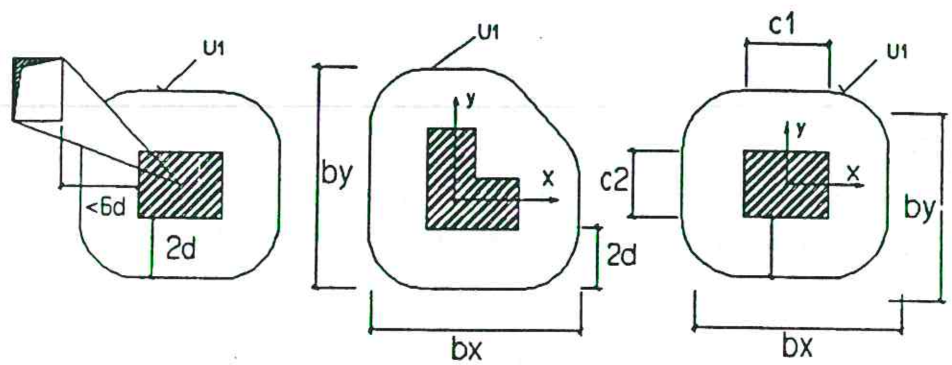


Figure 6.7.2 Critical perimeter u_1 at interior columns

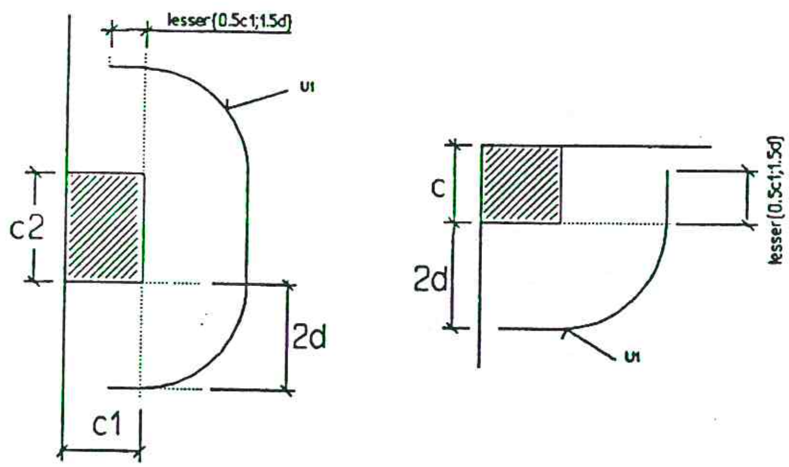


Figure 6.7.3 Critical perimeter u_1 at edge and corner column

6.7.3.3 Punching of slabs with transfer of moments to column

(1) The resistant punching load is reduced, if there is a moment transfer between the slab and the column. This may be taken into account by increasing the applied shear τ_{Sd} by a facor β giving an effective design value

$$\tau_{Sd,eff} = \beta \ \tau_{Sd} \tag{6.7.5}$$

where $\beta = 1.15$ for edge columns,
$\beta = 1.40$ for corner columns.

6.7.3.4 Slabs with punching shear reinforcement

(1) When the punching capacity according to eq. (6.7.4) or (6.7.5) is not sufficient, either a drop panel may be placed or shear reinforcement pe provided. If shear reinforcement is placed, the following checks have to be carried out.

(2) The maximum punching capacity is limited by the capacity of the concrete in compression of the node where the load is transferred. The maximum load transferred across the perimeter u_0 inmediately adjacent to the loaded area (see Fig. 6.7.4) is limited to:

$$P_{Sd,eff} \leq P_{Rd} = 0.50 \ u_0 \ d \ f_{cd1} \qquad (6.7.6)$$

where u_0 perimeter defined in Fig. 6.7.4.

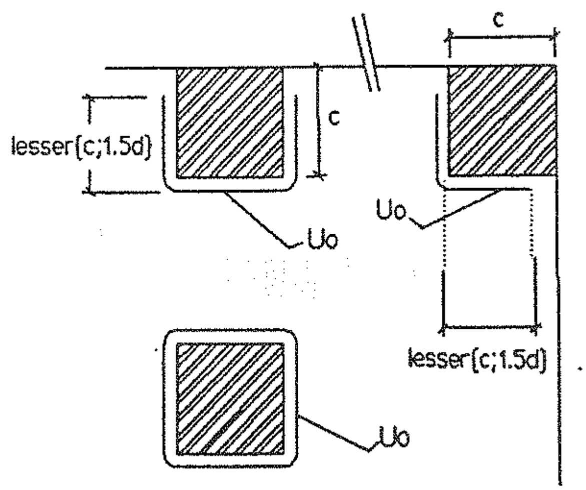


Fig. 6.7.4 Perimeter u_0 for maximum resistance, u_0

(3) At the zone in which the punching shear reinforcement is placed, the punching capacity is given by:

$$P_{Sd,eff} \leq 0.09 \ \xi \ (100 \ \rho \ f_{ck})^{1/3} \ u_1 \, d \ + 1.5 \, d \, (A_{sw}/s_r) \, f_{ywd} \sin\alpha \qquad (6.7.7)$$

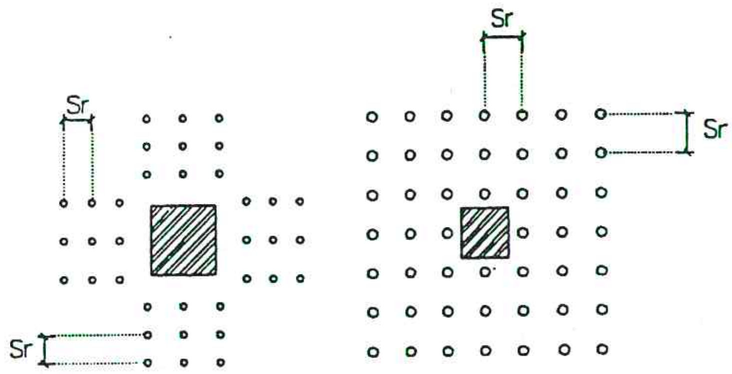
where

A_{sw} = area of shear reinforcement in a layer around the column.

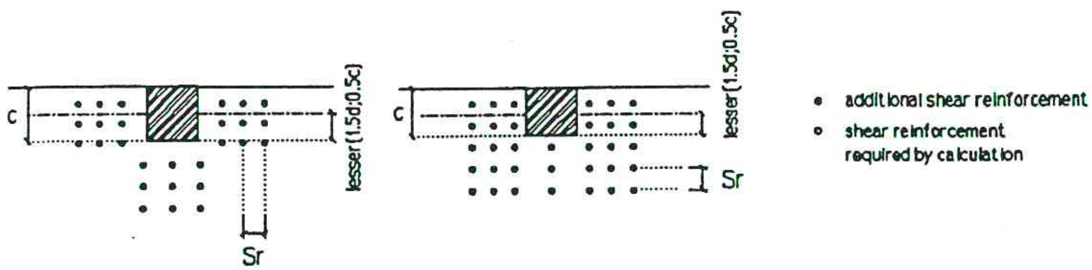
s_r = radial spacing of the layers of shear reinforcement, Fig.6.7.5

