University of Texas
Dept. of Civil, Arch. & Env. Eng.

Instructor: E. B. Williamson
CE 397

CE 397 Blast-Resistant Structural Design
Spring 2010

Course Purpose:

CE 397 focuses on designing structures to resist blast loads. This class builds upon fundamental concepts covered in courses on structural dynamics, structural analysis, reinforced concrete design, and steel design.

Course Objectives:

By the end of the course, you should be able to do the following:

- Develop mathematical expressions for the variation in load as a function of time for a given blast scenario.
- Compute the dynamic response of blast-loaded structural components, accounting for nonlinear effects.
- Size reinforced concrete and steel structural components to achieve a prescribed level of performance for a given blast scenario.
- Design blast-loaded structures for resistance to progressive collapse.

Topics:

- History and Overview of Blast-Resistant Structural Design
- Identification of Threats
- Computation of Blast Loads
- Blast Effects against Structural Components
- Structural Analysis Techniques for Blast-Loaded Structures
- Site Planning and Layout
- Design of Structures to Resist Blast Effects
- Progressive Collapse
- Special Interest Topics
  - Predicting Human Injury
  - Design of Blast Doors and Windows
  - Retrofitting of Existing Structures

Text:

There is no required text for the course. The instructor will provide handouts throughout the semester to address the topics covered in class.

References and Supplemental Reading

* ASCE Task Committee on Blast Resistant Design. *Design of Blast Resistant Buildings in Petrochemical Facilities*. ASCE, Reston, VA, 1997. — $30

Bulson, P. S. *Explosive Loading of Engineering Structures*. E & FN Spon, London, 1997.

Conrath, E. J., Krauthammer, T., Marchand, K. A., and Mlakar, P. F. *Structural Design for Physical Security: State of the Practice*. ASCE, Reston, VA, 1999. — $30

Krauthammer, T. *Modern Protective Structures*. CRC Press, Boca Raton, FL, 2008. – Univ. of Florida, >$100

Structures to Resist the Effects of Accidental Explosions. Unified Facilities Criteria (UFC) 3-340-02. U.S. Department of Defense, Washington, D.C., 2008. - free download

Office Hours:

Tu/Th 11:00 - 12:00
Office: ECJ 4.722
Phone: 475-6175
email: ewilliamson@mail.utexas.edu
**Note:* I maintain an "open door" policy outside of regularly scheduled office hours. If the door to my office is open, please feel free to stop in.

Prerequisites:

Students enrolled in CE 397 are expected to have taken (or be currently enrolled) in a course on structural dynamics, be able to carry out basic design of steel and reinforced concrete members, be familiar with basic concepts of nonlinear structural behavior, and be comfortable with computer applications such as SAP, Excel, etc.

Conduct of Course:

The course consists primarily of lectures, homework assignments, and student projects. Attendance is essential. Homework assignments are subject to the due dates stated when distributed. Late work will not be accepted. Grades will be computed using the following distribution: Homework (50%), term project (including final report and presentation) (50%).

Course Evaluation:

The students will evaluate the course and the instructor on forms provided by the Measurement and Evaluation Center.

Course Drop Dates:

From the 1st through the 4th class day, graduate students can drop or add a course on Rose or TEX. Beginning with the 5th class day, graduate students must initiate any adds or drops in their department. Graduate students can drop a class until the last class day with permission from the departmental Graduate Advisor and the Dean. Graduate students with GRA/TA/Grader positions or with Fellowships may not drop below 9 hours in a long session.

Academic Integrity:

As engineers you will be responsible for upholding the canons of ethics for the profession. A test of your ability to do so is to uphold the University's Academic Honesty Policy. While I do not anticipate problems of this nature, any instances of academic dishonesty will be dealt with immediately and severely in accordance with published procedures. Students who violate University rules on scholastic dishonesty are subject to disciplinary penalties, including the possibility of failure in the course and/or dismissal from the University. Because such dishonesty harms the individual, all students, and the integrity of the University, policies on scholastic dishonesty will be strictly enforced. For further information, visit the Student Judicial Services web site http://deanofstudents.utexas.edu/sjs/.

Additional Information:

Web-based, password-protected class sites will be associated with all academic courses taught at the University. Syllabi, handouts, assignments and other resources are types of information that may be available within these sites. Site activities could include exchanging email, engaging in class discussions and chats, and exchanging files. In addition, electronic class rosters will be a component of the sites. Students who do not want their names included in these electronic class rosters must restrict their directory information in the Office of the Registrar, Main Building, Room 1. For information on restricting directory information, see the Undergraduate Catalog or go to: http://www.utexas.edu/student/registrar/catalogs/gi00-01/app/appc09.html.

The University of Texas at Austin provides, upon request, appropriate academic adjustments for qualified students with disabilities. Any student with a documented disability (physical or cognitive) who requires academic accommodations should contact the Services for Students with Disabilities area of the Office of the Dean of Students at 471-6259 as soon as possible to request an official letter outlining authorized accommodations. For more information, contact that Office, or TTY at 471-4641, or the College of Engineering Director of Students with Disabilities at 471-4321.

University of Texas
Dept. of Civil, Arch. & Env. Eng.

Instructor: E. B. Williamson
due: 28 Jan or 02 Feb.

CE 397 Blast-Resistant Structural Design

Homework 1

Focusing on the past 20 years, investigate significant terrorist incidents involving the use of explosives. Prepare a summary report that describes the different types of buildings and other structures that have been subjected to such events. Your summary report should be brief (no longer than 3 pages), and it may include tables, graphs, or figures that show statistics or trends you feel are noteworthy.

Ronan Point, London (1968) – gas leak

Murrah Building, OK City (1995)

Tanzania/Nairobi Embassies (1998)

WTC (2001)

Air India 182 in 1985

Greenpeace vessel 1985

From Boyd + Sullivan

- 1980 Bologna train station
- 85-86 Tokyo subway (not bombing)
- 1995 Israel bus bombing
- 1993 world trade center attempted chemical bomb

Pg 25 – 29% transit attacks are bombing

"Since 1991, public trans. has been the target of 20 to 35% of worldwide terrorist attacks" DOT reference

10/10

CE 397: Blast-Resistant Structural Design — Catherine Hovell
Homework #1 — 28 January 2010

Introduction

To get an idea of the distribution and types of terrorist attacks over the last few decades, one must first be aware of the sheer number of events that could be considered under this headline. A quick glance at a simple source like Wikipedia shows that the events attributed to the actions of terrorists have been in the thousands for a single year. This is especially true in a time of war, when any incident against the current sitting government is counted as terrorism.

Using a more thorough search engine to comprehend the number of terrorist attacks, a graph such as is presented in Figure 1 can be drawn (from the University of Maryland).

Incidents per Year Grouped By Type of Attack

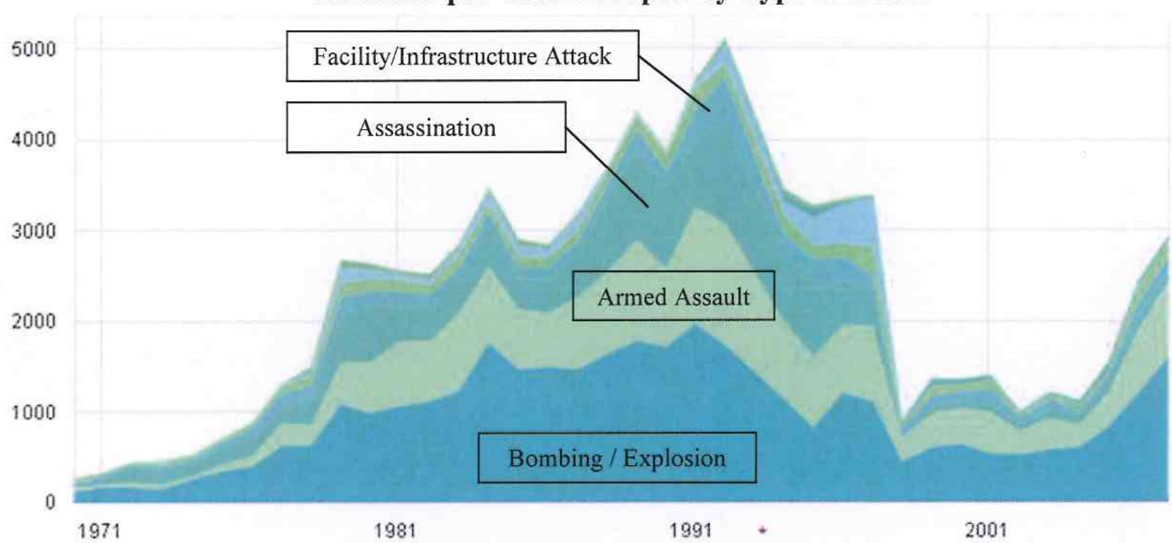


Figure 1: Summary of the number of terrorist attacks since 1970, broken down into the top eight categories (unlabeled are kidnapping, unarmed assault, barricading hostages, hijacking, and unknown). Data gathered by the University of Maryland START National Consortium.

A few observations can be made using ~~this~~ these data. First, as presented in class, explosions are the most common type of terrorist activity. Second, the relative number of type of attack tends to stay constant – in times where terrorism is high, all types of attacks increase together; there do not seem to be periods when one certain method of attack is more "popular" than another.

data → plural
datum → singular

One observation not to immediately fixate on is the dropoff in total number of incidents between 1997 and 1998. While it is true that through history, there is an ebb and flow of number of incidents (likely highlighted by the timeframe of major wars), the START website explains that the graphed data ~~comes~~ come from two different sources, one from 1970-1997, and one post-1997. Among the differences in these sources are how an act of terror is defined, thus changing the expected numbers graphed.

With the assumption that 1997 and 1998 would have had approximately the same number of incidences, had the same definitions been used, numbers in the post-1997 category could be approximated as being in the pre-1997 definitions by using a scaling factor of 4 (as 1998 showed approximately one quarter the number of incidences as 1997). Considering this proposed scaling, the most recent data looks even more severe in comparison with historical records – acts of terror are increasing with each year, surpassing numbers ever seen before. The bottom line is clear: the need for blast-resistant structural design may soon become a standard rather than an unnecessary extravagance.

When looking more closely at the events that have taken place in the past few decades, it is easy to get lost in the volume. Reports from the US government regarding the Iraq war indicate timeframes in which between 500 and 1000 improvised explosive devices (IEDs) were found each month (it is important to note that those numbers include bombs detonated successfully and those that were defused by the military) (globalsecurity.org, 2009). Rather than focusing on the multitude of small attacks, this report is going to describe several different types of targets and one or two specific events against each of those targets. Included are buildings, bridges, and transportation modes.

Buildings

The bombing of the Murrah Building in Oklahoma City is one of the most well-known (and deadly) terrorist activities in American history involving pure explosives. On April 19, 1995, a truck laden with an explosive equivalent to 4,000 lb of TNT was detonated 14 ft from an exterior column of the building. A study performed by Mlakar, Corley, Sozen, and Thornton (1997) estimated the blast loads on the building were in excess of 10,000 psi. The blast load analyses were performed assuming a uniform pressure of 140 psi and a duration of 5 msec.

The building was a reinforced concrete structure with a large transfer girder at the third floor. The blast load exceeded the shear capacity of the closest column holding the transfer girder and severely damaged two others. Without the support of the columns below, the transfer girder and slabs above failed. Forty-five percent of the occupants (163 people) of the building are reported to have died due to the explosion (Shariat, et al., 1998).

Bridges

Destroying infrastructure is a standard mode of attack in a time of war. Since Roman times, cities and groups of people have been cut off from essential supplies (e.g., food, water, and armaments) through sieges and the destruction of roads and bridges. Little has changed in a few thousand years; ongoing wars currently include the bombing of an airstrip or the physical destruction of a bridge by insurgents.

Bridges are particularly vulnerable because an explosive can get within very close range of the critical members. In most highway-over-highway applications, spacing from substructure columns to the travel lanes is minimized to maximize space for the road- or waterway below. This geometry allows space for a truck or boat to pull within a few feet of a column line. In areas where the substructure is protected, significant damage can still be done from the roadway above, which is easily accessed by a moving vehicle.

In June, 2007, a major concrete bridge connecting cities in Iraq was destroyed by a suicide bomber (BBC News, 2007). In February 2009, militants destroyed an iron bridge in Pakistan used by Western forces to access Afghanistan (Reuters, 2009). In October 2009, a bridge on the route between Iraq and Syria – used for military transport and refugees – was destroyed with a truck full of dynamite (Surk, 2009).

While plots have been uncovered regarding attempts at some of the major landmarks in the United States – e.g., the Golden Gate Bridge, the Brooklyn Bridge – the majority of actual attacks are on typical highway bridges, not those with high visibility (Williams, 2009). While the prominent bridges are frequently protected from attack – fencing around the anchors of a suspension cable, for instance, or video monitoring of stopped vehicles on the roadway – typical highway bridges are not often given the extra design consideration needed for adequate performance under extreme loading.

Transportation Modes

While the destruction of a bridge has a very obvious impact, when all transportation modes are considered, studies indicate that bridges are targeted significantly less than other methods civilizations use to move people. Jenkins and Gersten culled data from 1920 through 2000 and from 1997 through 2000; both time periods show bridges and tunnels constituting only 1 to 5% of the attacks (Jenkins and Gersten, 2001). The largest number of incidences occurred on buses – 42% of the attacks from 1997 to 2000. As a note, their data does not separate bombing attacks out from robberies, hijackings, and other forms of terrorism, and it is unclear if one mode of terrorism is easier to execute in a certain situation; however, bombings do constitute 55% of the attacks.

To highlight the damage that can be caused through transportation-related bombings, a few incidences are listed here.

- 2 August 1980: Bologna train station; 40 deaths.
- 23 June 1985: Air India Flight 182; 329 deaths.
- July, August 1995: Tel Aviv city buses; 11 deaths.
- 12 October 2000: USS Cole, docked in Aden, Yemen; 17 deaths (military personnel).
- 7 July 2005: London subway system: near simultaneous incidences in three locations as well as a fourth on a city bus within an hour; 56 deaths.

[Information from wikipedia, Boyd and Sullivan, and various news services]

The bombing of a bus or plane, one could assume, would have very different motivations from bombing a building or a bridge with regard to desired damage. Destroying a bridge does not need to cause casualties; the goal would be to disrupt the flow of goods and people. To bomb a bus or plane, however, assumes that those on board will die; it is likely the blast load will encounter humans before the confines of the structure. It seems that terrorist would target a transit system both for the disruption and financial cost incurred, but more to instill fear in civilians and force the government to act to protect their people.

Conclusions

This paper has presented the details of just a handful of the terrorist bombing incidences that have happened in recent history. These events and accompanying statistics were chosen to highlight a few main trends observed. First, as is shown in Figure 1, the number of terrorist attacks (primarily using bombs) has increased steadily over the last decade. Second, these attacks can occur with several different motivations related to the target – destruction of an important building, interruption of the flow of goods, or loss of life and installation of fear in civilians.

For the lattermost case, a structural engineer cannot necessarily change the outcome after the explosive is detonated. For buildings and bridges, however, considering the effects of blast loads during design can protect a structure and the occupants within. It may not be economically feasible to construct all structures with consideration of a blast load, but it seems inevitable that the subject will be raised more often than ever before during the design process.

nice job.

References:

BBC News (2005). "London Attacks." *BBC News*. Available at: <http://news.bbc.co.uk/2/shared/spl/hi/uk/05/london_blasts/html/>. Accessed: 26 January 2010.

BBC News (2007). "Three die in Iraq bridge bombing." *BBC News*. Available at: <http://news.bbc.co.uk/2/hi/6740131.stm>. Accessed: 26 January 2010.

Boyd, A. and J.P. Sullivan (1997). "Emergency Preparedness for Transit Terrorism." Transit Cooperative Research Program, Synthesis of Transit Practice 27. Available at: <onlinepubs.trb.org/Onlinepubs/tcrp/tsyn27.pdf>. Accessed: 22 January 2010.

globalsecurity.org (2009). "Iraq Significant Activities (SIGACTS)." *GlobalSecurity.org*. Available at: <http://www.globalsecurity.org/military/ops/iraq_sigacts.htm>. Accessed on: 27 January 2010.

Holland, C.E. (2008). "Blast-Resistant Design of Highway Bridge Columns." *The University of Texas at Austin*. MSE Thesis. August 2008.

Jenkins, B.M. and L.N. Gersten (2001). "Protecting Public Surface Transportation Against Terrorism and Serious Crime: Continuing Research on Best Security Practices." *MTI Report 01-07*. Mineta Transportation Institute, San Jose, CA.

Keel, P., Rose, D., Bose, A., and J. Joyce (1985). "Jumbo crashes killing 325." *Guardian*. Available at: <http://www.guardian.co.uk/print/0,,4954457-110875,00.html>. Accessed: 26 January 2010.

Lee, J. (2008). "Exploring Global Terrorism Data: A Web-Based Visualization of Temporal Data." *ACM Crossroads*, Vol. 15, No. 2, pp. 7-16.

Lumpkin, J. (2000). "USS Cole Bombing." *GlobalSecurity.org*. Available at: <http://www.globalsecurity.org/security/profiles/uss_cole_bombing.htm>. Accessed: 25 January 2010.

Mlakar, P.F. (2010). Structural Engineering Graduate Seminar. 04 November 2010.

Mlakar, P.F., Corley, W.G., Sozen, M.A., and C.H. Thornton (1997). "Blast Loading and Response of Murrah Building." *Forensic Engineering*, Proceedings from the First Forensic Engineering Congress, pp. 36-43.

Reuters (2009). "Militants in Pakistan Blow Up Khyber Pass Bridge." *Reuters*. <http://www.france24.com/en/20090203-militants-pakistan-blow-bridge-khyber-pass-western-supplies>. Accessed: 25 January 2010.

Shariat, S., Mallonee, S., and S. Stephens-Stidham (1998). "Oklahoma City Bombing Injuries." Oklahoma State Department of Health.

Surk, B. (2009). "U.S.-used bridge blown up in Iraq." *The Columbus Dispatch*. Available at: <http://www.dispatch.com/live/content/national_world/stories/2009/10/18/ap_iraq_daily_1018.ART_ART_10-18-09_A7_IIFDE69.html>. Accessed: 25 January 2010.

University of Maryland (2010). "Global Terrorism Database." *National Consortium for the Study of Terrorism and Responses to Terrorism (START)*. Available at: <http://www.start.umd.edu/start/>. Accessed: 25 January 2010.

Williams, G.D. (2009). "Analysis and Response Mechanism of Blast-Loaded Reinforced Concrete Columns." *The University of Texas at Austin*. PhD Dissertation. May 2009.

wikipedia.com (2010). "7 July 2005 London bombings." Available at: <http://en.wikipedia.org/wiki/7_July_2005_London_bombings>. Accessed: 26 January 2010.

wikipedia.com (2010). "Air India Flight 182" Available at: <http://en.wikipedia.org/wiki/Air_India_Flight_182>. Accessed: 26 January 2010.

wikipedia.com (2010). "August 1980 Bologna bombing." Available at: <http://en.wikipedia.org/wiki/August_1980_Bologna_bombing>. Accessed: 26 January 2010.

wikipedia.com (2010). "USS Cole bombing." Available at: <http://en.wikipedia.org/wiki/USS_Cole_bombing>. Accessed: 26 January 2010.

University of Texas
Dept. of Civil, Arch. & Env. Eng.

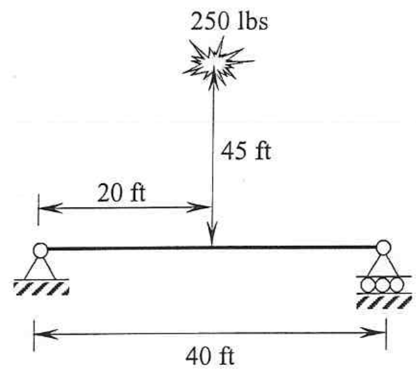
Instructor: E. B. Williamson

due: 04 February 2010

CE 397 Blast-Resistant Structural Design

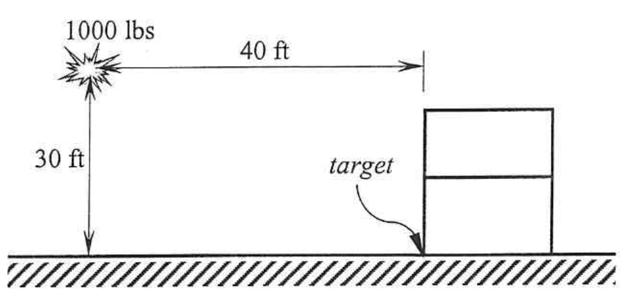
Homework 2

1. A spherical explosive charge weighing 250 lbs (TNT equivalent weight) is positioned 45 ft above the midspan of a 40-ft long simply supported beam as shown in the figure below. Determine the peak reflected pressure and the reflected impulse for the beam at the following locations: (a) at midspan and (b) at the beam ends. Also, describe how you would model the load acting on the entire beam. Use both the formulas derived in class and the charts to determine an answer.



2. A hemispherical explosive charge of 800 lbs of Composition C-4 is positioned 50 ft away from the face of a building. For this scenario, compute the following blast parameters: (a) peak incident pressure, (b) incident impulse, (c) peak reflected pressure, and (d) reflected impulse. In computing these values, use the formulas developed in class, and compare these results to those obtained from the chart. Describe the differences observed among your solutions and indicate how you would use the results you obtained to define the load history for design.

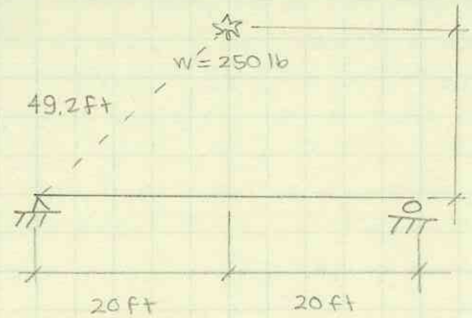
3. A spherical explosive charge weighing 800 lbs (TNT equivalent weight) is positioned 30 ft above the ground and 40 ft away from the front of a building (see figure). For a point at the base of the wall, compute the peak incident pressure, the peak reflected pressure, the incident impulse, and the peak reflected impulse.



STAEDTLER® No. 937 811E Engineer's Computation Pad

HOMEWORK #2

1.



subscript m refers to midspan, e refers to beam ends

$$Z_m = \frac{45\text{ ft}}{(250\text{ lb})^{1/3}} = 7.14\text{ ft/lb}^{1/3}$$

$$Z_e = \frac{\left[(45\text{ ft})^2 + (20\text{ ft})^2\right]^{1/2}}{(250\text{ lb})^{1/3}}$$

$$= 7.82\text{ ft/lb}^{1/3}$$

From the spherical blast chart,

$P_{r_m} = 34$ psi $\qquad P_{re} = 30$ psi

$i_{r_m} = 18 \cdot W^{1/3}$ $\qquad i_{re} = 17 W^{1/3}$

$= 113$ psi·ms $\qquad = 107$ psi·ms

Using equations,

$$P_{so} = \frac{4120}{Z^3} - \frac{105}{Z^2} + \frac{39.5}{Z} \qquad \text{check: } 3\text{ ft/lb}^{1/3} < Z < 20\text{ ft/lb}^{1/3}$$

$$P_{so_m} = \frac{4120}{(7.14)^3} - \frac{105}{(7.14)^2} + \frac{39.5}{(7.14)} = 15\text{ psi}$$ ✓

$P_{so_e} = 12$ psi

$$P_r \sim 2P_{so}\left[\frac{103 + 4P_{so}}{103 + P_{so}}\right]$$

$$P_{r_m} = 2(15\text{ psi})\left[\frac{103 + 4(15\text{ psi})}{103 + (15\text{ psi})}\right] = 41\text{ psi}$$ — not very close to chart value

$P_{re} = 31$ psi

[Grader's note, red:] $P_{so_{HENRYCH}} = 12.8 \Rightarrow P_R = 34$ WHICH AGREES VERY WELL w/ CHART VALUE

[Grader's note, red:] IT'S OK TO REPORT YOUR ANSWER WITH APPROPRIATE SIG. DIGITS, BUT DO NOT PROPAGATE ERROR w/ INACCURATE NUMBERS

Assume $t_o = 2.0 \cdot W^{1/3} = 12.6$ ms (from chart) to calculate:

$$i_{r_m} = \tfrac{1}{2} P_{r_m} t_o = \tfrac{1}{2}(41\text{ psi})(12.6\text{ ms}) = 258\text{ psi·ms}$$

$i_{re} = 195$ psi·ms

Now, confirm / make easier with computer automation...

CE 397: BLAST

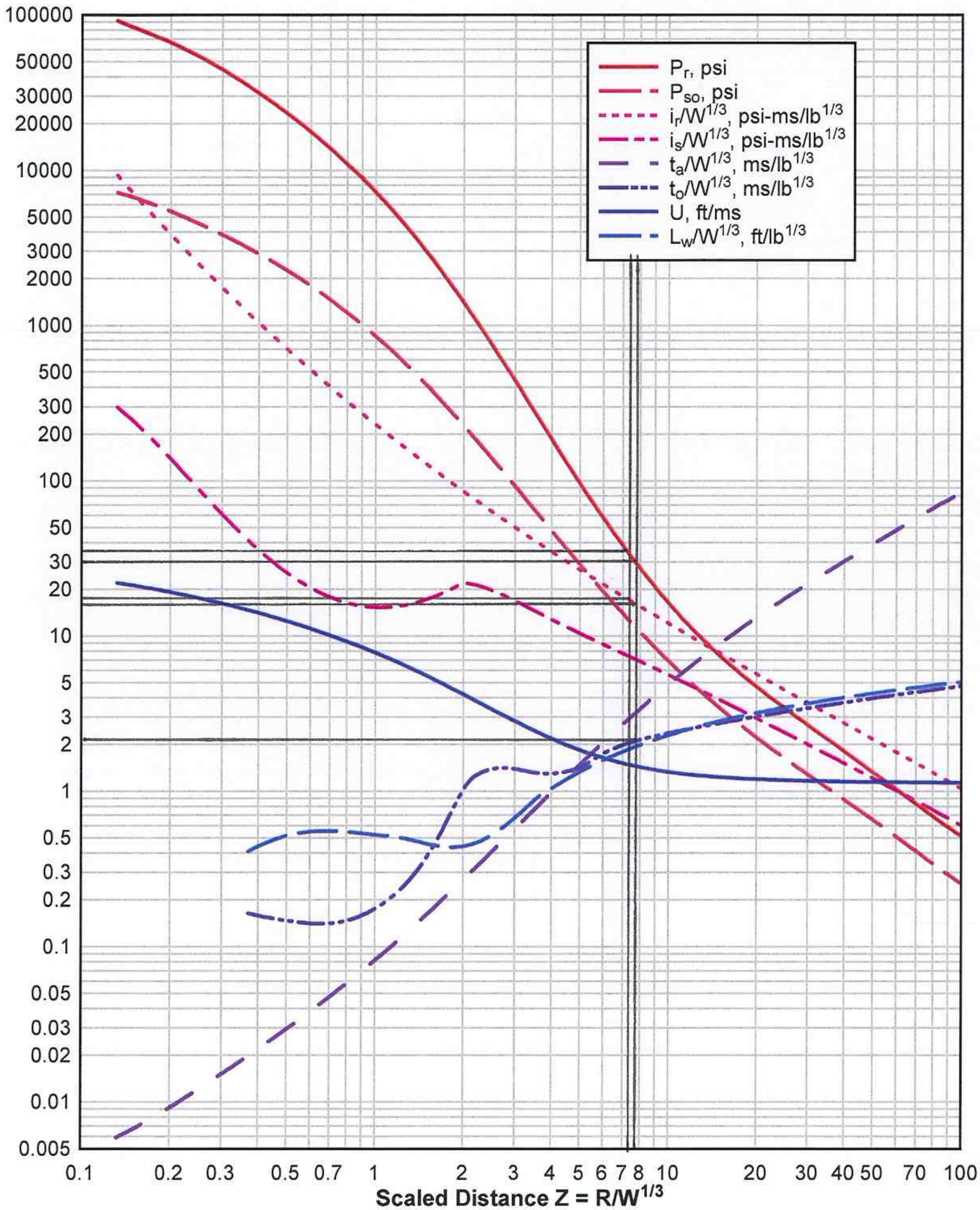
HOMEWORK #2

C. HOVELL
04 Feb 10

2/10

1. (cont'd)

Figure 2-7 Positive Phase Shock Wave Parameters for a Spherical TNT Explosion in Free Air at Sea Level



CE397: BLAST

C.HOVELL
04 Feb. 10

3/10

CE397: Blast-Resistant Design
Homework #2, Problem 1

28/30

Explosive material is TNT
Blast is assumed to be spherical
Input weight = 250 lb (equivalent TNT)

Part A - Beam Center
Input standoff = 45.0 ft
Scaled standoff = 7.14 ft/lb^1/3
= 2.83 m/kg^1/3

Calculation	TM 855	Henrych	Chart
Peak shock pressure	= 14.8 psi	12.8 psi	12 psi
Peak reflected pressure	= 40.7 psi	34.1 psi	34 psi
Positive phase duration	= 7.7 ms	7.7 ms	13 ms
Side-on impulse	= 57.2 psi-ms	49.6 psi-ms	47 psi-ms
Reflected impulse	= 157.5 psi-ms	132.1 psi-ms	113 psi-ms

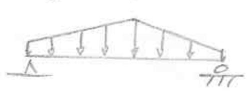
Part B - Beam End
Input standoff = 49.2 ft
Scaled standoff = 7.82 ft/lb^1/3
= 3.10 m/kg^1/3

✓

Calculation	TM 855	Henrych	Chart
Peak shock pressure	= 12.0 psi	10.8 psi	11 psi
Peak reflected pressure	= 31.4 psi	27.8 psi	30 psi
Positive phase duration	= 8.4 ms	8.4 ms	13 ms
Side-on impulse	= 50.0 psi-ms	45.2 psi-ms	44 psi-ms
Reflected impulse	= 131.2 psi-ms	116.1 psi-ms	107 psi-ms

Comments on Load Modeling

- assume load hits both points simultaneously
- model triangularly



check assumption by calculating loads at beam quarter points

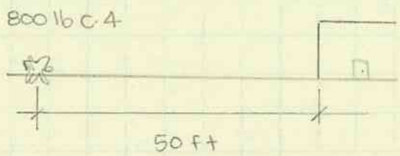
- assumed there is surface beyond beam ends; no longitudinal clearing effects considered ✓
- use TM calculated values as they are the most conservative

OK, but this level may not be needed. Difference in midspan load and load on ends is small. While conservative, a uniform load is probably easier to work with in subsequent calculations.

STAEDTLER® No. 937 811E Engineer's Computation Pad

HOMEWORK #2

2.



C.4:
equivalent weight
- pressure = 1.37
- impulse = 1.19

acceptable range: 10–100 psi

$W_p = (800\,lb)(1.37) = 1096\,lb\ TNT \times 1.8$
$W_i = (800\,lb)(1.19) = 952\,lb\ TNT \times 1.8$

Scaled standoff

$$Z_p = \frac{50\,ft}{(1096\,lb)^{1/3}} = 3.~~99~~\,ft/lb^{1/3}$$

4.85 math error

$$Z_i = \frac{50\,ft}{(952\,lb)} = 4.~~15~~\,ft/lb^{1/3}$$

5.08

formula ok, but your computed value for Z is not correct

Values from chart:

$P_{so} = 70\,psi$ ✗ $\quad i_s = 18\,(952\,lb)^{1/3} = 177\,psi\cdot ms$ ✗

$P_r = 300\,psi$ ✗ $\quad i_r = 41\,(952\,lb)^{1/3} = 403\,psi\,ms$ ✗

(too high)

Try running full calculations using each W value (attached).

CE397: BLAST

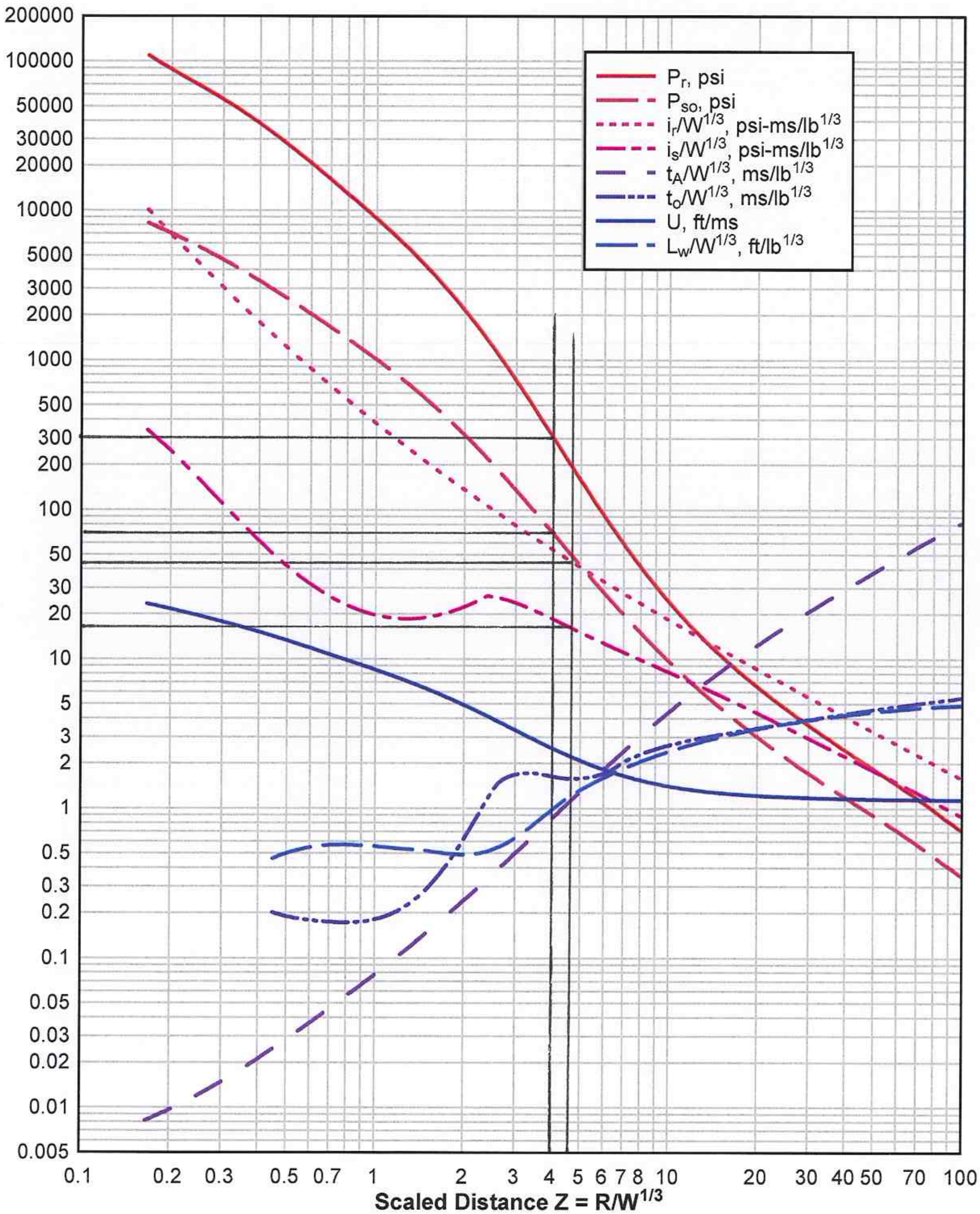
HOMEWORK #2

C. HOVELL
04 Feb 10

5/10

2. (cont'd)

Figure 2-15 Positive Phase Shock Wave Parameters for a Hemispherical TNT Explosion on the Surface at Sea Level



CE 397: BLAST

C. HOVELL
04 Feb 10

6/10

CE397: Blast-Resistant Design
Homework #2, Problem 2

Explosive material is C-4
Blast is assumed to be hemispherical
Input standoff = 50.0 ft

Using Pressure Conversion Factor, =1.37
Blast weight = 1096 lb (equivalent TNT, pressure)
Scaled standoff = 3.99 ft/lb^1/3
= 1.58 m/kg^1/3

Calculation	TM 855	Henrych ✓	Chart
Peak shock pressure	= 68.3 psi	41.6 psi	70 psi
Peak reflected pressure	= 300.1 psi	155.2 psi ✓	300 psi
Positive phase duration	= 7.4 ms	7.4 ms	18 ms
Side-on impulse	= 252.1 psi-ms	153.6 psi-ms	
Reflected impulse	= 1107.4 psi-ms	572.6 psi-ms	

Using Impulse Conversion Factor, =1.19
Blast weight = 952 lb (equivalent TNT, impulse)
Scaled standoff = 4.18 ft/lb^1/3
= 1.66 m/kg^1/3

Calculation	TM 855 ✓	Henrych ✓	Chart
Peak shock pressure	= 59.9 psi	37.7 psi	
Peak reflected pressure	= 252.1 psi	135.9 psi	
Positive phase duration	= 7.4 ms	7.4 ms	17 ms
Side-on impulse	= 221.1 psi-ms	139.0 psi-ms ✗	177 psi-ms
Reflected impulse	= 930.1 psi-ms	501.3 psi-ms ✗	403 psi-ms

Check duration formula. Your value is low

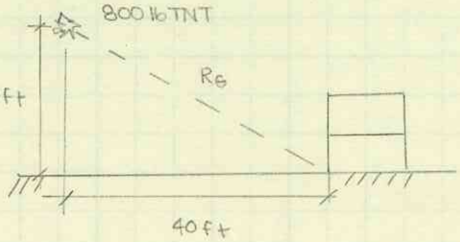
To define the loads on the structure, I would use P_{so} = 68 psi, P_r = 300 psi as both the charts and TM formulas produce those predicted values. Given the wide variation in impulse values, I would average the three results to get numbers for design — i_s = 179 psi·ms, i_r = 611 psi·ms.

if you are uncertain, why not use most conservative value?

The calculations of impulse were made using t_o and Z values calculated with W = 952 lb. While the chart does not match the calculations well, changing W by such a small amount does not noticably change t_o.