α = angle between shear reinforcement and the plane of the slab

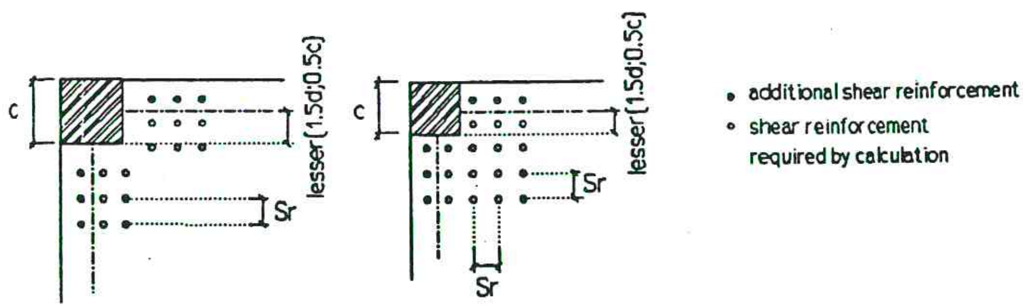
$f_{ywd} \leq 300$ MPa



a) Layout of shear reinforcement at interior columns (plan view)



b) Layout of shear reinforcement at edge columns (plan view)



c) Layout of shear reinforcement at corner columns (plan view)

Fig. 6.7.5 Layout of shear reinforcement for differnet cases

(4) A minimum shear reinforcement has to be provided as defined by the following equation:

$$A_{sw} / s_r = [\, 0.03 \;(100 \;\rho \; f_{ck})^{1/3} \, u_1 \,] \;/\; 1.5 \; f_{ywd} \; \sin\alpha \qquad (6.7.8)$$

(5) The shear reinforcement may consist of vertical or inclined bars or stirrups and must be arranged following the requirements defined in Fig. 6.7.6.

(6) At edge- and corner-column connections, the shear reinforcement required by calculation should be placed within the segments indicated in Fig. 6.7.5 b and c. Similar reinforcement at the same spacings should be provided in the areas between these segments and the slab edges, but should not be taken into account in calculations.

(7) Outside the area with the perimeter $u_{n,eff}$ as defined in Fig.6.7.7, where the shear reinforcement is required, the punching capacity must comply with eq. (6.7.3). In this case it may be assumed that the effect of unbalanced moment transmitted by shear has vanished.

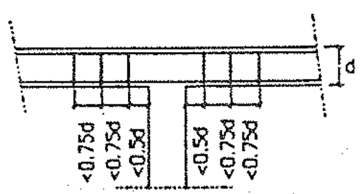


Figure 6.7.6 Layout of shear reinforcement (elevation)

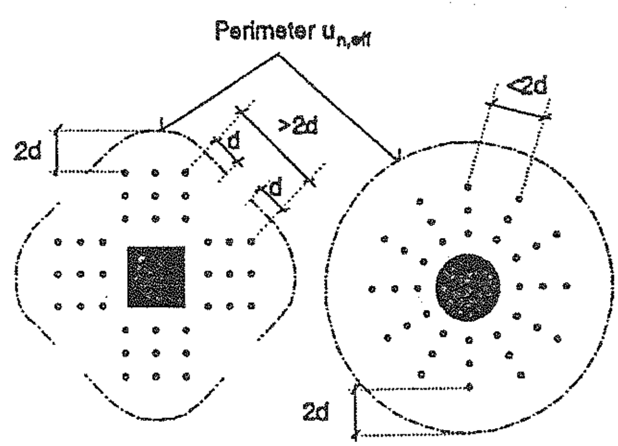


Figure 6.7.7 Definition of perimeter $u_{n,eff}$

6.8 Plate and shell elements

(1) A shell element is an element in the B-region of a slab, plate or shell, which is subjected to the combined action effects, i.e. axial forces and bending moments as well as in-plane and out-of-plane shear forces due to shear and torsion. The design model consists of two chords or outer layers connected by the web or inner layer in between. The outer layers represent plate elements subjected to in-plane normal and shear forces only, which result from the combined action effects. Generally, also the additional forces have to be considered, which result from the transfer of shear and torsion in the web due to the truss-action. The inner layer between these outer plate elements transmit the transverse shear forces as web between the chords.

(2) The distance between the outer layers, i.e. the inner lever arm z or the web height, is defined by the axes of the outer plate elements. The position of these axes may normally be assumed in the middle of the two reinforcing layers, unless large axial compressive forces act on the shell element, i.e. the compressive strength of the struts in the outer layers is exceeded. In the latter case $2h/3$ may be taken as an approximation.

(3) The design model for the plate elements under biaxial normal forces and shear forces consists of inclined struts equilibrated by ties in two orthogonal directions, normally. For biaxial tensile and compressive normal forces and for pure shear the angle of the struts may be assumed at 45° with respect to the directions of the reinforcement.

(4) For more refined considerations see MC 90 as well as the *CEB Bulletin for Bending and Compression*.

6.9 Fatigue

(1) Fatigue checks should be made for the effective fatugue action based on traffic measurements and by applying $\gamma_{Sd} = 1.00$.

(2) Stresses should be determined on basis of the elastic theory, taking into account cracking of concrete by modifying the stiffnesses accordingly.

(3) A verification of the fatigue strength under compression need not be carried out if the following condition holds (see Fig. 6.8.1):

$$\sigma_{c,max} / f_{cd} \leq 0.50 + 0.45\, \sigma_{c,min} / f_{cd} \leq 0.90 \qquad (6.9.1)$$

where: $\sigma_{c,max}$ = maximum compressive stress at fibre under frequent load combination

$\sigma_{c,min}$ = minimum compressive stress at the same fibre where $\sigma_{c,max}$ occurs, if $\sigma_{c,min} < 0$ (tension) then the condition $\sigma_{c,max} / f_{cd} \leq 0.50$ should be fulfilled

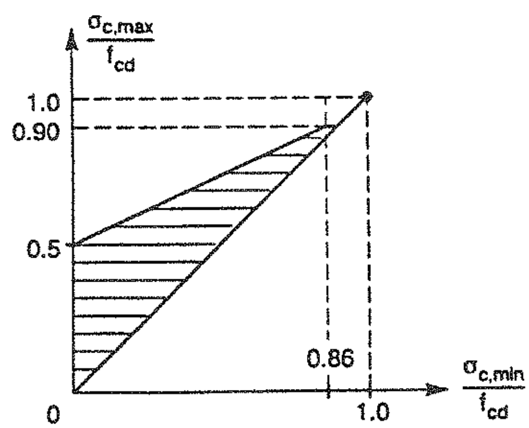


Fig. 6.9.1 Fatigue strength of concrete in compression

(4) For unwelded reinforcing bars and for prestressing steel subjected to tension, adequate fatigue resistance may be assumed, if under the frequent combination of actions the stress variation $\Delta\sigma_s$ does not exceed 70 N/mm^2.

(5) Slender and special structures may require special attention, and reference is made to MC 90, sect. 2.1.7.

7 Serviceability Limit State

7.1 Requirements

(1) It should be demonstrated that the structure and the structural elements perform adequately in normal use. To meet this requirements the serviceability limit states should be verified.

(2) Depending on the type and function of a structure or a structural element the verification of different serviceability limit states may be relevant, such as the limitation of

- stresses (see section 7.4),
- crack widths (see section 7.5),
- deformations (see section 7.6),
- vibrations (see section 7.7).

7.2 Actions and Action Effects

7.2.1 Permanent and variable actions

The permanent and variable actions are defined as for ULS (see sect. 6.2), but shall be applied with $\gamma_f = 1.0$.

Prestressing effects shall be considered with its mean value, as stated in section 3.4.2, and with a safety factor of $\gamma_p = 1.0$.

7.2.2 Load combinations

The combination of loads to be considered depends on the type of SLS and on the specific problem. It is suitable to utilize one of the combinations given in table 7.1, i.e:

- quasi-permanent combination,
- frequent combination,
- rare combination.

All direct and indirect actions such as loads or imposed or restrained deformations due to temperature effects, shrinkage, creep, changes of support conditions etc. should be considered.

The values in Table 7.1 are indicative, and whereever possible loads and corresponding ψ -values should be taken from national or international standards.

Table 7.1: Combination of actions for SLS

Quasi-permanent	$G + P$		$+ \Sigma\psi_2 Q_2$
Frequent	$G + P$	$+ \psi_1 Q_1$	$+ \Sigma\psi_2 Q_2$
Rare	$G + P$	$+ \quad Q_1$	$+ \Sigma\psi_1 Q_2$

with: Q_1 = basic variable action; Q_2 = other variable actions;

ψ_1 = coefficient for frequent value of an action

ψ_2 = coefficient for quasi-permanent value of an action

Action	ψ_1	ψ_2
Buildings		
dwellings	0.4	0.2
offices, retail stores	0.6	0.3
parking areas	0.7	0.6
Highway bridges*		
$l = 10$ m	0.7	0
$l = 100$ m	0.5	0
Wind **	0.2.- 0.5	0
Snow **	0.2 - 0.8	0 - 0.2
Temperature	0.5	0

* For intermediate values: linear interpolation

** Depending on geographic location

7.2.3 Material properties

The material properties shall be assumed with their mean value or their characteristic value depending on the particular application and the relevance of the behaviour. Partial safety factor γ_c shall be as given in section 2.1.

The following examples may be used as a guide line for which value may apply:

- deflections: mean value of elasticity, E_{cm}.
- onset of cracking for loads, crack widths: characteristic value of tensile strength, $f_{ctk,\,0,05}$.
- restraint forces in uncracked structures under imposed deformation: characteristic value of tensile strength, $f_{ctk,\,0,95}$.

7.3 Structural Analysis

7.3.1 Effective span

(1) Usually, the effective span l is equal to the distance between adjacent support axes.

(2) In case of wide support widths t the support axis may be assumed in a distance of $t / 3$ of the support face, but not more than $h / 2$ (with h = depth of the supported member), unless more refined considerations are made.

7.3.2 Effective width of flanges

(1) In the absence of more accurate methods, the effective width b_{eff} for compression flanges of beams with solid webs, and hollow box sections, may be taken as

$$b_{eff} = b_w + l_o / 5 \qquad \text{(T-beams)}$$

$$b_{eff} = b_w + l_o / 10 \qquad \text{(L-beams)} \qquad (7.1)$$

with: b_w = thickness of the web,
l_o =distance between points of zero moments.

(2) The effective width b_{eff} shall not be taken longer than the actual width of the flange see fig. 7.1. These effective widths may be assumed constant over the entire span, including the zones near intermediate supports.

(3) In general, for the effective width of a tension flange the same width may be assumed as for a compression flange.

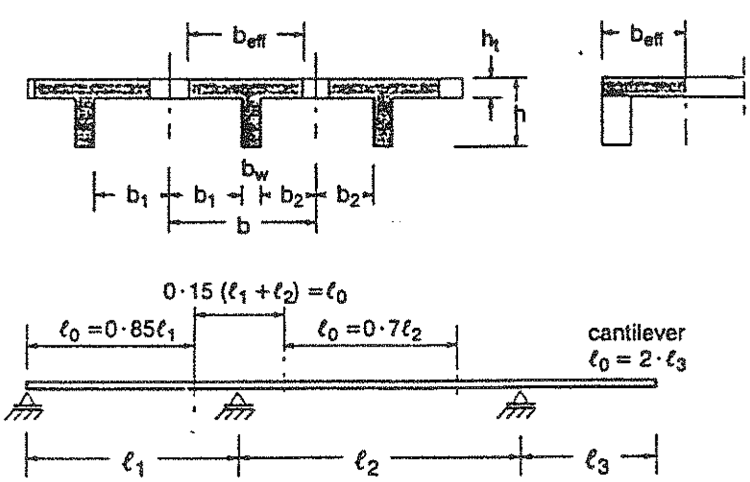


Fig. 7.1 Effective widths for the flanges of T-beams

7.3.3 Distribution of internal forces

(1) For SLS checks it is, in general, assumed that the structure as a whole behaves quasi-elastically: i.e. that the distribution of internal forces of hyperstatical systems may be calculated according to the theory of elasticity. Where relevant, non-elastic effects such as cracking and, in certain cases, creep and shrinkage are then accounted for by an appropriate reduction of stiffness.

(2) However, in many cases, it is sufficient to assume a plausible distribution of internal forces which satisfies the conditions of equilibrium and which, from experience, can be expected to differ little from the elastic one.

7.3.4 Redistribution of internal forces

(1) Relevant changes of statical systems due to different stages of construction should be considered. Owing to creep and concrete relaxation the final distribution of internal forces tends towards that of the one-mass system, as if the structure had been entirely cast in one operation.

(2) Detailed step-by-step calculations could be carried our in accordance with the methods given in the *CEB Manual on structural effects of time-dependent behaviour of concrete.* In most cases the associated changes in the distribution of action effects can be approximated by

$$S_\infty = S_o + (S_e - S_o) \frac{\phi}{1 + 0.8\,\phi} \qquad (7.2)$$

with: S_∞ = final action effect after redistribution,

S_o = initial action effect at the construction stage,

S_e = action effect for the one-mass system,

ϕ = creep coefficient.

Both, S_o and S_e, shall include the effects of prestressing forces. Fig. 7.2 illustrates the above equation.

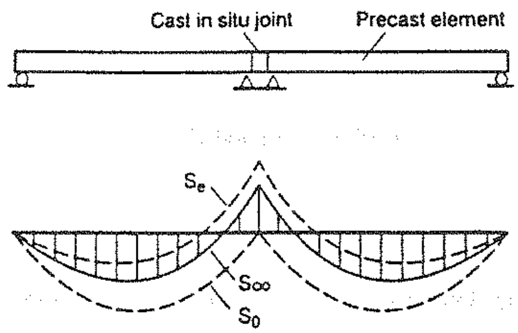


Fig. 7.2: Redistribution of internal forces for a beam on three supports

7.4 Stress Limitations

7.4.1 General and cases where stress limitations are not essential

(1) Under service load conditions the limitation of stresses may be required for

- tensiles stresses in concrete
- compressive stresses in concrete
- tensile stresses in steel.

(2) The stress limitations given in subsections 7.4.3 and 7.4.4 below may generally be assumed to be satisfied without further calculations provided the minimum reinforcement provisions of subsection 7.5.5 are satisfied.

7.4.2 Concrete in tension

Where assessing cracking according to section 2.1.4, the tensile strength $f_{ctk,\,0,05}$ should be used. In webs with shear and torsion, the principal tensile stress should be used to asses cracking.

7.4.3 Concrete in compression

(1) Excessive compressive stresses in the concrete under service load may lead to longitudinal cracks and high and hardly predictable creep, with serious consequences to prestressing losses. When such effects are likely to occur, measures should be taken to limit the stresses.

(2) If under the quasi-permanent load combination the stress exceeds $0.45\ f_{cm}(t)$ a non-linear model should be used for the assessment of creep (see MC 90, 2.1.6.4.3(d)).

7.4.4 Steel

(1) Tensile stresses in the steel under serviceability conditions should be limited to

$$\sigma_s \leq 0{,}8\ f_{yk} \tag{7.3}$$

This is to avoid inelastic deformation of the steel since this would load to large, permanently open cracks.

(2) For more stringent crack control it may be neccessary to restrict the stress level in the reinforcing steel, and the increase in the stress after decompression in the prestressing steel; further, see sect. 7.5.4.

7.5 Crack Control

7.5.1 Requirements

(1) It should be ensured that, with an adequate probability, cracks will not impair the function requirements, the durability, and the appearance of the structure.

(2) Cracks do not, per se, indicate a lack of serviceability or durability of a concrete structure. Cracks due to tension, bending, shear and torsion are often inevitable in reinforced structures, resulting from either direct loading or from restraint of imposed deformations.