HOMEWORK #2

3.



$$R_G = \sqrt{(30\,ft)^2 + (40\,ft)^2} = 50\,ft$$ ✓

$$Z = \frac{50\,ft}{(800\,lb)^{1/3}} = 5.4\,ft/lb^{1/3}$$ ✓

calculate P_{so} as if spherical:

$$P_{so} = \frac{4120}{(5.4)^3} - \frac{105}{(5.4)^2} + \frac{39.5}{(5.4)} = 30.1\,psi$$ ✓

angle of incidence

$$\alpha = \tan^{-1}\left(\frac{40\,ft}{30\,ft}\right) = 53.1°$$ ✓

scaled height

$$Z_h = \frac{30\,ft}{(800\,lb)^{1/3}} = 3.23\,ft/lb^{1/3}$$

get reflection coefficient from 2-193

$$C_{r\alpha} \sim 2.0$$ ✓

$$P_{r\alpha} = P_{so} \cdot C_{r\alpha} = 2.0(30.1\,psi) = 60.2\,psi$$ ✓ ✓

Now, using goalseek in excel, find Z needed to calculate $P_{so} = 60.2\,psi$

$$Z = 4.17\,ft/lb^{1/3}$$ ✓ based on TM 855 formula OK

keep standoff, $R_G = 50\,ft$

new $W = 1722\,lb$

└ use for calculations

↑ Conservative to adjust for computation of duration

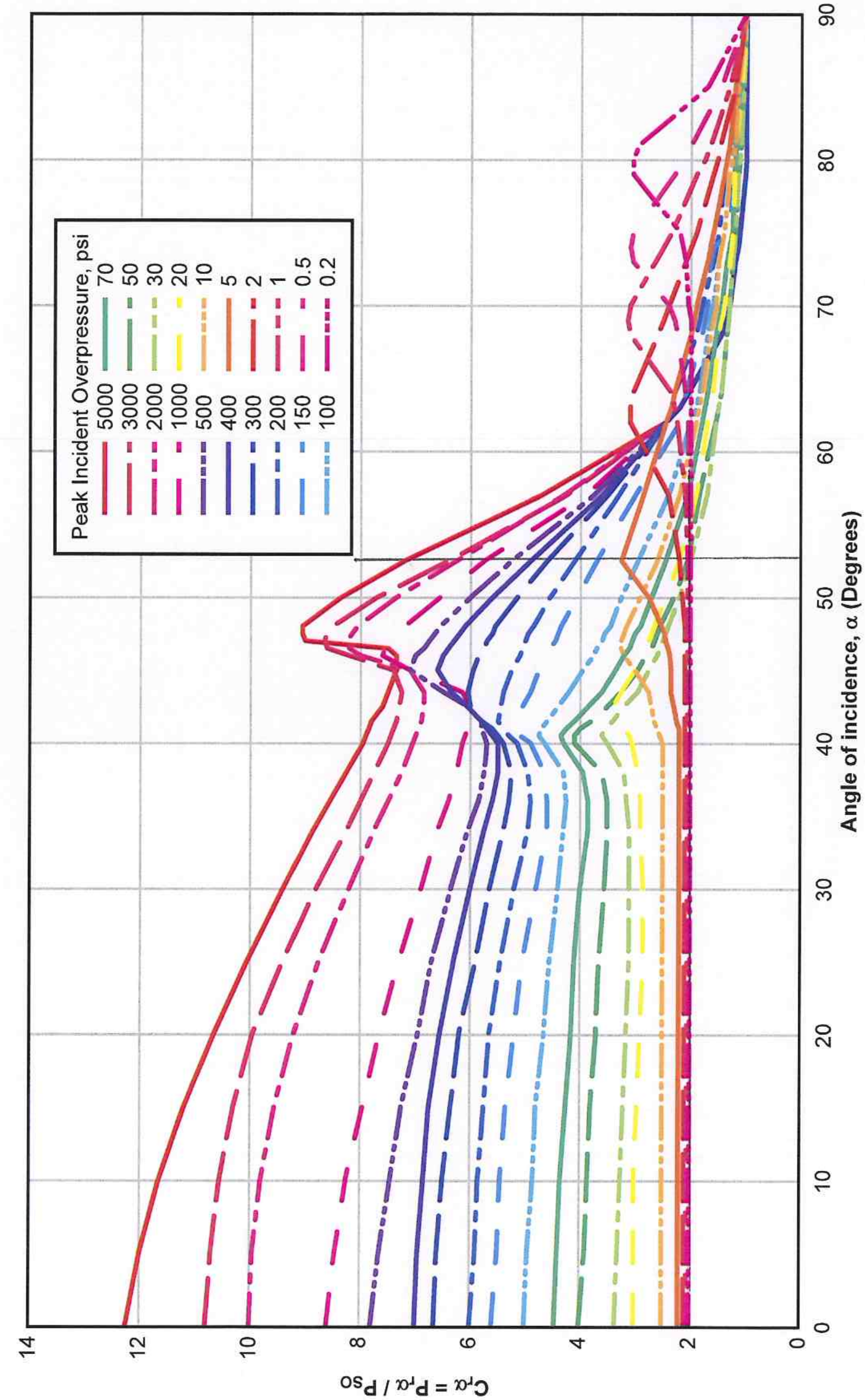
CE 397: BLAST

HOMEWORK #2

C. HOVELL
04 Feb 2010

8/10

Figure 2-193 Reflected Pressure Coefficient versus Angle of Incidence



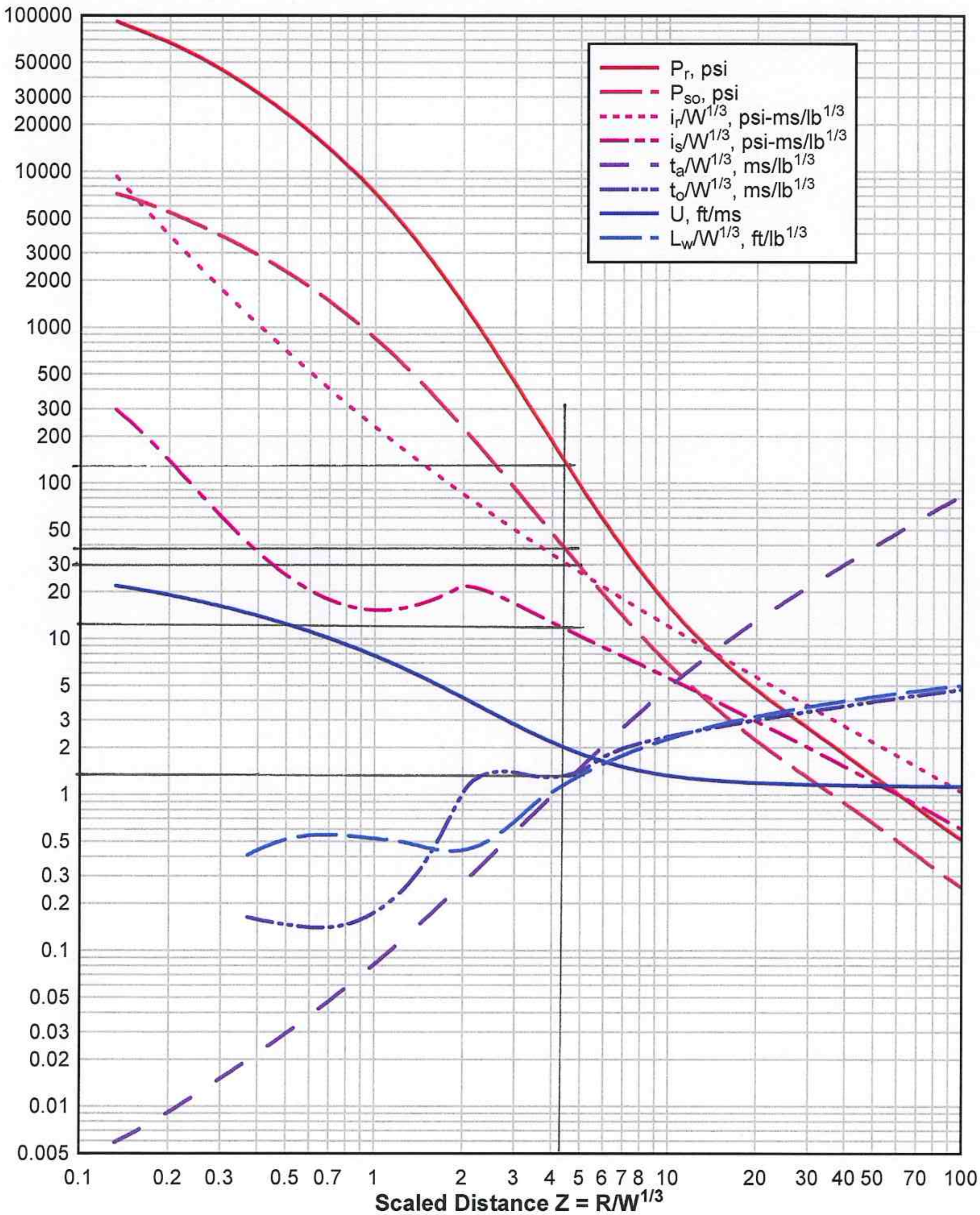
CE 397: BLAST

HOMEWORK #2

C. HOVELL
04 Feb 10

9/10

Figure 2-7 Positive Phase Shock Wave Parameters for a Spherical TNT Explosion in Free Air at Sea Level



C. HOVELL 10/10
04 Feb 10

CE397: Blast-Resistant Design
Homework #2, Problem 3

Explosive material is TNT
Blast is assumed to be spherical
Calculated weight = 1722 lb (equivalent TNT)

Input standoff = 50.0 ft
Scaled standoff = 4.17 ft/lb^1/3
= 1.65 m/kg^1/3

Calculation	TM 855	Henrych	Chart
Peak shock pressure	= 60.2 psi ✓	37.8 psi	39 psi
Peak reflected pressure	= 253.6 psi	136.5 psi	110 psi
Positive phase duration	= 9.0 ms	9.0 ms	13 ms
Side-on impulse	= 270.2 psi-ms	169.7 psi-ms	126 psi-ms
Reflected impulse	= 1138.3 psi-ms	612.7 psi-ms	360 psi-ms

Pso not correspond to new z value

While the TM855 values seem significantly more conservative than the Henrych or chart values, I would use them in design because the modified Z value was found using the TM855 equation for Pso. Alternatively, a second Z could be found from the Henrych calculations.

This approach is recommended. The z computed from before is valid only for the TM 855 formulas, you should find z = 3.8 for Henrych to give P = 44.5 = 2 × Pso Henrych

* Your use of the chart for this case is not correct. You need to find where P = 2 × Pso for z = 5.4 and use the values for new z to get desired value

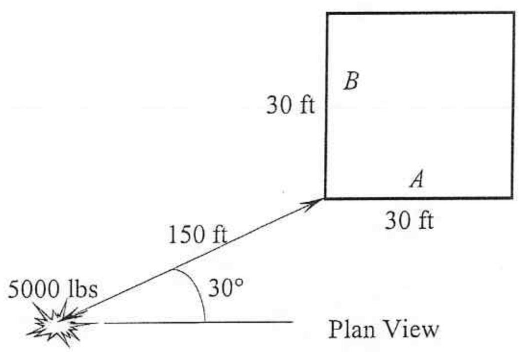
University of Texas
Dept. of Civil, Arch. & Env. Eng.

Instructor: E. B. Williamson

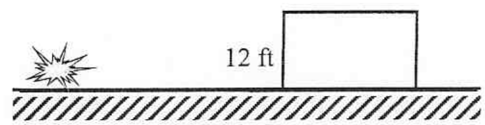
CE 397 Blast-Resistant Structural Design

Homework 3 **Due: 25 FEB**

A hemispherical explosive charge weighing 5000 lbs (TNT equivalent weight) is positioned 150 ft from the corner of a 30 ft × 30 ft building that is 12 ft in height. The geometry of the charge relative to the building orientation is given below.



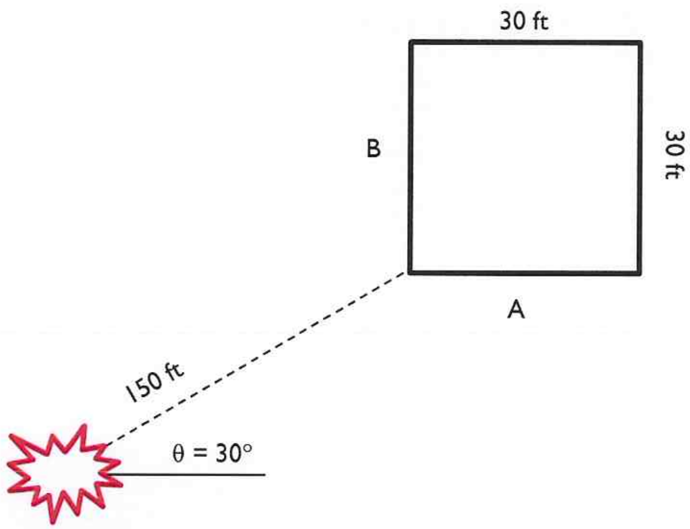
Plan View



Elevation View

Determine the blast pressure as a function of time that is acting on the roof and walls *A* and *B* (see figure above). For the final blast loading on each component, compute the corresponding impulse. Discuss any assumptions you make in the computation of the load histories for each component.

Homework #3



Begin calculations by considering three points on Wall A, at the middle of the wall, the front face, and the rear face. If the three calculations result in similar values, a representative single point can be used for Wall B.

Following pages include:
- Excel sheet showing calculations at three points for Wall A
- Excel sheet showing calculations for Wall B
- Fig. 2-193
- Fig. 2-194(b)
- Chart for C_r
- Time-history plots for Walls A and B

Values are marked on the charts for three points on Wall A and a single point (in the middle of the wall) for Wall B.

Given the small effects on impulse, load, and time of incident associated with moving the point of interest from the middle of Wall A to the front corner, I would use the load data obtained from the midpoint. If in design, I found those loads to cause a response in the structure close to the limits of acceptability, I would switch to the load history found at the front corner.

28/30

Homework #3

Calculations for Wall A (at front edge, middle, and rear edge)

Weight of Explosive			MATERIAL FACTORS			
Raw weight =	5000	lb	Material Type =	TNT		
Type of blast =	H		Pressure =	1.00		
			Impulse =	1.00		
Weight for Calcs =	9000	lb	* includes 1.8 modification for hemisphereical			
	4086	kg	charge, if appropriate, and material factors			
Building Dimensions						
Height =	12.0	ft	Length = 30.0	ft		
Standoff	Front		Middle		Rear	
x =	129.9	ft	144.9	ft	159.9	ft
y =	75.0	ft	75.0	ft	75.0	ft
Straight Line =	150.0	ft	163.2	ft	176.6	ft
	45.7	m	49.7	m	53.8	m
Angle of incidence, α =	60.0		62.6		64.9	
Initial Calculations						
Z for chart =	8.77	$lb/ft^{1/3}$	9.54	$lb/ft^{1/3}$	10.33	$lb/ft^{1/3}$
Z =	7.21	$lb/ft^{1/3}$	7.84	$lb/ft^{1/3}$	8.49	$lb/ft^{1/3}$
	2.86	$kg/m^{1/3}$	3.11	$kg/m^{1/3}$	3.37	$kg/m^{1/3}$
In range, TM 855?	OK		OK		OK	
In range, Henrych?	OK	RANGE 3	OK	RANGE 3	OK	RANGE 3
Pso, TM 855 =	14.45	psi	11.87	psi	9.93	psi
Pso Henrych =	12.58	psi	10.74	psi	9.28	psi
Pso used from here	14.45	TM 855	11.87	TM 855	9.93	TM 855
* use higher value						
Find Values in Charts						
Cr_alpha =	1.90		1.82		1.80	* 2-193, need Pso
$ir\_alpha/W^{1/3}$ =	13.00		12.00		9.50	* 2-194, need Pso
Cr =	1.36		1.31		1.28	* chart in notes, need Pso
C_D =	1.00		1.00		1.00	
Calculate Pressure on Wall						
Pr_alpha =	27.45	psi	21.60	psi	17.87	psi
ir_alpha =	270.41	psi-ms	249.61	psi-ms	197.61	psi-ms
to (from ir, Pr) =	19.71	ms	23.12	ms	22.12	ms
tc =	19.61	ms	20.36	ms	20.83	ms
consider clearing?	YES		YES		YES	tc < to
qs =	4.44	psi	3.06	psi	2.18	psi
P_stagnation =	18.89	psi	14.93	psi	12.11	psi
P_tc =	0.09	psi	1.78	psi	0.70	psi
ir_fromcurve =	269.99	psi-ms	240.41	psi-ms	193.90	psi-ms

Is shock front assumed to be planar?

Choose which formula matches the chart value better

why not use value that most closely matches chart?

$t_c = \frac{3S}{U}$ for specific point

How is clearing time computed? WHAT IS NEAREST FREE EDGE DISTANCE?

CE397: BLAST

C. HOVELL
25 Feb. 2010

Homework #3

Calculations for Wall B (at middle of wall)

Weight of Explosive			**MATERIAL FACTORS**		
Raw weight =	5000	lb	Material Type =	TNT	
Type of blast =	H		Pressure =	1.00	
			Impulse =	1.00	
Weight for Calcs =	9000	lb	* includes 1.8 modification for hemisphereical		
	4086	kg	charge, if appropriate, and material factors		
Building Dimensions					
Height =	12.0	ft	Length =	30.0	ft
Standoff	Middle				
x =	129.9	ft			
y =	90.0	ft			
Straight Line =	158.0	ft			
	48.2	m			
Angle of incidence, α =	34.7				
Initial Calculations					
Z for chart =	9.24	lb/ft$^{1/3}$			
Z =	7.60	lb/ft$^{1/3}$			
	3.01	kg/m$^{1/3}$			
In range, TM 855?	OK				
In range, Henrych?	OK	RANGE 3			
Pso, TM 855 =	12.77	psi			
Pso Henrych =	11.40	psi			
Pso used from here	12.77	TM 855			
* use higher value					
Find Values in Charts					
Cr_alpha =	2.80	✓			* 2-193, need Pso
ir_alpha/W$^{1/3}$ =	16.00	✓			* 2-194, need Pso
Cr =	1.34				* chart in notes, need Pso
C_D =	1.00				
Calculate Pressure on Wall					
Pr_alpha =	35.77	psi			
ir_alpha =	332.81	psi-ms			
to (from ir, Pr) =	18.61	ms			
tc =	19.90	ms			
consider clearing?	NO				tc < to

← IS SHOCK FRONT PLANAR?

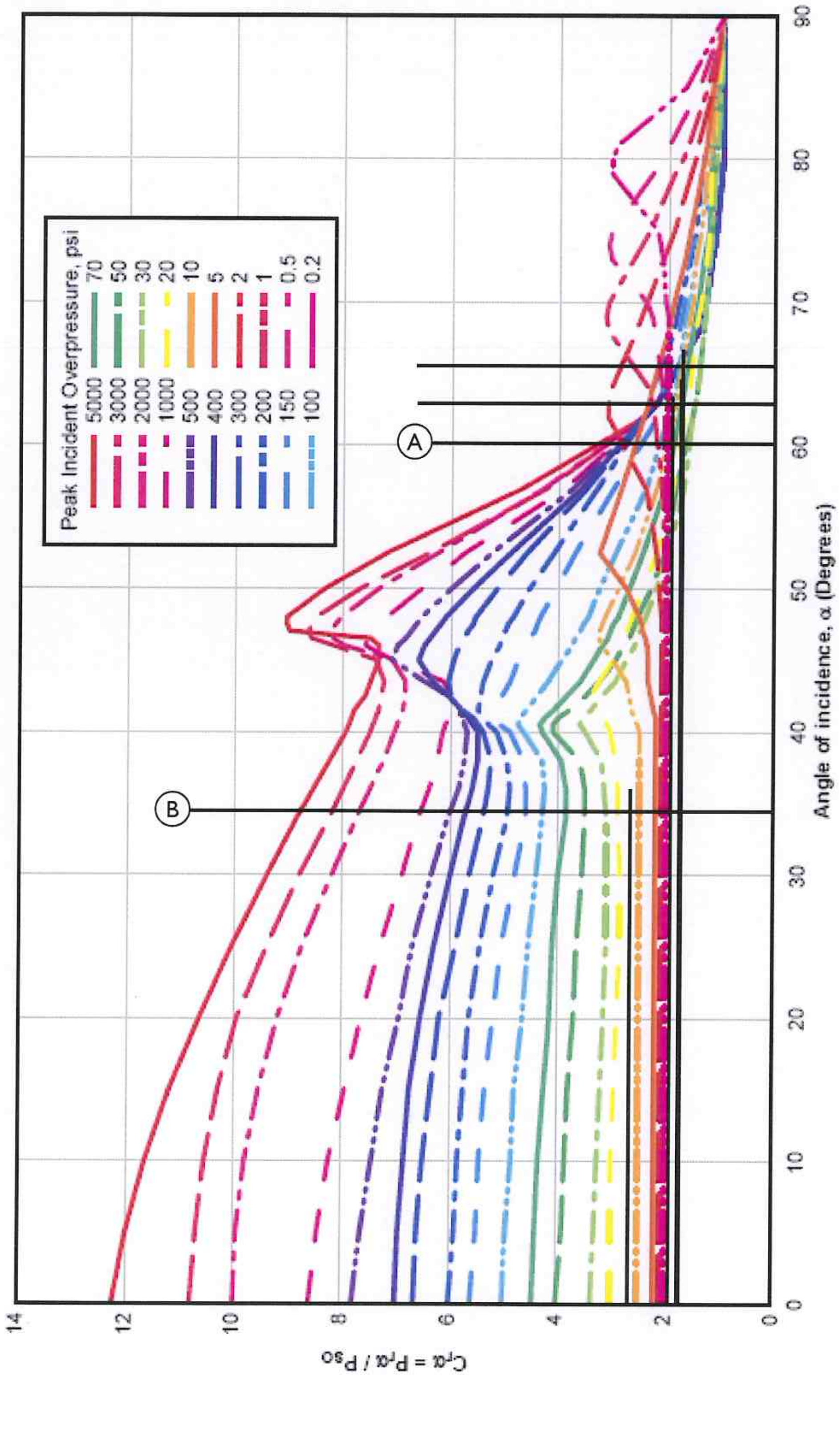
if it's planar, θ stays constant!

show calculation

average clearing used if entire wall is loaded at the same time; else, clearing at a point.

Homework #3

Figure 2-193 Reflected Pressure Coefficient versus Angle of Incidence



Homework #3

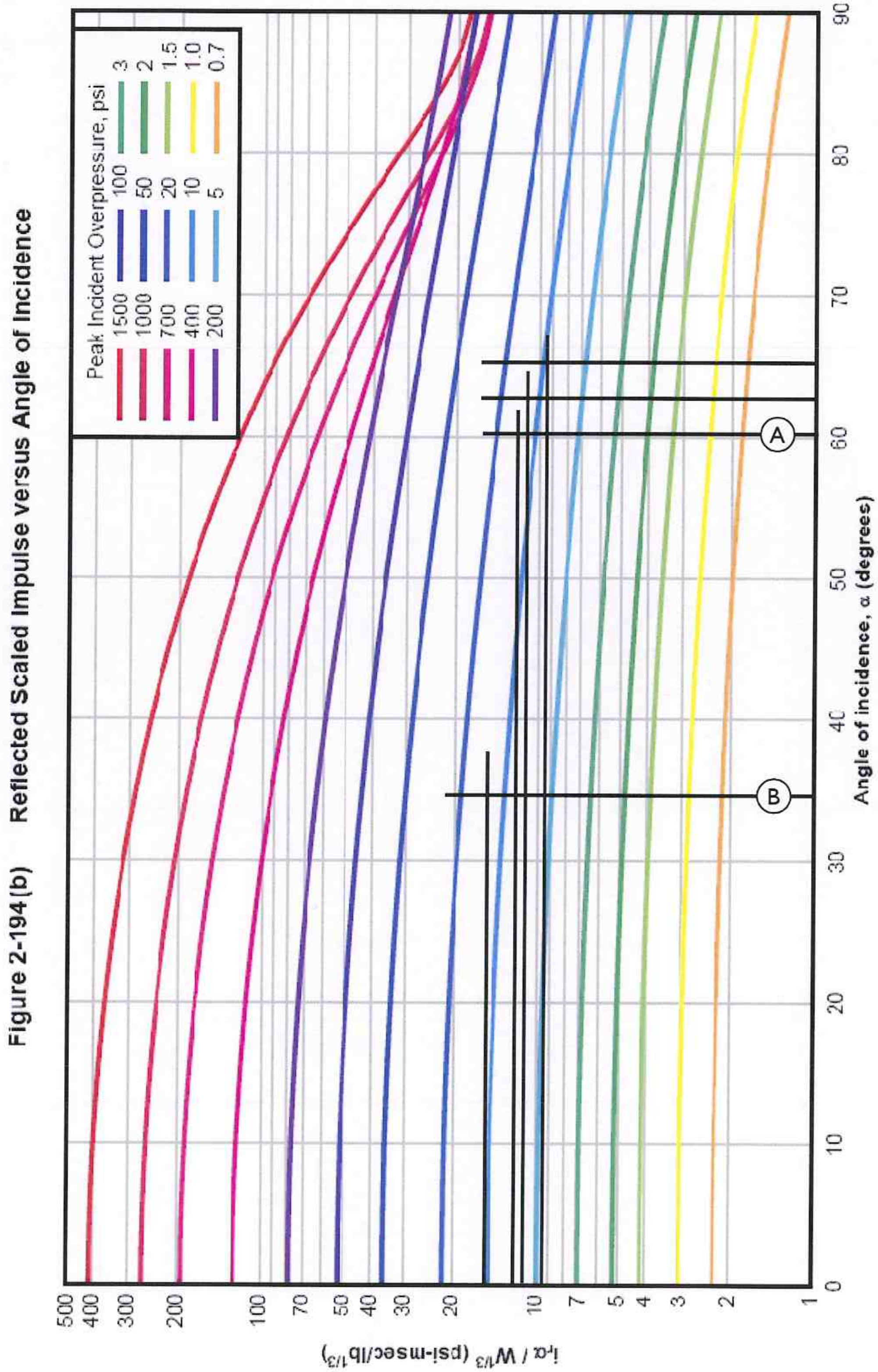
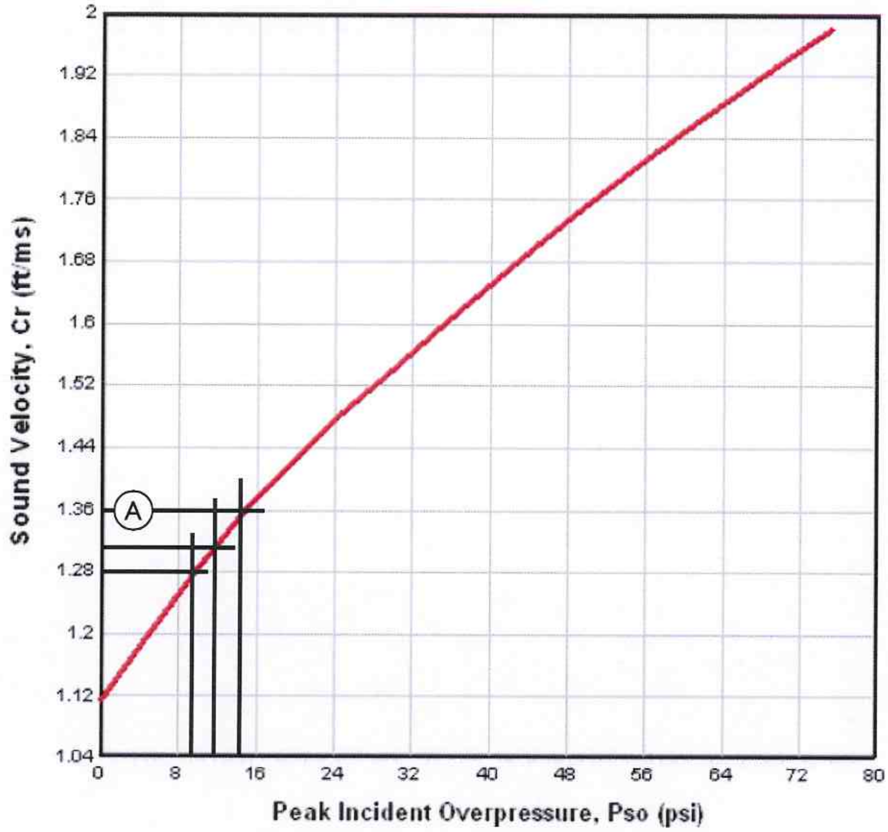
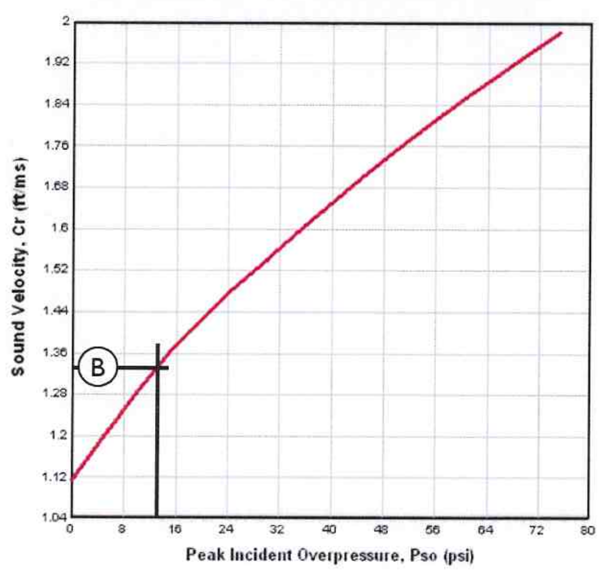


Figure 2-194 (b) Reflected Scaled Impulse versus Angle of Incidence
Peak Incident Overpressure, psi
1500
1000
700
400
200
100
50
20
10
5
3
2
1.5
1.0
0.7
A
B
Angle of incidence, α (degrees)
$i_{r\alpha}$ / $W^{1/3}$ (psi-msec/lb$^{1/3}$)

Homework #3

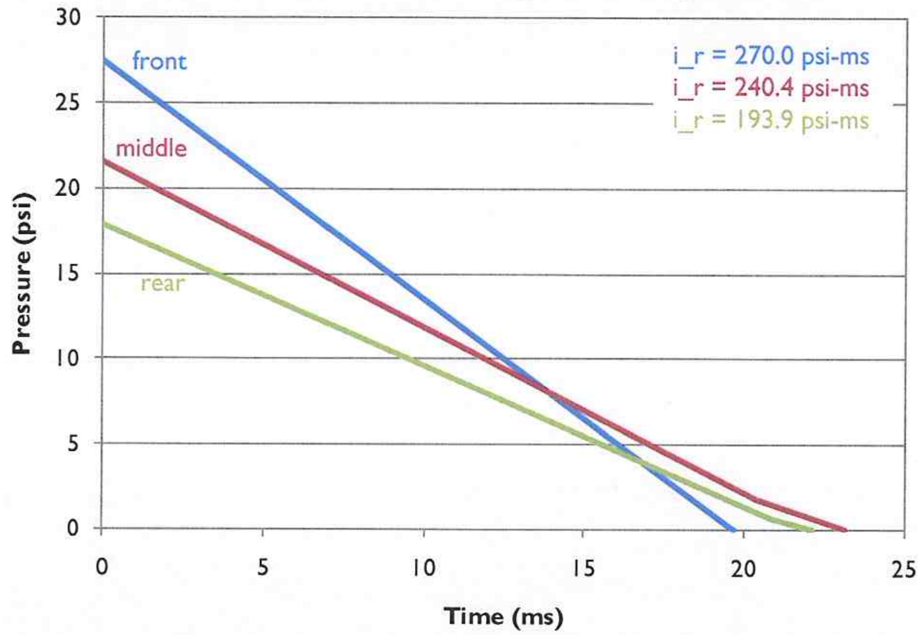




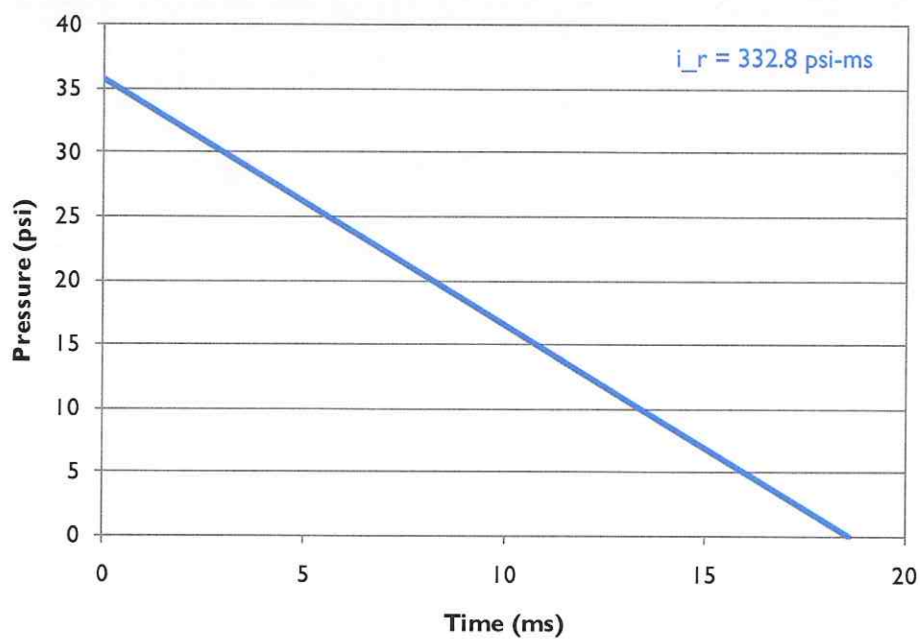
C_r comes into play for average clearing time

Homework #3

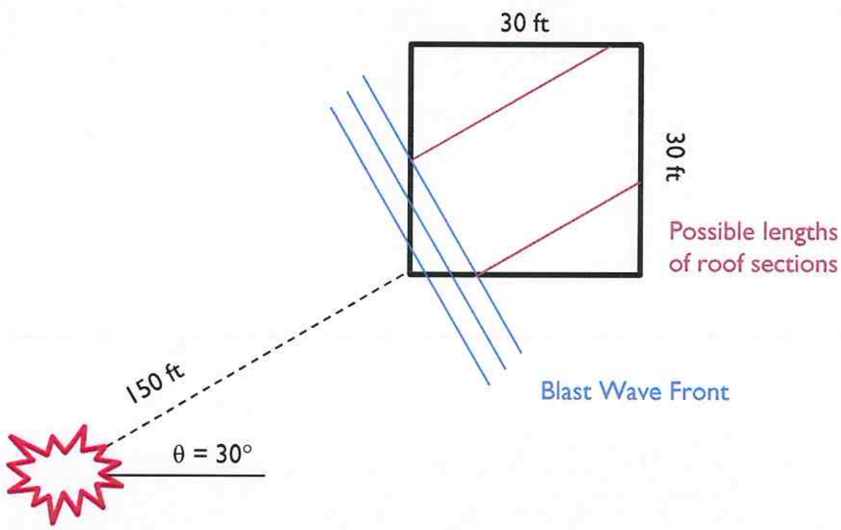
Time-History for Wall A, as measured at center, front edge, and rear edge of the wall



Time-History for Wall B, as measured at the center



Homework #3



Roof loads are highly dependent on the length over which the load will be acting. It seems appropriate to either take this length as the length of the structural members being loaded, or as the length of the building perpendicular to the wave front. If the latter is used, the length would vary dependent on the point of initial contact of the blast wave, as shown in the figure above.

Logically, however, the roof members are going to respond along their length, which would be 30 ft. As a smaller value for length results in a larger C_E and thus a larger roof pressure load, that number will be used. As with the wall sections, if upon calculating response of the roof, I learn that I am at or near capacity of the member, these calculations can and should be performed in more detail to find a more accurate loading history.

As well, the point of initial contact of the blast with the roof will be considered at the corner of Walls A and B, as opposed to the other points that exist as marked above.

Following pages include:

- Excel sheet showing calculations for Roof loads
- Fig. 2-15
- Fig. 2-196
- Fig. 2-197
- Fig. 2-198
- Time-history plot for roof loading

Homework #3

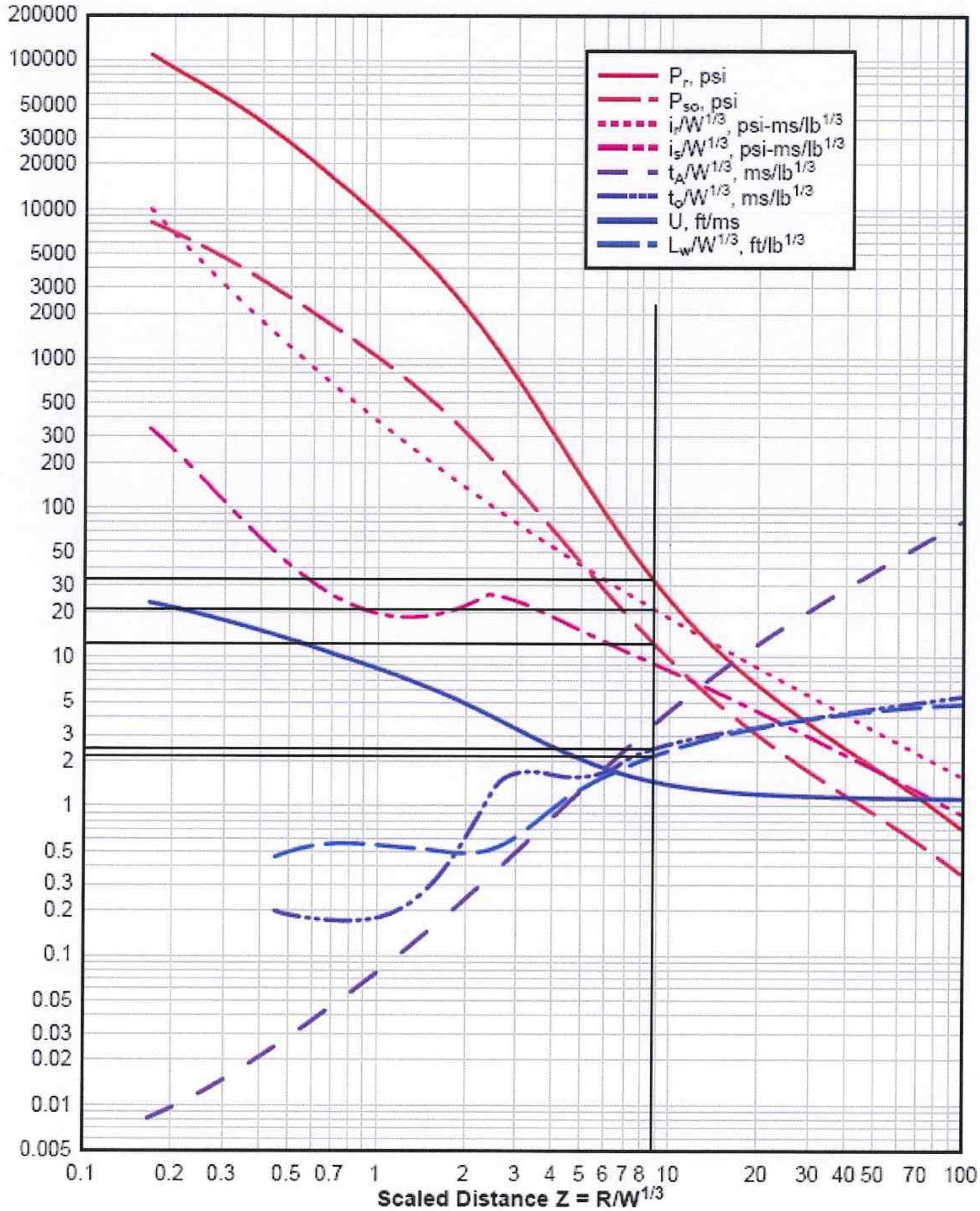
Calculations for Roof

Weight of Explosive			MATERIAL FACTORS		
Raw weight =	5000	lb	Material Type =	TNT	
Type of blast =	H		Pressure =	1.00	
			Impulse =	1.00	
Weight for Calcs =	9000	lb	* includes 1.8 modification for hemisphereical		
	4086	kg	charge, if appropriate, and material factors		
Building Dimensions					
Height =	12.0	ft	Length =	30.0	ft
Standoff	Middle				
x =	129.9	ft			
y =	75.0	ft			
Straight Line =	150.0	ft			
	45.7	m			
Angle of incidence, α =	30.0				
Initial Calculations					
Z for chart =	8.77	$lb/ft^{1/3}$			
Z =	7.21	$lb/ft^{1/3}$			
	2.86	$kg/m^{1/3}$			
In range, TM 855?	OK				
In range, Henrych?	OK	RANGE 3			
Pso, TM 855 =	14.45	psi			
Pso Henrych =	12.58	psi			
Pso used from here	14.45	TM 855			
* use higher value					
Find Values in Charts					
Cr_alpha =	1.82				* 2-193, need Pso
$ir\_alpha/W^{1/3}$ =	12.00				* 2-194, need Pso
$L\_wf/W^{1/3}$ =	2.10				* 2-15, need hemiZ
Given L =	30.00	ft			
L_wf/L =	1.20				
C_E =	0.52				* 2-196, need L_wf/L, Pso
C_D =	-0.40				* from table, need qs
scaled rise time =	1.10				* 2-197, need L_wf/L, Pso
scaled duration =	2.85				* 2-198, need L_wf/L, Pso
Calculate Pressure on Roof					
qs =	4.44	psi			
rise time =	22.88	ms			
duration =	59.28	ms			
P_R =	5.66	psi			
impulse =	167.84	psi-ms			

(Handwritten annotations: Cr_alpha and ir_alpha circled — "not needed for roof"; check marks beside L_wf/W, C_E, scaled rise time, scaled duration; beside qs: "← show calculations")

Homework #3

Figure 2-15 Positive Phase Shock Wave Parameters for a Hemispherical TNT Explosion on the Surface at Sea Level



Homework #3

Figure 2-196 Peak Equivalent Uniform Roof Pressures

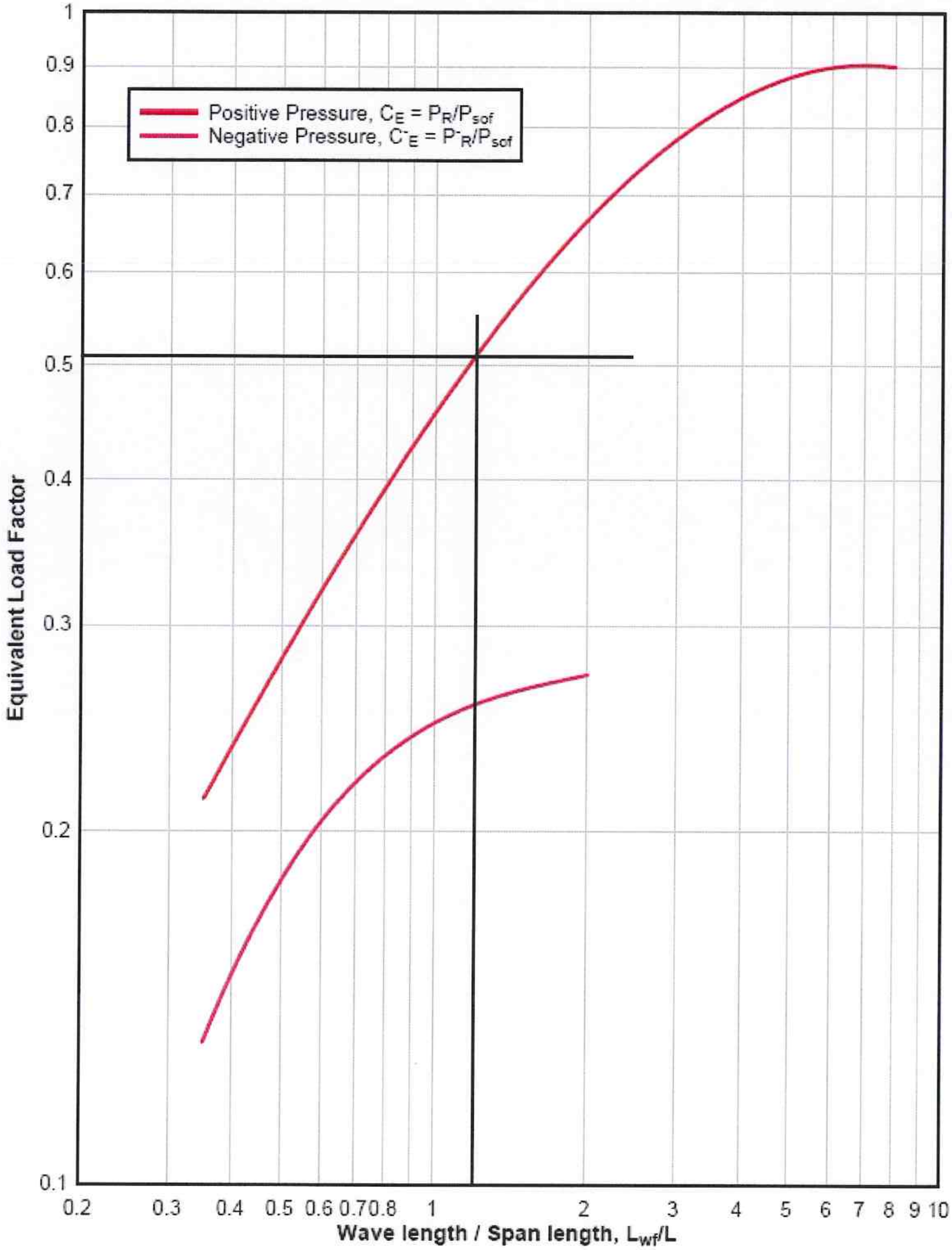
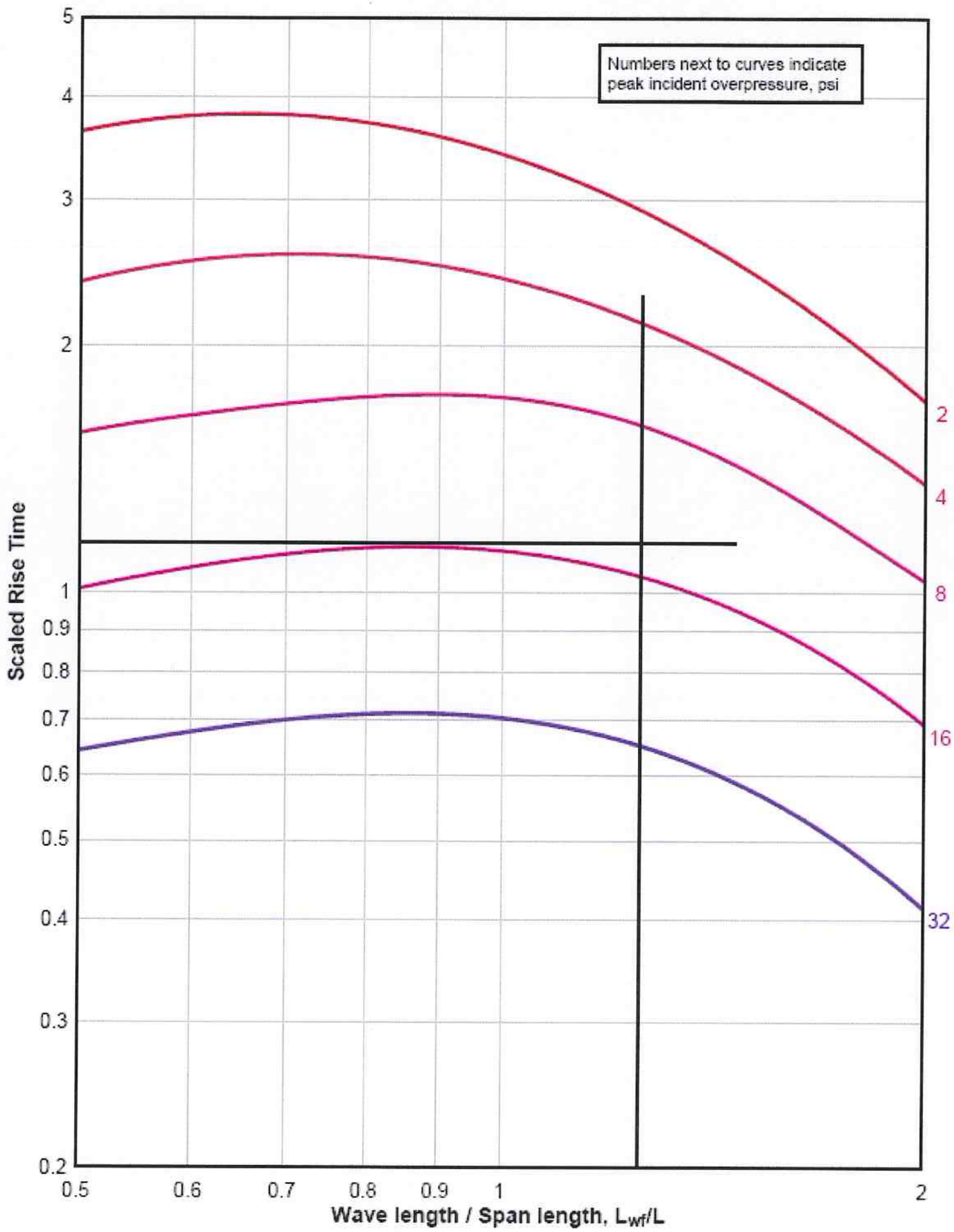


Figure 2-197 **Scaled Rise Time of Equivalent Uniform Positive Roof Pressures**



Homework #3

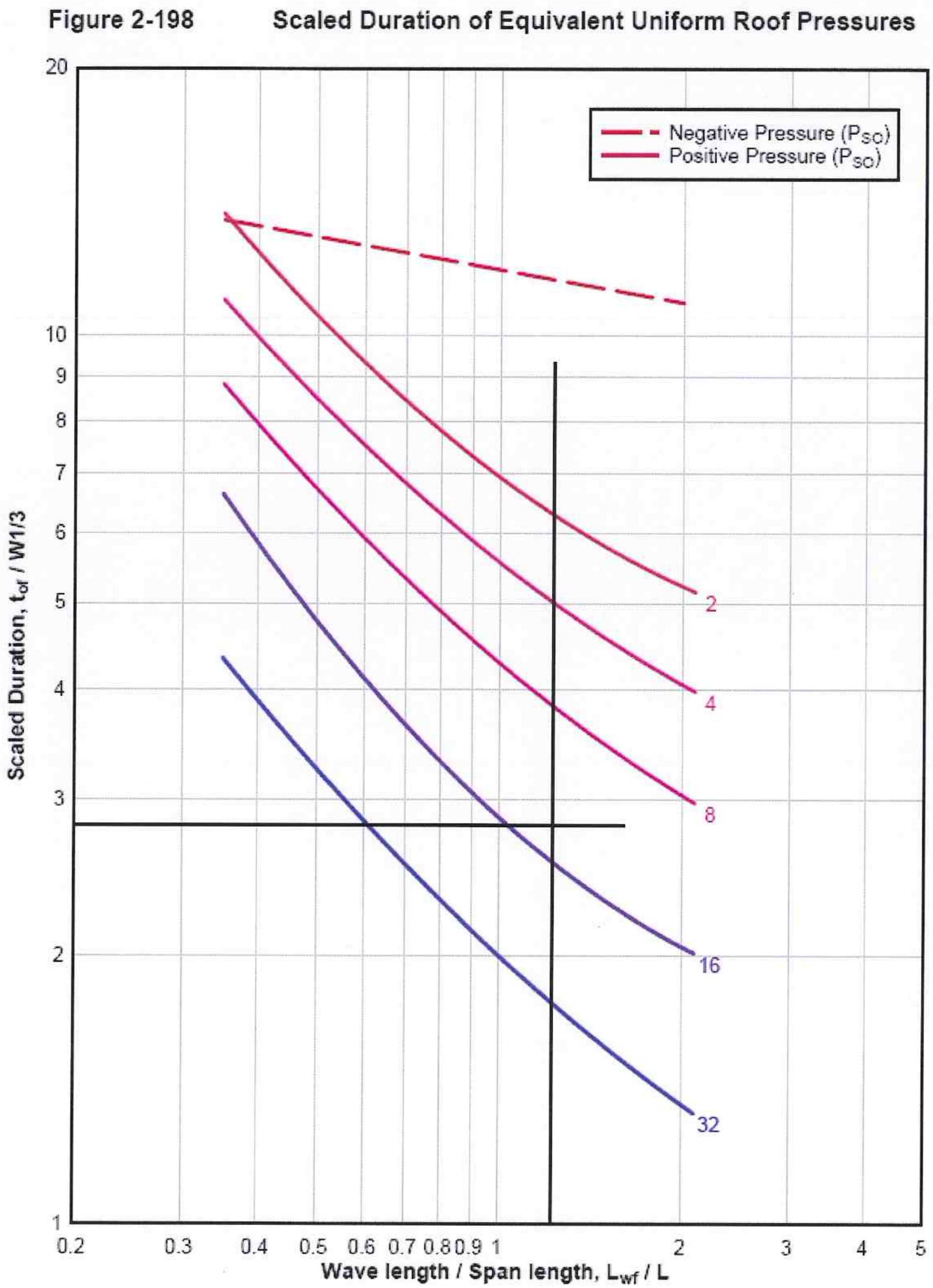


Figure 2-198 Scaled Duration of Equivalent Uniform Roof Pressures
Negative Pressure (Pso)
Positive Pressure (Pso)
Scaled Duration, tof / W1/3
Wave length / Span length, Lwf / L
20
10
9
8
7
6
5
4
3
2
1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
3
4
5
2
4
8
16
32

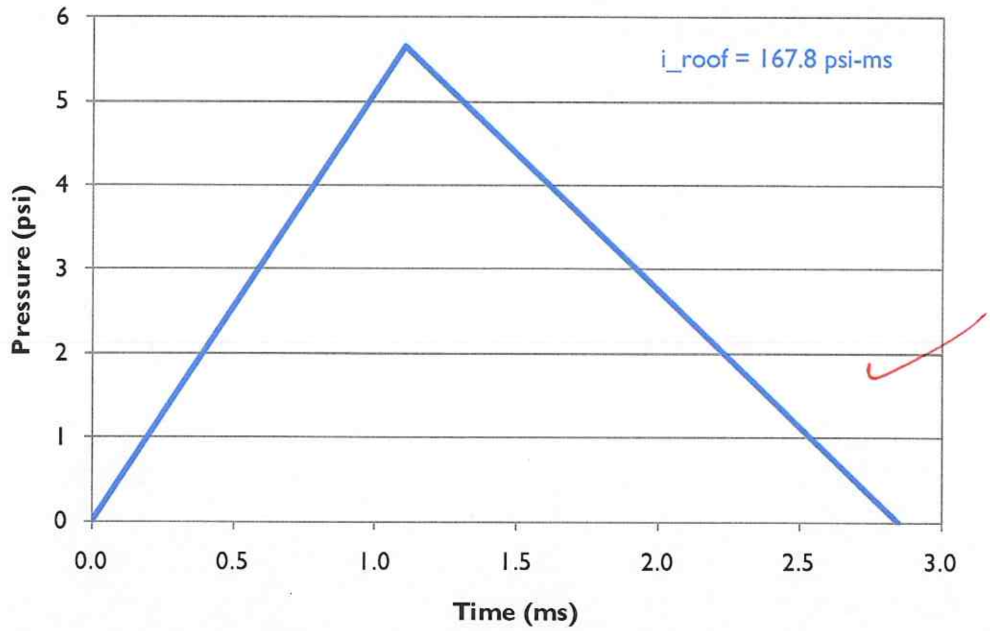
CE397: BLAST

C. HOVELL
25 Feb. 2010

14/14

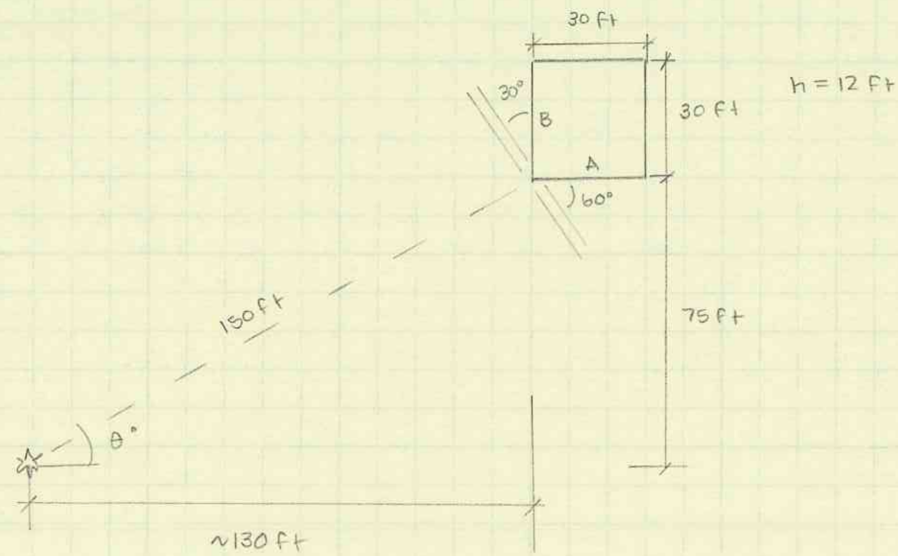
Homework #3

Time-History for Roof



STAEDTLER® No. 937 811E Engineer's Computation Pad

HOMEWORK #3



Begin calculations by considering a point in the middle of the wall. Later, check difference between those loads and if the front corner were used (front corner should yield larger forces than middle, which are both then larger than the far corner).

Calculations shown for side A; same procedure followed for B

$$R_G \text{ (to middle of side A)} = \left[(130\,ft + 15\,ft)^2 + (75\,ft)^2\right]^{1/2} = 163.2\,ft$$

$$Z = \frac{163.2\,ft}{[1.8(5000\,lb)]} = 7.84\,ft/lb^{1/3}$$

for comparison, $Z_{\text{near corner}} = 7.2$, $Z_{\text{far corner}} = 8.5$

To account for angle of incidence, we need P_{so} for Fig. 2-193

$P_{so_{chart}} = 10$ psi $\quad P_{so_{TM\,855}} = 11.9$ psi $\quad P_{so_{Henrych}} = 10.7$ psi

checking other corners, $P_{so} = 11$ psi, 9 psi

To be (theoretically) conservative, use largest P_{so} value (TM855).

Using Fig 2-193, $\alpha = 60°$, $P_{so} = 12$ psi $\longrightarrow C_{r\alpha} = 1.98$

$P_{so} = 10$ psi $\quad C_{r\alpha} = 2.0$

Note: a higher pressure P_{so} results in a lower $C_{r\alpha}$. Using the middle of the wall is thus likely going to provide a suitable estimate for load across the wall.

STAEDTLER® No. 937 811E Engineer's Computation Pad

HOMEWORK #3

Now use $C_{r\alpha}$ to calculate P_r on wall

$$P_{r\alpha} = C_{r\alpha} \cdot P_{so} = (1.98)(11.9\ \text{psi}) = 23.7\ \text{psi}$$

↑ while $C_{r\alpha}$ was larger using a smaller P_{so}, this pairing of $C_{r\alpha}$ and P_{so} provides a higher (more conservative) load.

Calculate clearing time:

$$t_c = \frac{4HW}{(W+2H)\,C_r} = \frac{4(12\text{ft})(30\text{ft})}{[30\text{ft} + 2(12\text{ft})] \cdot C_r}$$

C_r is found using chart provided in notes
[use original $P_{so} = 12$ psi]

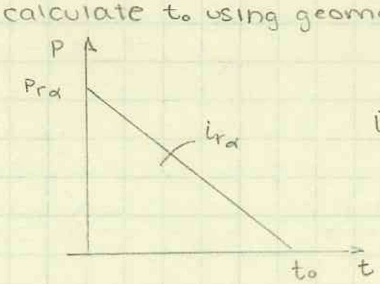
$$C_r = 1.31$$

$$t_c = \frac{(48\text{ft})(30\text{ft})}{(54\text{ft})(1.31)} = 20.36\ \text{ms}$$

Find i_r to calculate t_o using 2-194(b)

$$\frac{i_{r\alpha}}{W^{1/3}} = 11, \quad i_{r\alpha} = (11)(1.8 \cdot 5000\,\text{lb})^{1/3} = 228.8\ \text{psi}\cdot\text{ms}$$

back calculate t_o using geometry



$$i_{r\alpha} = \tfrac{1}{2} P_{r\alpha} \cdot t_o$$

$$t_o = \frac{2(228.8\ \text{psi}\cdot\text{ms})}{23.73\ \text{psi}}$$

$t_o = 19.3\ \text{ms} < t_c$

no clearing takes place

If $t_o > t_c$, next, calculate stagnation pressure

$$P_{stag} = P_{so} + C_D \cdot q_s$$

(C_D = 1.0)

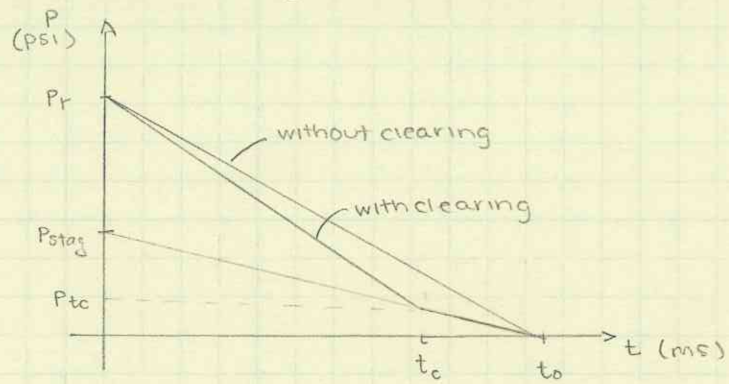
$$q_s = \frac{P_{so}^2}{0.4P_{so} + 41.2}$$

$$P_{t=t_c} = (t_o - t_c)\frac{P_{stag}}{t_o}$$

STAEDTLER® No. 937 811E Engineer's Computation Pad

HOMEWORK #3

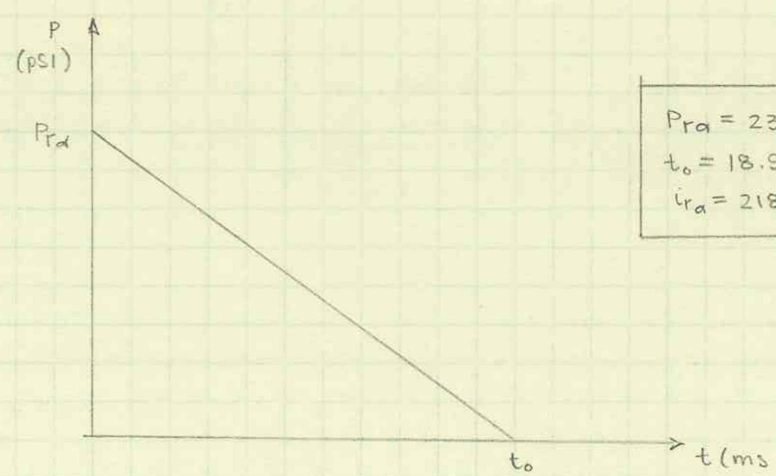
Generalized time history



calculate impulse, area beneath the curve, for case with clearing

$$i_r = \frac{1}{2}\left[P_{tc} \cdot (t_o - t_c) + (P_{tc} + P_r) \cdot t_c \right]$$

Time history for wall A



$P_{ra} = 23.1$ psi
$t_o = 18.9$ ms
$i_{ra} = 218.4$ psi·ms

when evaluated at midspan of the wall section

Now, return to question of if midpoint is reasonable.

decrease in Z → increase in P_{so}

12 psi to 14.5 psi

- decrease in C_{ra}: 1.95 to 1.9 → increase in P_{ra}: 23 to 27.5 → slight decrease in t_o, 18.9 to 18.2 ms
- increase in $\frac{i_{ra}}{W^{1/3}}$: 10.5 to 12 → i_r: 218 to 250
- increase in C_r: 1.31 to 1.33 → $t_c > t_o$ in both cases

Conclusions on next page

No. 937 811E Engineer's Computation Pad STAEDTLER®

HOMEWORK #3

Given the small effects on impulse, load, and time of incident associated with moving the point of interest from the middle of wall A to the front corner, I would use the load data obtained from the mid·point. If in design, I found those loads to cause a response in the structure close to the limits of acceptability, I would switch to the load history found at the front corner.

Calculations for wall B are presented in the attached pages.

Roof loading

$$P_R = C_E P_{sof} + C_D q_{of}$$

P_{sof}, q_{of} come from first point of contact

use corner of walls A and B

use h in R_G calculation to reduce amount of conservativism.

$$Z = \frac{[(150ft)^2 + (12ft)^2]}{(1.8 \cdot 5000lb)^{1/3}} = 7.23\ ft/lb^{1/3}$$

using TM855 equation, $P_{so} = 14.34$ psi

(TM855 equation: provides the highest value at this Z)

$$q_{of} = \frac{P_{sof}^2}{0.4P_{sof} + 41.2} = 4.38\ psi$$

use chart 2-15 (using $Z = 8.8 ft/lb^{1/3}$) to get L_{wf}

$$\frac{L_{wf}}{W^{1/3}} = 2.1, \quad L_{wf} = 35.9\ ft$$

(W = 5000 lb)

use chart 2-196 to find C_E

$$\frac{L_{wf}}{L} = \frac{35.9ft}{30ft} = 0.85$$

$C_E = 0.41$

using 30 ft is more appropriate than a diagonal distance, given the likely orientation of the roof beams. As well, a small L results in a larger C_E and larger P_R, and is thus more conservative.

HOMEWORK #3

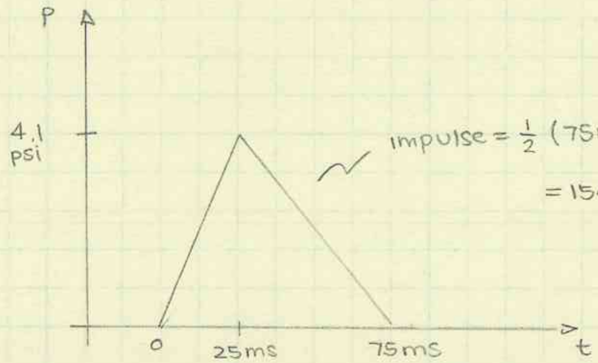
From notes, $C_D = -0.40$ given $q_o < 25$ psi

$$P_R = (0.41)(14.34\text{ psi}) + (-0.40)(4.38\text{ psi}) = 4.13\text{ psi}$$

To plot time-history, find scaled rise time and duration from 2-197 and 2-198

scaled rise time $= 1.2W^{1/3} = 24.5$ ms

scaled duration $= 3.6W^{1/3} = 74.9$ ms



impulse $= \frac{1}{2}(75\text{ms})(4.1\text{ psi})$

$= 154.5$ psi·ms

As with the wall systems, if this load produces a response near the limit of what is acceptable, it could be tightened up (e.g. change the roof length for different points of impact) to confirm or improve accuracy of numbers.

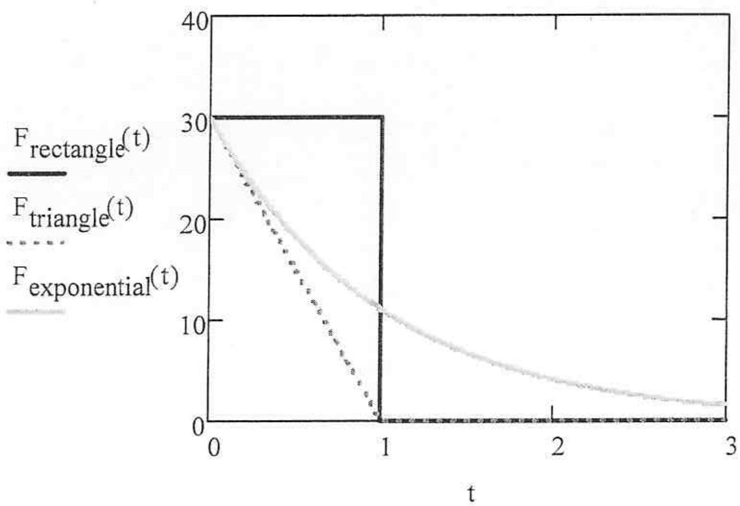
University of Texas
Dept. of Civil, Arch. & Env. Eng.

Instructor: E. B. Williamson

CE 397 Blast-Resistant Structural Design

Homework 4 **Due: 25 March 2010**

Using your choice of programming language (e.g., C, Fortran, Visual Basic, etc.) or available software (e.g., Mathcad, Matlab, Excel, etc.), develop the ability to analyze the dynamic response of a single-degree-of-freedom system. You should develop two different numerical solution procedures: one based on the Newmark-β method, and the other based on the Central Difference method. To validate the performance of your program, you are to develop a P-I diagram that shows the effect of load variation on response. In particular, you are to consider three different load inputs: (a) a rectangular pulse, (b) a triangular pulse, and (c) an exponential pulse. Schematics of these functions are shown in the figure below. Normalize the computed results in a manner that allows the P-I diagrams for all three load cases to be presented on the same graph.



29/30

HOMEWORK #4

Several steps were taken to create a force-impulse (F-I) diagram for the system. First, the equations for the asymptotes expected in the quasi-static and impulsive regions were derived by hand. From these equations, the values that were needed to plot the response were determined. Two numerical methods were used (Newmark's Method and the Central Difference Method) to approximate the maximum deflection under different loading conditions. Finally, the data were plotted, normalized to show each forcing function on one plot.

Axes and Asymptote Derivation

The response of a structural system under load is highly dependent on the relationship between the duration of loading and the natural period of the system. When the duration is relatively small compared to the natural period, the system responds impulsively. When the duration is relatively large compared to the natural period, the system responds quasi-statically. The X- and Y-axis of the plot are scaled to the impulsive and static responses, respectively. The exact relationship depends on the nature of the system; specifically, the shape of the force-displacement curve.

To begin formulation of the F-I diagram, the relationship between applied load and response of the system can be found. The X-coordinate of any point will be calculated using the impulsive relationship while the Y-coordinate will be calculated using the quasi-static relationship.

The quasi-static asymptote was found by equating the external work done by the load to the internal energy of the system (Equations 1(a) through (c)). The internal energy can be found by calculating the area beneath the force-deflection plot. For a system with a constant stiffness, the force-deflection plot is a line of constant slope, *k*.

$$\text{External Work [done by load]} = \text{Internal Energy} \qquad \text{Equation 1(a)}$$

$$F_o x_{max} = \tfrac{1}{2} k x^2_{max} \qquad \text{Equation 1(b)}$$

$$\frac{2F_o}{k x_{max}} = 1.0 \qquad \text{Equation 1(c)}$$

✓

where:

F_o = Maximum force applied, kip
x_{max} = Maximum displacement due to applied load, in.
k = Stiffness of system, kip/in.

When the forces applied occur over a long duration, the dynamic response should nearly match the static response, and thus the relationship given in Equation 1(c) should be true. When the force applied occurs over a shorter duration, the response will be impulsive rather than quasi-static. In this case, the internal kinetic energy of the system can be equated to the strain energy (or internal energy) of the system (Equations 2(a), (b), and (c)). When the duration of load is very short, the response will essentially be impulsive and Equation 2(d) will be appropriate.

$$\text{Initial Kinetic Energy} = \text{Strain Energy} \quad \text{Equation 2(a)}$$

$$\tfrac{1}{2} mv^2 = \tfrac{1}{2} kx^2_{max} \quad \text{Equation 2(b)}$$

$$\frac{I^2}{2m} = \tfrac{1}{2} kx^2_{max} \quad \text{Equation 2(c)}$$

$$\frac{I}{x_{max}\sqrt{km}} = 1.0 \quad \text{Equation 2(d)}$$

✓

where:

m	=	Mass of the system
v	=	Initial velocity of the system
I	=	Impulse applied

In Equations 1(c) and 2(d), there are several constants and several variables. The constants are generally provided by the problem statement, whereas the variables are calculated. The source of each number is provided in Table 1.

Table 1: Summary of sources for values used in calculations and graphing.

Reference	Source of Value
k	Constant, assumed to be 1.0 ✓
m	Constant, assumed to be 1.0 ✓
F_o	Constant, assumed to be 30 kip ✓
I	Calculated using F_o and duration; duration varied to capture entire spectrum of response. ✓
x_{max}	Calculated using Newmark's Method or the Central Difference Method. Dependent on k, m, F_o, and duration of loading. ✓

Response Calculation

The response of the system was estimated using two numerical methods, Newmark's Method and the Central Difference Method. A program was written in Matlab to perform the necessary operations. As the response was desired across a range of loading schemes, two loops were programmed. The general structure of the program is given in Figure 1. In the outer loop, the duration of loading was varied from 0 to $3T_n$. The inner loop existed for the numerical calculation methods; an increment of time was established relative to the natural period of the system and then was used to accurately estimate the response of the system. ✓

quasi-static asymptote approached for duration → 40 Tn

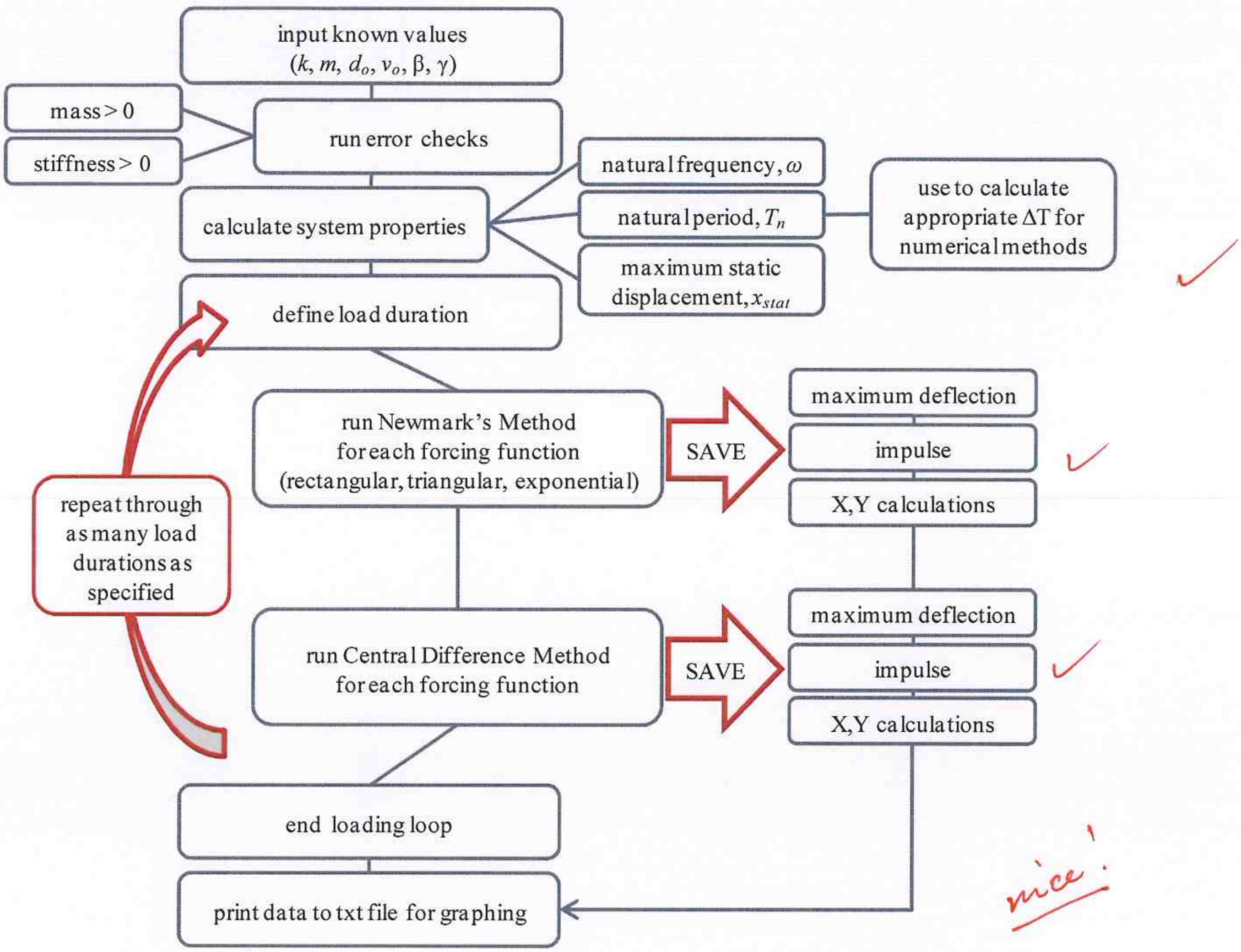


Figure 1: Schematic of program written in Matlab to determine F-I diagram data points.

The importance of using an appropriate time step was most evident in the rectangular loading scheme. With ΔT = T_n / 200, the F-I diagram that resulted was very jagged in the impulsive region (Figure 2(a)). When ΔT was reduced to T_n / 1000, the approximation improved (Figure 2(b)). However, the time needed for calculations increased significantly. To improve the performance of the program, the larger timestep was maintained, even though it was less accurate than desired, because the error was only seen in the region where the behavior can be estimated with a straight line. The visible error could also be decreased by increasing the difference in duration of loading between cycles of calculation, as each point of maximum and minimum error would not be used in formulating the plot.

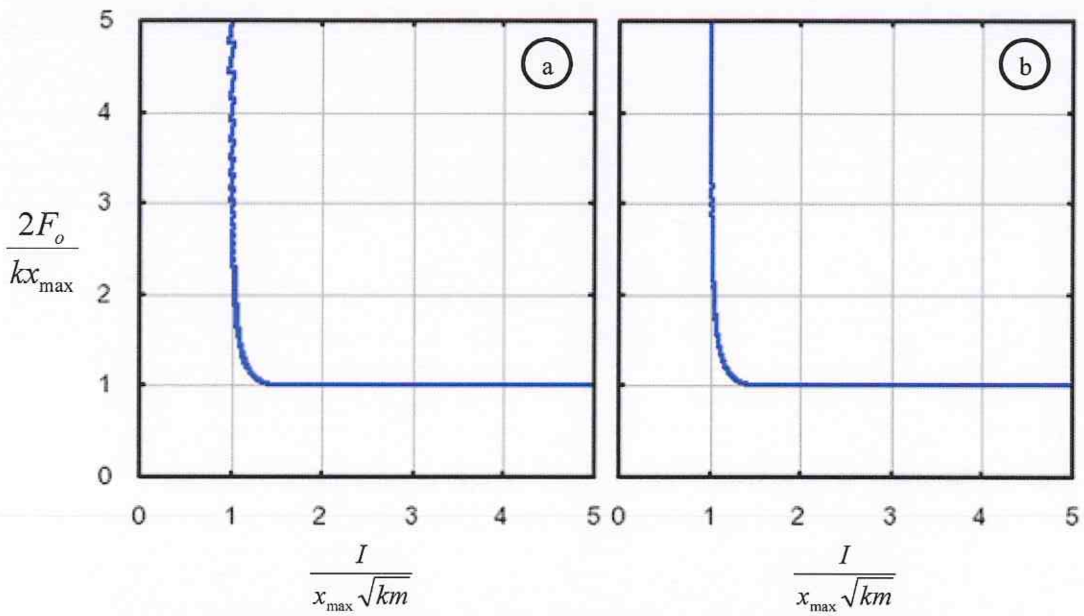


Figure 2: (a) A large time step results in poor approximations in the impulsive region, which can be improved by using a smaller time step (b).

The size and length of the time steps used for varying the loading and performing numerical approximations are summarized in Table 2.

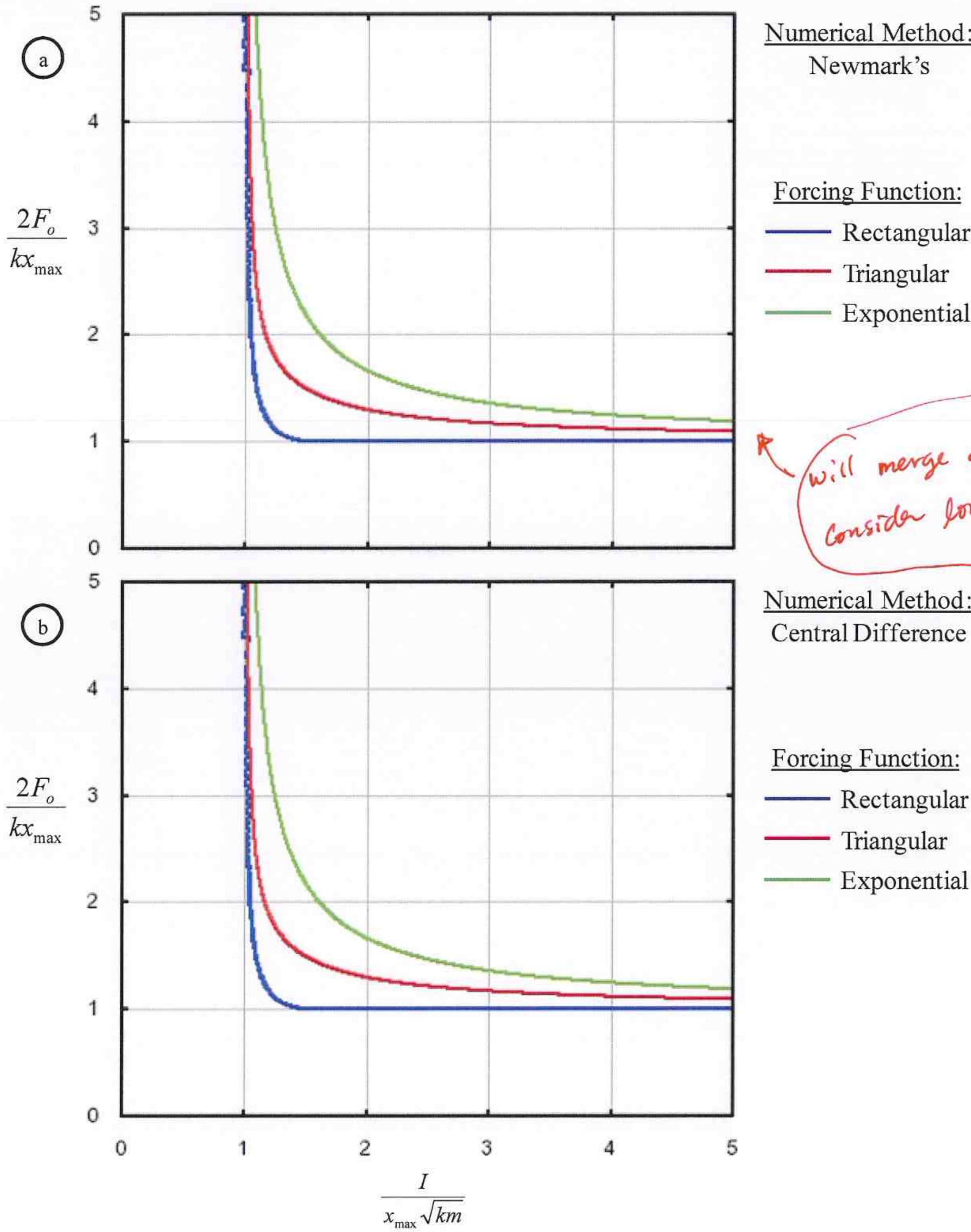
Table 2: Size and length of time steps used for calculations.

Reference	Time
minimum duration	0
maximum duration	$3T_n$
duration step size	0.01
ΔT for numerical methods	minimum of: $T_n/300$, $T_n/(10\pi)$
range of calculation	$3T_n$

small, but not 0
should be larger
make relative to Tn

Force-Impulse Diagram

The force-impulse diagram for a generalized system was plotted using the results from both methods for approximating the response of the system. The two diagrams are given in Figure 3(a) and (b). The difference between the two approximations was extremely small. The average difference in calculated x_{max} across all load durations was on the order of 10^{-4}. The maximum difference in calculated X- and Y-values used in the graphs was on the order of 10^{-2}.



will merge as you consider longer durations

Figure 3: Force-impulse diagrams found using (a) Newmark's Method and (b) the Central Difference Method to approximate the maximum displacement of the system under load.

nice write-up.

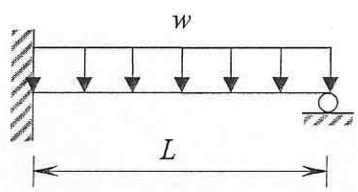
University of Texas
Dept. of Civil, Arch. & Env. Eng.

Instructor: E. B. Williamson

CE 397 Blast-Resistant Structural Design

Homework 5 **Due: 15 April 2010**

Extend the homework solution you prepared for HW 4 to compute the response of an SDOF system that undergoes inelastic deformation. You may choose to implement either the Central Difference method or the Newmark-β method (i.e., you are not required to use both as in HW 4). Your program need only consider one change in stiffness; thus, you can define an effective spring constant for those cases in which the true behavior would suggest a transition from an elastic response to an elastic-plastic response prior to forming a mechanism. Using your program, compute the response of the system shown in the figure below.



$E = 29,000$ ksi $\quad I = 1240$ in^4 $\quad S = 173$ in^3 $\quad Z = 192$ in^3 $\quad L = 20$ ft $\quad F_y = 50$ ksi

$I = 1240$ in^4 $\quad w = 27$ k/ft $\quad duration = 30$ msec $\quad self\text{-}weight = 109$ lb/ft

Graph the following response quantities: (a) Displacement versus time, (b) resistance versus time, (c) resistance versus displacement, and (d) reaction force versus time at each end of the beam.