(3) Thus, the designer should specify, in agreement with the client, the relevant criteria to be fulfilled for the finished structure and for intermediate construction phases. Such criteria may be in terms of limitation of tensile stresses or of crack widths.

(4) Crack width limitation may be verified either by calculation of crack widths or by appropriate detailing. In cases where the ULS design leads to low reinforcement ratios, minimum reinforcement may have to be provided.

(5) Due to many uncertainties in the assumptions, actual crack widths in the structure may be larger than the ones assumed in the design.

7.5.2 Crack width limits

7.5.2.1 Reinforced concrete members

(1) For reinforced members, and in the absence of specific equirements (e.g. watertightness), it may be assumed that for exposure classes 2 to 4 a crack width limit w_{lim} equal to 0.30 mm under the quasi-permanent combination of actions is satisfactory.

(2) For exposure class 1, this limit may be relaxed provided that it is not neccessary for reasons other than durability.

(3) When de-icing agent are expected to be used on top of tensioned zones, appropriate values should be specified in accordance with the client, depending on the thickness and quality of the concrete, and of additional protective layers.

7.5.2.2 Prestressed concrete members

(1) For prestressed members if more detailed data are not available, the crack width limiting values presented in Table 7.2 may be used.

(2) For corrosion protected tendons the crack width limits of reinforced concrete members shall apply. Corrosion protected tendons shall mean multistrand tendons encapsulated in a thick-walled plastic tube, or monostrand tendons protected with grease and extruded sheathing, or equivalent systems.

Table 7.2 Crack width limits

Exposure class	w_{lim} under the frequent load combination post-tensioned	pre-tensioned
1	0.20	0.20
2	0.20	No tension within c_{nom} distance from tendon is allowed
3 and 4	(a) No tension allowed within c_{nom} of prestressing steel; b) if tension is accepted, impermeable ducts or coating of the tendons should be applied; in this case w_{lim} = 0.20	

7.5.3 Calculation of crack widths

7.5.3.1 Introduction

(1) The following inequality should be observed

$$w_k \leq w_{lim} \tag{7.4}$$

where: w_k denotes the characteristic crack width calculated under the appropriate combination of actions.

w_{lim} denotes the nominal limit value of crack width which is specified for cases of expected functional consequences of cracking or for cases related to durability.

(2) The formation, propagation and width of cracks depend on a great number of parameters, some of which (e.g. casting and curing procedure, climatic conditions, temperature, etc.) are not known at the design stage. Elaborate crack-width-calculations are thus only warranted in special cases and if the relevant parameters can be reliably predicted.

7.5.3.2 Basic crack width formula

(1) For all stages of cracking, the design crack width may be calculated according to

$$w_k = s_r \ \varepsilon_{sm} \tag{7.5}$$

where: s_r = average crack spacing
ε_{sm} = average steel strain

The crack spacing and average steel strain may be calculated for a concrete tie around a reinforcing ribbed bar as follows:

$$s_r = 2\,c + \alpha_b \ Ø / \rho \tag{7.6 a}$$

$$\varepsilon_{sm} = \varepsilon_s - 0.40 \ \varepsilon_{sr1} \tag{7.6 b}$$

where: c = concrete cover ; $Ø$ = bar diameter
$\rho = A_s / A_{c,eff}$
$A_{c,eff}$ = effective concrete area defined in Fig. 7.6
α_b = 0.125 for good bond; α_b = 0.200 for other areas
$\varepsilon_{sr1} = f_{ct,min} / \rho \ E_s$

(2) For more details see MC 90, 7.4.3.

7.5.4 Crack control by detailing

(1) Applying sect. 7.5.3 with some simplifying assumptions, and assuming crack widths of 0.30 mm for reinforced concrete members, and 0.20 mm for prestressed concrete members, the following simplified rules for detailing are obtained.

(2) For cracking caused mainly by restraint, crack widths will not generally be excessive provided that the bar sizes given in Fig. 7.3 are not exceeded. Thereby the σ_s-value of Fig. 7.3 is that calculated at cracking of the element.

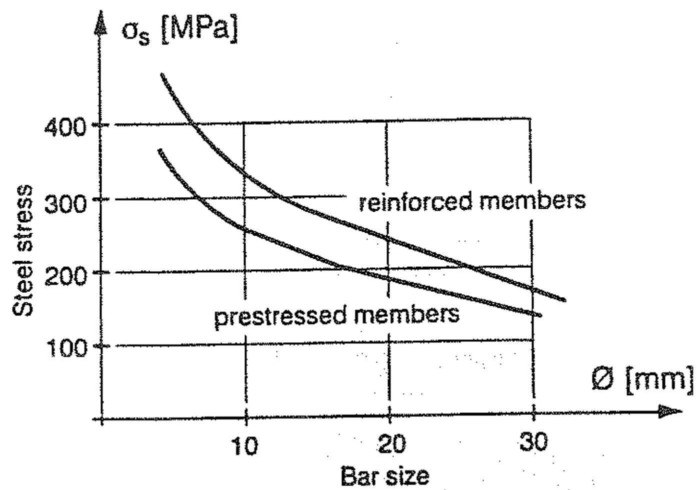


Fig. 7.3: Maximum bar diameter

(3) For cracks caused mainly by loads, crack widths will not generally be excessive provided either the provisions of Fig. 7.3 or those of Fig. 7.4 are satisfied.

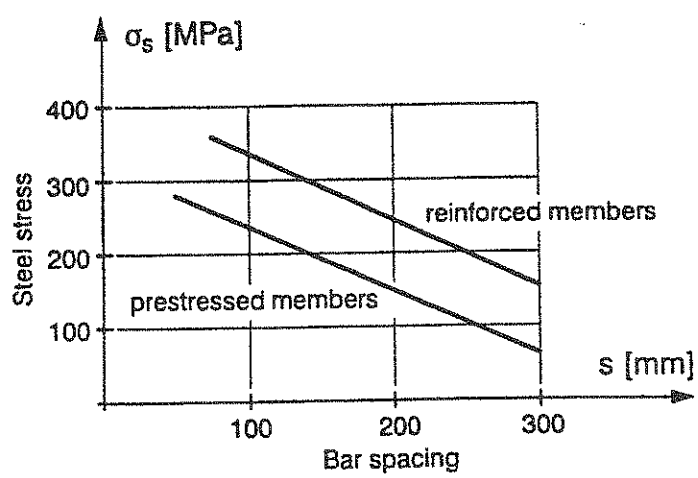


Fig. 7.4: Maximum bar spacing

7.5.5 Minimum reinforcements

(1) In every area where under SLS conditions the tensile strength of concrete may be exceeded, a minimum amount of reinforcement should be provided to assure a predictable behaviour of the member.

(2) For the combination of pure tension and flexure, and in the absence of more rigorous methods, a minimum amount of reinforcement, $A_{s,min}$, should be provided within tensioned concrete zones of all load-bearing members

$$A_{s,min} = \rho_{r,min}\ A_{c,eff}$$

where: $\rho_{r,min}$ = minimum percentage acc. to Fig. 7.5

$A_{c,eff}$ = efffective concrete area as defined in Fig. 7.6

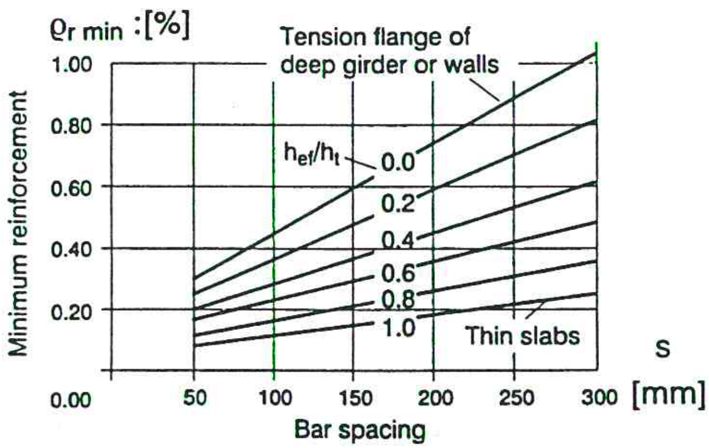


Fig. 7.5 Minimum percentage $\rho_{r,min}$
(the graph is valid for f_{ctm} = 2.9 MPa and f_{yk} = 460 MPa. Other values may be extrapolated with $\rho_{r,min} \times (f_{ctm}/2.9) \times (460/f_{yk})$

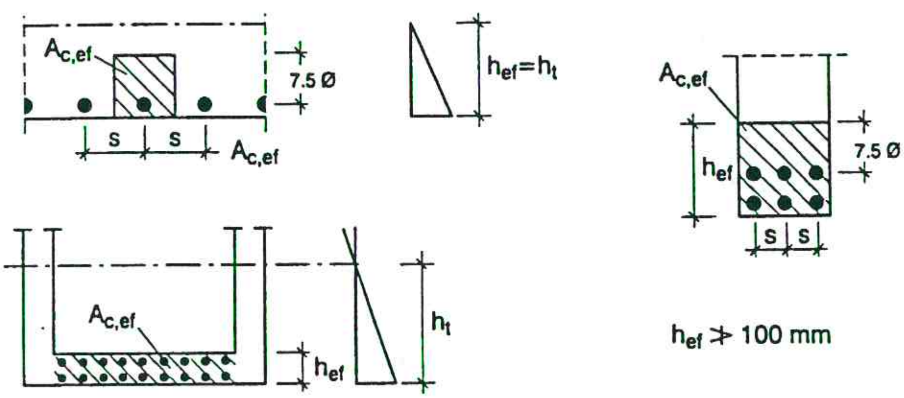


Fig. 7.6 Effective concrete area for minimum reinforcements

(3) In tension zones with large diameter bars or with bundles of bars requiring large concrete cover, a skin reinforcement according to Fig. 7.7 is required for adequate crack control. This reinforcement may be taken into account for the flexural and shear design, if appropriately detailed.

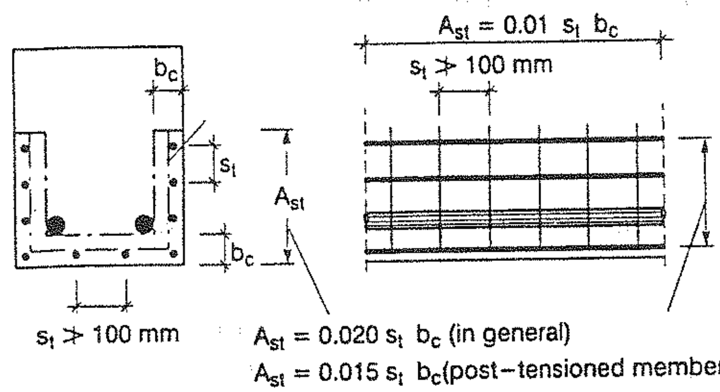


Fig. 7.7 Skin reinforcement in case of large concrete covers

(4) In prestressed members or in reinforced concrete members subject to compressive normal force, the minimum reinforcement area may be reduced below that neccessary for ordinary reinforced concrete due to the influence of

- the increased flexural stiffness fo the compression zone,
- the contribution of the prestressing tendons,
- the effect of prestress or compressive normal force contributing to crack width limitation of single cracks.

(5) In prestressed members, the minimum reinforcement for crack control is not neccessary in areas where, under the rare combination of actions the concrete remains in compression.

(6) Prestressing tendons may be taken into account as minimum reinforcement within a 300 mm square surrounding the tendon, provided that the different bond behaviour of the tendons and reinforcment are considered. In the absence of better information, the prestressing tendons may be assumed 50 % effective.

7.5.6 Crack control for D-regions

7.5.6.1 Definition of the model

A D-region may be modelled with a strut-and-tie model for the verification of cracking at the Serviceabilty Limit State. The model should be orientated by the stress fields determined from a linear-elastic analysis.

7.5.6.2 Crack control

For the verification of the crack control the requirements of sect. 7.5.1 apply.

The verification of crack control is carried out by checking the crack width of the tie with the maximum force. The area of the tie is defined in Fig. 7.6 according to MC 90. Based on the area and stress in the tie, the verification can be carried out in accordance with sect. 7.5.4.

7.6 Deformations

7.6.1 Requirements

(1) In service deformations (deflections and rotations) may be harmful to

- the appearance of the structure,
- the integrity of non-structural parts,
- the proper function of the structure or its equipment.

To avoid harmful effects of deformations appropriate limiting values should be respected.

(2) Deformation limits should normally be agreed between the designer and the client. In the absence of such agreements, the following values can be used as indications:

(a) total deflection below level of supports under quasi-permanent loads
span / 200 to span / 300

(b) deflection that occurs after addition of partitions
span / 500 to span / 1000

More detailed guidance is given in ISO 4356.

7.6.2 Means of limiting deformations

(1) Under certain conditions, the checking of deformations by calculations may not be necassary. Conditions generally contributing to reduction of deformations are:

a) Prestressing, even to a low degree,

b) Using high-strength concrete,

c) Careful curing of the concrete,

d) Removing the falsework as late as possible or else supporting the structure by temporary props,

e) Proper dimensioning and detailing of the reinforcement,

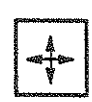
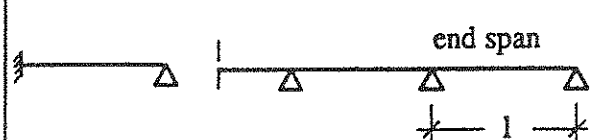
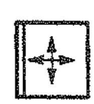
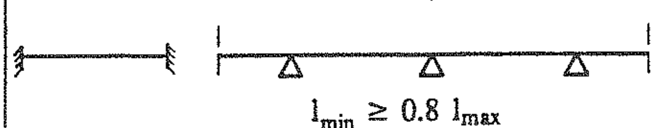
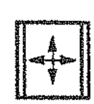
f) Avoiding high span/depth ratios $(\alpha l) / h$,

g) Compensating by initial camber.

(2) For case f) the α–values of different structural systems which yield approximately the same deflections are given in Table 7.3.

Deflections rarely become critical for spans smaller than 5 m or when the $(\alpha l) / h$ - values do not exceed 25 for beams or 30 for slabs.

Table 7.3: Values for α (ratio of notional and actual spans)

Beams	Slabs	α
		1.0
		0.8
		0.6
		2.4

(3) Special attention should be given to the support conditions (continuity) in hyperstatic structures: a small reduction in end restraint due to cracking at the supports may lead to a considerable increase in deflection in the span.

7.6.3 Deformations due to bending

(1) For building members, long-term deflections can be evaluated by the following relations based on a bilinear relationship between load and deflection:

$$a = (1 + f)\, a_e \qquad \text{for } M_d < M_r$$

$$a = (h / d)^3\, \eta \; (1 - 20\, \rho_{cm}) \cdot a_e \qquad \text{for } M_d \geq M_r \qquad (7.7)$$

with: M_r = cracking moment assessed according to sect.2.1.4 (4) and based on $f_{ct,m}$

a_e = elastic deflection calculated with the rigidity $E_c\, I_c$ of the cross-section (neglecting the reinforcement)

M_d = bending moment at mid-span of a beam or a slab, or at the fixed end of a cantilever under frequent actions

ρ_{tm} = geometrical mean ratio of tensile reinforcement

ρ_{cm} = geometrical mean ratio of compressive reinforcement

η = correction factor (see Table 7.4), which includes the effects of cracking and creep

ϕ = creep coefficient

Table 7.4: Correction factor η for estimate of deflections

ρ_{tm} (%)	0.15	0.2	0.3	0.5	0.75	1.0	1.5
η	10	8	6	4	3	2.5	2

(2) The mean percentage ρ_{tm} of tensile reinforcement is determinded according to the bending moment diagram (see Fig. 7.5)

$$\rho_{tm} = \rho_a \cdot \frac{l_a}{l} + \rho \frac{l_o}{l} + \rho_b \frac{l_b}{l} \tag{7.8}$$

where

ρ_a, ρ_b = percentages of tensile/compressive reinforcement at the left and rigth supports, respectively,

ρ = percentage of tensile reinforcement at the M_{max}-section

An estimate of the lengths l_a and l_b is generally sufficient.