CATHERINE HOVELL
20 April 2010

HOMEWORK #5

28/30

To analyze the system shown in Figure 1, a program was written in Matlab, based off the program presented in Homework #4. The properties of the equivalent single degree of freedom system used were taken from Table 5.3 and are reproduced in Table 1.

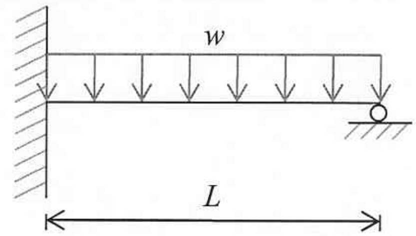


Figure 1: System to be analyzed.

Table 1: Equivalent single degree of freedom properties for system shown in Figure 1.

Single DOF Property	Value
Load-Mass Factor, K_{LM}	
Elastic	0.78
Elastic-Plastic	0.78
Plastic	0.66
Maximum Resistance, R_m	$12M_p/L$
Effective Spring Constant, k_E	$\frac{160EI}{L^2}$
Dynamic Reaction: Elastic Range	
at Wall	$0.26R+0.12F$
at Support	$0.43R+0.19F$
Dynamic Reaction: Plastic Range	
at Wall	$0.39R+0.12F+M_p/L$
at Support	$0.39R+0.12F-M_p/L$

REACTIONS ARE SWITCHED. REACTION @ WALL GREATER THAN REACTION @ PIN

CORRECT IN YOUR PROGRAM

The loading applied to the beam, w, was assumed to decay linearly over the duration of loading, 30 ms (0.03 sec). The Central Difference Method was used to approximate the displacements at a time $(t + 1)$ by solving for equilibrium at time t. As the resistance, $R(d_t)$, varied with the displacement d_t and was necessary for solving for d_{t+1}, a necessary step within the computational process was the determination of R_t. The following logic sequence was used:

```
if (Ans.D(CD) > d_max)
        d_max  = Ans.D(CD);
        d_cent = d_max - cfg.dy;
        if  (d_cent < 0)
                d_cent = 0;
        end
end

Ans.R(CD) = cfg.keff*(Ans.D(CD)-d_cent);
test.EP   = 1;

if (Ans.R(CD) >= cfg.Rm)
        Ans.R(CD) = cfg.Rm;
        test.EP   = 2;
end
if (Ans.R(CD) <= -cfg.Rm)
        Ans.R(CD) = -cfg.Rm;
        test.EP   = 2;
end
```

After computing deflection at each step (`Ans.D(CD)`), the deflection was compared against the maximum deflection, originally defined as the deflection at time 1. If deflection is increasing, a new maximum is set. With that new maximum comes a new center around which an elastic curve would exist, so long as the center is greater than zero. The maximum resistance is set at R_m, that which would occur when the system is yielding. The `test.EP` value dictates whether the system is yielding (2) or not (1), for use in determining support reactions.

The following plots show the resulting displacement versus time (Figure 2(A)), resistance versus time (Figure 2(B)), resistance versus displacement (Figure 3(A)), and reaction force versus time (Figure 3(B)). The logic used to determine resistance as a function of displacement shows very little residual error; while the calculated resistance follows the linear path between -200 and 200 kip several times, the line still appears minimally wider than if a single line had been drawn.

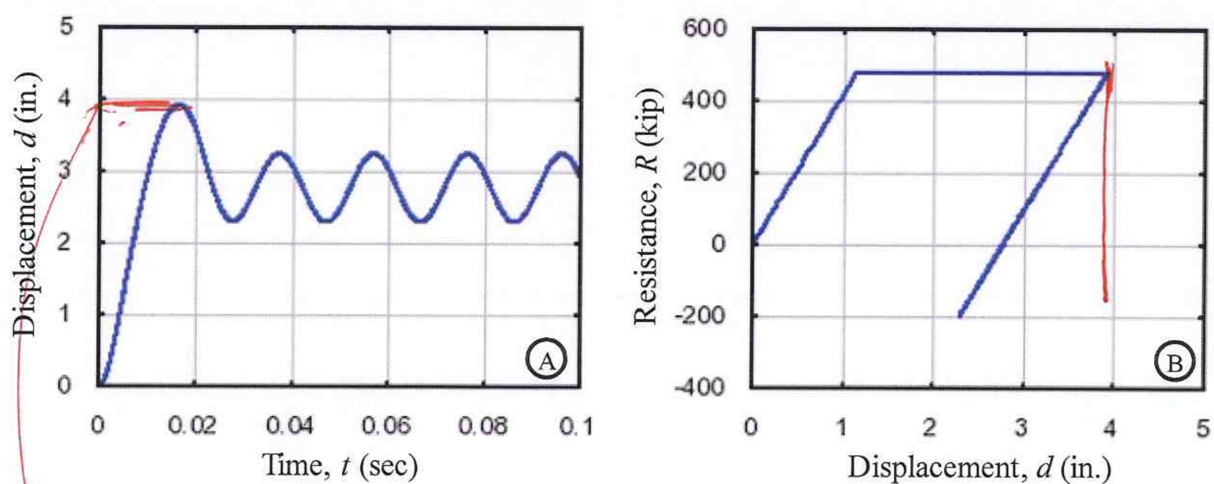


Figure 2: (A) Displacement versus time and (B) resistance versus displacement.

You have over-estimated the displacement by a small amount – hard to tell exact value based on graph

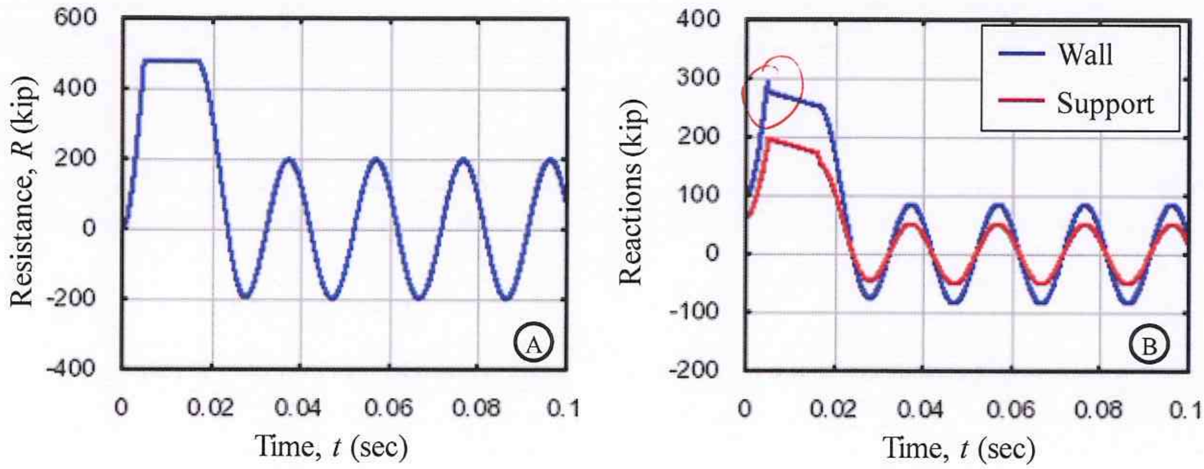


Figure 3: (A) Resistance versus time and (B) reaction forces versus time.

```
clear all
clc

% Define inputs - UNITS are KIPS and INCHES
        cfg.g   = 386.4;                  % gravity                         [in/s^2]
        cfg.m   = 0.109/(12*cfg.g);       % mass per unit length            [k-s^2/in^2]
        cfg.L   = 20*12;                  % length of beam                  [in]
        cfg.mT  = cfg.m*cfg.L;            % total mass of system            [k-s^2/in]

        cfg.I   = 1240;                   % moment of inertia               [in^4]
        cfg.S   = 173;                    % section modulus                 [in^3]
        cfg.Z   = 192;                    % plastic section modulus         [in^3]

        cfg.E   = 29000;                  % modulus of elasticity           [ksi]
        cfg.Fy  = 50;                     % yield stress                    [ksi]
        cfg.Mp  = cfg.Z*cfg.Fy;           % plastic moment capacity         [k-in]
        cfg.Ms  = cfg.Mp;                 % moment capacity of support      [k-in]
        cfg.Rm  = 12*cfg.Mp/cfg.L;        % maximum spring resistance       [k]

        cfg.w   = 27/12;                  % max applied distributed load    [k/in]
        cfg.Fo  = cfg.w*cfg.L;            % max resultant load              [k]
        cfg.dur = 0.030;                  % duration of triangular load     [msec]

        cfg.KLM.E       = 0.78;           % load-mass factor, elastic       [-]
        cfg.KLM.EP      = 0.78;           % L-M factor, elastic-plastic     [-]
        cfg.KLM.P       = 0.66;           % load-mass factor, plastic       [-]
        cfg.KLM.Use     = 0.5*(0.5*(cfg.KLM.E+cfg.KLM.EP)+cfg.KLM.P);
                                          % effective load-mass factor      [-]
        cfg.mUse        = cfg.mT*cfg.KLM.Use;
                                          % effective mass for calcs        [k-s^2/in]

        cfg.keff        = 160*cfg.E*cfg.I/(cfg.L^3);
                                          % effective spring constant       [k/in]
        cfg.dy  = cfg.Rm/cfg.keff;        % displacement at first yield     [in]

        d_o     = 0;                      % initial displacement            [in]
        v_o     = 0;                      % initial velocity                [in/s]

% Error checking
        if (cfg.mT<=0)
                error('Mass less than or equal to zero.');
        end
        if (cfg.keff<=0)
                error('Stiffness less than or equal to zero.');
        end
        if (d_o>cfg.dy)
                error('System at yield at onset of loading.');
        end

% System properties
        omega   = sqrt(cfg.keff/cfg.mUse);        % natural frequency of the system
        Tn      = 2*pi/omega;                     % natural period of the system
        x_stat  = 2*cfg.Fo/cfg.keff;              % maximum static displacement

% Dynamic range calculation requirements
        DT      = Tn/300;                 % deltaT for central difference calculations
        t_end   = 6*Tn;                   % duration of calculations
        steps   = t_end/DT+1;             % number of iterations
        Time    = [0:DT:t_end];           % vector of time steps

% Define forcing functions
nonzero = find(Time<cfg.dur);             % find location of points < duration of load
zeroes  = find(Time>cfg.dur);             % find location of points > duration of load
        f_tri(nonzero)  = cfg.Fo*(1-Time(nonzero)/cfg.dur);
        f_tri(zeroes)   = 0;
```

check to see loading is positive

```
% Define vectors into which data will be stored
Ans.D   = zeros(size(Time));                 % deflection               [in]
Ans.V   = zeros(size(Time));                 % velocity                 [in/s]
Ans.A   = zeros(size(Time));                 % acceleration             [in/s^2]
Ans.R   = zeros(size(Time));                 % resistance               [lb/in]
Ans.VW  = zeros(size(Time));                 % reaction at wall         [lb]
Ans.VS  = zeros(size(Time));                 % reaction at support      [lb]

a_o = (cfg.Fo-cfg.keff*d_o)/cfg.mUse;       % initial acceleration
Ans.D(1)        = d_o;                       %
Ans.V(1)        = v_o;                       % assign initial conditions to vectors
Ans.A(1)        = a_o;                       %
Ans.R(1)        = 0;                         %

% Calculate behavior using Central Difference Method
        d_neg1  = d_o-DT*v_o+DT^2*a_o/2;
        Ans.D(2)= DT^2/cfg.mUse*(f_tri(1)-Ans.R(1)) + 2*d_o - d_neg1;
        if (Ans.D(2)>cfg.dy)
                error('System at yield at d_1.');
        end
        Ans.R(2)= Ans.D(2)*cfg.keff;
        d_max   = Ans.D(1);
        d_cent  = 0;

        CD = 3;
        while (CD<=steps)
                Ans.D(CD)         = DT^2/cfg.mUse*(f_tri(CD-1)-Ans.R(CD-1))...
                                    + 2*Ans.D(CD-1) - Ans.D(CD-2);
                Ans.V(CD-1)       = (Ans.D(CD) - Ans.D(CD-2)) / (2*DT);
                Ans.A(CD-1)       = (Ans.D(CD-2) - 2*Ans.D(CD-1) + Ans.D(CD)) / DT^2;

                if (Ans.D(CD) > d_max)
                        d_max   = Ans.D(CD);
                        d_cent  = d_max - cfg.dy;
                        if (d_cent < 0)
                                d_cent = 0;
                        end
                end

                Ans.R(CD) = cfg.keff*(Ans.D(CD)-d_cent);
                test.EP = 1;

                if (Ans.R(CD) >= cfg.Rm)
                        Ans.R(CD) = cfg.Rm;
                        test.EP = 2;
                end
                if (Ans.R(CD) <= -cfg.Rm)
                        Ans.R(CD) = -cfg.Rm;
                        test.EP = 2;
                end

                if (test.EP == 1)
                        Ans.VW(CD) = 0.43*Ans.R(CD)+0.19*f_tri(CD);
                        Ans.VS(CD) = 0.26*Ans.R(CD)+0.12*f_tri(CD);
                else if (test.EP == 2)
                        Ans.VW(CD) = 0.38*cfg.Rm+0.12*f_tri(CD)+cfg.Ms/cfg.L;
                        Ans.VS(CD) = 0.38*cfg.Rm+0.12*f_tri(CD)-cfg.Ms/cfg.L;
                end
                end

                CD = CD+1;
        end
        Ans.All = [Time'        Ans.D'  Ans.R'  Ans.VW' Ans.VS' Ans.V' f_tri'];
```

effective mass factor needs to be updated depending upon whether response in elastic or plastic

ONLY ALLOWS FOR "POSITIVE" LOADING. IF USER DEFINES LOADING < 0, WILL CAUSE PROBLEMS w/ YOUR SOLUTION

✓ CORRECT - NOT CONSISTENT w/ TABLE ON P.1

```
% Publish data into a file
dlmwrite('CE397_HW5_Output.txt', Ans.All, ' ');
```

EXPERIMENTAL PROJECT

Shock Tube Testing of Structural Components

Structural component testing in a shock tube is used in practice to study the response of blast-loaded elements and to validate analysis and design standards. To determine the accuracy of structural response calculations for blast effects, as well as to determine the suitability of design procedures covered in class, physical tests are to be conducted in the shock tube testing facility furnished by ABS Consulting in San Antonio, TX. A schematic of the end of the shock tube where specimens are placed, along with several photos, are provided in Figures 1-3 below.

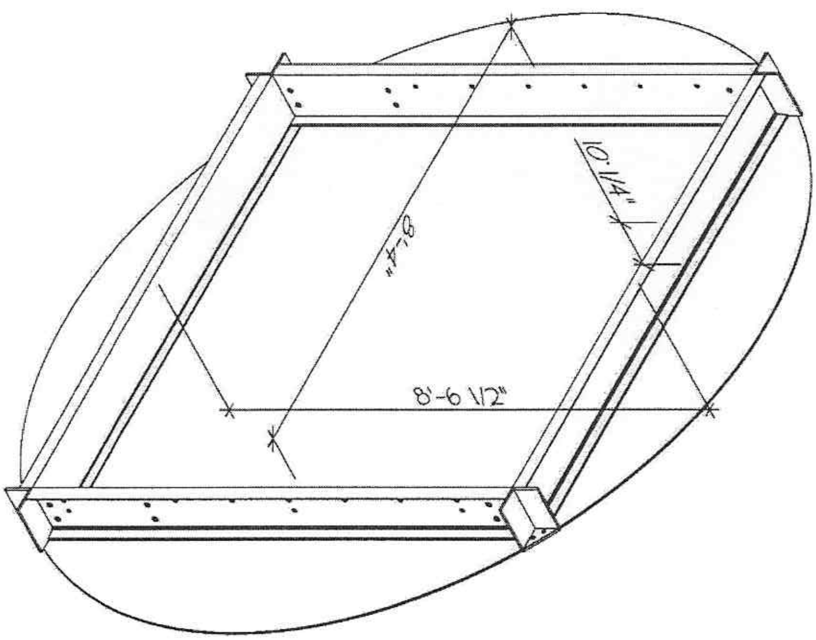


Figure 1: Test Specimen Opening Detail

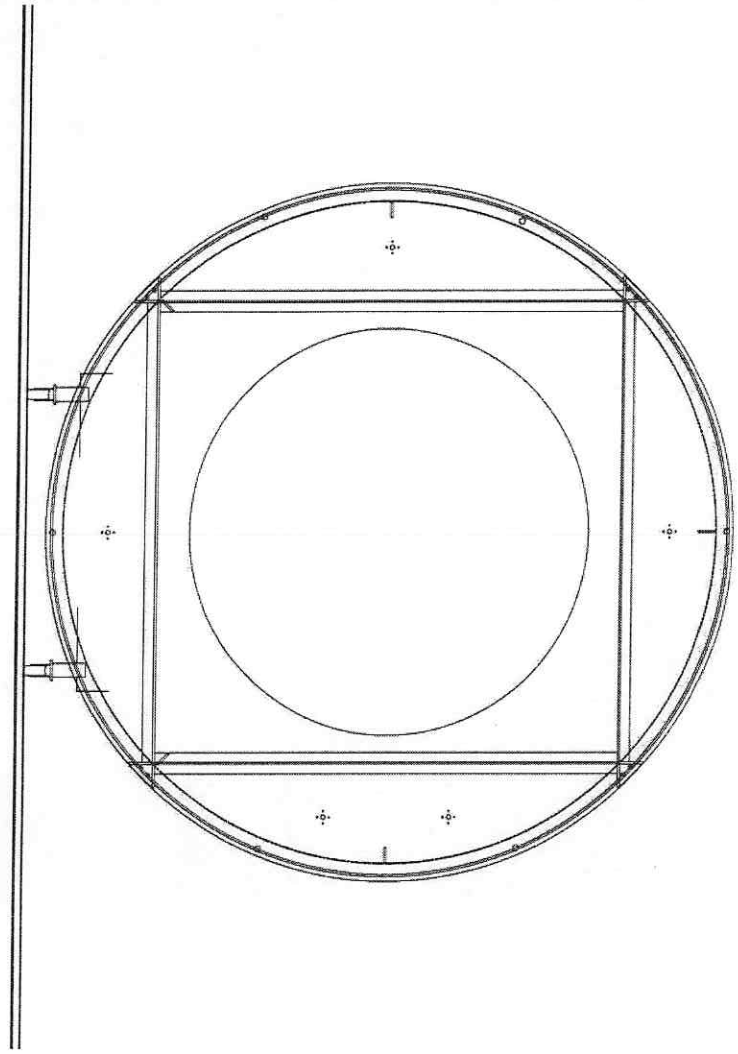


Figure 2: Shock Tube Specimen Opening Detail

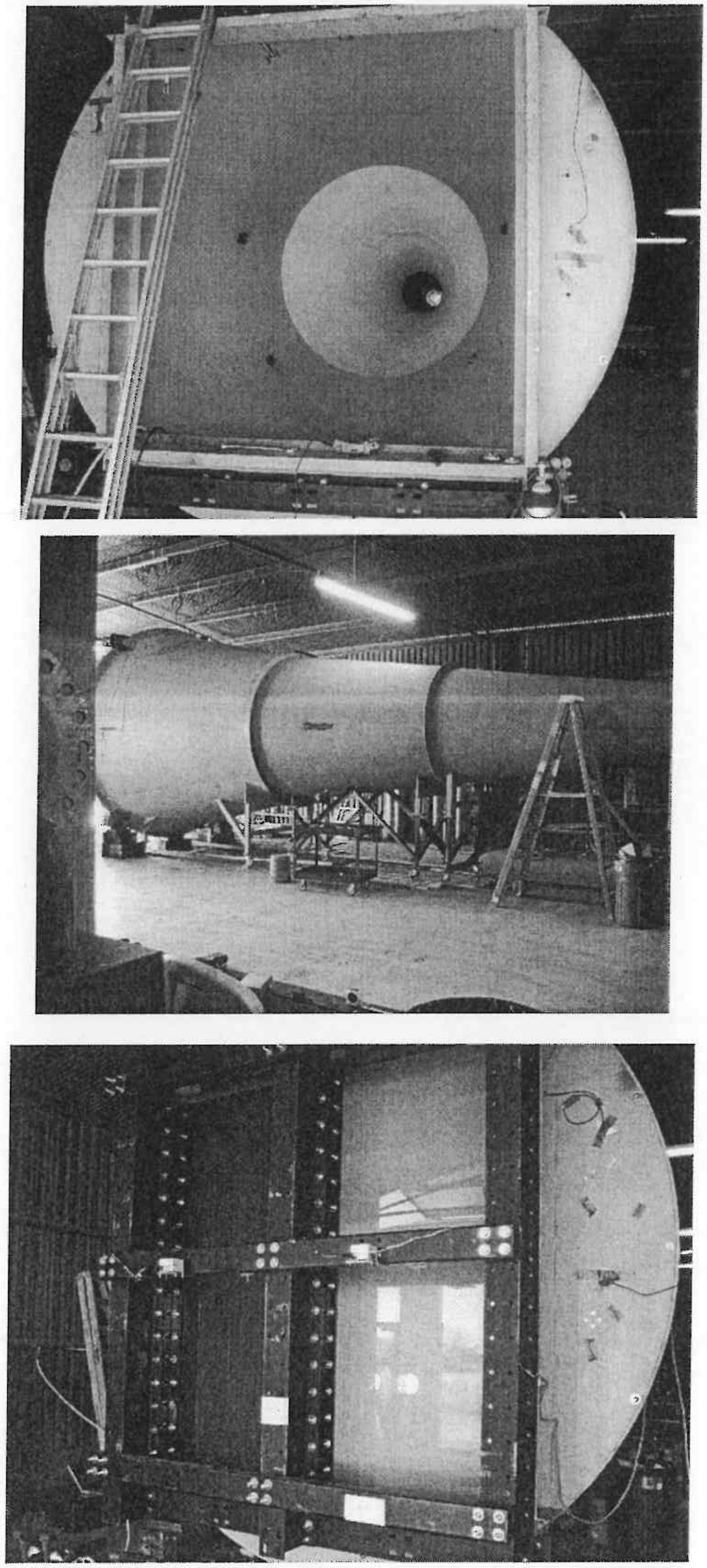


Figure 3: Shock Tube Photos

Shock Tube Testing Considerations:

- The largest blast load capability is P=16 psi and i = 250 psi-msec. It is **strongly recommended** that specimens be designed to fail at 8-10 psi and 150 psi-msec.
- Test items over 150 lbs need to have lifting attachments on the blast face to facilitate loading in the test buck.
- Filler panels can be used to take up the extra space for test items that are smaller than the minimum opening dimensions.
- Load cells can potentially be located on the top half of one side and across one half of the head.
- It is advisable to build the test specimens within a frame so that they can be easily mounted within the shock tube and so that they are less likely to be damaged during transport from Austin.
- Sixteen channels of data acquisition are available, but four are typically reserved for collection of blast load data. The remaining channels can be used to capture strain gage measurements and other test data. A string potentiometer is available for measuring displacements.

Project Description:

The performance of cold-formed steel stud wall systems subjected to blast loads is not well understood. In order to develop a better understanding of how such wall systems behave under blast loads, the class is to design, construct, and test three to four wall units that will be tested in the shock tube at ABS Consulting in San Antonio, TX. Prior to testing, the class will be responsible for predicting the response of each specimen. Based on data collected from the tests, a report will be required for each specimen that describes in detail the pre-test predictions and the accuracy with which the response was predicted. The final report should document all calculations and describe the details of the test program. Oral presentations will be given at the end of the semester (specific date to be announced later). As lead time is needed to purchase the necessary materials and to construct the specimens, development of a testing plan describing specific details of the specimens must be submitted with sufficient time for review and for incorporation of any required modifications. Accordingly, **drafts of project proposals are due 4 March 2010**. As a minimum, project proposals are to include the following information:

- Preliminary drawings of specimens to be tested
- Costs needed for materials and transport
- Preliminary instrumentation plan
- Schedule

Performance of Steel Stud Walls Subjected to Blast Loads

Bryan Bewick[1], John Hoemann[2], Eric Williamson[3]

[1] *Air Force Research Laboratory, Air Base Technologies Division, Engineering Mechanics and Explosion Effects Research Group, Tyndall AFB, FL, USA, Bryan.Bewick@tyndall.af.mil*

[2] *Research Structural Engineer, U.S. Army Engineer Research & Development Center, CEERD-GS-V, 3909 Halls Ferry Road, Vicksburg, MS 39180. (formerly Air Force Research Laboratory Support Contractor, Applied Research Associates, Inc., Tyndall AFB, FL, USA)*

[3] *University of Texas at Austin, Austin, TX, USA, ewilliamson@mail.utexas.edu*

ABSTRACT

Past research has demonstrated that steel stud walls can perform well when subjected to large blast events. The construction methods needed to achieve good performance that take advantage of the inherent ductility offered by steel, however, have been costly and have often required the use of specialized connection details that allow a stud to reach its full flexural and/or tensile capacities prior to connection failure. The goal of the current study is to develop techniques for mitigating large blast loads acting against steel stud walls using conventional construction materials and techniques. Two issues of concern for the current research are: 1) the performance under blast loads of typical connections, either commercial clips or the standard screwed-stud-to-track, has yet to be fully examined, and 2) current methods of design do not incorporate the mechanical interaction of veneer layers for potentially increasing the blast resistance of steel stud walls. To better understand the role played by connection design details and wall system construction details, research for this project includes laboratory testing, field testing, and computational modeling. In this paper, the authors provide an overview of the research program and a summary of the findings that have been developed to date. From the data collected during this project, designs that exhibit a balance of simplistic, economic, and adequate protection will be developed.

INTRODUCTION

Construction trends have brought about an increase in the use of cold-formed steel studs in Air Force facilities. Furthermore, previous research by the Department of State (DOS) and the Engineer Research and Development Center (ERDC) of the U.S. Army Corps of Engineers (DiPaolo and Woodson, 2006) and by the Air Force Research Laboratory (AFRL) (Salim, Dinan, and Townsend, 2005 and Salim, Muller, and Dinan, 2005) have shown that steel stud walls have significant potential for mitigating large blast events. The current state of steel stud research, however, has not addressed all the variables that can influence the behavior of typical wall systems. These previous steel stud research programs were aimed at protection of facilities designed to withstand threats that are more demanding than the typical Unified

Facility Criteria 4-010-01 (UFC, 2007) threats that standard DOD and government facilities are designed to withstand. As a result, there is a research gap that exists in the blast-resistant design of conventional steel stud wall systems. Figure 1 illustrates a typical resistance function for steel stud wall behavior (Salim, Muller, and Dinan, 2005). In order to withstand high demand blast threats, previous efforts have focused on designing the connections to get the full tensile membrane behavior of wall system components. For standard military and government structures, this level of response is far beyond the required capacity and is quite costly. Thus, two areas of research can be identified as still having potential for changing the behavior of these wall systems: 1) the lack of data on connections, either commercial clips or the standard screwed-stud-to-track, and their influence on the allowable response under a blast load have yet to be fully examined; 2) current methods of design do not incorporate the mechanical interaction of veneer layers for potentially increasing the resistance of a steel stud wall.

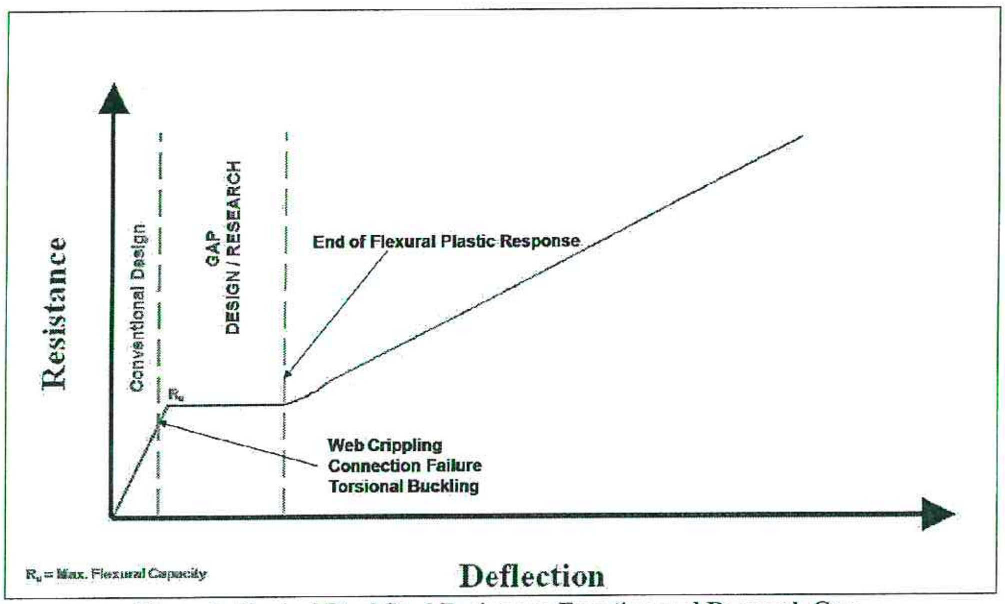


Figure 1: Typical Steel Stud Resistance Function and Research Gap

The first area of research recognizes the development of commercial connectors, intended for hurricane, seismic or large load reversal applications, as having potential in retrofit or new construction applications for blast mitigation. These technologies could form the bridge between the standard screwed-stud-to-track method and the findings of previous work by DOS, ERDC, and AFRL. The previous work led to connection designs for fully developing the axial capacity of steel studs prior to connection failure, but they were large and expensive. Aside from commercial connectors, the issue of ductility limits for traditional connectors has also been recognized by the ASCE Committee on Cold-Formed Steel as a topic that, to date, has not been fully researched.

The second area of research is based on observations from a recent experiment performed by AFRL at Tyndall AFB. A forensic and analytical post-test analysis led to the hypothesis that the non-structural veneer of a steel stud wall system had acted compositely, increasing the overall stiffness of the system (Grumbach, Naito, and Dinan, 2007). In the past, such veneers as stucco or brick have been ignored in calculating the resistance of a wall system, only utilizing their potential as added mass for dynamic analysis and not for providing any strength.

OBJECTIVES

Building from previous research, a main objective of the current study is to create an analytical methodology—validated against test data—that can accurately predict response limit states of various types of steel stud wall assemblies. Another objective is the development of a standard that will allow engineers to have the option of adding the increased resistance of veneers that can perform compositely with cold-formed steel studs. To meet these objectives, the current research project includes detailed finite element analyses and a series of laboratory tests that are intended to measure fundamental aspects of steel stud behavior including connection response. Primary factors in the selection of steel stud wall systems for use in Air Force facilities will be performance under blast loads, system cost, ease of construction, and availability of materials.

CHARACTERIZATION OF STEEL STUD PERFORMANCE

To characterize the response of steel stud walls for use in Air Force facilities, it is important to recognize the wide range of construction practices that exist. Steel stud walls can be used as load-bearing components or non-load-bearing components, and a variety of exterior finishes and internal sheathing may be used. From an economic perspective, it is desirable to select wall configurations that are commonly used along with materials that are readily available. While it is possible to develop significant blast resistance with steel stud walls, tests to date have shown that specially designed fasteners that attach the studs to the structural floor and floor/roof beams are needed to develop this capacity (Dinan, 2005 and Shull, 2002). The use of these special fasteners is costly and requires experienced workers for correct installation. Thus, currently available methods for developing adequate blast resistance are expensive. To meet the objectives of the current project, it is desirable to utilize wall construction techniques that use readily available materials so that costs are kept to a minimum. Accordingly, the test program aims to characterize how standard cold-formed steel stud walls, using common sheathing materials such as drywall, oriented strand board (OSB), stucco, etc., utilizing conventional structural connections (e.g., slip track) and potentially proprietary connection devices, perform under blast loads.

Experimental Test Program. Laboratory experiments have been proposed to assist in the objectives of characterizing the capacity and response behavior of cold-form steel studs. Three component-level experiments have been devised before full-scale static experiments will be performed. The component level experiments are comprised of the following: 1) Tensile Membrane Action (TMA); 2) Bending and Prying Action (BPA) and 3) Crippling and Crushing Action (CCA).

The first series of component experiments, TMA, is for exploring the axial-tensile capacity of the steel-stud-to-track connection. Figure 2 shows the experimental setup. Steel studs are placed back-to-back, for symmetry, and then attached by various screw configurations to the track. As described previously, connection designs have been developed for achieving the full capacity of the steel stud; however, the aim of the TMA experiments is to explore the spectrum between full capacity and the single conventional screw installation (Figure 2(b)). Using this setup, seventy-three samples have been tested in an MTS load frame under quasi-static loading—0.5 inches per minute—to record each scenario's load versus deflection response. The specimens have included combinations of various track and stud thicknesses with different screw diameters and quantities. At the lower end of the spectrum was a 20-gauge track and stud screwed together by a single #8 self-tapping framing screw per flange/stud intersection. On the higher end of the spectrum, a 12-gauge track and stud were similarly placed together with six #12 self-tapping screws per flange/stud intersection. An additional nine samples were examined at an increasing loading rate up to 2.0 inches per minute.

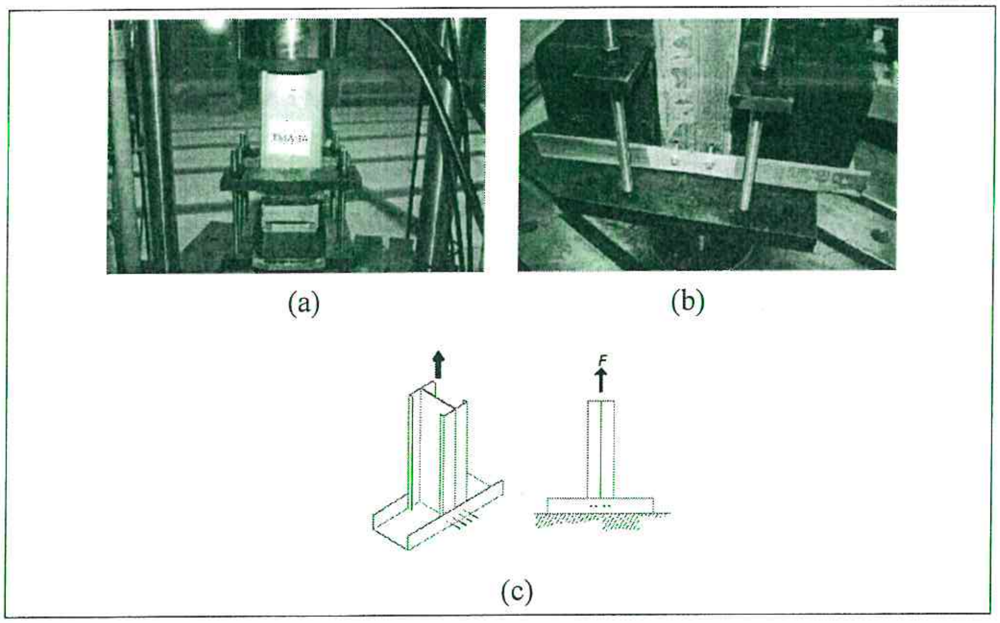


Figure 2: TMA Experimental Setup

The second series of component tests is the BPA experiments. This series examines the identical test matrix as the TMA series but subjects the samples to rotation and shear through a cantilever loading condition (Figure 3). The objective is to investigate rotational capacity of the stud in the track. The track is assumed to be held rigidly to the support with the focus of the testing to determine the degree of rotation at which the track and stud disconnect. Similarly to the TMA series, seventy-three samples have been tested in an MTS load frame under displacement control at a loading rate of 0.5 inch per minute. An additional nine samples were examined at varying rates up to 2.0 inches per minute.

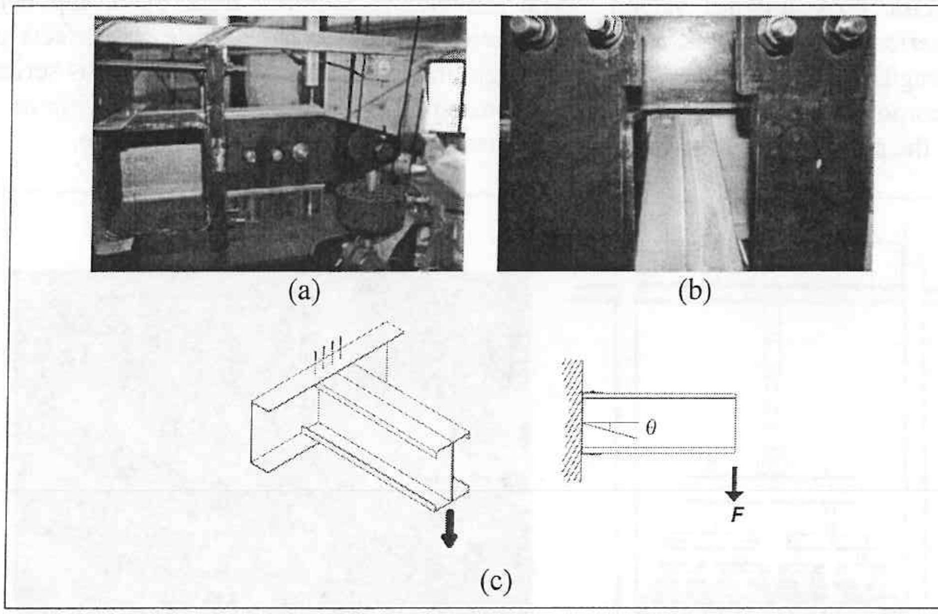


Figure 3: BPA Experimental Setup

The third and final component series exploring cold-form steel capacity is the CCA experiments. The purpose of this test series is to evaluate the shear or crippling capacity of the studs inside of the track channel. It is hypothesized that studs with deep webs and/or thin gauge sections have additional absorption capabilities not mathematically accounted for in current blast design procedures. Current procedures focus only on the flexural absorption of the steel stud and use the shear or connection capacity as a limit state (SBEDS 2008). At the time of this writing, the BPA series was still in progress. Figure 4 shows the generalized schematic of the test setup; the experiment utilizes an MTS load frame under displacement control and is patterned similarly to a four-point bending experiment.

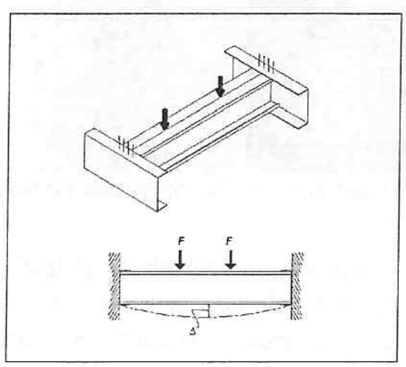


Figure 4: CCA Experimental Setup

Full-Scale Experimental Series. With knowledge acquired from the component level series of experiments, a full-scale series is planned to evaluate the effects of span length, materials, and connection design on wall system behavior. In this series, the incorporation of veneer will be studied as a point of additional capacity. Figure 5 shows the proposed setup with a 16-point loading tree and an arbitrary sample.

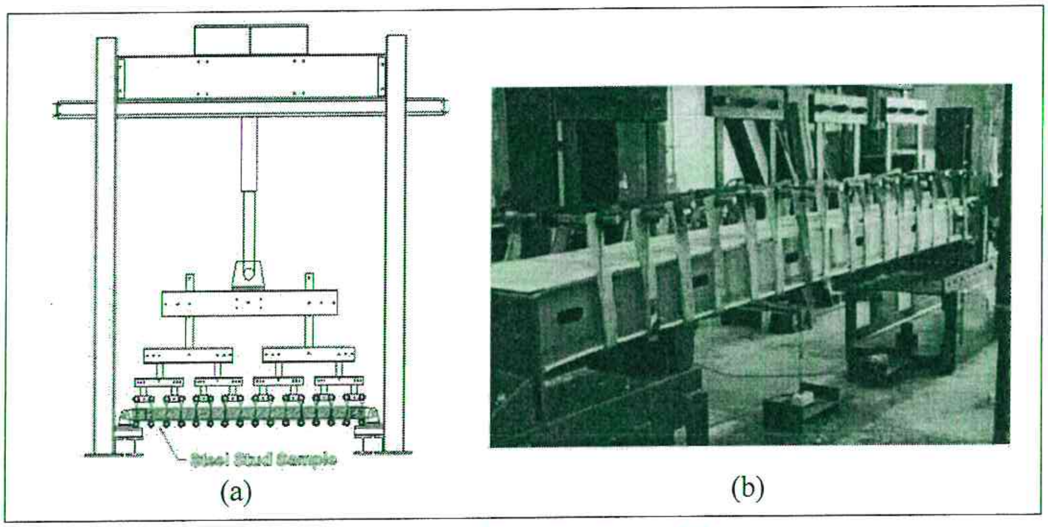


Figure 5: Loading Tree Experimental Setup

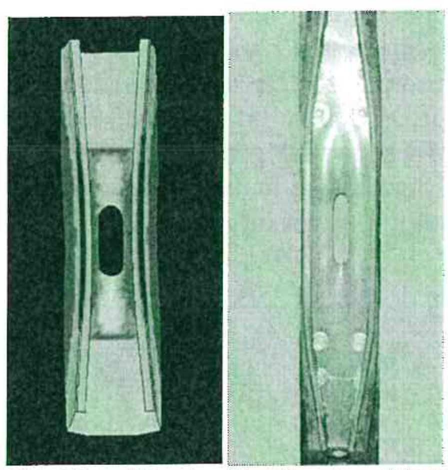


Figure 6: Comparison of FE Model and Tension Test Specimen Tested at University of Missouri

Computational Modeling. To compliment the physical testing program, computer-based simulations using detailed finite element models are a major component of the ongoing research. Such models are needed to carry out parametric studies and to extend the test data beyond the range of specimens that can be physically tested during the research program. Because of the complicated failure mechanisms observed in past blast tests on steel stud walls, it is important to understand the role played by individual components in controlling the overall behavior of a typical wall assembly. Thus, the development of detailed finite element models parallels the physical testing program. To date, several different types of finite element models have been developed. Simple tension specimen models have been developed, and computed results show good agreement with past tests at the University of Missouri (Figure 6) (Shull, 2002).