(3) For other types of deformations, reference is made to the MC 90.

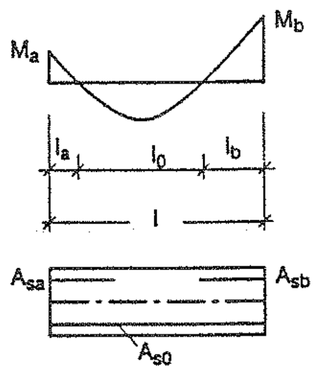


Fig. 7.5: Bending moment diagram defining l_o, l_a and l_b

7.7.3 Deformation control of D-regions

(1) For the verification of the deformationcontrol the requirements of sect. 7.6.1.1 apply.

(2) The deformations of the struts may be assessed with an average area over the entire length, because the struts are stiffer than the ties and therefore mostly have not much influence on the deformations of the D-region.

(3) The deformations of the ties should be calculated considering the tension stiffening effect provided by the of concrete between the cracks. This effect may be calculated according to section 3.2 of MC 90, and in particular the expressions given in sect. 3.2.3.

7.7 Vibrations

7.7.1 General

Vibrations of structures may affect the serviceability of a structure as follows:

- functional effects (discomfort to occupants, affecting operation of machines, etc.),
- structural effects (mostly on non-structural elementes as cracks inpartition loss of cladding etc.).

7.7.2 Vibrational behaviour

(1) To secure satisfactory behaviour of a structure subject to vibrations, the fundamental natural frequency of vibration of the relevant structures should be kept sufficiently apart from critical values ($f > f_{crit}$), which depend on the function of the corresponding building, see Table 7.5.

(2) The vibrational behaviour of structures can be influenced by the following measures:

- changing the dynamic actions,
- changing the natural frequencies by changing the rigidity of the structure or the vibrating mass,
- increasing the damping features, etc.

Table 7.5 Recommended lower bounds of the fundamental frequency f_{crit} of floors

Structures	**Frequency f_{crit} [Hz]**
Gymnasia and sports halls	8.0
Dance rooms and concert halls without permanent seating	7.0
Concert halls with permanent seating	3.4
Structures for pedestrians and cyclists	see below*

* Natural frequencies between 1.6 and 2.4 Hz and between 3.5 and 4.5 Hz are to be avoided in structures for pedestrians and cyclists. Joggers can also cause vibrations in structures with natural frequencies between 2.4 and 3.5 Hz.

(3) Calculations of natural frequencies should always be carried out with careful thought being given to the structural contribution of floor finish, the dynamic modulus of elasticity, and the cracking including tension stiffening effect of concrete between cracks. It is advisable to carry out sensitivity analyses by varying these parameters.

8. Structural Members

8.1 General

(1) Detailing of reinforcement should in general follow from the design model adopted. The axes of the reinforcement should coincide with the axes of the corresponding ties in the model. Particular care should be given to the reinforcement anchorages at the nodes. Some complementary rules for some structural elements are given in this section.

(2) A minimum amount of reinforcement shall be provided to ensure a proper behaviour of the members under the effects of all actions. Special attention should be given to restraints respectively imposed deformations, which are not explicitly considered in the analysis.

(3) It should be remembered that prestressing is a very effective way to counteract applied loads. Very important loads (at the scale of the structure), should in principle be balanced by prestressing, improving serviceability and detailing.

8.2 Beams

8.2.1 Longitudinal reinforcement

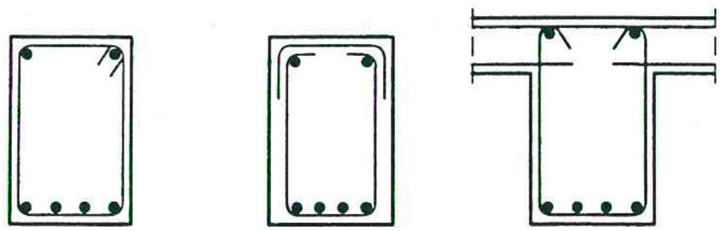
(1) The minimum area of the tensile reinforcement is defined in section 7.5.5.

(2) The area of the tensile reinforcement and the area of the compressive reinforcement should not exceed 0.04 A_c. One-third of the maximum reinforcement needed in the span should be extended to the end supports, and one-quarter to the intermediate supports. Continuity of bottom reinforcement is recommended to resist accidental positive moments.

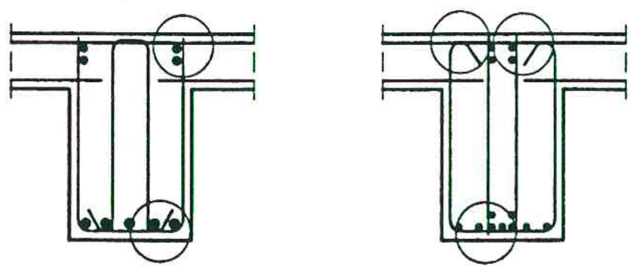
(3) In flanged cross-sections, at least 50% of the logitudinal reinforcement should be located in the flange-web connection.

8.2.2 Transverse reinforcement

(1) Transverse reinforcement should form an angle of 45° to 90° with the axis of the beam. In most cases it consists of vertical stirrups (see Fig. 8.1 a), well anchored by the anchor elements given in sect. 5.6.4 (5), see Fig. 5.12. It may also consist of a combination of stirrups and high bond bars shear assemblies (Fig. 8.1 b).



a) possible layout of stirrups



b) examples for combinations of stirrups and shear assemblies

Fig. 8.1 Possible layout of transverse reinforcement

(2) A minimum transverse reinforcement should be provided, the amount of which should correspond to a mechanical ratio of transverse reinforcement of 0.20, i.e.:

$$A_{sw,min} = 0.20 \; b_w \, s_w \; \sin\alpha \;\; f_{ctm} \, / \, f_{yk} \qquad (8.1)$$

where s_w = spacing of stirrups measured along member axis

(3) The maximum spacing of the stirrup legss either in the longitudinal or in the transverse direction shall not exceed the values given in section 6.4.3.

8.2.3 Torsional Reinforcement

The transverse reinforcement in members subjected to torsion should be detailed as shown in Fig. 8.1 a.

At least one longitudinal bar should be placed in each corner of the stirrup. The other bars should be uniformly distributed along the internal perimeter of the stirrups, at a spacing not exceeding 350mm.

8.3 Columns

8.3.1 Longitudinal Reinforcement

(1) The area of longitudinal reinforcement should normally not be less than $0.008\ A_c$ and not be more than $0.04\ A_c$. In lapped joints the area of reinforcement should not exceed $0.08\ A_c$.

(2) The minimum number of longitudinal bars should be four for rectangular columns and six for circular columns. The diameter of the bars should not be less than 12 mm.

8.3.2 Transverse Reinforcement

(1) The diameter of the transverse reinforcement should not be less than 6 mm or one-quarter of the diameter of the largest longitudinal bars. Their spacing should not exceed the lesser of the following values:

- 12 - times the minimum diameter of the longitudinal bars;
- the least lateral dimension of the column;
- 300 mm.