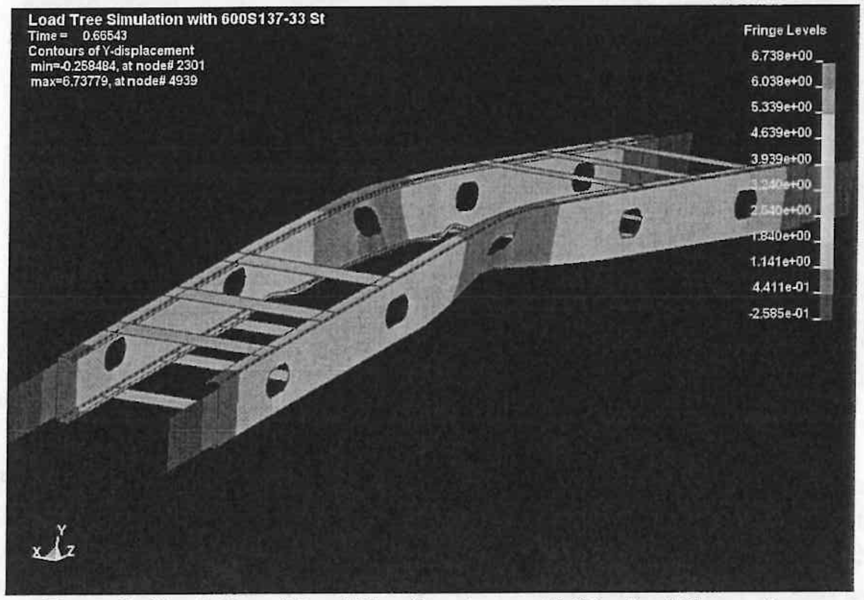


Figure 7: Simulation of Load Tree Specimen with Idealized Connections

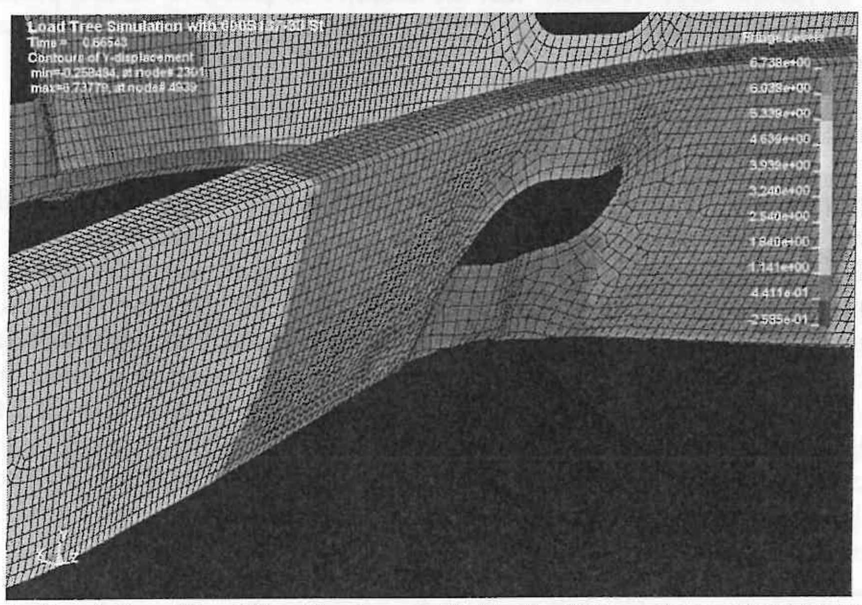


Figure 8: Simulation of Load Tree Specimens with Idealized Connections - Close-Up View of Critical Mid-Span Region

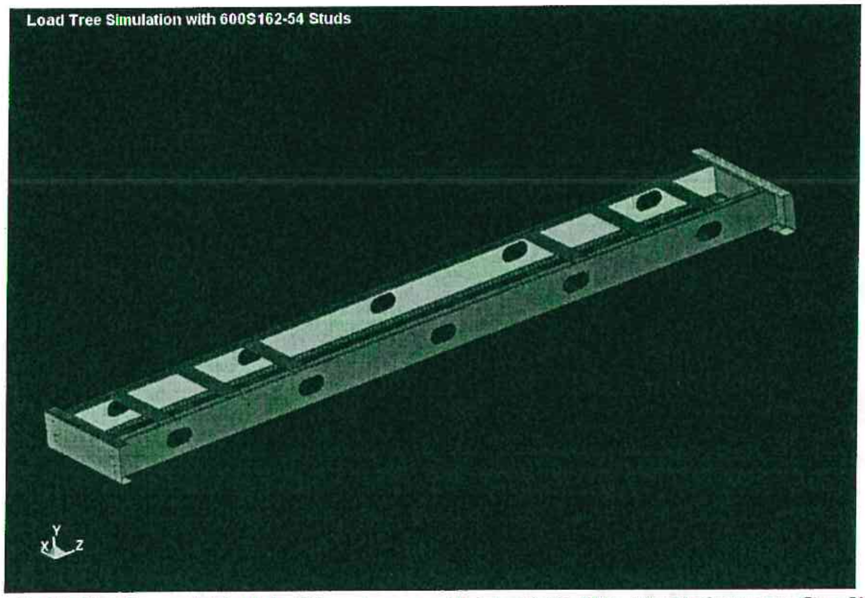


Figure 9: Finite Element Model of Specimens with Realistic Slip-Track Support Conditions

As indicated by Figures 7 and 8 above, simulation models to date have been able to capture the local buckling and yielding of material that occurs at the critical mid-span region of uniformly loaded studs. While the general trend in response agrees well with observations from similar tests in the past, detailed data are needed to validate the predictions of the finite element models. These data will become available once the physical testing program described above is completed. Figure 9 shows the modeling of realistic support details that are common in practice. In this particular figure, a specimen representing a non-load-bearing wall is shown; it utilizes a slip track connection.

SUMMARY AND CONCLUSIONS

Laboratory experiments isolate structural behaviors, allowing for theoretical analyses to be developed that describe localized behaviors. For conventional steel stud construction, TMA, BPA, and CCA experiments isolate behaviors building up to wall component experiments to predict the blast response for conventional designs. Knowledge gained by the TMA series will assist in setting limit states for a selection of connection designs utilizing an inexpensive addition of screws above the common single screw used in practice. The BPA series defines the rotation of a stud track connection in order to define the rotation limits and the connection behavior under bending stresses. The CCA series defines the behavior of a conventional connection design subjected to stresses that induce web crippling and helps define how much of the applied loading is absorbed through shear in the studs. Full-scale component experiments as outlined in this paper provide the knowledge to predict which of the isolated connection response mechanisms will occur within a steel stud wall design based on span, connection detail, and sheathing detail.

The methodologies produced by this work will be validated against measured blast data. Any gaps in the data set will be supplemented with computational experiments. The results of the research are expected to be improved methodologies for the design of conventional steel stud structures against typical blast threats as outlined in the UFC (UFC, 2007). This research bridges the gap between conventional fully-elastic based design and the full tensile membrane capacity blast design to provide guidance for construction details that meet the anti-terrorism UFC requirements in an economical manner.

ACKNOWLEDGEMENTS

This work is sponsored by the Air Force Civil Engineering Support Agency (AFCESA). All laboratory work was performed by the Engineering Mechanics and Explosives Effects Research Group, Force Protection Branch, Airbase Technologies Division, of the Air Force Research Laboratory (AFRL) at Tyndall Air Force Base Florida. The finite element analyses were done using DoD supercomputers maintained by the DoD High Performance Computing Modernization Program (HPCMP).

REFERENCES

Dinan, R. (2005). "Blast Resistant Steel Stud Wall Design." Doctoral Dissertation, University of Missouri, Department of Civil Engineering, Columbia, MO 65211-2200.

DiPaolo, Beverly P. and Stanley C Woodson (2006). "An Overview of Research at ERDC on Steel Stud Exterior Wall Systems Subjected to Severe Blast Loading." *ASCE Structures Congress*, May 18-20, 2006, St. Louis, MO, USA.

Grumbach, Steven D., Clay Naito, and Robert J. Dinan (2007). "Use of Precast Concrete Walls for Protection of Steel Stud Construction." *78th Shock and Vibration Symposium*, November 4-8, 2007, Philadelphia, PA, USA.

Salim, H., R. Dinan, and P. T. Townsend (2005). "Analysis and Evaluation of In-Fill Steel-Stud Wall Systems under Blast Loading." *J. of Structural Engineering*, ASCE, 131(8), 1216-1225.

Salim, H., P. Muller, and R. Dinan (2005). "Response of Conventional Steel Stud Wall Systems under Static and Dynamic Pressure." *J. of Performance of Constructed Facilities*, ASCE, 19(4), 267-276.

Shull, J. (2002). "Steel Stud Retrofit Connection Development and Design." Masters Thesis, University of Missouri, Department of Civil Engineering, Columbia, MO 65211-2200.

SBEDS (2008), Single-degree-of-freedom Blast Effects Design Spreadsheet, U.S. Army Corps of Engineers, Protective Design Center (PDC).

Unified Facilities Criteria (UFC) 04-010-01, (2007). *DoD Minimum Antiterrorism Standards for Buildings,* 22 January 2007.

STAEDTLER® No. 937 811E Engineer's Computation Pad

CE 397: BLAST 19 January 2010

INTRODUCTION TO BLAST

Comments on references

- ASCE books are ~ $30, compilations of articles
  - simple, easy-to-get into
  - 1997 book is about to be re-released
- Blast and Ballistic Loading of Structures
- Blast Effects on Buildings

  } hardcover, good, maybe not worth the cost
- Bulson book
- Krauthammer book is good, detailed, expensive
  - if a real book (text) is wanted, that might be the one to buy

Term Project

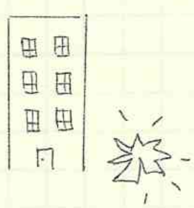
Design a structural component to resist blast

- predict response
- compare to measured response
- present findings with interpretation of results

Designing for blast: general principles

- no factors of safety used
- realistic (not design) material strengths used
- strain rates considered in load factor and strength
- goal is to prevent casualty
  - some structures require greater serviceability
    - └ hospitals, military operations, etc.

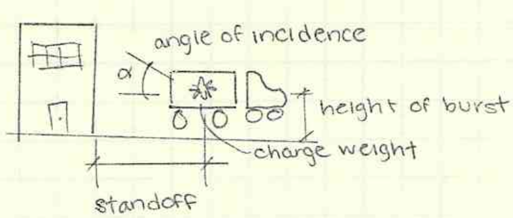
Blast loads

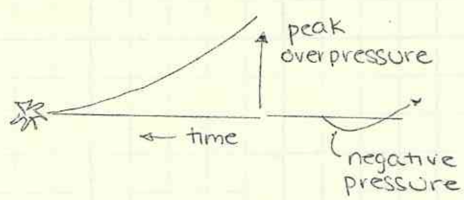


estimating or predicting blast load requires many assumptions that could be wrong

- blast waves
- propulsion of fragments or missiles
- thermal radiation (nuclear)

Variables:





negative pressure can be important; usually in lightweight structures (not concrete, steel)

INTRODUCTION

Blast load calculations

$$Z_G = \frac{R_G}{W^{1/3}}$$

R_G ← standoff

W ← weight of explosive (in TNT equivalence)

Ex: $R_{G1} = 10$ ft, $W_1 = 10$ lb
$R_{G2} = 20$ ft, $W_2 = 80$ lb for same force

Fragments:
- primary: casing of a weapon
- secondary: things thrown by blast (dirt, rocks, barriers...)

Strain rate effects

on strength: load is trying to yield material faster than material can keep up
increase in yield of ~35%, strength by ~10%.

generally handled by simple multiplication factors

Blast vs. seismic loading

similarities:
- ductility is important
- load reversal
- inelastic behavior
- life safety is critical
- damage to nonstructural components

differences:
- duration of load
- direction preference in blast
- hemispherical vs. lateral loads
- magnitude of load is high; difficult to predict
- localized vs. widespread loads
- debris hazard

Earthquakes resist lateral loads system-wide; blast requires ductile responce of a few local members
(Earthquakes: plastic behavior can occur almost anywhere)
(Blast: define location of plastic hinges)

NOT THE SAME. — EBW notes have good chart (copy in)

PHYSICS OF EXPLOSIONS

TNT Equivalency

ANFO — used a lot in quarrying and mining

Ammonium Nitrate Fuel Oil

most common in simplistic homemade bombs

Equivalency factors

- same pressure, P_{eq}
- same impulse, I_{eq}

Values are only good through certain pressure ranges (see handout)

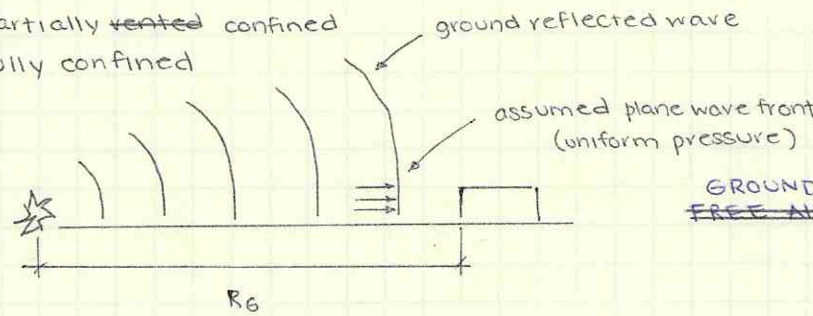
Types of Blasts

Unconfined

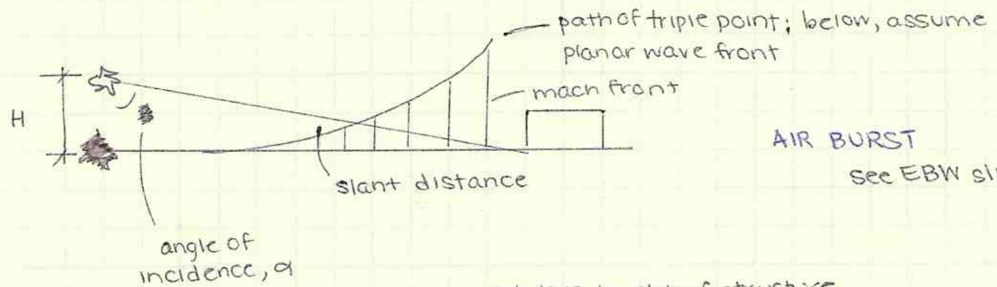
- free air burst
- air burst
- surface burst

Confined

- fully vented
- partially ~~vented~~ confined
- fully confined



GROUND ~~FREE AIR~~ BURST



AIR BURST

See EBW slide

relative height of structure to level of triple point line is important

PHYSICS OF EXPLOSIONS

Geometric Scaling Principle

how to use information from one geometry to predict behavior with a different geometry

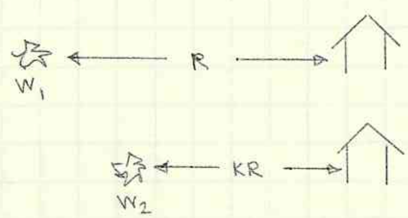
$$Z = \frac{R}{W^{1/3}}$$

third root is related to the volume of a sphere

Hopkinson-Cranz scaling,
or just cube-root scaling
doesn't work well close in to something that is not spherical
e.g. cylinder, shaped charge, fused explosive

Z: scaled standoff – physical standoff divided by charge weight to the 1/3rd

R in ft
W in lbs



$$K = \left(\frac{W_2}{W_1}\right)^{1/3}$$

pressure scales directly – P = P
time scales – t_1, $t_2 = Kt_1$ (if K < 1, impulse is smaller)

Predicting P_{so}

$$P_{so} = \frac{kE}{R^3}$$

R: standoff distance, ft
k: dimensionless calibration constant (akin to a material factor)
E: instantaneous energy release (related to W)

$$P_{so} = \frac{k_1 W}{R^3} \leftarrow W$$

TM 5-855 (Army Technical Manual) of 1965 suggested an equation for P_{so} using Z (scaled standoff).

$$P_{so} = \frac{4120}{Z^3} - \frac{105}{Z^2} + \frac{39.5}{Z}$$

$160 > P_{so} > 2$ psi
$20 > Z > 3$ ft/lb$^{1/3}$

very limiting constraints

21 January 2010

NOT exact numbers.
Averaged.
Don't consider surroundings and other factors.

same pressure caused — same impulse created — range of applicability

Table 2-6. Averaged Free-Air Equivalent Weights

Explosive	Equivalent Weight, Pressure (lbm [1])	Equivalent Weight, Impulse (lbm [1]) not measured	Pressure Range (psi [2])
ANFO (diesel fuel and fertilizer)	0.82	—	1-100
Composition A-3	1.09	1.076	5-50
Composition B	1.11 1.20	0.98 1.3	5-50 100-1,000
Composition C-4	1.37	1.19	10-100
Cyclotol (70/30)	1.14	1.09	5-50
HBX-1	1.17	1.16	5-20
HBX-3	1.14	0.97	5-25
H-6	1.38	1.15	5-100
Minol II	1.20	1.11	3-20
Octol (70/30, 75/25)	1.06	—	E
PBX - 9404	1.13 1.7	— 1.2	5-30 100-1,000
PBX - 9010	1.29	—	5-30
PETN	1.27	—	5-100
Pentolite	1.42 1.38 1.50	1.00 1.14 1.00	5-100 5-600 100-1,000
Picratol	0.90	0.93	—
Tetryl	1.07	—	3-20
Tetrytol (Tetryl/TNT) (75/25, 70/30, 65/35)	1.06	—	E
TNETB	1.36	1.10	5-100
TNT	1.00	1.00	Standard
TRITONAL	1.07	0.96	5-100

> 1 means more efficient (less needed)

To convert pounds (mass) to kilograms, multiply by 0.454
To convert pounds (force) per quare inch to kilopascals, multiply by 6.89

$$100 \text{ lbs of TNT} = \frac{100}{\text{value}} \text{ lb of other product}$$

ex. 73 lb CF = 100 lb TNT

→ wt. product × value = equivalent weight TNT

PHYSICS OF EXPLOSIONS

Airblast

In general, consider:

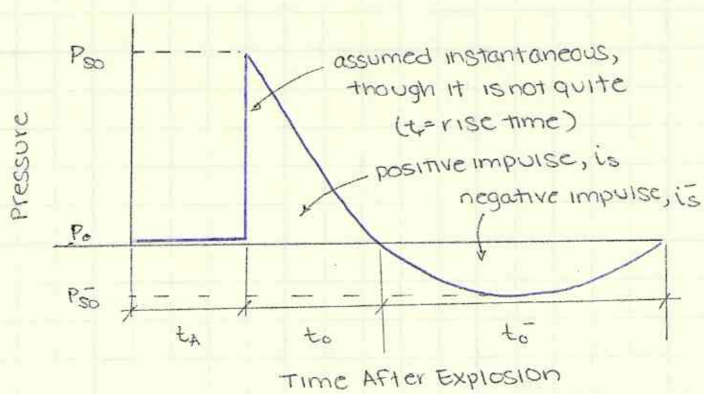
1. blast overpressure ← most important
2. fragment loading
3. shock transmitted through the ground
4. thermal loads (nuclear)

Definitions

detonation: process of supersonic combustion that involves a shock wave and a reaction zone behind it. The shock compresses the material, thereby increasing the temperature to the point of ignition. The ignited material burns behind the shock front and releases energy that supports shock propagation (22,000–28,000 ft/s for most high explosives)

deflagration: chemical reaction that propagates with subsonic speed and without a shock front

(subsonic: reaction occurs slower that the speed of sound through the material (supersonic is faster))



P_o : atmospheric pressure
t_A : arrival time — time from charge going off to you feeling it
P_{so} : side-on pressure, or free field pressure
t_o : positive phase duration
t_o^- : negative phase duration
P_R : reflected pressure

of interest is the area under the curve
"impulse" ~ $\int_t P\,dt$
we want both positive and negative impulse values

STAEDTLER® No. 937 811E Engineer's Computation Pad

COMPUTATION OF BLAST LOADS

General principles

- shock waves are nonlinear, whereas sound waves are linear
- reflection and diffraction occur upon encountering a surface, but shock waves behave differently
- simplifications are necessary; validate off test data

// See shockwave quote handout

Rankine - Hugoniot Relations

Ambient Fluid	shock front	Shock Fluid
$P_o, \rho_o, T_o, \gamma_o$ $\mu_o = 0$	in front of the shock front ← | U_s behind the shock front	$P_{sh}, \rho_s, T_s, \gamma_s$ $U_s - \mu_s$

P_o = ambient atmospheric pressure

other variables are the same, but for ambient

U_s = shock front velocity

P_{sh} = peak shock pressure
ρ_s = shock density
μ_s = peak particle velocity
T_s = shock air temperature
γ_s = shock air specific heat ratio

Assumption: assume air as an ideal gas.
This allows ρ_s, T_s, P_{sh} to be related to pre-shock conditions using a single variable such as shock strength (i.e., peak overpressure)

Specific heat ratio, γ

$$\gamma = \frac{C_p}{C_v}$$

C_p ← specific heat at constant pressure
C_v ← ... at constant volume

specific heat: the amount of heat required to change a unit of mass by one degree

Conservation Equations

... of mass

$$\rho_o U_s = \rho_s (U_s - \mu_s)$$

... of momentum

$$P_{sh} - P_o = \rho_o \mu_s U_s$$

... of specific internal energy

$$\frac{1}{2} U_s^2 + \frac{\gamma_o}{\gamma_o - 1} \cdot \frac{P_o}{\rho_o} = \frac{1}{2} (U_s - \mu_s)^2 + \frac{\gamma_s}{\gamma_s - 1} \frac{P_{sh}}{\rho_s}$$

Shock Waves

A shock wave is the dividing surface between moving material and stationary material. When a force is applied to a material surface, the material adjacent to that surface begins to move, while the material farther from that surface is still at rest. Necessarily, the material in motion is compressed, and occupies less space than it did initially. This statement seems self-evident, but when it is applied to real cases it often leads to configurations far removed from usual experience. In almost all usual experience, a force applied to a metal object can be thought of moving the object as a rigid body; the far end of the object moves as soon as the force is applied. This common sense intuitive model is an approximation valid most of the time. The sound speed, or shock-wave speed, through the object is very fast, and the delay between application of the force and motion of the far end can easily go undetected. Only when the delay is important, or when the compression of the material is considerable, or when the deformation of the material is large, do we need to consider shock waves. Common experience that rigid-body mechanics, or mechanics with small deflections, describes the real world is deeply engrained in all of us, and study of shock-wave mechanics requires a suspension of disbelief.

Zukas, J. A. and Walters, W. P. (1998). *Explosive Effects and Applications.* Springer-Verlag, New York.

COMPUTATION OF BLAST LOAD

Assumptions for shock propagation
- ideal gas law is valid

$$\frac{P}{\rho^{\gamma}} = \text{constant}$$

- adiabatic flow → changes in pressure and volume occur with no change in temperature "adiabatic flow"

Useful relationships

Peak overpressure

$P_{so} = P_{sh} - P_o$ pressure above atmospheric

for low overpressures ($P_{so} < 300$ psi), it can be assumed that:

$$\gamma_s \sim \gamma_o = 1.4 \text{ for air}$$

density ratio

$$\frac{\rho_s}{\rho_o} = \frac{6\left(P_{so}/P_o\right) + 7}{P_{so}/P_o + 7}$$

– only variable is P_{so}

shock velocity

$$U_s = c_o\left(\frac{P_{so}(\gamma+1)}{P_o(2\gamma)} + 1\right)^{1/2}, \quad c_o = \left(\frac{\gamma P_o}{\rho_o}\right)^{1/2}$$

at sea level, $c_o = 1116$ ft/s, $P_o = 14.7$ psi,

$$U_s = 1116\,(0.0583 P_{so} + 1)^{1/2}$$

peak particle velocity

$$u_s = \frac{224.8 P_{so}}{(P_{so} + 17.5)^{1/2}}$$

peak dynamic pressure

$$q_s = \frac{P_{so}^2}{(0.4 P_{so} + 41.2)}$$

pressure variation with time ← Friedlander equation

$$P_s(t) = P_{so}\left[1 - \left(\frac{t - t_a}{t_o}\right)\right] e^{-k(t-t_a)/t_o} \quad \text{for } t_a \le t \le t_a + t_o$$

k = rate of decay, waveform factor

CE 397: BLAST 26 January 2010

COMPUTATIONS...

Estimating Peak Shock Pressure

- Army Technical Manual (1965)

$$P_{so} = \frac{4120}{Z^3} - \frac{105}{Z^2} + \frac{39.5}{Z}$$

$160 > P_{so} > 2$ psi

$20 > Z > 3$ ft/lb$^{1/3}$

multiply W by 1.8 to estimate hemispherical case, not spherical

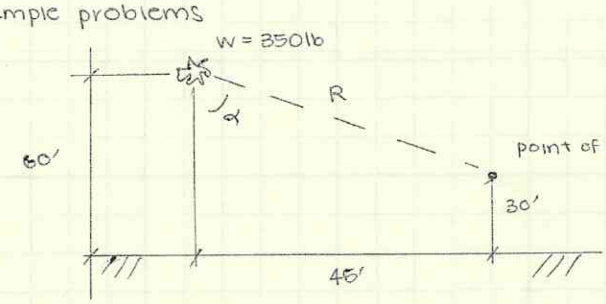
- notes include a metre/kilogram version, pressure in bar (Henrych)

$$P_{so} = \frac{14.072}{Z} + \frac{5.540}{Z^2} - \frac{0.357}{Z^3} + \frac{0.00625}{Z^4} \qquad 0.05 < Z < 0.3$$

$$P_{so} = \frac{6.194}{Z} - \frac{0.326}{Z^2} + \frac{2.132}{Z^3} \qquad 0.3 \le Z < 1$$

$$P_{so} = \frac{0.662}{Z} + \frac{4.05}{Z^2} + \frac{3.288}{Z^3} \qquad 1 \le Z < 10$$

Example problems



Free air burst – only consider direct impact; no interaction with the ground

$$\text{standoff} = \left[(30\text{ ft})^2 + (45\text{ ft})^2\right]^{1/2} = R = 54.1\text{ ft}$$

$$Z = \frac{54.1\text{ ft}}{(350\text{ lb})^{1/3}} = 7.67\text{ ft/lb}^{1/3}$$

take value to chart for a spherical charge

$P_r = 30$ psi $\qquad U = 1.4$ ft/ms

$P_{so} = 12$ psi

using Army TM equation, $P_{so} = 12.5$ psi

classified program says $P_{so} = 11.5$ psi

using equation from notes, $U = 1.47$ ft/ms

program, $U = 1.44$ ft/ms

$$t_o = W^{1/3}\left[\frac{980\cdot\left(1+(Z/0.54)^{10}\right)}{\left(1+(Z/0.02)^3\right)\cdot\left(1+(Z/0.74)^6\right)\cdot\left(1+(Z/0.69)^2\right)^{1/2}}\right]$$

(wrong)

IN SI UNITS

not a great estimation, but works to calculate impulse.

arrival time of shock

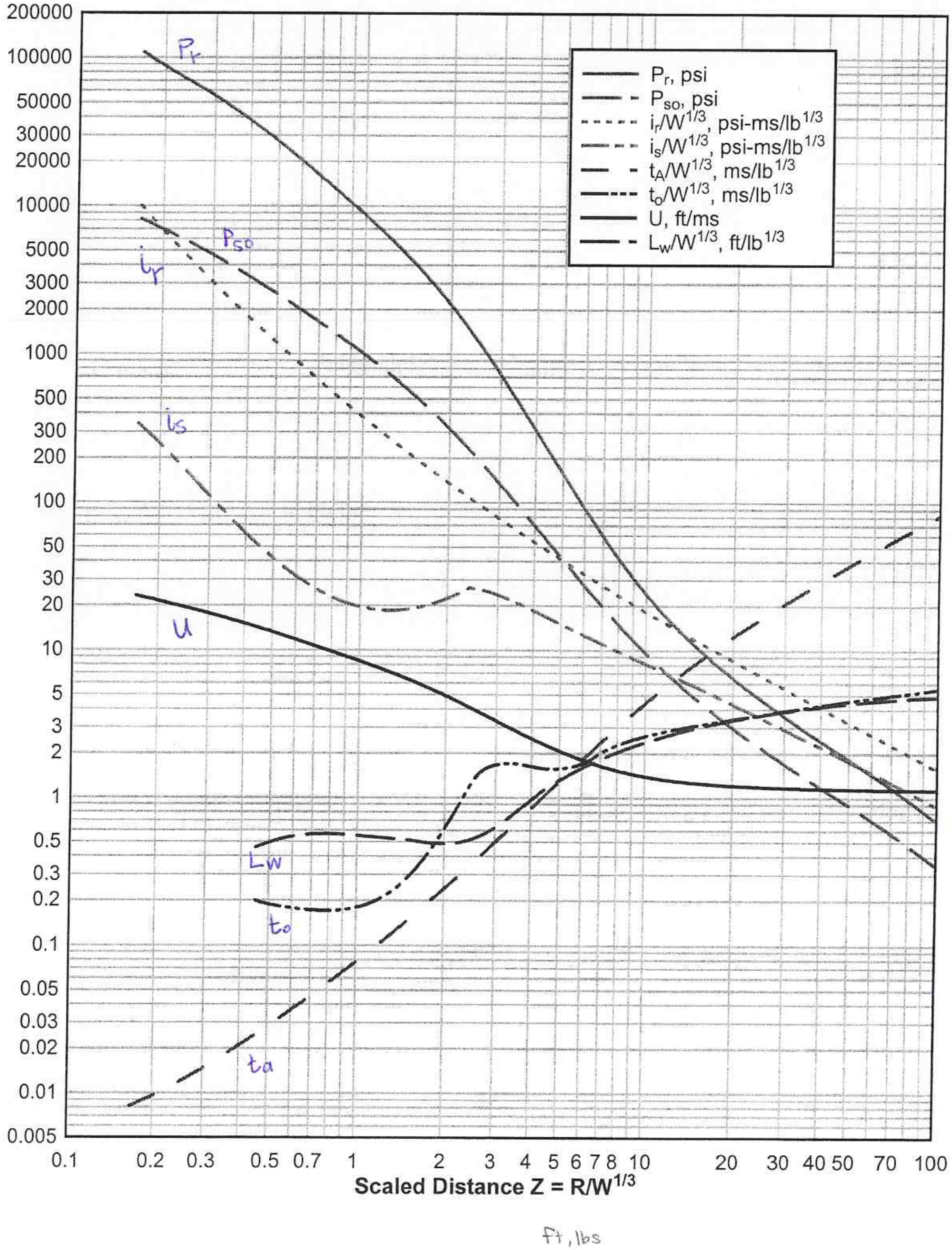
$t_a = R/\text{average } U \leftarrow Z=3, Z=20$ (or actual Z)

$i_s \approx \frac{1}{2} P_{so} t_o$

STAEDTLER® No. 937 811E Engineer's Computation Pad

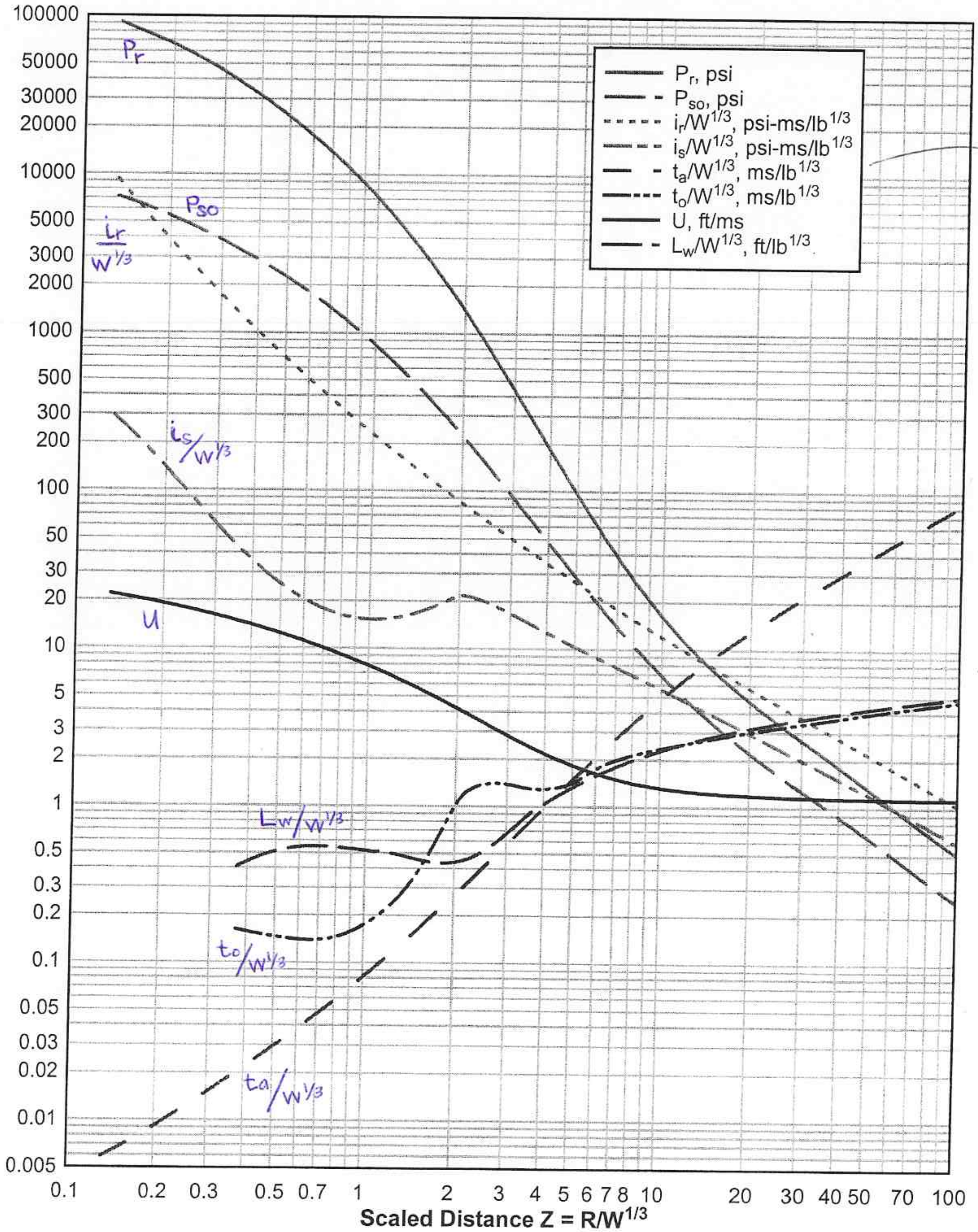
26 January 2010

Figure 2-15 Positive Phase Shock Wave Parameters for a Hemispherical TNT Explosion on the Surface at Sea Level ← multiply by 1.8



26 January 2010

Figure 2-7 Positive Phase Shock Wave Parameters for a Spherical TNT Explosion in Free Air at Sea Level



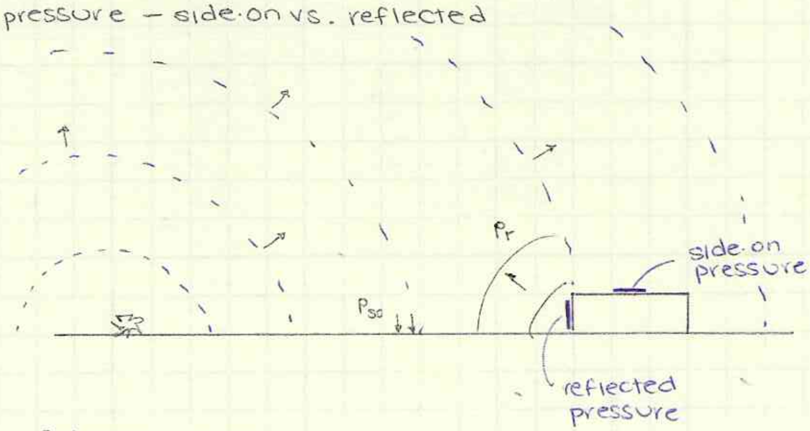
some values are P, others are scaled or other direct quantity

ft, lbs

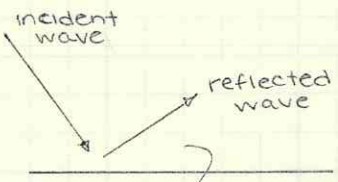
STAEDTLER® No. 937 811E Engineer's Computation Pad

COMPUTATIONAL EXAMPLES

Blast pressure – side-on vs. reflected



Reflected waves



pressure, etc. increase in reflected wave.

increase factor = 2.0 (theoretical)
1.8 (actual)

in confined spaces, factor can be as high as 14; can be very significant

$$RF = \frac{P_{ro}}{P_{so}} \approx 2\left[\frac{103 + 4P_{so}}{103 + P_{so}}\right]$$

min = 2
max = 8, theoretically; however, assumptions STOP holding at some point

Angle of incidence

$$C_{r\alpha} = \frac{P_{r\alpha}}{P_{so}}$$ vs. α chart available

many curves, corresponding to P_{so} value

- from $0 < \alpha < 40°$, not much change
- $40 < \alpha < 55°$, value increases

rules for sonic waves don't apply

at 40°, angle of reflection equals angle of incidence

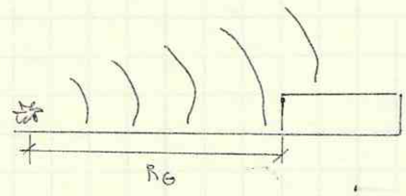
(research done by airplane engineers)

STAEDTLER® No. 937 811E Engineer's Computation Pad

COMPUTATIONS

Effect of burst position

- when a blast occurs against a perfect reflecting surface, the outgoing wave is hemispherical.



- under these conditions, free-air calculations can be used by multiplying W by 1.8 (2.0 by theory).

* note that computed scaled standoff is different from what is used in hemispherical handout. [can't just use spherical handout ¨]

Hemispherical burst

W = 1000 lb

R = 200 ft, or 20 ft

W = 1000 lb
R = 200 ft

$$Z = \frac{200\text{ ft}}{(1000\text{ lb})^{1/3}} = 20\ \text{ft/lb}^{1/3}$$

fairly large value; not a huge threat. chart, equations match

reflected pressure should be > 2 × P_{so}

	chart values	Equations	comp.
P_{so}	~3 psi	~3 psi	3 psi
P_r	~6.5 psi	6.6 psi	6.5 psi
i_s	$4.2W^{1/3}$ = 42 psi/ms	41 psi·ms	43 psi·ms
i_r	$8.5W^{1/3}$ = 85 psi·ms	89 psi·ms	85 psi·ms

Now reduce R_G to 20 ft, not 200 ft

$Z = 2\ \text{ft/lb}^{1/3}$

	chart	equations	program
P_{so}	320 psi	238 psi	
P_r	2100 psi	1470 psi	
i_s	240 psi·ms	162 psi·ms	
i_r	1400 psi·ms	1000 psi·ms	

Henrych formula tends to underestimate at low Zs, overestimate at high Zs.

↑ all low because P_{so} value was low;

override P_{so} value and equations match charts. t_o equation also can be off.

COMPUTATIONS

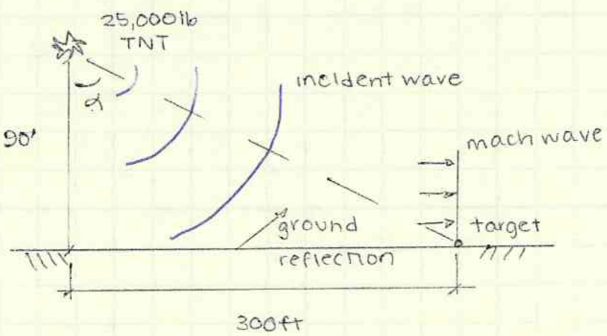
Clearing Effects

- for a reflector, where flow around an edge or edges occur, the duration of the reflected pressures is controlled by the size of the reflecting surface.
- the high reflected pressure seeks relief toward the lower pressure regions, and this tendency is satisfied by the propagation of rarefaction (relief) waves from the low to the high-pressure region.
- these waves, traveling at the velocity of sound in the reflected pressure region, reduce the reflected pressures to the stagnation pressure.

No. 937 811E Engineer's Computation Pad STAEDTLER®

EXAMPLE PROBLEMS

Spherical blast with ground reflection



incident wave and ground reflection combine into a planar wave called a mach wave

How much does ground reflection add?