(2) The transverse reinforcement should be detailed such that each bar or each group of bars placed in a corner is held by transverse reinforcement (Fig. 8.2); the same principle applies to every two intermediate bars of the outer layer of reinforcement. No bar that is not held shall be located at a distance of more than 150 mm away from a bar that is held.

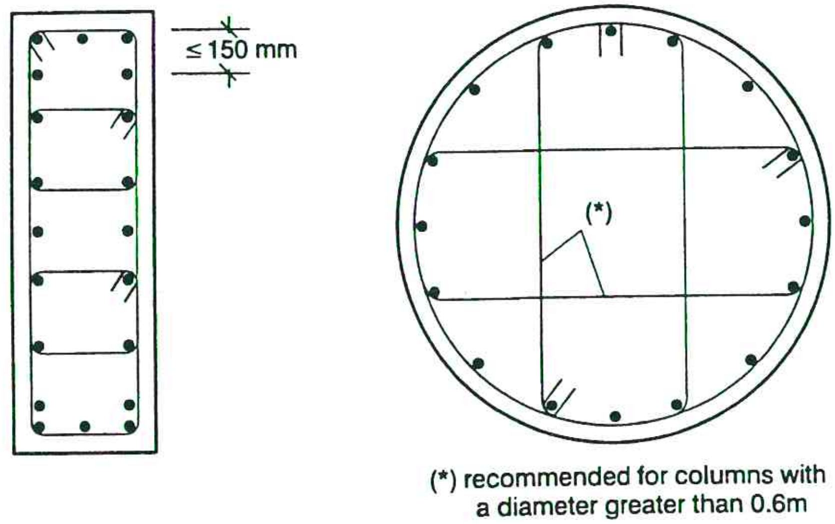


Fig. 8.2 Examples of transverse reinforcement in columns

(3) In general all transverse reinforcement should be appropriately anchored by hooks.

(4) Local effects in column D-regions (beam-column connections, cross section variations) should be studied according to the chapter 6.5. In general it is recommended to decrease the spacing of transverse reinforcement, e.g. by a factor of about 0.6, in the regions located above and below a beam or slab connection, as illustrated in Fig. 8.3.

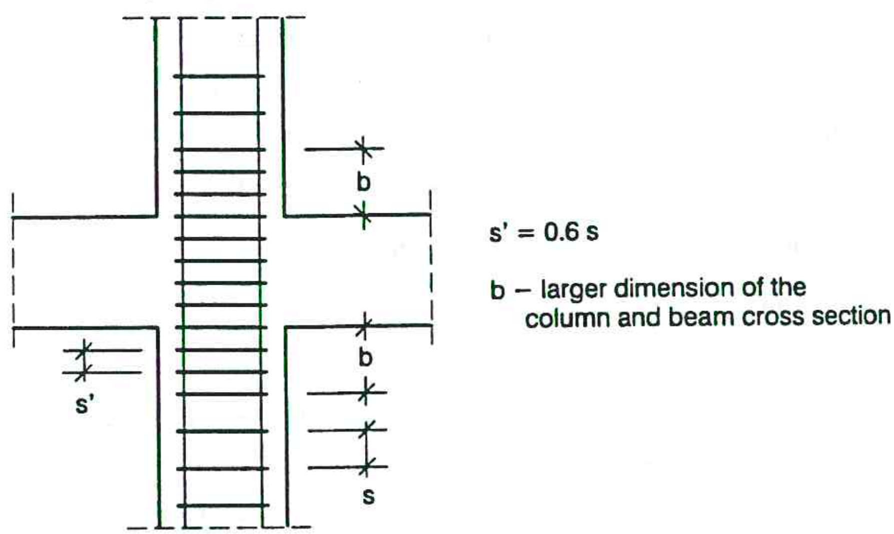


Fig. 8.3 Illustration of transverse reinforcement detailing for a beam-column connection

8.4 Slabs

8.4.1 Flexural Reinforcement

(1) The minimum reinforcement is given in section 7.5.5. One-half of the reinforcement needed in the span should be extended to the end supports, one-third to intermediate supports.

(2) The ratio of secondary to main reinforcement areas should be at least equal to 0.2 at any point. For high concentrated loads this minimum ratio should be increased to 0.33.

(3) The lesser of the following values for the maximum spacing of the bars are recommended:

- for main reinforcement: $s_{max} < 1.2\ h$ or $< 350mm$
- for secondary reinforcement $s_{max} < 2.0\ h$ or $< 350mm$

(4) If the corner of a slab formed by two simple supported edges is prevented from lifting and such restraint is not taken into account in the analysis, then top and bottom reinforcement capable of resisting a moment at least equal to the value of the maximum moment in the span should be provided at the corner. For a corner with one edge simply supported and the other restrained, this reinforcement should be capable to resist a moment equal to at least one-quarter of the maximum moment in the span. The corner reinforcement should extend from the face of the support up to a distance at least equal to 0.2-times the smaller span.

If the edge of a slab is partially restrained and this restraint has not been considered in the analysis a minimum top reinforcement should be provided according to sect. 7.5.5. This reinforcement should extend from the face of the support up to a distance at least equal to 0.2 times the corresponding span.

(5) Particular attention should be given to free edges (see Fig. 8.4). At a free edge the slab should contain:

- longitudinal reinforcement running parallel to the edge and consisting of at least two bars, one in top corner and the other in the bottom corner;

- transverse U-shaped reinforcement running perpendicular to the edge and enclosing the longitudinal reinforcement with the free ends extending up to a distance of at least 2 h from the edge.

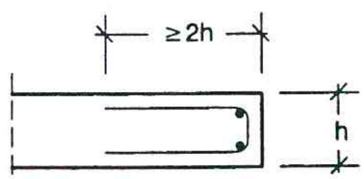


Fig. 8.4 Reinforcement along a free edge of a slab

8.4.2 Shear reinforcement

(1) Shear reinforcement should be provided in zones where $V_{Sd} > V_{rd}$. This reinforcement should contain stirrups with the same minimum amount as defined for beams in sect. 8.2.2. The stirrups should enclose at least 50% of the longitudinal reinforcement at the bottom and top. In general, the shear reinforcement should be inclined between 45° and 90° to the middle plane of the slab.

(2) The distance between the face of a support and the first layer of shear reinforcement should not exceed 0.5d. The transverse spacing of bars in the same layer should not exceed 1.5d or 400 mm, whichever is less.

(3) The requirements for punching-shear reinforcement are given in the section 6.5.3.4.

8.5 Walls

8.5.1 Vertical Reinforcement

The area of the vertical reinforcement should lie between 0.004 A_c and 0.04 A_c; generally half of this reinforcement should be located at each face. The distance between two adjacent vertical bars should not exceed twice the wall thickness or 300 mm, whichever is less.

8.5.2 Horizontal Reinforcement

Horizontal reinforcement running parallel to the faces of the wall should be provided at each surface with minimum area of 30% of that of the vertical reinforcement. The spacing should not be greater than 300 mm and the diameter not be less than one-quarter of vertical bars. If the area of vertical reinforcement exceeds 0.02 A_c. then the clause in sect. 8.3.2 applies.

8.6 Deep Beams

(1) The requirements and criteria for modelling of deep beams are given in the section 6.9.3.

(2) The main longitudinal reinforcement corresponding to the bottom tie in the design model should be distributed over a depth of about 0.12 h or 0.12 l from the lower face of the beam, whichever is less (see Fig. 8.5). This reinforcement should extended from one support to the other and should be thoroughly appropriate anchored at the supports, e.g. by horizontal hooks or loops, or best by means of anchorage plates.

(3) A mesh of orthogonal reinforcement with minimum area of 0.1% in each direction should be provided at each face.