Approximate approach — accounts for interaction of incident and ground reflected wave

Approach 1 — based on UFC manual

- calculate angle of incidence and scaled height

  α, $h/W^{1/3}$
  └ 73.3° └ 3.08

Fig. 2-9 • use chart with α on x, $P_{r\alpha}$ on y, lines for scaled height

for example, $P_{r\alpha} = 10.1$ psi

└ side-on pressure at point of interest (not reflected)

Fig. 2-10 • use second chart to get impulse value

$$\frac{i_s}{W^{1/3}} = 9.2 = 269 \text{ psi}\cdot\text{ms}$$

Reflected pressure

- look at spherical chart
- find Z value such that $P_{so} = 10.1$ psi

  value will not equal $R_G/W^{1/3}$

  $Z = 7.8 \text{ ft/lb}^{1/3}$
- read off P_r
- now find Z value for previously found i_s

  $Z = 5.7 \text{ ft/lb}^{1/3}$
- read i_r from new Z value

  $i_r = 643.3$ psi·ms

EXAMPLE PROBLEMS

Approach # 2

- calculate P_{so} for a point just before target, ignoring ground reflection

$$Z = \frac{R}{W^{1/3}} = 10.7\ ft/lb^{1/3}$$

$$P_{so} = 6.1\ psi$$

- now multiply by reflection coefficient (fig. 2-193)
  - from chart, factor = 1.65
  - $C_{r\alpha} = 1.65$
- get reflected side-on pressure
  - $P_{so} \cdot C_{r\alpha} = 10.1\ psi = P_{r\alpha}$
- second chart for impulse
  - use original P_{so} value, not new version

Williamson modifications

mathcad:

given

P(guess) = value known

sol.Z = find(guess)

sol.Z = answer

$Z = 8.4\ ft/lb^{1/3}$

using formulas with fake Z,

$P_r = 25.6$ psi

$i_s = 208.3$ psi·ms

$i_r = 528.3$ psi·ms

- calculate P_{so} directly
- use multiplication factor
- back-calculate Z

"Correct" Solution

$P_{so} = 12.4$ psi

$i_s = 203$ psi·ms

$P_r = 20.9$ psi

$i_r = 378$ psi·ms

Compare:

	Hemispherical (R = 300 ft)	Spherical (R = 313 ft)	Ground Interaction (R = 313 ft)
P_{so}	9.1 psi	6.0 psi	10 to 12 psi
P_r	22.6 psi	14.0 psi	20.9 psi
i_s	231.5 psi·ms	153.6 psi·ms	203 psi·ms
i_r	515.1 psi·ms	324.4 psi·ms	378 psi·ms

Figure 2-9 Variation of Reflected Pressure as a Function of Angle of Incidence

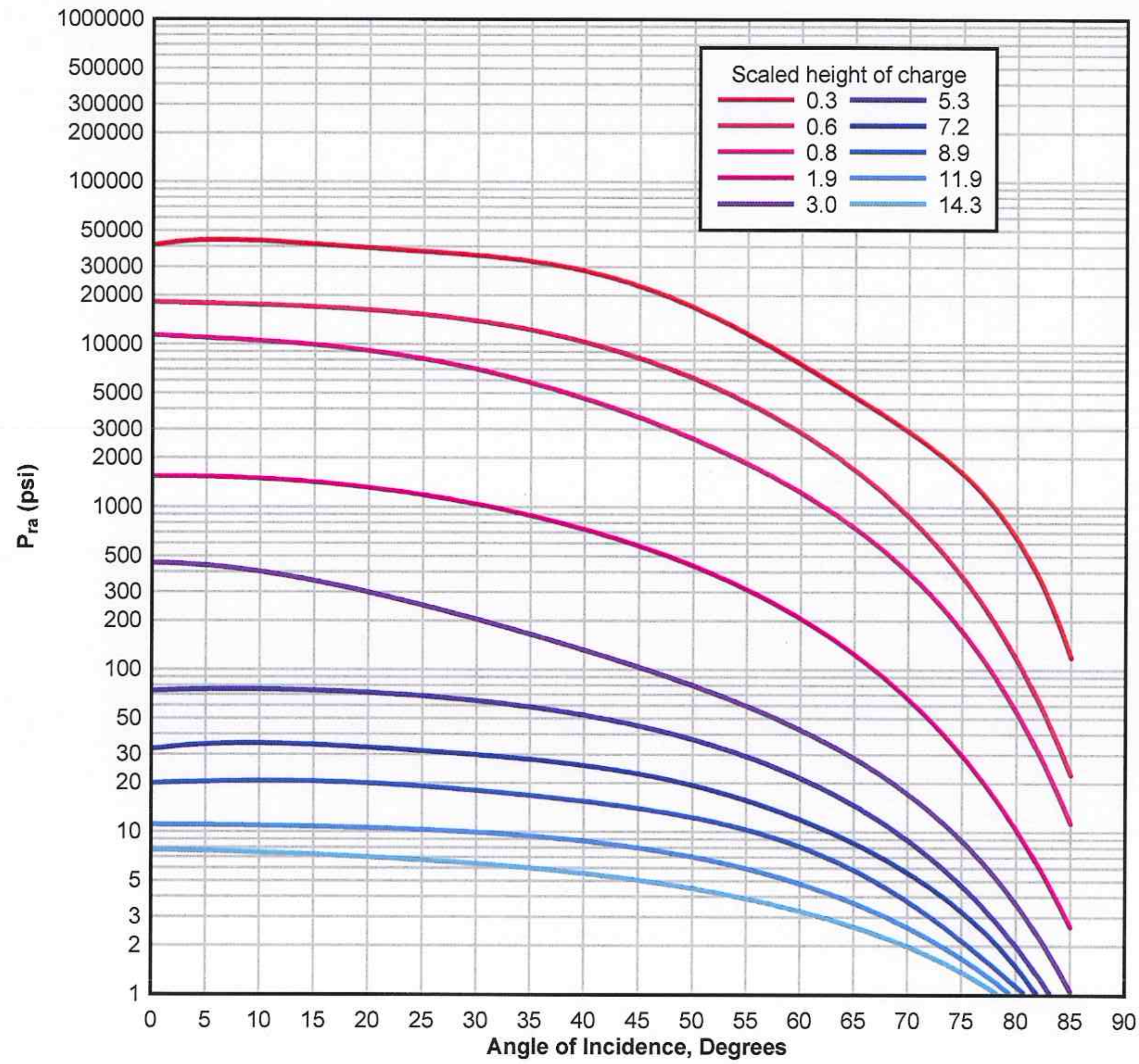


Figure 2-10 Variation of Scaled Reflected Impulse as a Function of Angle of Incidence

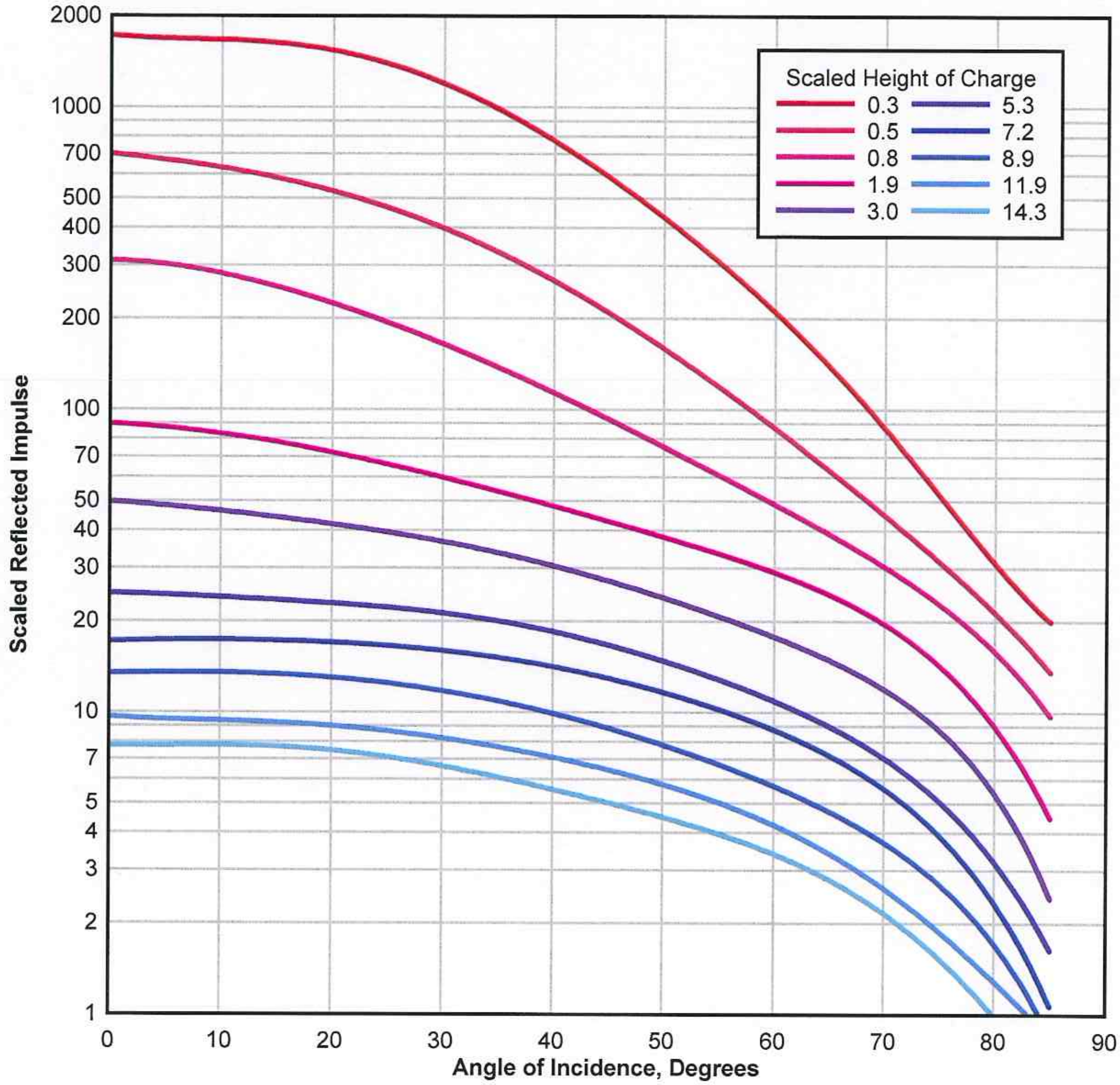
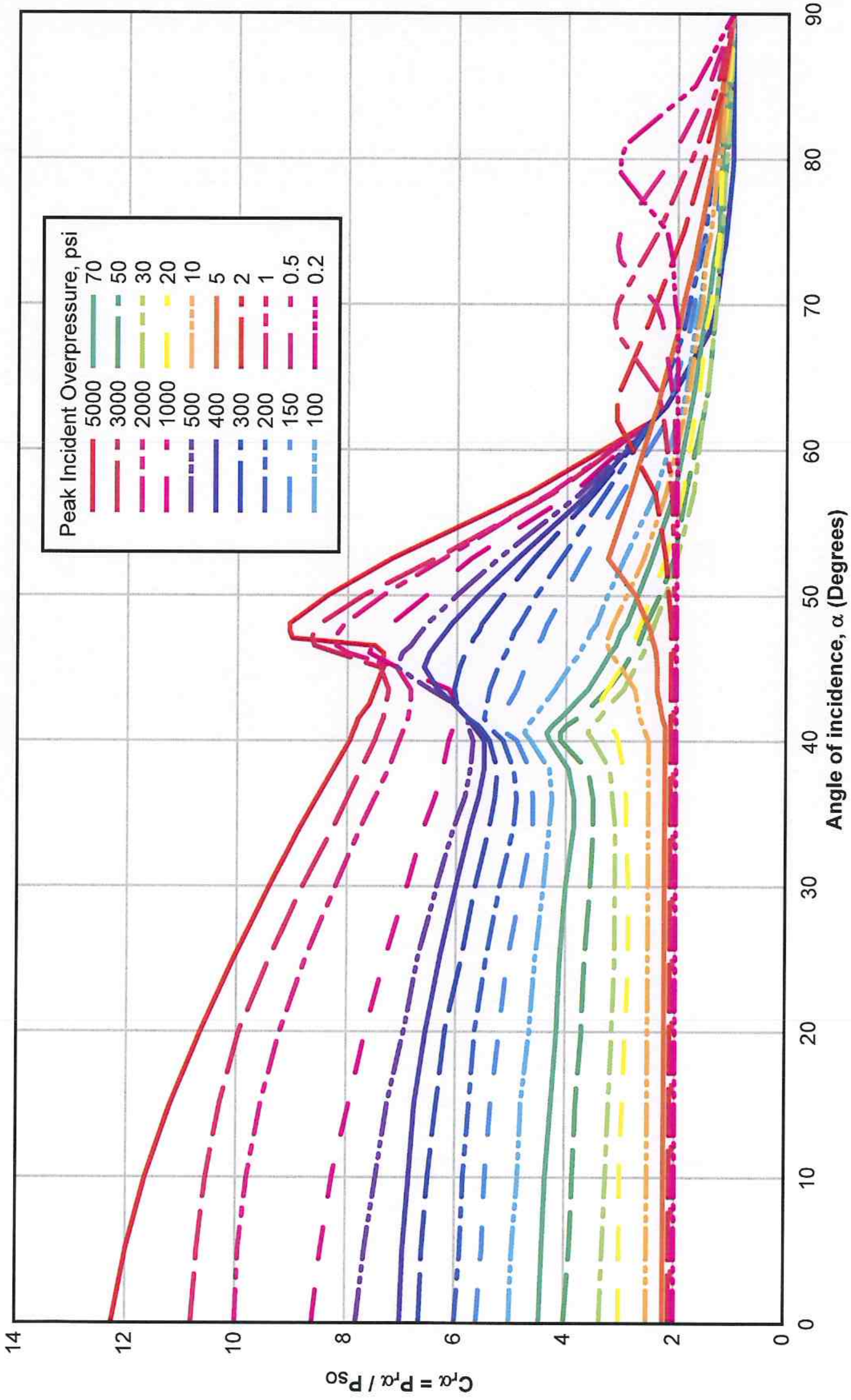


Figure 2-193 Reflected Pressure Coefficient versus Angle of Incidence

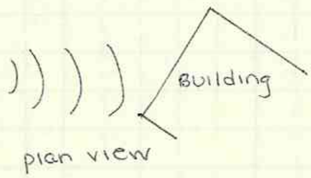


STAEDTLER® No. 937 811E Engineer's Computation Pad

CALCULATION OF LOAD

Angle of incidence

angle charts may need to be used twice



wave doesn't hit perpendicular

Additional issues with predicting loads

Assumptions

- ideal gas behavior, constant γ
- hemispherical or spherical charge
  normal to the side of a cylindrical charge, loads can be amplified significantly
- nothing interferes with propogation of the blast (no obstacles)
- bare explosive – no casing or confinement effects
- TNT equivalency; ignore unique chemical properties

internal Explosions

– multiple reflections

"shock addition" rules are mostly emperical, simple addition is inappropriate,
military "ray tracing",
Lamb methods (British)

non-linear effects (emperical) / constantly changing conditions (inappropriate)

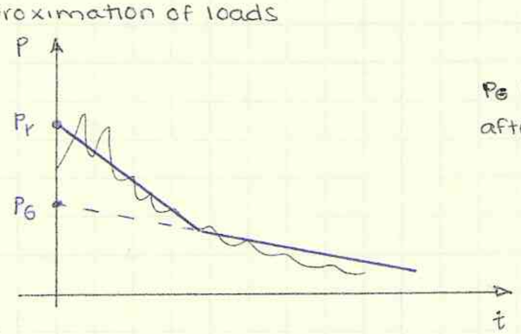
– confinement effects

hot gasses trapped, change air pressure
in addition to shock pressure
gas pressure depends on:

- volume of the room
- type of explosive (chemical used)
- how quickly gas cools
- room geometry
- openings / ventilation
- frangible – something that will be destroyed in a blast

long duration for build-up, quasi-static

– approximation of loads



P_G is built-up gas pressure
after blast passes, gas pressures still exist

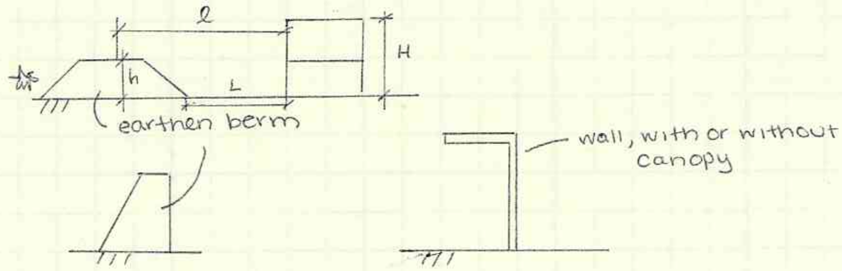
– infiltration of external explosion into a building

STAEDTLER® No. 937 811E Engineer's Computation Pad

CALCULATIONS & APPROXIMATIONS

Blast walls and revetments

goal is to provide protection to structure behind



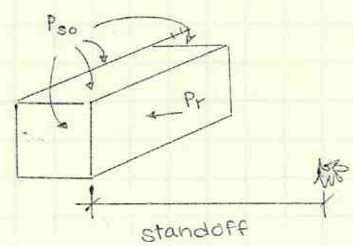
factors affecting loads on target

- H/h ratio
- if wall is frangible / debris hazard
- distance behind the wall, ℓ and L
- blast wave reformation behind wall

Loads on structures

Three pressure components of load

1. incident (side-on)
2. reflected
3. dynamic (drag forces)



Duration of approximated positive phase, t_{of}, is computed by:

$$t_{of} = \frac{2i}{P}$$

Loading categories

- contact or near contact (high intensity, non-uniform loads) $Z < 1$
- close-in (spherical shock wave) $1 < Z < 3$
- plane wave (planar waves) $Z > 3$

Directly loaded surface (front wall)

the average time needed to relieve the reflected pressure on the wall by clearing around the sides and over the roof is given by:

$$t_c = \frac{4HW}{(W+2H)C_r}$$

t_c = avg. reflected pressure clearing time
H = structure height
W = structure width
C_r = sound velocity (depends on P_{so})
└ get from a chart as a function of P_{so}

the clearing time for a specific point on the front wall is given by:

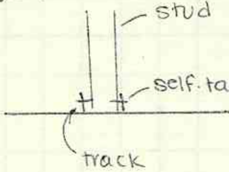
$$t_{cp} = \frac{3S_p}{U_s}$$

S_p = shortest distance from point of interest to a free edge
U_s = shock front velocity

"three transits to the edge" rule

COMPUTATION of LOADS

Term project



8-10 psi, 150 psi·ms
design on the smaller side of what is available (4 in.)

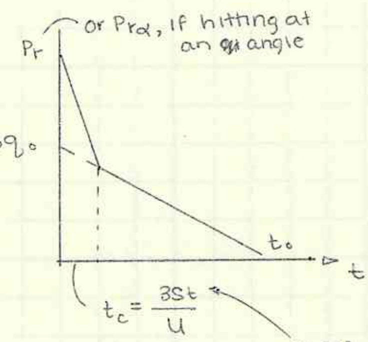
- clading (corrugated metal) on one side; no "interior"
- SSMA: steel stud manufacturing association – for info
- standard 36ksi, generally higher
- rent flatbed or trailer

consider connection to shock tube
- fixed?
- pinned?

leave space if necessary

Loads on Buildings

equivalent triangular load



$t_c = \frac{3S}{U}$ — S (?) – shortest of H/2 and W/2

clearing time

if $t_c > t_o$, clearing doesn't occur

stagnation pressure

$$P = P_{so} + C_D q_o$$

drag coefficient (usually 1.0); q_o: dynamic pressure, from chart or equation

directly loaded surfaces

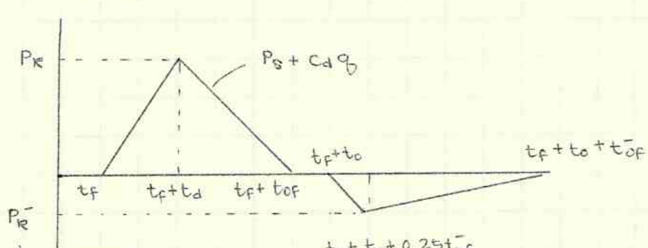
the average time... $t_c = \frac{4HW}{(W+2H)C_r}$ (see previous page of notes)

└ get value from a chart, P_{so} on x (on blackboard)

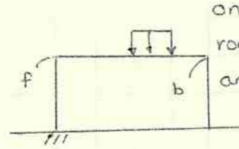
compare calculated impulse and i_r from chart

i_r is max value, use i_r, P_r to calculate t

roof and side wall loading



"pressure on the roof", not reflected pressure



only part of the roof is loaded at any one time

two important points: f and b. act as though load is uniform over the whole roof

COMPUTATION of LOADS

Roof and side wall loading

$$P_R = C_E P_{sof} + C_D q_{of}$$

(C_D: negative value; chart is in notes)

intended to incorporate the fact that load moves across the roof

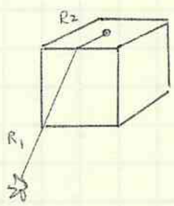
- calculate wave length over length of structure
- use on chart for C_E

$$\frac{L_{wf}}{L_{roof}}$$

(L_{wf}: wave front length; in chart)

t_d is on a chart too, using scaled length
"scaled duration"
total duration chart

Simplified load calculation



$R = R_1 + R_2$, distance from blast to point of interest

- using the blast load at the middle of the span is conservative
- averaging the load over the panel can reduce conservatism

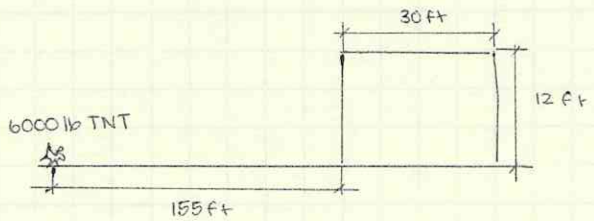
General rules

- use unfactored loads
- consider long-term dead loads (1.0 or 1.1), but not wind, earthquake
- consider half of live loads (0.5LL)

STAEDTLER® No. 937 811E Engineer's Computation Pad

EXAMPLE CALCULATIONS

Problem setup



- square building, 30 ft × 30 ft
- no windows or doors

compute:
- front wall loading
- roof loading

Front wall loading

hemispherical load

$$Z = \frac{155\ \text{ft}}{(6000\ \text{lb})^{1/3}} = 8.5$$ use hemispherical chart

or,

$$Z = \frac{155\ \text{ft}}{(1.8 \cdot 6000\ \text{lb})^{1/3}} = 7.0$$ for calculations

how well can we estimate side-on pressure, P_{so}? everything depends on that number!

calculated:

$P_{so_H} = 13.26$ psi

$P_r = 35.59$ psi

$q = 3.78$ psi $i_s = 177.3$ psi·ms

$t_o = 26.75$ msec $i_r = 475.9$ psi·ms

Step ① Get Quantities.

② compute clearing time

$$t_c = \frac{4HW}{(2H+W)\,C_r} = \frac{4(12\,\text{ft})(30\,\text{ft})}{[2(12\,\text{ft})+(30\,\text{ft})]\,C_r}$$

get C_r from chart in notes (using P_{so})

$C_r = 1.34$ ft/msec

$t_c = 19.9$ msec

$t_c < t_o$, so consider clearing

③ calculate stagnation pressure

assume $C_D = 1$ for front wall

$P_{stag} = P_{so} + C_D \cdot q = 17.04$ psi for this example

General Procedure

calculate Z for chart, use to get P_{so}

assign P_{so} to calculation closer to chart (855 vs. Henrych)

→ use P_{so} to calculate other values

→ compute clearing time
  use table to get C_r (chart)

→ calculate stagnation pressure

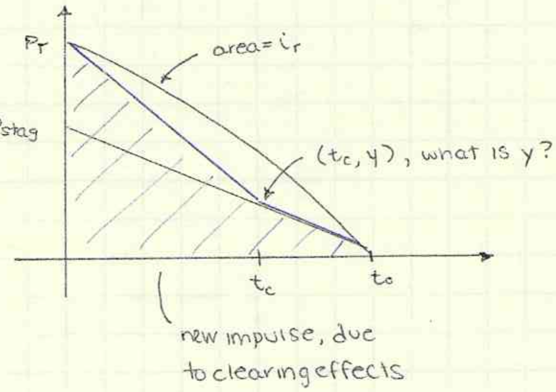
→ calculate impulse

→ compare i_r to chart
  (chart most conservative)

EXAMPLE CALCULATIONS

continuation

(4)



$$y = (t_o - t_c)\frac{P_{stag}}{t_o} = 4.36 \text{ psi}$$

calculate impulse,

$$t_c \cdot y + \tfrac{1}{2} y (t_o - t_c) + \tfrac{1}{2} t_c (P_r - P_{stag}) = \text{new impulse}$$

$= 412.4$ psi·msec

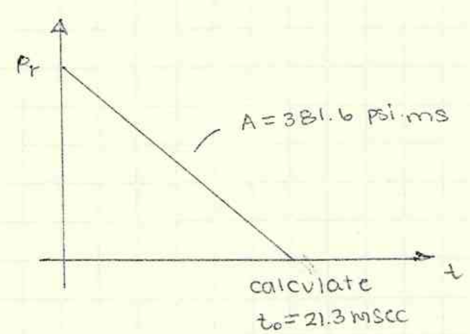
(compare to 475.9 psi·ms)

(5) Now compare calculations to chart

$i_r = 21 W^{1/3} = 381.6$ psi·ms > calculated value, so use chart value because our t_o calc. isn't so great.

(381.6 psi·ms: use for design)

(6) For design,



if new t_o and t_c are far apart, can calculate clearing effects

STAEDTLER® No. 937 811E Engineer's Computation Pad

EXAMPLE CALCULATION

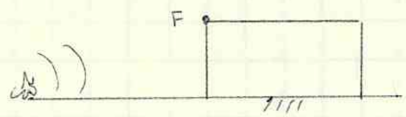
Continued problem: Roof Load

same calculations for side, back walls

key points:

- since load is moving, only part of the roof is loaded at one time; negate effect with equivalency factor
- planar wave, triple point higher than roof

Simplified Procedure



use parameters at point F

planar wave means load history at point F is the same as front wall

① calculate roof pressure

$$P_R = C_E P_{so} + C_D q$$

- C_E: calculate using chart 2-196
- P_{so}: from front wall calcs
- $C_D q$: not the same as for front wall

calculate wave length / span length

↑ chart value, $L_w / W^{1/3} = 2.1$, $L_w = 38.16$ ft

$$\frac{L_w}{\text{span}} = \frac{38.2\text{ ft}}{30\text{ ft}} = 1.27, \quad C_E = 0.52 \text{ from 2-196}$$

② compute q associated with $P_{so} = 6.9$ psi $[C_E P_{so\,front}]$

$$q = \frac{P_{so}^2}{0.4P_{so} + 41.2} = 1.08 \text{ psi}$$

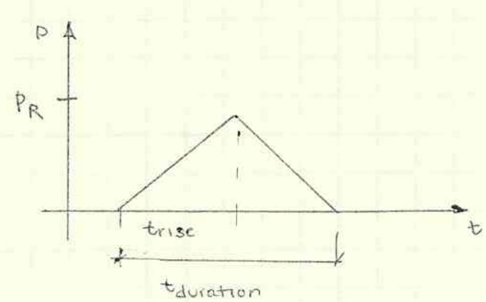
③ compute drag coefficient (get from table)

$$C_D = -0.4$$

④ return to step 1

$$P_R = C_E P_{so} + C_D q = 6.47 \text{ psi}$$

⑤



scaled rise time comes from 2-197

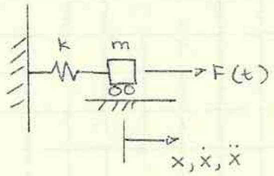
use P_{so} from point F

scaled duration from 2-198

then calculate impulse

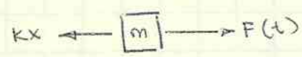
STRUCTURAL DYNAMICS

Single DOF system



Equation of motion

$$\Sigma F = ma = m\ddot{x}$$



$$F(t) - kx = m\ddot{x}$$

$$\boxed{F(t) = m\ddot{x} + kx}$$ linear 2nd order ODE

$$x(t) = x_h(t) + x_p(t)$$

$x_h(t)$ = homogeneous solution

$x_p(t)$ = particular solution

$$\ddot{x}(t) + \omega^2 x(t) = \frac{F(t)}{m}$$

↑ natural frequency

$$\omega = \sqrt{\frac{k}{m}}$$

homogeneous comes from when $\Sigma F = 0$

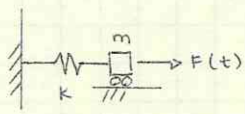
$$\ddot{x}(t) + \omega^2 x(t) = 0$$

$$x(t) = A\sin\omega t + B\cos\omega t$$

STAEDTLER® No. 937 811E Engineer's Computation Pad

STRUCTURAL DYNAMICS

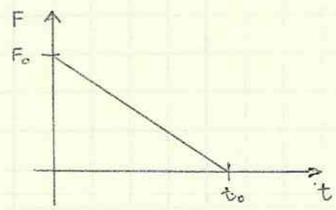
Single DOF systems



$$F(t) = m\ddot{x} + kx$$

> homogeneous response ($\Sigma F = 0$)
> $x(t) = A\sin\omega t + B\cos\omega t, \quad \omega^2 = k/m$

Triangular load



start by guessing form of particular solution

$$x_p(t) = C + Dt$$

$$x(t) = A\sin\omega t + B\cos\omega t + C + Dt$$
$$\dot{x} = \omega A\cos\omega t - \omega B\sin\omega t + D$$
$$\ddot{x} = -\omega^2 A\sin\omega t - \omega^2 B\cos\omega t$$

use these definitions in original equation of motion

$$\ddot{x}(t) + \omega^2 x(t) = \frac{1}{m}F(t)$$

$$-\omega^2(A\sin\omega t + B\cos\omega t) + \omega^2(A\sin\omega t + B\cos\omega t) + \omega^2(C + Dt) = \frac{F_o}{m}(1 - t/t_o) = \frac{1}{m}F(t)$$

simplifies to

$$\frac{\omega^2(C + Dt)}{\omega^2} = \frac{F_o}{m}(1 - t/t_o) = \frac{F_o}{\omega^2 m}(1 - t/t_o)$$

$$C + Dt = \frac{F_o}{\frac{k}{m}\cdot m}(1 - t/t_o)$$

$$C = \frac{F_o}{k}, \quad D = \frac{-F_o}{kt_o}$$

$$x(t) = A\sin\omega t + B\cos\omega t + \frac{F_o}{k}(1 - t/t_o)$$

solve for A, B using initial conditions

Assume:

- system starts from rest
- $x(t=0) = 0$, $\dot{x} = 0$

$$x(0) = B + \frac{F_o}{k} = 0, \quad B = \frac{-F_o}{k}$$

$$\dot{x}(0) = \omega A + \frac{-F_o}{kt_o}, \quad A = \frac{F_o}{k\omega t_o}$$

$$\boxed{x(t) = \frac{F_o}{k}\left[\frac{1}{\omega t_o}\sin\omega t - \cos\omega t + 1 - t/t_o\right]}$$

valid when $0 < t < t_o$; afterwards, system is in free vibration.

STRUCTURAL DYNAMICS

Triangular load, cont'd

when $t > t_o$, load stops and system is in free vibration

$$x(t) = \frac{F_o}{k}\left[\frac{1}{\omega t_o}\left(\sin\omega t - \sin\omega(t-t_o)\right) - \cos\omega t\right]$$

$x_{static} = \frac{F_o}{k}$, if load is applied and held constant

Dynamic load factor (DLF)

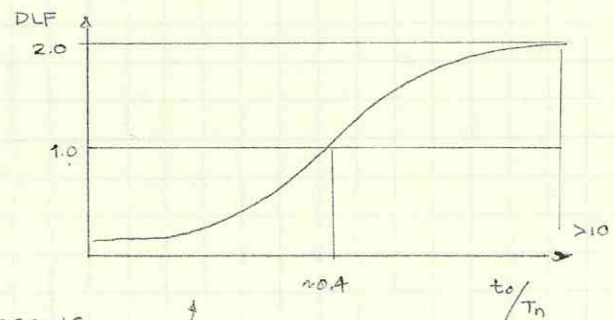
or dynamic amplification factor

$DLF = \frac{x_{max}}{x_{static}}$, basically, the term in parentheses, as $x_{static} = F_o/k$

$$t < t_o,\ DLF = \frac{\sin\omega t}{\omega t_o} - \cos\omega t + 1 - t/t_o$$

in many cases, we simply want to find maximum displacement, and will not need to carry out a time-history analysis.

x_{max}



with such short durations, system literally can't respond in time to flex.

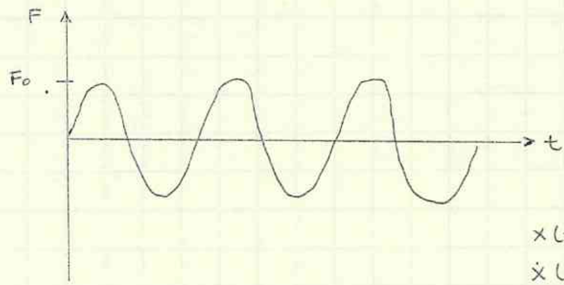
dynamic amplification plot

logrithmic scale

STAEDTLER® No. 937 811E Engineer's Computation Pad

STRUCTURAL DYNAMICS

Sinusoidal load



$$F(t) = F_o \sin \Omega t$$

$$x(t) = A\sin\omega t + B\cos\omega t + C\sin\Omega t + D\cos\Omega t$$
$$\dot{x}(t) = A\omega\cos\omega t - B\omega\sin\omega t + \Omega C\cos\Omega t - \Omega D\sin\Omega t$$
$$\ddot{x}(t) = -A\omega^2\sin\omega t - B\omega^2\cos\omega t - \Omega^2 C\sin\Omega t - \Omega^2 D\cos\Omega t$$

$$\ddot{x} + \omega^2 x = \frac{F(t)}{m}$$

$$-\omega^2(A\sin\omega t + B\cos\omega t) - \Omega^2(C\sin\Omega t + D\cos\Omega t) + \omega^2(A\sin\omega t + B\cos\omega t) + \omega^2(C\sin\Omega t + D\cos\Omega t)$$

$$(-\Omega^2 + \omega^2)(C\sin\Omega t + D\cos\Omega t) = F_o \cdot \frac{1}{m}\sin\Omega t$$

$D = 0$, no cos term to match

$$C(\omega^2 - \Omega^2)\sin\Omega t = \frac{F_o}{m}\sin\Omega t$$

$$C = \frac{F_o}{m(\omega^2 - \Omega^2)}, \quad A = -C\frac{\Omega}{\omega}, \quad B = 0$$

$$\boxed{x(t) = \frac{F_o}{m(\omega^2 - \Omega^2)}\left[-\frac{\Omega}{\omega}\sin\omega t + \sin\Omega t\right]}$$

$$x_o = \frac{F_o\omega^2}{k(\omega^2 - \Omega^2)}\left[-\frac{\Omega}{\omega}\sin\omega t + \sin\Omega t\right], \quad DLF = \frac{\omega^2}{(\omega^2 - \Omega^2)}\left[-\frac{\Omega}{\omega}\sin\omega t + \sin\Omega t\right]$$

maximum value of DLF

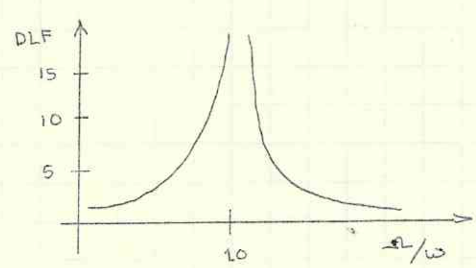
$$\frac{d}{dt}(DLF) = \frac{\omega^2\Omega}{\omega^2 - \Omega^2}\left[-\cos\omega t + \cos\Omega t\right]$$, max occurs at

$$\cos\omega t = \cos\Omega t$$

when $\Omega = \omega$, system resonates to an infinite level (bad news)

$\overline{T}$ = period of the applied load

$\omega\overline{T}$ = scaled time, or $2\pi \cdot \overline{T}/T$ or $\omega/\Omega \, 2\pi$

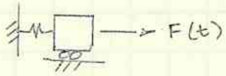


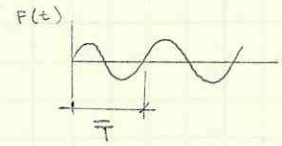
form of load applied has great impact on response.

STAEDTLER® No. 937 811E Engineer's Computation Pad

DYNAMIC RESPONSE

Review

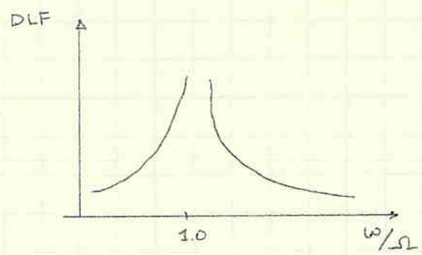


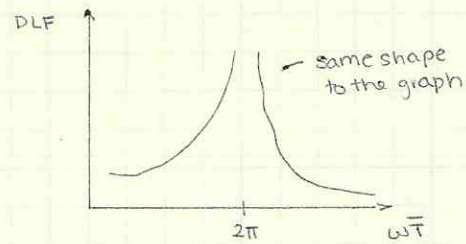


$F(t) = F_0 \sin \Omega t$

dynamic load factor (DLF) spikes as Ω approaches ω

↳ $\frac{x_{max}}{x_{static}}$



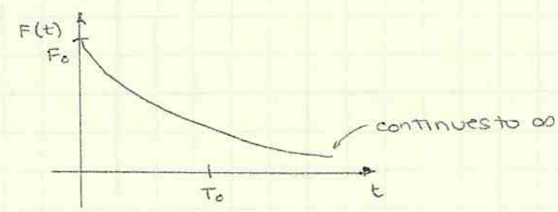


"resonance plot"

scaled time $= 2\pi \cdot \frac{\bar{T}}{T_n}$

Exponential Load

$F(t) = F_0 e^{-t/t_0}$ ← T_0

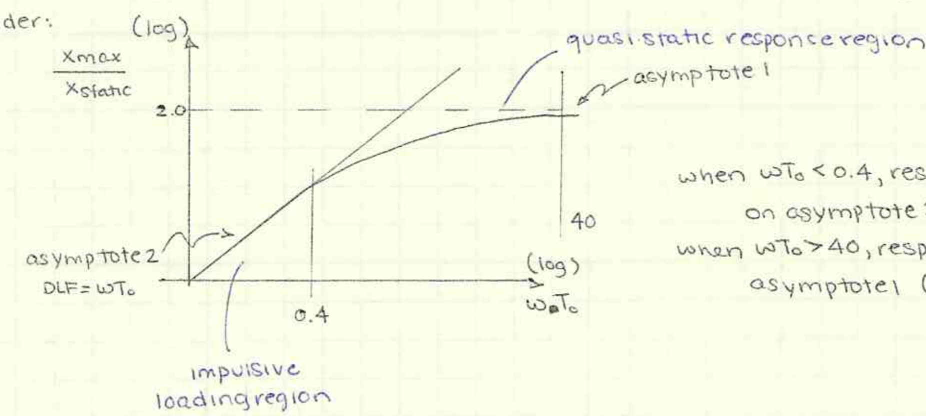


Response

$$x(t) = \frac{F_0}{k} \frac{(\omega T_0)^2}{[1 + (\omega T_0)^2]} \left[\frac{\sin \omega t}{(\omega T_0)^2} - \cos \omega t + e^{-t/T_0} \right]$$

continuous function; does not have a second half (as triangular loading had).

consider:



when $\omega T_0 < 0.4$, response is on asymptote 2 ($DLF = \omega T_0$)

when $\omega T_0 > 40$, response is on asymptote 1 ($DLF = 2$)

STAEDTLER® No. 937 811E Engineer's Computation Pad

DYNAMIC RESPONSE

Exponential load

For $\omega T_o > 40$

- $\frac{x_{max}}{x_{static}} = 2.0$

$x_{max} = 2\frac{F_o}{k}$ quasi-static response region

realize: $\omega T_o = \frac{2\pi T_o}{T_n}$

— for large value, applied load decays slowly relative to natural period ($T_o > T_n$)

For $\omega T_o < 0.4$

- $\frac{x_{max}}{x_{static}} = \omega T_o$ impulsive loading region

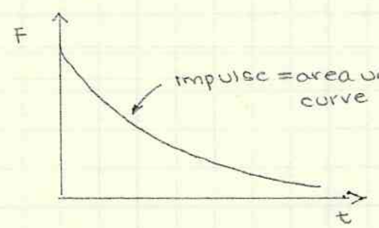
— for small value, applied load decays quickly relative to natural period

$x_{max} = \omega T_o \cdot x_{static}$, max displacement is less than the static displacement

$x_{max} = \omega T_o \cdot \frac{F_o}{k} = F_o T_o \cdot \frac{\omega}{k}$

$x_{max} = \frac{I\left[k/m\right]^{1/2}}{k}$

$= \frac{I}{[km]^{1/2}}$



$I = \int_0^\infty F(t)\,dt = \int_0^\infty F_o e^{-t/T_o}\,dt$

$= F_o T_o$

For $0.4 < \omega T_o < 40$, dynamic loading region
dynamic analysis is needed here

Energy-based procedure to compute asymptotes

Quasi-static asymptote: $x_{max} = 2\frac{F_o}{k} = 2x_{static}$

external work done by the load $= F_o x_{max}$

internal strain energy $= \frac{1}{2}kx_{max}^2$

$F_o x_{max} = \frac{1}{2}kx_{max}^2$

$F_o = \frac{1}{2}kx_{max}$, $x_{max} = 2\frac{F_o}{k}$ yay, same answer!

impulse asymptote: $x_{max} = \frac{I}{[km]^{1/2}}$

strain energy: $\frac{1}{2}kx_{max}^2$

initial kinetic energy: $F\,dt = m\,dv$ (precise form of $F=ma$)

impulse ↗ $F\,dt$; $m\,dv$ ↖ mv_o

v_o: initial velocity of system $= \frac{I}{m}$

kinetic energy, $KE = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(\frac{I}{m}\right)^2 = \frac{1}{2}\frac{I^2}{m}$

now, $\frac{1}{2}kx_{max}^2 = \frac{1}{2}\frac{I^2}{m}$

$x_{max}^2 = \frac{I^2}{km}$, $x_{max} = \frac{I}{[km]^{1/2}}$, woo!

STAEDTLER® No. 937 811E Engineer's Computation Pad

DYNAMIC RESPONSE

Pressure · Impulse (PI) diagram

technically, sometimes a force · impulse diagram

① invert the y · axis of DLF vs. ωT_o plot

$\frac{x_{max}}{x_{static}}$ becomes $\frac{x_{static}}{x_{max}} = \frac{F_o}{k \cdot x_{max}}$

now y · axis is a scaled force instead of a scaled displacement

② Take x · axis (ωT_o), multiply by scaled force to make impulse

$$\omega T_o \cdot \frac{F_o}{k \cdot x_{max}} = (F_o T_o)\frac{\omega}{k \cdot x_{max}}$$

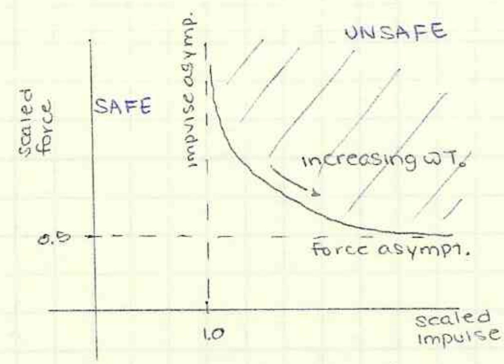
↑ impulse

$$= \frac{I}{[km]^{1/2} \cdot x_{max}} \sim \text{new x·axis}$$

③ plot with new axes

$$x: \frac{I}{[km]^{1/2} \cdot x_{max}}, \quad y: \frac{F_o}{k \cdot x_{max}}$$

note that x_{max} occurs in the denominator of both



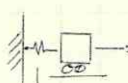
Every point on curve experiences the exact same max deflection

└ "isodeformation" or "iso·damage" curve

some people multiply y by 2.0 to get asymptote at 1.0 instead of 0.5 (1/2)

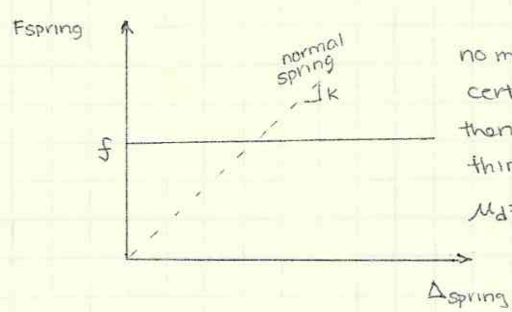
If max displacement is known, you can find what F, I combination are acceptable

Develop a P.I diagram for a rigid plastic system



$\rightarrow F(t) = F_o e^{-t/T_o}$

no longer a spring; now a rigid plastic mechanism



no movement until a certain load point, then easy movement. think of friction with $\mu_d = 0$ (dynamic)

develop asymptotes

quasi · static asymptote

$$F_o x_{max} = f \cdot x_{max}$$

[external work done by load] [internal energy; area under F · Δ plot]

$$\frac{F_o}{f} = 1 \leftarrow \text{asymptote}$$

STAEDTLER® No. 937 811E Engineer's Computation Pad

DYNAMIC RESPONSE

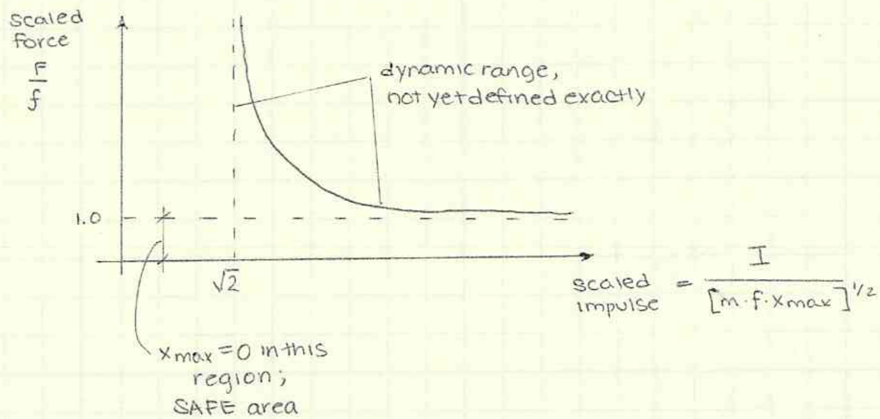
Develop PI diagram

impulse asymptote

$$\frac{I^2}{2m} = f \cdot x_{max}$$

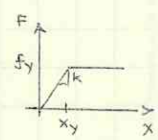
($f \cdot x_{max}$: area under F.A curve; $\frac{I^2}{2m}$: $\frac{1}{2}mv^2$, $v = \frac{I}{m}$)

$$\frac{I^2}{2mf\,x_{max}} = 1, \text{ or } \frac{I}{[m \cdot f \cdot x_{max}]^{1/2}} = [2]^{1/2}$$



More realistic case

spring is now elastic, perfectly plastic



quasi-static:

$$F_o \cdot x_{max} = \frac{1}{2} k x_y^2 + f_y (x_{max} - x_y)$$

(or $\frac{1}{2} f_y x_y$; or $k x_y (x_{max} - x_y)$)

$$= \frac{1}{2} k x_y^2 (1 - 2) + k x_y x_{max} = k \cdot x_y \left(x_{max} - \frac{1}{2} x_y\right)$$

if $\mu = n$, $x_y n = x_{max}$

$$F_o x_{max} = k \cdot x_{max} \cdot \frac{1}{n}\left(x_{max} - \frac{1}{2n} x_{max}\right)$$

$$= \frac{k}{n} x_{max}^2 \left(1 - \frac{1}{2n}\right), \quad F_o = \frac{k}{n} x_{max}\left(1 - \frac{1}{2n}\right)$$

or

$$\boxed{\frac{F_o}{k \cdot x_{max}} = \frac{1 - 1/2n}{n} = \frac{2n-1}{2n^2}}$$

impulse asymptote

$$\frac{I^2}{2m} = k \cdot x_y \left(x_{max} - \frac{1}{2} x_y\right)$$

consider specific cases of x_{max} relating to ductility

ductility ratio $\mu = \frac{x_{max}}{x_y}$

$$\frac{I^2}{\cancel{2}m} = k \cdot x_{max}^2 \left(\frac{2n-1}{\cancel{2}n^2}\right), \quad \boxed{\frac{I}{[km]^{1/2}} = x_{max}\left[\frac{2n-1}{n^2}\right]^{1/2}}$$

DYNAMIC RESPONSE

P-I curves

Generalized equations:

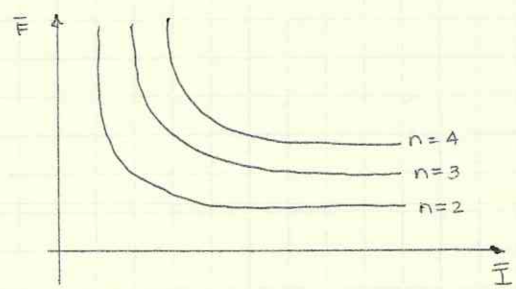
$$\frac{F_o}{k \cdot x_{max}} = \frac{2n-1}{2n^2} \quad \text{quasistatic}$$

$$\frac{I}{x_{max}[km]^{1/2}} = \left[\frac{2n-1}{n^2}\right]^{1/2} \quad \text{impulsive}$$

Assume:

$k=1$, $m=1$, $x_y=1$ ($n = x_{max}$)

$$\bar{F} = \frac{2n-1}{2n}, \quad \bar{I} = \sqrt{2n-1}$$

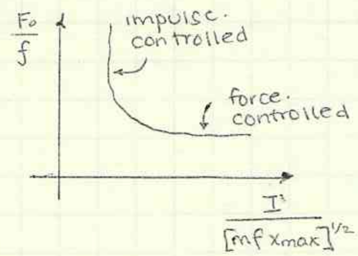


as n increases, more damage can be tolerated before failure

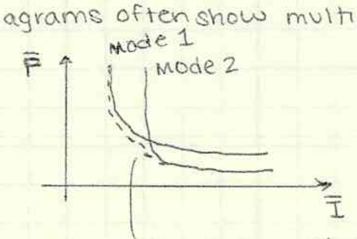
DYNAMIC RESPONSE

More on Pressure-Impulse Diagrams

- developed computationally or experimentally
  - inelastic systems show lots of scatter in maximum deformation
  - more scatter seen in force-controlled range (quasi-static load cases)



- real systems may have multiple failure modes
  - P-I diagrams often show multiple failure modes



Human Injury

Three modes of human injury characterization

1. Primary – direct blast exposure
2. Secondary – due to fragmentation
3. Tertiary – human becomes the projectile

Max pressure recommended for human exposure to avoid tertiary injury is:

2.3 psi (from UFC document)

much smaller than values for lung or ear damage

Dynamic Response Region

Numerical solution to governing equation of motion

- direct integration techniques
  - ↳ deals directly with equation of motion (no coordinate transformation...)
- ignore damping (assumption)
  - doesn't affect Δ_{max} significantly
  - effect is small relative to energy dissipated through inelastic material response
- solution of continuous process and compute response at discrete points in time involves assumptions regarding how motion varies over a given time step

DYNAMIC RESPONSE

Numerical Solution

Newmark's Method

- implicit integration procedure
  → equilibrium at time t is used to solve for the Δ, v, a at time t (or, d_t, v_t, a_t)
  - d_t = displacement at time t [as compared to $x, \dot{x}, \ddot{x}$, the "exact" solution]
  - v_t = velocity at time t
  - a_t = acceleration at time t

vs. explicit, which uses equilibrium from the time step previous (d_{t-1}), etc.

- assumptions

$$v_{t+1} = v_t + \Delta t\left[(1-\gamma)a_t + \gamma a_{t+1}\right]$$

(γ ← weighting factor)

$$d_{t+1} = d_t + \Delta t v_t + \Delta t^2\left[\left(\tfrac{1}{2}-\beta\right)a_t + \beta a_{t+1}\right]$$

consider equilibrium at time $t = t+1$

$$m a_{t+1} + k d_{t+1} = F_{t+1} \quad \text{(damping term not included)}$$

$$m a_{t+1} + k\left[d_t + \Delta t v_t + \Delta t^2\left[(1/2-\beta)a_t + \beta a_{t+1}\right]\right] = F_{t+1}$$

(a_{t+1}: one variable in equation; F_{t+1}: known)

key is that solution is progressed step-wise: information from time t is know, t+1 is unknown

$$\boxed{(m + \Delta t^2 \beta k)\, a_{t+1} = F_{t+1} - k\left[\Delta t v_t + d_t + \Delta t^2 (1/2-\beta) a_t\right]}$$

in nonlinear problems, k is not a constant

one equation, one unknown

in multi DOF problems, m and k can be matrices which must then be inverted to solve for a_{t+1}

- disadvantages
  - factorization of stiffness at every time step
- advantages
  - unconditionally stable (with good selection of β and γ) ← numerical procedure, not structure
  - accurate, even with fairly large time steps

Note: a negative k could create a denominator of zero!

NUMERICAL INTEGRATION

Newmark's Method

Assumptions:

$$v_{t+1} = v_t + \Delta t\left[(1+\gamma)a_t + \gamma a_{t+1}\right] \quad \text{(minus, not plus)}$$

$$d_{t+1} = d_t + \Delta t\, v_t + \Delta t^2\left[(1/2-\beta)a_t + \beta a_{t+1}\right]$$

Resulting equation:

$$(m + \Delta t^2 \beta k)a_{t+1} = F_{t+1} - k\left[\Delta t\, v_t + d_t + \Delta t^2(1/2-\beta)a_t\right]$$

$\gamma = 0.5$, $\beta = 0.25$ average acceleration method unconditionally stable

$\gamma = 0.5$, $\beta = 1/6$ linear acceleration method (not used in this class)

Solved by establishing equilibrium at $t = t+1$

$$m a_{t+1} + k d_{t+1} = F_{t+1}$$

equation above is found by plugging into this equation

using average acceleration method,

$$(m + 0.25k\,\Delta t^2)a_{t+1} = F_{t+1} - k\left[\Delta t\, v_t + d_t + \Delta t^2 a_t(0.25)\right]$$

for multi-degree of freedom systems, m, k, etc. would be a matrix; v, d, a would be vectors.

division of right by left not possible; use inverse matrices (no fun if K changes at each time step).

assuming SDOF,

$$a_{t+1} = \frac{F_{t+1} - k\left[\Delta t\, v_t + d_t + \Delta t^2 \frac{a_t}{4}\right]}{m + \frac{k}{4}\Delta t^2}$$

computer implementation

1. specify mass, stiffness, Δt (maybe not), β, γ, force (as a function of time), duration, initial displacement and velocity (generally zero), d_0, v_0
2. Error checking
   - no negative numbers in inputs (m, k ...)
   - $\gamma > 0.5$, $\beta = 0.5\gamma - 0.25$
3. Solve for a_0 based on initial conditions
   $$a_0 = \frac{F_0 - k d_0}{m}$$
4. while $t < t_{end}$ (duration),
   $t = t + \Delta t$
   a_t = equation from above → use to get d_t, v_t until $t = t_{end}$.

STAEDTLER® No. 937 811E Engineer's Computation Pad

NUMERICAL INTEGRATION

Newmark's Method

computer implementation

Selection of Δt — choose using natural period

$$\omega = \left[\frac{k}{m}\right]^{1/2}, \quad T_n = \frac{2\pi}{\omega}$$

$$\Delta t = \frac{T_n}{200}$$ could be acceptable (typical = 100)

↑ smaller number, less precision

errors:

- period elongation
- amplitude decay

NUMERICAL INTEGRATION

Central Difference Method ← explicit procedure (system of equations does not require inversion of k at each t.)

Fundamental assumptions

$$a_t = \frac{d_{t-1} - 2d_t + d_{t+1}}{\Delta t^2}$$

$$V_t = \frac{d_{t+1} - d_{t-1}}{2\Delta t}$$

Dynamic equilibrium

$$m a_t + k d_t = F_t$$ ← equilibrium at time t

subbing in,

$$\frac{m}{\Delta t^2}\left[d_{t-1} - 2d_t + d_{t+1}\right] + k d_t = F_t$$

↑ only unknown in this equation

[solve for d_{t+1} based on equilibrium at t]

$$\boxed{\frac{m}{\Delta t^2}(d_{t+1}) = F_t - d_t\left[\frac{2m}{\Delta t^2} - k\right] - \frac{m}{\Delta t^2} d_{t-1}}$$

again, can't just divide if m were a matrix (MDOF).

upside: since matrix on left doesn't include k, matrix does not have to be inverted at each time step (constant value!) | easy to account for elements that fail; no longer in the model

downside: not unconditionally stable

Δt must be small enough ($\Delta t_{crit} < \frac{T_n}{\pi}$)

comparison to Newmark:

implicit — Newmark; ABAQUS, ANSYS, SAP...

explicit — central difference; ABAQUS Explicit, LS-DYNA...

usage:

- method is not self-starting

given d_0, v_0 (initial conditions)

$$d_1 = \frac{\Delta t^2}{m}\left[F_0 + \left(\frac{2m}{\Delta t^2} - k\right)d_0 - \frac{m}{\Delta t^2} d_{-1}\right]$$

↖ what happend before t=0?

need d_{-1}:

- compute a_0

$$m a_0 + k d_0 = F_0, \quad a_0 = \frac{1}{m}(F_0 - k d_0)$$

- solve for d_{-1} using a Taylor series expansion

$$d(-\Delta t) = d(0) + (-\Delta t)\underbrace{\frac{d}{dt}d(0)}_{V_0} + \frac{\Delta t^2}{2} a_0 \ldots$$

$$\boxed{d_{-1} = d_0 - \Delta t v_0 + \frac{\Delta t^2}{2} a_0}$$

NUMERICAL INTEGRATION

Central Difference Method

Computer Implementation

1. Specify input parameters: m, k, $F(t)$, d_0, v_0, a_0 (calculated), duration
2. Error checking
   - $m > 0$, $k > 0$
   - stability – calculate $\Delta t < \frac{T_n}{\pi}$
3. Solve for d_{-1} (start with a_0)
4. compute d_1

   could use to calculate a_0, v_0, but we know those already
5. Begin loop.
   - compute $d_2 = \frac{\Delta t^2}{m}\left[F_1 + \left(\frac{2m}{\Delta t^2} - k\right)d_1 - \frac{m}{\Delta t^2}d_0\right]$
   - use d_2 to calculate $a_1 = \frac{d_2 - 2d_1 + d_0}{\Delta t^2}$

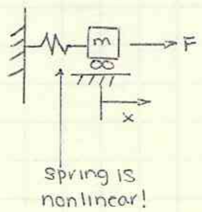
     and $v_1 = \frac{d_2 - d_0}{2\Delta t}$
   - $i = i + 1$

     $t = t + \Delta t$

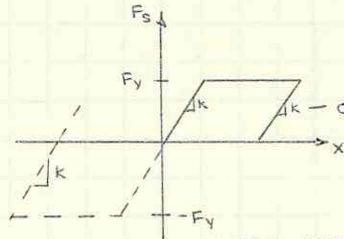
STAEDTLER® No. 937 811E Engineer's Computation Pad

NON-LINEAR SYSTEM RESPONSE

introduction



Force-deformation for spring



k — assumed to have the same slope on loading and unloading (e.g., steel)

again assumed; typical for steel, kinda

SIMPLEST INELASTIC MATERIAL MODEL!

General points of importance:

- loading + unloading stiffness
- history dependence
- hardening / softening behavior
- damage accumulation
- strength in tension vs. compression
  - └ and stiffness

Nonlinear equation of motion

$$m \cdot a + R(t) = F(t)$$

↑ < resistance in the spring > includes x term (unit of force)

Use Newmark's method first (more difficult), $\gamma = 1/2$, $\beta = 1/4$

① $d_{t+1} = d_t + \Delta t \cdot v_t + \frac{\Delta t^2}{4}(a_t + a_{t+1})$

② $v_{t+1} = v_t + \frac{\Delta t}{2}(a_t + a_{t+1})$

now, equation of motion has changed

$$m \cdot a_{t+1} + R(d_{t+1}) = F_{t+1}$$

↑ non-linear function

Modify Eq. 1 so the unknown is d, not a

$$a_{t+1} = \frac{4}{\Delta t^2}\left[d_{t+1} - d_t - \Delta t \cdot v_t\right] - a_t$$

plug into Eq. 2, get new v_{t+1} (not important now)

Revised Equilibrium equation

$$\boxed{\frac{4m}{\Delta t^2}\left[d_{t+1} - d_t - \Delta t\, v_t\right] - m a_t + R(d_{t+1}) = F_{t+1}}$$

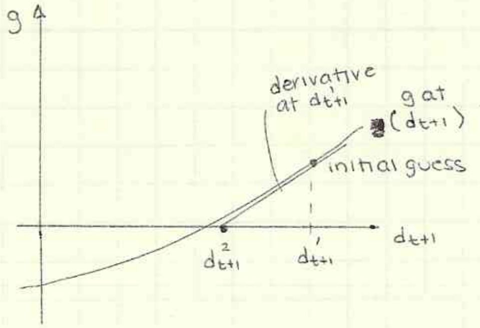
NON-LINEAR SYSTEM RESPONSE

Solve using Newton's method

$$\frac{4m}{\Delta t^2}\left[d_{t+1} - d_t - \Delta t \cdot v_t - \frac{\Delta t^2}{4} a_t \right] + R(d_{t+1}) - F_{t+1} = 0$$

$g(d_{t+1}) = 0$ residual function

done correctly, $g(d_{t+1})$ does equal zero



subscript: moment in time

superscript: iteration number

NON-LINEAR SYSTEM RESPONSE

Newmark's Method for nonlinear systems

- $a_{t+1} = \frac{4}{\Delta t^2}\left[d_{t+1} - d_t - \Delta t \cdot v_t - \frac{\Delta t^2}{4} a_t \right]$
- $v_{t+1} = -v_t + (x_{t+1} - x_t)\frac{2}{\Delta t}$ (x → d)

subscripts refer to time steps in Newmark method iterations.

Equilibrium

$$m \cdot a_{t+1} + R(d_{t+1}) = F_{t+1}$$

$R(d_{t+1})$: force in the spring (nonlinear)

F_{t+1}: applied force

$$\frac{4m}{\Delta t^2}\left[d_{t+1} - d_t - \Delta t \cdot v_t - \frac{\Delta t^2}{4} a_t \right] + R(d_{t+1}) - F_{t+1} = 0 = g(x_{t+1})$$

(g: non-linear function)

solve using Newton-Raphson Method to solve

add superscript that refers to solution iteration

$$d^u_{t+1}$$

new loop in solving – advance time, iterate to solve; then advance time again.

$$g(x^{u+1}_{t+1}) = g(x^u_{t+1}) + \Delta x^u_{t+1} \cdot g'(x^u_{t+1}) + \ldots \quad \text{or, } f(\Delta x) = f(0) + \Delta x \cdot f'(0) + \ldots$$

solve for Δx

$$\boxed{\Delta x^u_{t+1} = \frac{-g(x^u_{t+1})}{g'(x^u_{t+1})}}$$

what is g'?

$$\frac{dg(d_{t+1})}{d(d_{t+1})} = \frac{4m}{\Delta t^2} + \frac{dR(d_{t+1})}{d(d_{t+1})}$$

(evaluated at point u)

for the elastic-perfectly plastic system assumed, this is either <u>k</u> or <u>0</u>.

STAEDTLER® No. 937 811E Engineer's Computation Pad

NON-LINEAR RESPONSE

Procedure for implementation

1. input parameters / error check

   include 2 of 3: f_y, d_y, k (third is calculated); tolerance

2. compute a_0 using equilibrium at $t=0$

$$a_0 = \frac{F_0 - k d_0}{m}$$

← remember that $k(d)$; stiffness may $= 0$ at d_0

   increment the time step

3. while $t <$ duration of calculations

   a. need initial guess

$$d^{0}_{t+1} = d_t + \Delta t v_t + \Delta t^2 \cdot \frac{a_t}{4}$$

   b. compute $R(d^{0}_{t+1})$, $\frac{dR(d^{0}_{t+1})}{d(d^{0}_{t+1})}$

   ↑ spring force ↑ spring stiffness

   | generally done in a subroutine (elastic-plastic, elastic, etc.)

   c. update a_{t+1}, v_{t+1}

   d. evaluate $g(d^{0}_{t+1})$, compare to tolerance

   if abs $|g(d^{0}_{t+1})| <$ tolerance, move on: $d_t = d^{0}_{t+1}$

   if $|g(d^{0}_{t+1})| >$ tolerance,

$$d^{1}_{t+1} = d^{0}_{t+1} + \Delta d^{0}_{t+1}$$

$$\Delta d^{0}_{t+1} = \frac{-g(d^{0}_{t+1})}{g'(d^{0}_{t+1})} \leftarrow \frac{4m}{\Delta t^2} + \frac{dR(d^{0}_{t+1})}{d(d^{0}_{t+1})}$$

   iterate as necessary

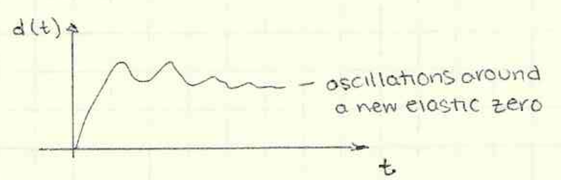
   e. increment time, continue

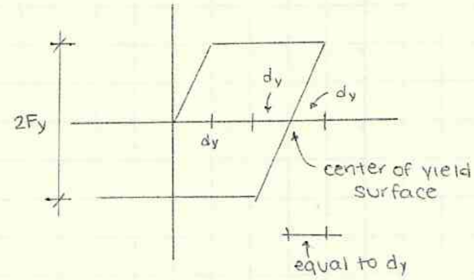
Loading vs. Unloading

- test $= v_{t+1} \cdot v_t$ to determine if x_{max} was reached

  if test > 0, motion is in same direction

  if test ≤ 0, x_{max} has been passed (change in direction)





NON-LINEAR RESPONSE

Central Difference Method for nonlinear systems

Assumptions:

$$a_t = \frac{d_{t-1} - 2d_t + d_{t+1}}{\Delta t^2}$$

unlike Newmark method, we don't have to rework equation to depend on d; already does.

$$v_t = \frac{d_{t+1} - d_{t+1}}{2\Delta t}$$

Nonlinear equation of motion at time = t,

$$m \cdot a_t + R(d_t) = F_t$$

↑ resistance at t

$R_t = R(d_t)$

subbing in,

$$\frac{m}{\Delta t^2}\left[d_{t-1} - 2d_t + d_{t+1}\right] + R_t = F_t$$

(d_{t+1}: unknown value; R_t: known, as d_t is known)

$$\boxed{d_{t+1} = \frac{\Delta t^2}{m}(F_t - R_t) + 2d_t - d_{t-1}}$$

No iterations! This is the solution!

Numerical implementation

1. Specify input data; check validity of input
2. solve for initial acceleration

   $$a_o = \frac{F_o - Kd_o}{m}$$ ← if $d_o > d_y$, $kd_o = f_y$, as system has yielded

3. solve for $d_{-1} = d_o - \Delta t v_o + \frac{\Delta t^2}{2} a_o$ ← d_{t-1}; $d_t = d_o$, d_{t+1} ...
4. step through algorithm
   - now need to keep track of nonlinear spring response store velocity to check direction of movement

Resistance Function

$v_{old} \cdot v_{new} > 0$ – loading (tension or compression) — max deflection, R

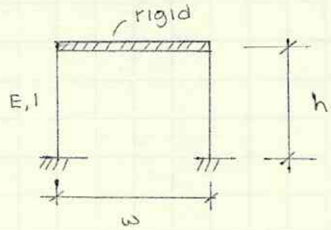
< 0 – change of direction (unloading)

new xmax, center of yield surface

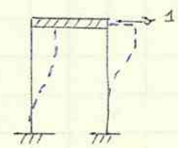
STAEDTLER® No. 937 811E Engineer's Computation Pad

NON-LINEAR RESPONSE

Example



SDOF system because beam is rigid



$\frac{12EI}{L^3} = k$ per column

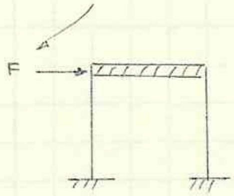
$k_{elastic} = \frac{24EI}{L^3}$ ignore axial stiffness because it is so much larger (less response)

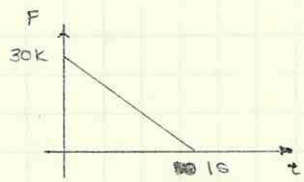
$E = 29{,}000$ ksi

$I = 110$ in^4

$h = L = 16$ ft , $k = 10.8$ k/in

Now, apply blast load



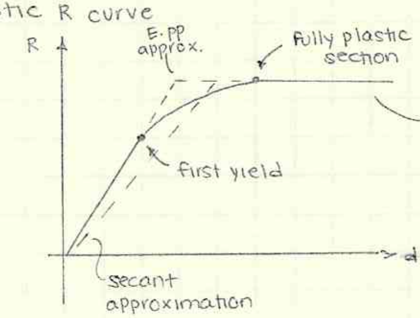


need mass (not weight!) = 0.25 k/ft/s^2 ← g (in inches/s^2) 386.4 in/s^2

$F_y = 15$ kip

beware of stiffening systems — T_n goes down, as does critical time step (oh noes!)

More realistic R curve

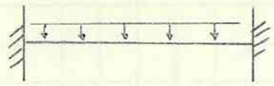


reasonable to assume k=0, as hinges form at top and bottom of each column

STAEDTLER® No. 937 811E Engineer's Computation Pad

EQUIVALENT SDOF SYSTEM ANALYSIS

Examples



Assumption:

vibration (dynamic response) of the continuous component can be described by a single deformed shape.

what shape?

1. fundamental mode shape — does not depend on the applied load
2. select the displaced shape corresponding to the static application of the applied load

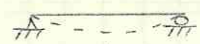
$$EIv^{iv}(x,t) + m\ddot{v}(x,t) = p(x,t)$$

solving,

$$v(x,t) = A_o \sin\left(\frac{\pi x}{\ell}\right)\sin(\omega t)$$

separation of variables x, t

] for a pin-pinned beam

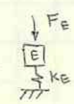


$\sin\left(\frac{\pi x}{\ell}\right)$; amplitude varies with time

Method:

1. Identify a key location on the structure
   e.g. midspan

   at key location, the displacement of the real system will match that of the equivalent system ←



real system:

$$V(x,t) = \phi(x)\cdot z(t)$$ assumed displaced shape, $\phi(x)$; variation in time, $z(t)$

2. Equivalent mass

   Based on the idea of preserving kinetic energy

   KE of real, equivalent system should be the same

$$\frac{1}{2} m_E \dot{z}(t)^2 = \int_0^L \frac{1}{2} m\,(\dot{v}(x,t))^2\,dx$$

m: mass / unit length; $\dot{v}$: velocity

$$\frac{\partial v(x,t)}{\partial t} = \frac{\partial}{\partial t}\left[\phi(x)\cdot z(t)\right] = \phi(x)\cdot\frac{\partial z(t)}{\partial t}$$

$$\frac{1}{2} M_E (\dot{z})^2 = \int_0^\ell \frac{1}{2} m\,(\phi(x))^2\,dx\cdot(\dot{z})^2$$

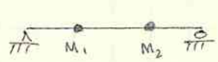
$$\boxed{M_E = \int_0^\ell m\,\phi(x)^2\,dx}$$

how much of the total mass is contributing to response?

Mass transformation factor

$$\alpha_M = \frac{M_E}{M_{total}} = \frac{\int_0^L m\,\phi(x)^2\,dx}{\int_0^L m\,dx}$$

Lumped masses



$$M_E = \sum_{i=1}^{2} M_i\,\phi(x_i)^2$$

EQUIVALENT SYSTEMS

Beam example, cont'd

3. Equivalent force

"conservation" of work done by forces (applied load vs. equivalent force)

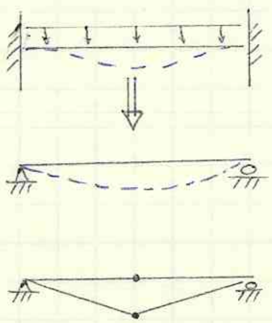
$$F_E \cdot z(t) = \int_0^{\ell} w(x,t) \cdot \phi(x)\,dx \cdot z(t)$$

$$\boxed{F_E = \int_0^{\ell} w(x,t) \cdot \phi(x)\,dx}$$

Load transformation factor

$$\alpha_L = \frac{F_E}{F_{tot}} = \frac{\int_0^L w \cdot \phi\,dx}{\int_0^L w \cdot dx}$$

4. Equivalent stiffness



elastic range

as loads increase, hinges form and boundaries change

elastic-plastic range

plastic range

Equate strain energy for real and equivalent systems

$$\frac{1}{2} k_E \cdot z^2 = \int_0^L \frac{1}{2} EI \left[\phi''(x)\right]^2 dx \cdot z^2$$

↖ area under the moment-curvature diagram

$$\boxed{k_E = \int_0^L EI \left[\phi''(x)\right]^2 dx}$$

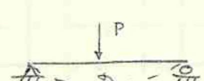
Stiffness transformation factor

$$\alpha_K = \frac{k_E}{k_{total}} = \frac{k_E}{\text{load}/\text{deflection}}$$

(~~stiffness~~) to move unit point through deflection

↖ depends on loading and boundary condition

ex.



$$\Delta = \frac{PL^3}{48EI}$$

$$P = k\Delta$$

$$k = \frac{48EI}{L^3}$$

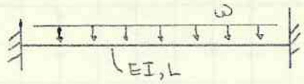
$$\boxed{\alpha_L = \alpha_K}$$

due to assumption on shape of idealized system

STAEDTLER® No. 937 811E Engineer's Computation Pad

EQUIVALENT SYSTEM

Example



Elastic Range

1. Get the mode shape, $\phi(x)$

$$v(x) = \frac{\omega L^2}{24EI}\left[x^2 - \frac{2x^3}{L} + \frac{x^4}{L^2}\right]$$ from AISC.

to get $\phi(x)$, evaluate $v(L/2)$ and set it $= 1$

$$v(L/2) = \frac{\omega L^2}{24EI}\cdot\frac{L^2}{16} = \frac{\omega L^4}{384EI}$$

$$\phi(x) = \frac{384EI}{\omega L^4}\cdot\frac{\omega L^2}{24EI}\left[x^2 - \frac{2x^3}{L} + \frac{x^4}{L^2}\right]$$

$$\phi(x) = \frac{16}{L^2}\left[x^2 - \frac{2x^3}{L} + \frac{x^4}{L^2}\right] = 16\left[\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^4\right]$$

2. Equivalent mass

$$M_E = \int_0^L m\,\phi(x)^2\,dx$$

$$M_E = \frac{128}{315}mL \quad , \quad \alpha_M = \frac{128}{315}$$

in words, only 128/315 of the total mass contributes to equivalent system $= 0.406$

3. Equivalent force

$$F_E = \omega\int_0^L \phi(x)\,dx = 8/15\,\omega L$$

$$\alpha_L = 0.533$$

4. Equivalent stiffness

$$K_E = \int_0^L EI\left[\phi''(x)\right]^2 dx$$

$$= \frac{1024EI}{5L^3}$$

$\alpha_K = 8/15$

or, $$\frac{\frac{1024EI}{5L^3}}{\frac{384EI}{\omega L^4}\cdot\omega L} = 8/15$$

usage:

$$M_E\ddot{x} + K_E x = F_E$$

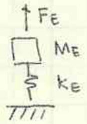
$$(\alpha_M M_T)\ddot{x} + (\alpha_L K_T)x = \alpha_L \cdot F_T$$

$$\boxed{\frac{\alpha_M}{\alpha_L}M_T\ddot{x} + K_T x = F_T}$$

($K_T x$ → $R(x)$ in non-linear system)

EQUIVALENT SYSTEMS

Example (cont'd)



Three ranges of response
- elastic
- elastic-plastic
- plastic

Elastic range

1. Assume displaced shape: based on deflected shape associated with static application of loading

   evaluate equation at point of interest (midspan), then normalize original equation

2. Find equivalent mass and mass transformation factor

$$M_E = \int_0^L m\,\phi(x)^2\,dx \quad , \quad \alpha_M = \frac{M_E}{M_{total}}$$

(m = mass per unit length.)

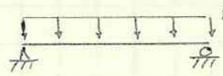
3. Find equivalent force

$$F_E = w\int_0^L \phi(x)\,dx$$

4. Find equivalent stiffness

$$k_E = \int_0^L EI\left[\phi''(x)\right]^2 dx$$

Elastic-plastic range



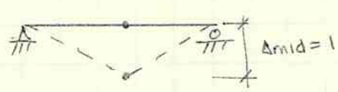
not a precise approximation of boundaries, but this works for design purposes

no longer fixed-fixed, so $\phi(x)$ changes for each range.

$$\phi_{EP}(x) = \frac{16}{5}\left[\frac{x}{\ell} - 2\left(\frac{x}{\ell}\right)^3 + \left(\frac{x}{\ell}\right)^4\right] \quad \text{(from AISC, normalized)}$$

calculate M_E, F_E, k_E in the same manner with a new $\phi(x)$

Plastic range



displacement function is not continuous

$$v(x) = \frac{2x}{L} \quad , \quad x \le L/2$$

$\alpha_M = 0.33$
$\alpha_L = 0.50$ — from tables

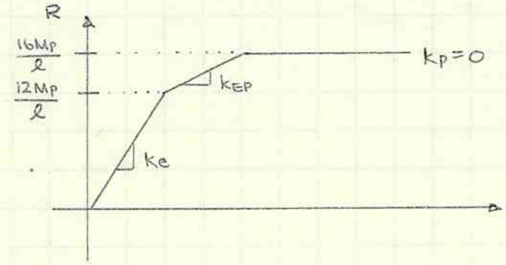
STAEDTLER® No. 937 811E Engineer's Computation Pad

STAEDTLER® No. 937 811E Engineer's Computation Pad

EQUIVALENT SYSTEMS

Develop spring resistance function

assume tension and compression are the same



for fixed-fixed beam example,

elastic to elastic-plastic occurs when:

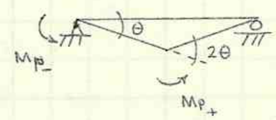
$$\frac{w\ell^2}{12} = M_p \quad \text{or} \quad w\ell = \frac{12M_p}{\ell}$$

↑ applied force

second joint occurs:

$\frac{16M_p}{\ell}$, assuming $M_{p+} = M_{p-}$

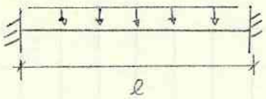
↑ from chart 5.2



moments are not in the same direction, so they don't necessarily equal one another

EQUIVALENT SYSTEM

Full example



W14 × 109 beam with continuous lateral support
(no LTB, buckling, etc.)

$A = 32\ in^2$

$d = 14.32\ in$ $E = 29{,}000\ ksi$

$I = 1240\ in^4$ $F_y = 50\ ksi$

$S = 173\ in^3$ $m = 109\ lb/ft \cdot g$

$Z = 192\ in^3$ $\ell = 20\ ft = 240\ in.$

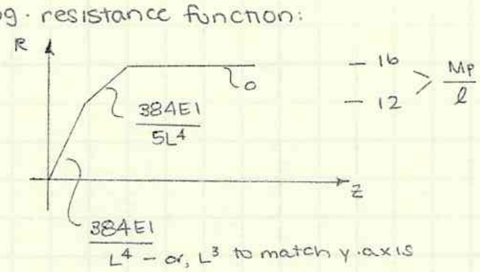
Analysis:

Mass:

weight = (109 lb/ft)(20 ft) = 2180 lb

$$\text{mass} = \frac{2180\ lb}{386.4\ in/s^2} = 5.6462\ lb \cdot s^2/in.$$

spring · resistance function:



$M_P = Z \cdot f_y = (192 in^3)(50 ksi)$

$M_P = 9600\ k \cdot in$

or 9,600,000 lb · in

Loading:

w comes from blast pressure (psi) multiplied by tributary width of loading to get lb/in

Dynamic analysis

$$M_E \ddot{z} + R(z) = F_E$$

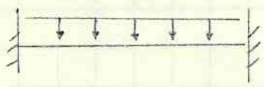
$$\frac{\alpha_M}{\alpha_L} M_T \ddot{x} + k_T x = F_T$$

↑ load · mass factor

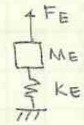
$$\boxed{\alpha_{LM} M_T \ddot{x} + R(x) = F_T}$$

EQUIVALENT SYSTEMS

Dynamic Reactions



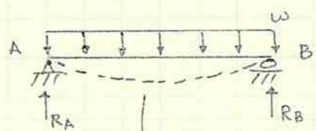
becomes



which is then analyzed dynamically

problem is set up with the displacement of the mass, z, matches the displacement of a critical point on the real structure (generally the centerpoint of the beam).