(4) In case of suspended loading additional stirrups shall be provided to hang-up these loads up to a level of h or l (see Fig. 8.6).

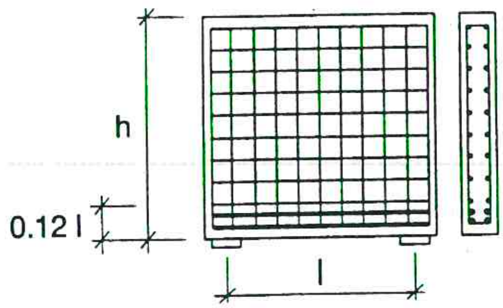


Fig. 8.5 Distribution of longitudinal reinforcement in a deep beam

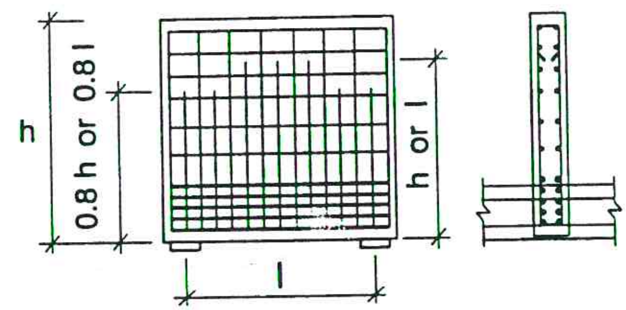


Fig. 8.6 Recommended reinforcement layout for a deep beam with suspended loading (schematic)

APPENDIX 1. CHARACTERISTIC VALUES OF VARIABLE ACTIONS

This short appendix on variable actions certainly does not constitute a complete load specification; such an endeavour would indeed go well beyond the scope and the competency of these Recommendations on practical design. There are other national and international bodies dealing with the question of adequate load assumptions.

However, it seems somewhat futile to specify combinations and partial safety coefficients for actions, without also specifying the actions. Thus, the purpose of this appendix is to indicate the order of magnitude of the loads which served as guidelines for the present Recommendations

For variable actions of natural origin, such as wind, temperature, snow, earthquake, etc., local conditions are a dominant factor; therefore no generally valid regulations can be proposed.

A 1. Highway bridges: live loads

Two types of loading should be considered, as follows.

H1 is a normal traffic load representation that includes three categories of loading to cater for different local conditions and requirements.

H2 is an abnormal vehicular load requirement, the details of which are the prerogative of the appropriate transport authority.

A 1.1 Type H1: normal traffic loading

Type H1 loading consists of a uniformly distributed load (see section A 1.1.1) and an additional concentrated load (see section A 1.1.2). A longitudinal force due to traction or braking of vehicles is also to be considered (see section A 1.1.3).

The vertical loading includes an allowance for dynamic effects. In cases where vibrations need to bc investigated explicitly, the loading will require separate consideration.

A 1.1.1 Uniformly distributed load (UDL)

The UDL should be taken from Fig. A1, according to the loading category. The UDL consists of joined sections of lane with constant intensity of loading.

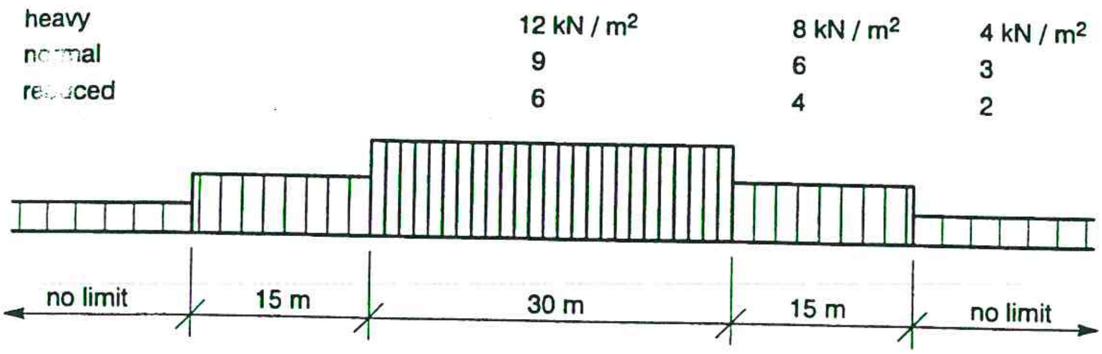


Fig. A1 UDL for type H1 loading in longitudinal direction

For multi-lane bridges, the maximum lane loads to be considered as acting simultaneously are given in Fig. A2 as percentages of the lane loading obtained from Fig. A1. Longitudinal and transverse effects should be determined for the most unfavourable arrangements of lane loadings.

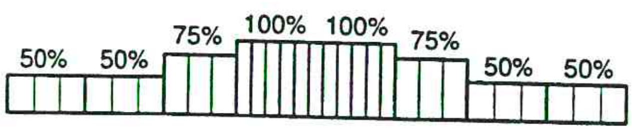


Fig. A2 Multi-lane loading percentages in transverse direction

A 1.1.2 Concentrated load

In addition to the UDL, a single concentrated load should be considered, acting in any position and distributed over either a square contact area of side 500 mm or a circular contact area of diameter 550 mm. The load should be taken, according to the loading category, as 400 kN (heavy), 300 kN (normal) or 200 kN (reduced).

A 1.1.3 Longitudinal force

A longitudinal force resulting from traction or braking of vehicles should be considered, acting at the road surface and parallel to it on one lane only. The force should be taken, according to the loading category, als 400 kN (heavy), 300 kN (normal) or 200 kN (reduced).

A 1.1.4 Footway and cycle track load

The UDL appropriate to the reduced loading category should be taken from Fig. A1. Special consideration should be given for loaded lengths in excess of 30 m where exceptional crowds may be expected (as, for example, where a footbridge serves a sports stadium).

Where the footway or cycle track is not protected from higway traffic by an effective barrier, the concentrated load appropriate to the reduced loading category should also be considered.

A 1.2 Type H2: abnormal vehicle loading

Where the controlling local authority requires the bridge to be designed for an abnormal vehicle loading, the loading specification should be obtained from the appropriate transport authority.

A 2 Buildings: imposed floor loads

The UDL to be considered for floor slabs and their supporting members should be taken from Table A1, according to the building category or function. This load provides for the normal effects of impact and vibration.

The concentrated load given in Table A1 should also be considered for floor slabs. A single load should be considered, acting alone at any position and distributed over a square contact area of side 300 mm.

Table A1 Imposed floor loads in buildings

Building category or function	UDL, kN/m²	Concentrated load, kN
Apartment houses, private rooms	1·5	2·0
corridors	4·0	5·0
Hotels, guest rooms	2·0	2·0
corridors	4·0	5·0
Schools, classrooms	3·0	3·0
corridors	4·0	5·0
Office buildings, offices	2·5	5·0
filing and storage areas	5·0	5·0
computing etc. areas	3·5	5·0
Hospitals, private rooms	2·0	2·0
Assembly halls, fixed seats	4·0	—
movable seats	5·0	4·0
Stairs, private buildings	1·5–3·0	2·0–3·0
public buildings	5·0	4·0
Restaurants, dining rooms	2·0	3·0
Libraries, stock rooms (4.8 kN m⁻² m⁻¹)	9·6 min.	7·0
Garages, passenger cars only	2·5	9·0
Grandstands, stadia	5·0	4·0
Dance halls (without increase due to resonance)	5·0	4·0
Shops	4·0	4.0
Storage warehouses, light (2.4 kN m⁻² m⁻¹)	6·0 min.	7·0
Storage warehouses, heavy (4.0 kN m⁻² m⁻¹)	12·0 min.	9·0
Terraces, vehicular access	5·0	9·0
pedestrian access	4·0	5·0

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