How are reaction forces calculated?



need to consider, the inertial force

$$y(x,t) = \phi(x) \cdot z(t)$$

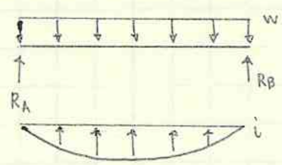
$$\text{vel.}(x,t) = \frac{\partial}{\partial t} y(x,t) = \phi(x) \cdot \dot{z}(t)$$

$$\text{accel.}(x,t) = \frac{\partial}{\partial t} \text{vel}(x,t) = \phi(x) \cdot \ddot{z}(t)$$

$$i(x,t) = m \cdot \phi(x) \cdot \ddot{z}(t)$$

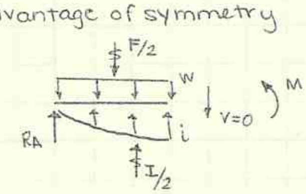
↑ inertial term

inertial force varies with position (x) in exactly the same manner as the displacements.



$$\phi(x) = \frac{16}{5}\left[\frac{x}{\ell} - 2\left(\frac{x}{\ell}\right)^3 + \left(\frac{x}{\ell}\right)^4\right]$$

Take advantage of symmetry



from handouts, $F = w\ell$ ← load resultant

$$I = \text{resultant of } i = \int_0^{\ell} m \cdot \phi(x)\, dx$$

located at:

$$\bar{x} = \frac{\int_0^{\ell/2} x \cdot i(x)\, dx}{\int_0^{\ell/2} i(x)\, dx} = \frac{61}{192}\ell \text{ from left end}$$

STAEDTLER® No. 937 811E Engineer's Computation Pad

EQUIVALENT SYSTEMS

Dynamic Reactions

Solving methods

1. $\sum F_y = 0$

$$R_A = \frac{F}{2} - \frac{I}{2}, \quad \text{or,} \quad \boxed{R_A(t) = \frac{1}{2}\left[F(t) - I(t)\right]}$$

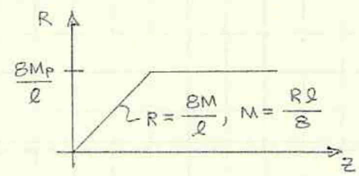
2. locate a point, o, located at centroid of inertia

$\sum M_o = 0$ ↻+

$$R_A \cdot \bar{x} - M - F/2 \cdot (\bar{x} - L/4) = 0$$

↑ (M): new variable that can be related to the spring resistance

$$M = \frac{R\ell}{8}$$



$$F_{ult} = \frac{8M_p}{\ell} = F_y$$

$$R_A = \frac{1}{\bar{x}}\left[F/2(\bar{x} - L/4) + \frac{R\ell}{8}\right]$$

$$\boxed{R_A = 0.39R + 0.11F}$$

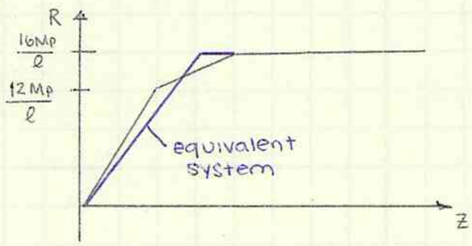
(0.39R: resistance in spring; 0.11F: applied load)

$R_A = C_R \cdot R + C_F \cdot F$; $C_R + C_F = 0.5$, as half the load goes to each support (for symmetric beams)

EQUIVALENT SYSTEMS

Returning to beam example

W14 x 109, fixed-fixed, distributed load ω



Equivalent system underestimates then overestimates; area under curve is maintained.

$$k = \frac{307\,EI}{L^3}$$ for this system

maximum displacements match; system may calculate to oscillate around a different x, but peak value is maintained.

Values available on charts (k_E)

$$\alpha_{LM_E} = \frac{\frac{1}{2}\left(\alpha_{LM\,elastic} + \alpha_{LM\,E.P}\right) + \alpha_{LM\,P}}{2} = 0.718 \text{ for this case}$$

Table 4-2. Typical Failure Criteria for Structural Elements

Element Type	Material Type	Type of Failure	Criteria	Light Damage	Moderate Damage	Severe Damage
Beams	Reinforced Concrete ($\rho >$ 0.5%/face)	Global Bending/ Membrane Response	Ratios of Center-line Deflection to Span, δ/L	4%	8%	15%
		Shear	Average Shear Strain Across Section, γ_v	1%	2%	3%
	Steel	Bending/ Membrane	δ/L	5%	12%	25%
		Shear	δ/L	2%	4%	8%
Slabs	Reinforced Concrete ($\rho >$ 0.5%/face)	Bending/ Membrane	δ/L	4%	8%	15%
		Shear	γ_v	1%	2%	3%
Columns	Reinforced Concrete ($\rho >$ 0.5%/face)	Compression	Shortening/ Height	1%	2%	4%
	Steel	Compression	Shortening/ Height	2%	4%	8%
Load-Bearing Walls	Reinforced Concrete ($\rho >$ 0.5%/face)	Compression	Shortening/ Height	1%	2%	4%
Shear Walls	Reinforced Concrete ($\rho >$ 0.5%/face)	Shear	Average Shear Strain Across Section	1%	2%	3%

APPENDIX 5.B
SUMMARY TABLES FOR RESPONSE CRITERIA

The following descriptions apply to the response ranges mentioned in the tables:

Low Response: Localized building/component damage. Building can be used, however repairs are required to restore integrity of structural envelope. Total cost of repairs is moderate.

Medium Response: Widespread building/component damage. Building cannot be used until repaired. Total cost of repairs is significant.

High Response: Building/component has lost structural integrity and may collapse due to environmental conditions (i.e. wind, snow, rain). Total cost of repairs approach replacement cost of building.

TABLE 5.B.1: Response Criteria for Reinforced Concrete

Element Type	Controlling Stress	μ_a	Support Rotation, θ_a (2)		
			Low	Medium	High
Beams	Flexure	N/A	1	2	4
	Shear: (1)				
	Concrete Only	1.3			
	Concrete + Stirrups	1.6			
	Stirrups Only	3.0			
	Compression	1.3			
Slabs	Flexure	N/A	2	4	8
	Shear: (1)				
	Concrete Only	1.3			
	Concrete + Stirrups	1.6			
	Stirrups Only	3.0			
	Compression	1.3			
Beam-Columns	Flexure:		1	2	4
	Compression (C)	1.3			
	Tension (T)	(3)			
	Between C & T	10.0			
	Shear (1)	1.3			
Shear Walls, Diaphragms	Flexure	3	1	1.5	2
	Shear (1)	1.5			

(1) Shear controls when shear resistance is less than 120% of flexural resistance.
(2) Stirrups are required for support rotations greater than 2 degrees.
(3) Ductility ratio = $0.05\ (\rho - \rho') < 10$

TABLE 5.B.2: Response Criteria for Reinforced Masonry

Element Type	μ_a (1)	Support Rotation, θ_a (2)		
		Low	Medium	High
One-Way	1	0.5	0.75	1
Two-Way	1	0.5	1	2

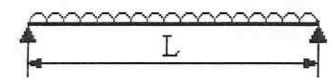
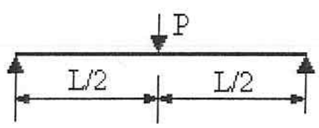
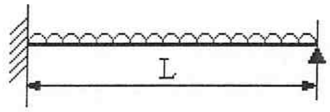
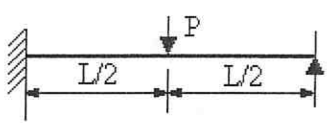
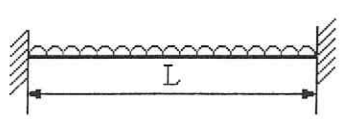
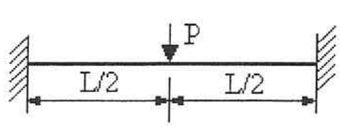
(1) Ductility ratio values (μ_a) apply to low response range.

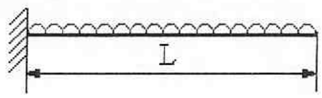
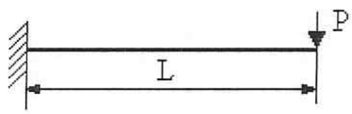
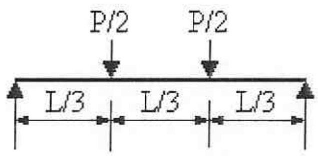
TABLE 5.B.3: Response Criteria for Structural Steel

Element Type	Response Range					
	Low		Medium		High	
	μ_a	θ_a	μ_a	θ_a	μ_a	θ_a
Beams, Girts, Purlins	3	2	10	6	20	12
Frame Members (1)	1.5	1	2	1.5	3	2
Cold-Formed Panels	1.75	1.25	3	2	6	4
Open-Web Joists	1	1	2	1.5	4	2
Plates	5	3	10	6	20	12

(1) Sidesway limits for frames: low = H/50, medium = H/35, high = H/25

Table 4-6 Ultimate Shear Stress at Distance d_e from Face of Support for One-Way Elements

Edge Conditions and Loading Diagrams	Ultimate Shear Stress V_U
	$\dfrac{r_u\left(\dfrac{L}{2}-d_e\right)}{d_e}$
	$\dfrac{R_u}{2d_e}$
	$LEFT\,SUPPORT \quad r_u\left(\dfrac{5L}{8}-d_e\right)/d_e$ $RIGHT\,SUPPORT \quad r_u\left(\dfrac{3L}{8}-d_e\right)/d_e$
	$LEFT\,SUPPORT \quad \dfrac{11R_u}{16d_e}$ $RIGHT\,SUPPORT \quad \dfrac{5R_u}{16d_e}$
	$\dfrac{r_u\left(\dfrac{L}{2}-d_e\right)}{d_e}$
	$\dfrac{R_u}{2d_e}$

Edge Conditions and Loading Diagrams	Ultimate Shear Stress V_U
	$\frac{r_u(L-d_e)}{d_e}$
	$\frac{R_u}{d_e}$
	$\frac{R_u}{2d_e}$

Design Requirements for Shear Stresses in Reinforced Concrete Components

1. The ultimate shear stress v_u must not exceed $10\sqrt{f'_{dc}}$ in sections using stirrups. The thickness of such sections must be increased and/or the quantity of flexural reinforcement reduced in order to bring the value of v_u within tolerable limits.

2. The minimum design stress (excess shear stress v_u - v_c) used to calculate the required amount of shear reinforcement, must conform to the limitations of the table shown below (UFC 3-340-02).

3. When stirrups are required, the area A_v should not be less than 0.0015 bs.

4. When stirrups are provided, the required area A_v is determined at the critical section, *and this quantity of reinforcement must be uniformly distributed throughout the element.*

5. Single leg stirrups should be used for slabs. At least one stirrup must be located at each bar intersection. Beams must use closed ties that completely enclose all flexural bars.

6. The maximum spacing of stirrups s is limited to $d/2$ for Type I cross sections and $d_c/2$ for Type II sections, but not greater than 24 inches.

Table 4-4 Minimum Design Shear Stresses for Slabs

Design Range	Type of Cross-Section	Type of Structural Action	Type of Shear Reinforcement	Excess Shear Stress v_u - v_c		
				$v_u \le v_c$	$v_c < v_u \le 1.85v_c$	$v_u > 1.85v_c$
Z ≥ 3.0 (US units) for	Type I	Flexure	Stirrups	0	v_u - v_c	v_u - v_c
	Type II	Flexure	Stirrups	0.85v_c	0.85v_c	v_u - v_c
	Type II & Type III	Tension Membrane	Stirrups	0.85v_c	0.85v_c	v_u - v_c
Z < 3.0 close-in	Type I*	Flexure	Stirrups or Lacing	0.85v_c	0.85v_c	v_u - v_c
	Type II & Type III	Flexure or Tension Membrane	Stirrups or Lacing	0.85v_c	0.85v_c	v_u - v_c

*Verify that spall is prevented.

DESIGNING FOR BLAST

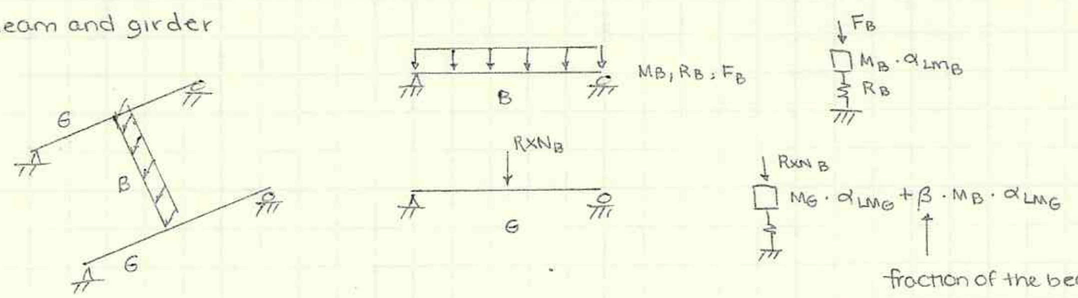
Design Process

- member-by-member design approach
- each member is treated as an SDOF system

Alternative: FEA

	SDOF	FEA	
(+)	· simple · fast / cheap · generally accurate · repeatability of analyses	· "accurate" (if used correctly) · prettier plots / better resolution of stress/strain variation in structure · automatically includes coupling of element response	(+)
(−)	· dynamic response among members is uncoupled [not so bad if $T_{n1}/T_{n2} > 2$] · controlling mode of response needs to be known to get $\phi(x)$	· more computationally demanding · accuracy may not be necessary given uncertainty of loading · defining the loading is very challenging └ over time and space · material modeling is difficult (damn concrete.) · difficult to account for localized failures	(−)
(+)	· localized failures can be readily accounted for		

Design of beam and girder



↑ (β) fraction of the beam mass contributing to response of the girder

~ 0.2 for RC structures [depends on rigidity of the beam]

$= 0.5$ for a rigid beam

Design Process

1. Design for non-blast loads first — size members initially (← elastic, conservative response)
2. Develop threat scenarios
3. Compute response to blast
   - ↑ inelastic, non-factored response
   - accurate load and resistance (material) properties
4. compare response to desired response; modify design as necessary.

STAEDTLER® No. 937 811E Engineer's Computation Pad

DESIGNING FOR BLAST

Reinforced concrete Design

Material properties – Guidelines

- use normal, Grade 60 steel
  - limit use of high-strength steel, unless there is supporting test data
  - we desire good ductility of steel
  - no data on epoxy-coated bars
- concrete: "normal" concrete
  - f'_c in 4–8ksi strength range
  - avoid high strength due to ductility concerns

"Actual" material properties used in design

- Static Increase Factor (SIF)
  - accounts for the in situ strength being greater than the specified material strength.
  - Table 5.A.1 on handout (ASCE doc)
  - SIF = 1.1 for concrete and steel (EBW numbers)
- Age Increase Factor
  - $K_A = 1.10$ for Type I cement concrete < 6 months
  - $K_A = 1.15$ after 6 months
- Strain Rate Effect or Dynamic Increase Factor
  - see table
  - values depend on scaled standoff and type of response (e.g. flexure vs. shear vs. compression, etc.)

STAEDTLER® No. 937 811E Engineer's Computation Pad

DESIGNING FOR BLAST

Reinforced concrete Design: shear and Flexure

Normal (not deep) beam members

→ clearspan = L

depth = h

ACI says $L/h > 4$ (shear), > 2 (flexure)

military says $L/h > 5$ (shear), 2 (flexure, simple)

2.5 (flexure, cont.)

Design goals

1. determine dynamic ultimate capacity to develop flexural resistance function.
2. determine design requirements to prevent shear failure.

For concrete:

- dynamic compressive strength

$$f'_{dc} = f'_c \cdot \alpha_s \cdot \alpha_A \cdot DIF$$

(f'_{dc}: d for dynamic; α_s: static strength factor; α_A: age factor; DIF: dynamic increase factor)

For rebar:

- dynamic yield strength

$$f_{dy} = f_y \cdot \alpha_s \cdot DIF$$

- dynamic ultimate strength

$$f_{du} = f_u \cdot \alpha_s \cdot DIF$$

(α_s, DIF are different)

- dynamic design strength for rebar

f_{ds}	group 1 member	group 2 member
f_{dy}	$0° \le \theta \le 3°$	$0° \le \theta \le 5°$ (θ: rotation at support)
$f_{dy} + \frac{1}{4}(f_{du} - f_{dy})$	$3° \le \theta \le 5°$	$5° \le \theta \le 8°$
$f_{dy} + \frac{1}{2}(f_{du} - f_{dy})$	$\theta > 5°$	$\theta > 8°$

accounts for strain hardening; should only be used for that steel.

group 1: 1 or 2 fixed supports, $L/h \le 10$
simple supports, $L/h \le 6$

group 2: 1 or 2 fixed supports, $10 < L/h \le 15$
simple supports, $6 < L/h \le 10$

Beyond that, beam is more like a cable or membrane, with behavior governed by direct tension. f_y is conservative.

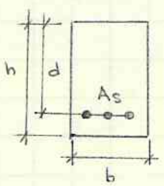
STAEDTLER® No. 937 811E Engineer's Computation Pad

DESIGNING FOR BLAST

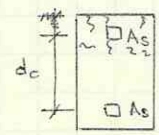
Reinforced concrete Design

cross-section types

Type I



Type II



compression concrete is crushed and no longer effective

Type II must have compression steel to resist initially loading

$\theta \le 2°$ (Type I)

$\theta > 2°$ (Type II) ← support rotation

Type I section

$$M_{du} = \frac{A_s f_{ds}}{b}\left[d - \frac{a}{2}\right], \quad a = \frac{A_s f_{ds}}{0.85\, b\, f'_{dc}}$$

M_{du} ← per unit width

only changes from standard design are the material properties: $f_y \rightarrow f_{ds}$, etc.

compression steel is added to resist rebound, but is generally ~~agreed~~ ignored in moment capacity calculations.

$\rho = \frac{A_s}{bd}$. Required $\rho \le 0.75\rho_b$ ← 75% balanced condition

$$\rho_b = 0.85\beta_1 \frac{f'_{dc}}{f_{ds}}\left[\frac{87{,}000}{87{,}000 + f_{ds}}\right]$$

$$\beta_1 = 1.05 - 0.05\frac{f'_{dc}}{1000}$$

$4000\text{ psi} \le f'_{dc} \le 8000\text{ psi}$

Type II section

No concrete contribution

$$M_{du} = \frac{A_s f_{ds} d_c}{b}$$

d_c ← lever arm between tension and compression steel

A_s ← minimum of A_s and A'_s

M_{du} ← again, per unit width

DESIGNING FOR BLAST

Design Recommendations in concrete

- $0.6\% \leq \rho \leq 0.8\%$ (reinforcement ratio)
  generally lower than typical designs.
- ideal to select more, smaller bars than few larger bars
- bundled bars are a very bad idea (UFC limits to 3 bars)
- typically, use the same steel top and bottom
  - avoid a dropoff in capacity when transitioning from type I to type II
  - needed for rebound capacity
  - specification requires $A_s' \geq \frac{1}{2} A_s$

DESIGNING FOR BLAST

Reinforced concrete: diagonal tension or sectional shear

for a beam,

$$\text{shear stress} = \frac{V_{cr}}{bd}$$

V_{cr} ← shear force at critical section

b ← beam width

d ← "depth"

UFC: d or d_c for Type I and Type II, respectively (very conservative calculation)

other documents say to use d always.

critical section

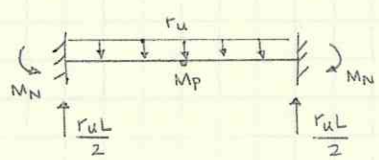
V_{cr} for design

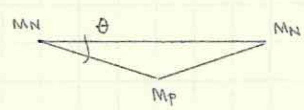
- based on static analysis
- requires sufficient shear capacity to ensure a flexural response

Method:

- compute shear based on static application of R_u or R_m

Example:





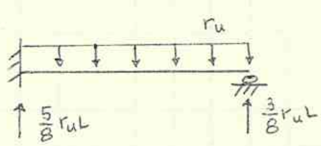
$$2M_N(\theta) + M_P(2\theta) = \left(r_u \cdot \frac{L}{2}\right)\left(\frac{L}{4}\cdot\theta\right)\cdot 2$$

$$r_u = \frac{8}{L^2}(M_N + M_P)$$

↑ load beam can take before forming three hinges

Now design for support reactions, $r_u L/2$, so that shear at support does not control response.

Example:



- calculate r_u to cause hinging
- design for shear at each support

DESIGNING FOR BLAST

Reinforced concrete shear capacity

$V_{capacity} = v_c + v_s$

→ $A_v = (v_u - v_c)\dfrac{b\,s}{f_{dy}}$ – area of steel needed for adequate performance

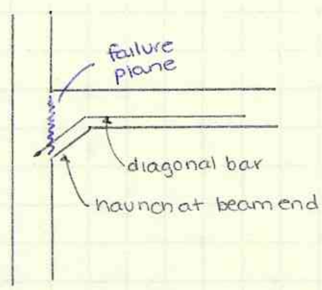
– may or may not include ϕ in denominator

→ $2\sqrt{f'_{dc}}$ or $(1.9\sqrt{f'_{dc}} + 2500\rho) \le 3.5\sqrt{f'_{dc}}$

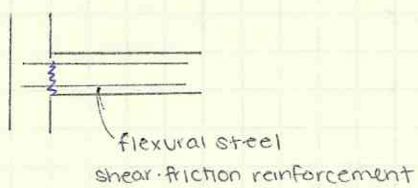
($2\sqrt{f'_{dc}}$ ↑) different from in flexural calcs – different dynamic increase factor

(2500ρ ↑) reinforcement ratio

Direct Shear in R/C structures



- increased capacity at beam end
- more concrete, more steel
- recommended for a slab, not a beam
- prefered in UFC document



- can't use steel stressed in flexural tension (only compression steel is okay)
- ignores aggregate interlock component

Direct shear capacity of concrete

$V_{c_{direct}} = 0.16\sqrt{f'_{dc}}\,b\,d$ (force; stress is divided by bd)

↑ another new f'_{dc}

- concrete can't be in tension
- $\theta \le 2°$ (support rotation)

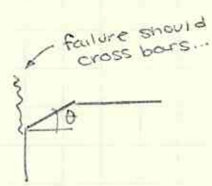
Capacity of bars / area required

$A_d = (v_{crit} \cdot b - v_c) \cdot \dfrac{1}{f_{ds} \cdot \sin\alpha}$

α = angle of bar off of ⊥ to failure plane

(v_{crit} ↑) force per unit length; (at face of the support)

(v_c ↑) can be zero



STAEDTLER® No. 937 811E Engineer's Computation Pad

STAEDTLER® No. 937 811E Engineer's Computation Pad

DESIGNING FOR BLAST

other constants:

modulus of elasticity

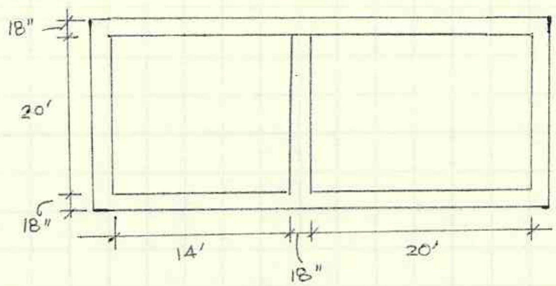
$E = 57{,}000\sqrt{f'_c}$, without modification (static value)

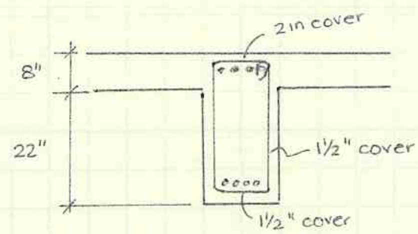
moment of inertia

$I = \frac{I_{gross} + I_{crack}}{2}$ use upon initial loading, through formation of initial plastic hinge ⟶ mechanism formation (all hinges formed)

$I = I_{crack}$ beyond that point (after plastic response has occured)

Example: response of a roof

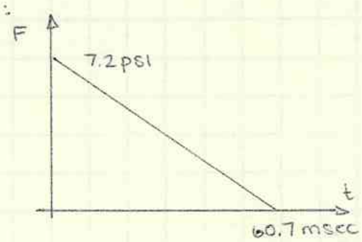




$f'_c = 4000$ psi

Gr. 60 rebar

loading:



Criteria:

- $\theta \le 2°$ (type I member)
- $L/h = 240\text{in}/18\text{in} = 8$, Group I

1. Identify DIF

	REBAR		CONCRETE
	f_{dy}/f_y	f_{du}/f_u	f'_{dc}/f'_c
bending	1.17	1.05	1.19
diagonal tension	1.0	1.0	1.0
direct shear	1.1	1.0	1.10

2. Identify other α

$\alpha_S = 1.1$ for rebar and concrete

$\alpha_A = 1.1$ (concrete only)

3. Calculate material properties

$f'_{dc_{bend}} = (4000\text{psi})(1.1)(1.1)(1.19) = 5760$ psi

$f'_{dc_{diag}} = (4000\text{psi})(1.1)(1.1)(1.0) = 4840$ psi

$f'_{dc_{direct}} = (4000\text{psi})(1.1)(1.1)(1.1) = 5324$ psi

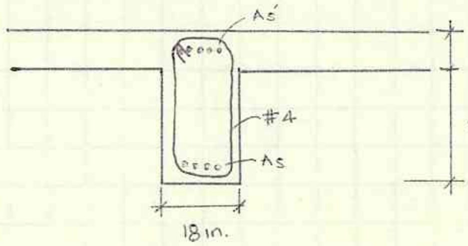
$f_{ds} = f_{dy}$

$f_{ds_{bend}} = 77.22$ ksi

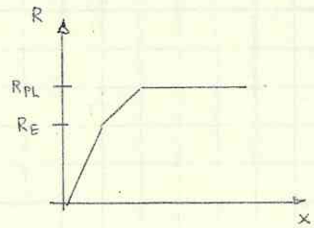
$f_{ds_{diag}} = 66$ ksi

$f_{ds_{direct}} = 72.6$ ksi

EXAMPLE PROBLEM



1. identify DIF
2. identify α values
3. calculate material properties



5-#6 bars

$M_N \neq M_P$ because, while $A_s = A_s'$, the cover and thus d are different

$$R_E = 12 \frac{M_N}{L}$$

$$R_{PL} = \frac{8}{L}(M_P + M_N)$$

4. calculate K, resistance functions

$A_s = A_s' = 2.2 \text{ in}^2$

$d_p = 30\text{ in} - 1.5\text{ in} - 0.5\text{ in} - 0.75\text{ in}/2 = 27.625\text{ in}$

(1.5 in = cover; 0.5 in = stirrup; 0.75 in/2 = half of bar)

$d_n = 27\tfrac{1}{8}$ in. (cover = 2 in., not 1.5 in.)

→ check reinforcement ratio

$$\rho_p = \frac{2.20 \text{ in}^2}{(18\text{ in})(27\tfrac{5}{8}\text{ in})} = 0.0045 \qquad \rho_n = 0.0044 \qquad [0.4\text{–}0.7\% \text{ is good}]$$

Moment capacity

$$a = \frac{A_s f_{dy}}{0.85 f'_{dc} b} = \frac{(2.2\text{ in}^2)(77{,}200 \text{ psi})}{0.85(5760 \text{ psi})(18 \text{ in})} = 1.93 \text{ in.}$$

(f_{dy} ← f_{ds})

$$M_N = A_s f_{dy}\left[d_n - a/2\right] = (2.2\text{ in}^2)(77.2 \text{ ksi})\left[27\tfrac{1}{8}\text{ in.} - 1.93\text{ in}/2\right]$$

$$= 4{,}443.252 \text{ k·in}$$

$$M_p = 4{,}528.172 \text{ k·in}$$

5. Calculate remaining constants, etc.

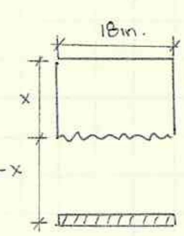
- $E_c = 57{,}000\left[4000 \text{ psi}/1000\right]^{1/2} = 3605 \text{ ksi}$

  could be increased by ~20%, but UFC says not to.

- $E_s = 29000$ ksi

- $I = \tfrac{1}{12} bh^3 = \tfrac{1}{12}(18\text{ in})(30\text{ in})^3 = 40{,}500 \text{ in}^4$

  I_{cr_p} = found around neutral axis



$$A_s = \frac{29000 \text{ ksi}}{3605 \text{ ksi}}(2.2\text{ in}^2) = 17.7 \text{ in}^2$$

$$bx \cdot \frac{x}{2} = nA_s(d_p - x)$$

$$x = 6.45 \text{ in}^2$$

$$I_{crp} = \tfrac{1}{3}bh^3 + Ad^2 = 9546 \text{ in}^4$$

(A ← A_s)

STAEDTLER® No. 937 811E Engineer's Computation Pad

EXAMPLE PROBLEM

5. Calculations for remaining properties (cont'd)
   - I_{cr_n} follows same procedure

$$I_{cr_n} = 9536.1 \text{ in}^4$$

   - I for use in calculations of stiffness

$$\frac{1}{2}\left[I_{gross} + \frac{1}{2}\left(I_{cr_p} + I_{cr_n}\right)\right] = 25{,}021 \text{ in}^4$$

   - Elastic Range of Response
     - $K_{LM} = 0.77$
     - $R_E = \frac{12 M_N}{L} = \frac{12(370.3 \text{ k}\cdot\text{ft})}{20 \text{ ft}} = 222.2 \text{ kip}$
     - $k_E = \frac{384EI}{L^3} = \frac{384(3605 \text{ ksi})(25{,}021 \text{ in}^4)}{(240 \text{ in})^3} = 2505.6 \text{ k/in}$

   - Elastic-Plastic Range
     - $K_{LM} = 0.78$
     - $R_{EP} = \frac{8}{L}(M_P + M_N) = 299.0 \text{ k}$
     - $k_{EP} = \frac{384EI}{5L^3}$ ← use cracked moment of inertia as section is obviously cracked $I_{avg} = 9541.3 \text{ in}^4$ | makes sense when using an R plot with three distinct regions (no equivalent E-EP k).

$k_{EP} = 191.1 \text{ k}\cdot\text{in}$

(using $I_{cr} + I_g$, $k_{EP} = 501 \text{ k}\cdot\text{in}$, a significant increase)

   - Plastic Range
     - $K_{LM} = 0.66$
     - $R_{EP} = R_{EP} = 299.0 \text{ K}$
     - $k_{EP} = 0$

6. Calculate mass quantities
   - unit weight of concrete = 150 lb/ft³
   - include influence (weight) of part of the slab (20%, per ACI)

$$weight_{beam} = (18 \text{ in})(30 \text{ in})(240 \text{ in})(150 \text{ lb/ft}^3) / (12 \text{ in/ft})^3$$

$$= 11.25 \text{ k}$$ ← divide by gravity to get mass = 29.115 lb·s²/in.

$$weight_{slab} = (20\%)(20 \text{ ft} + 14 \text{ ft})(8 \text{ in})(20 \text{ ft})(1/2)(150 \text{ lb/ft}^3)(1/12 \text{ ft/in}) = 6800 \text{ lb}$$

$$= 17.6 \text{ lb}\cdot\text{s}^2/\text{in.}$$

(20%) ↑ contribution; (20 ft + 14 ft) ↑ spans; (8 in) ↑ thickness; (20 ft) ↑ length; (1/2) ↑ half the span, not all (oops); 17.6 lb·s²/in. ↑ significant contribution

No. 937 811E Engineer's Computation Pad STAEDTLER®

EXAMPLE PROBLEM

7. Figure load history

$P = 7.2$ psi

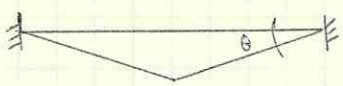
$$\text{Load into beam} = (7.2\text{ psi})\left[\begin{matrix}(10\text{ ft} + 7\text{ ft})(20\text{ ft})(144\text{ in}^2/\text{ft}^2) \\ +18\text{ in.}\end{matrix}\right] = 383{,}616\text{ lb}$$

(resultant load on beam, wL, to match previous units)

At this point, calculations in matlab, mathcad, excel are appropriate

consider assumptions

- support rotation < 2°



$$\theta = \frac{2 \cdot \Delta max}{L} \cdot \frac{180}{\pi} = 1° \text{ for this scenario}$$

fairly small, design is more than is needed to meet peformance criteria

Left to consider:

- shear: diagonal tension at support
  - → at critical section d away

$$V_u = \frac{V_d}{b\,d} \leq 10\sqrt{f'_{dc}} \qquad V_d = \frac{(L/2 - d_{bot})\,r_{plastic}/2}{L/2}$$

from EBW's mathcad sheet

$r_{plastic}$ = load to cause mechanism statically

$$V_c = 1.9\sqrt{f'_{dc}} + 2500\rho \quad \text{or} \quad V_c = 2\sqrt{f'_{dc}}$$

$$V_{s_{needed}} = \max(V_u - V_c,\ V_c)$$

require $V_s \geq V_c$, at least.

- find stirrup spacing needed
- check minimum steel required (table)
  - + $A_v \geq 0.0015\,b \cdot \text{spacing}$
- other standard ACI checks

- direct shear at support
  - dowel action from flexural steel
  - direct shear capacity of concrete (not in tension)

$$V_{c_{direct\ shear}} = 0.16 f'_{dc}\, b \cdot d$$

$= 415.989$ K for this example, compare to reaction force

PROGRESSIVE COLLAPSE

Design and Analysis

- Timeline and History
  - ANSI A58 — precursor to ASCE 7
    - 1972: "consider unanticipated loads"
  - 1976 — PCI recommendations
    - highly influenced by Dr. Breen
    - mainly considered a precast concrete problem
      - └ and construction
  - 1968: Ronan Point
    - by 1970, UK standards existed
    - 1974: ties were "created"/"discovered"
      - previously not used
  - 1973: Skyline Plaza
- Info learned ~~for~~ from Pentagon
  - Good things:
    - — redundant load paths
    - — short span beams
    - — substantial continuity across columns
    - — designed for 150 psf (psi?) live load
    - — spiral reinforcing ties in square columns
- what is progressive collapse?
  - Local collapse by UK standards
    - 15% of floor or roof ~~of~~ or 1000 sq ft
- ASCE 7 Design Guidelines
  - Direct design ~ consider a specific scenario and design for it
  - Indirect design — follow principles of good design — continuity, etc.
    - (ability to develop catenaries, large tie forces...)

(Direct design →) assumes a vulnerable column or beam to be able to not fail; thus maintaining structural integrity. ← only used by DOD and GSA (general services administration)

- designing for a known threat is difficult
  - because of the need to define the threat.

- Major design guidelines
  - GSA guidelines — only partially publically available
  - UFC guidelines — totally available
    - • "threat-independent" design guideline
    - • occupancy category — importance of building and who occupies it
    - • Requirements:
      - — tie forces → [must be held by the floor/roof, unless beam is crazy-flexible] floors hold up beams, not beams holding floors.
      - — alternate path analysis → primary and secondary members; deformation or force-controlled; see ASCE 41
      - — enhanced local resistance (strengthen exterior columns)

PROGRESSIVE COLLAPSE

Structural Analysis

- linear static – acceptable for simple structures
  - must consider deformation and load-controlled cases

  recognize ability to form hinges → loads magnified to approximate dynamic loading effects.

  "immaculate removal" of a member

- nonlinear static
- dynamic (nonlinear)

General Notes

- refer to ASCE 41
- consult the UFC document ← contains examples for wood, steel, and concrete buildings (multiple analyses).
- download EBW's notes
- is going to be incorporated into ASCE 7 in the coming years (IBC is still resisting heavily).
- EBW + Engelhardt to start PC project in 2010

STAEDTLER® No. 937 811E Engineer's Computation Pad

Gmail by Google

Catherine Hovell <cghovell@gmail.com>

blast figures and uses

Williamson, Eric B <eric.williamson@engr.utexas.edu> **Sat, Feb 20, 2010 at 5:27 PM**

To: Catherine Hovell <cghovell@gmail.com>

Catherine,

Figs. 2-9 and 2-10 are specifically for airblast load cases, and the angle on the x-axis in those graphs corresponds to the height of burst relative to the standoff from the target of interest. These charts are used only for computing Mach region loads that develop for cases in which the charge is positioned at some height above the ground (i.e., an airblast load case).

The other charts are for oblique shocks in which the shock front does not interact perpendicularly with the target of interest. These charts are the ones you will need for your current HW assignment. In addition, I just added a few charts on Blackboard that can be used for estimating an equivalent uniform roof load. Because the charge is assumed to be hemispherical for your HW, Figs. 2-9 and 2-10 should not be used.

Finally, the P_r_alpha is the reflected pressure for an oblique shock front. When computing loads in the Mach region, however, rather than using Figures 2-9 and 2-10 (like you did on HW 2), you can take the P_r_alpha that accounts for the ground reflection and then treat that value as a side-on pressure for subsequent loads against other surfaces. Thus, additional reflections can occur after the ground reflection if the Mach region interacts with a structure. When the charge is on the ground, however, only reflections with a structure can occur because the ground reflection is assumed to happen instantaneously. Thus, under these conditions, when you compute P_r_alpha, it is the reflected pressure for a pressure wave interacting with a surface at an oblique angle to the wave.

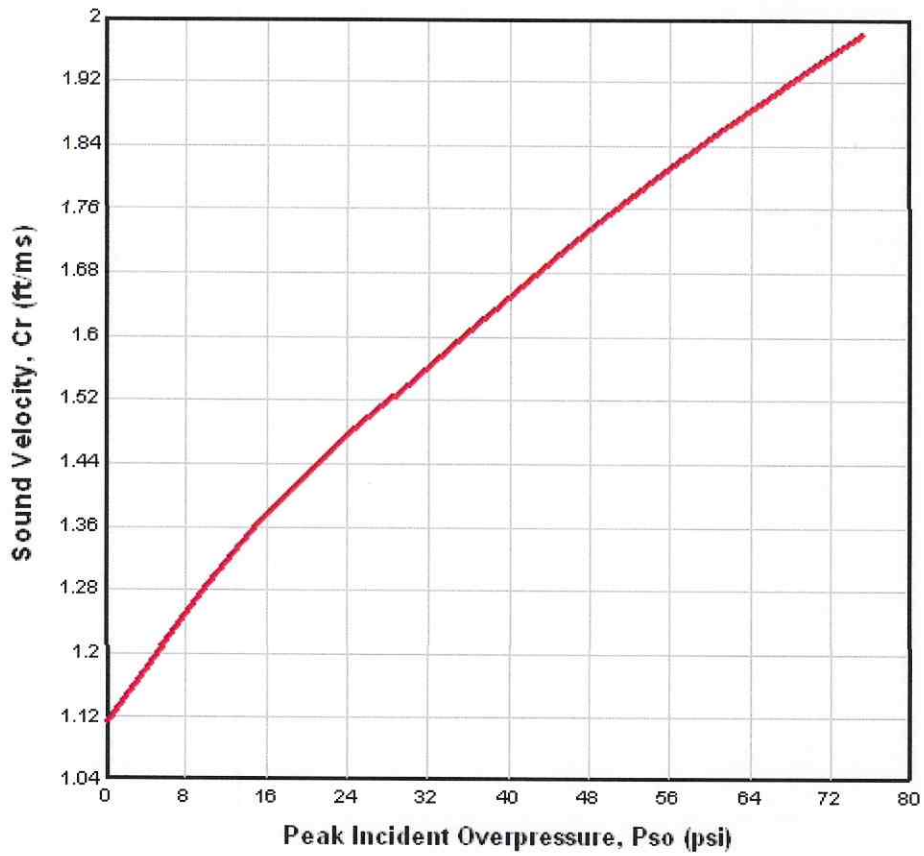
I know all this information is pretty confusing, and I hope that my explanation helps clear up some of the confusion. If you still have questions, however, please do not hesitate to contact me.

Best regards,

EBW

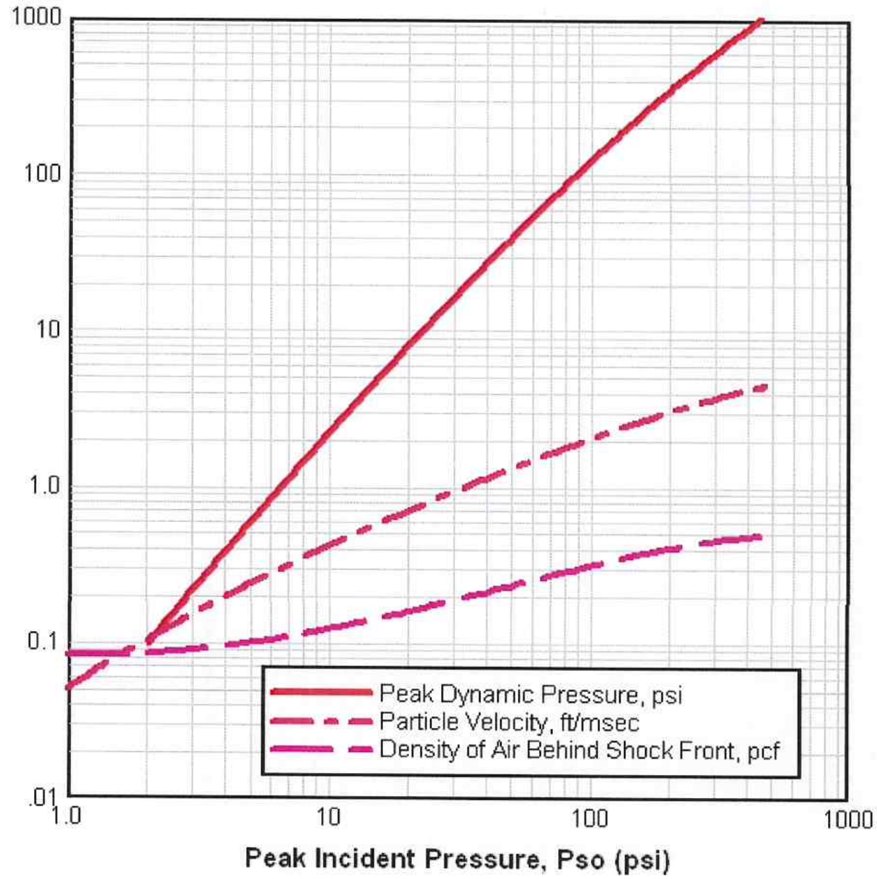
From: Catherine Hovell [mailto:cghovell@gmail.com]

Determination of c_r



Use graph to determine the sound velocity c_r as a function of the peak incident (side-on) overpressure P_{so}

Peak Dynamic Pressure



Use graph to determine the peak dynamic pressure q_o as a function of the peak incident (side-on) overpressure P_{so}

Figure 2-9 Variation of Reflected Pressure as a Function of Angle of Incidence

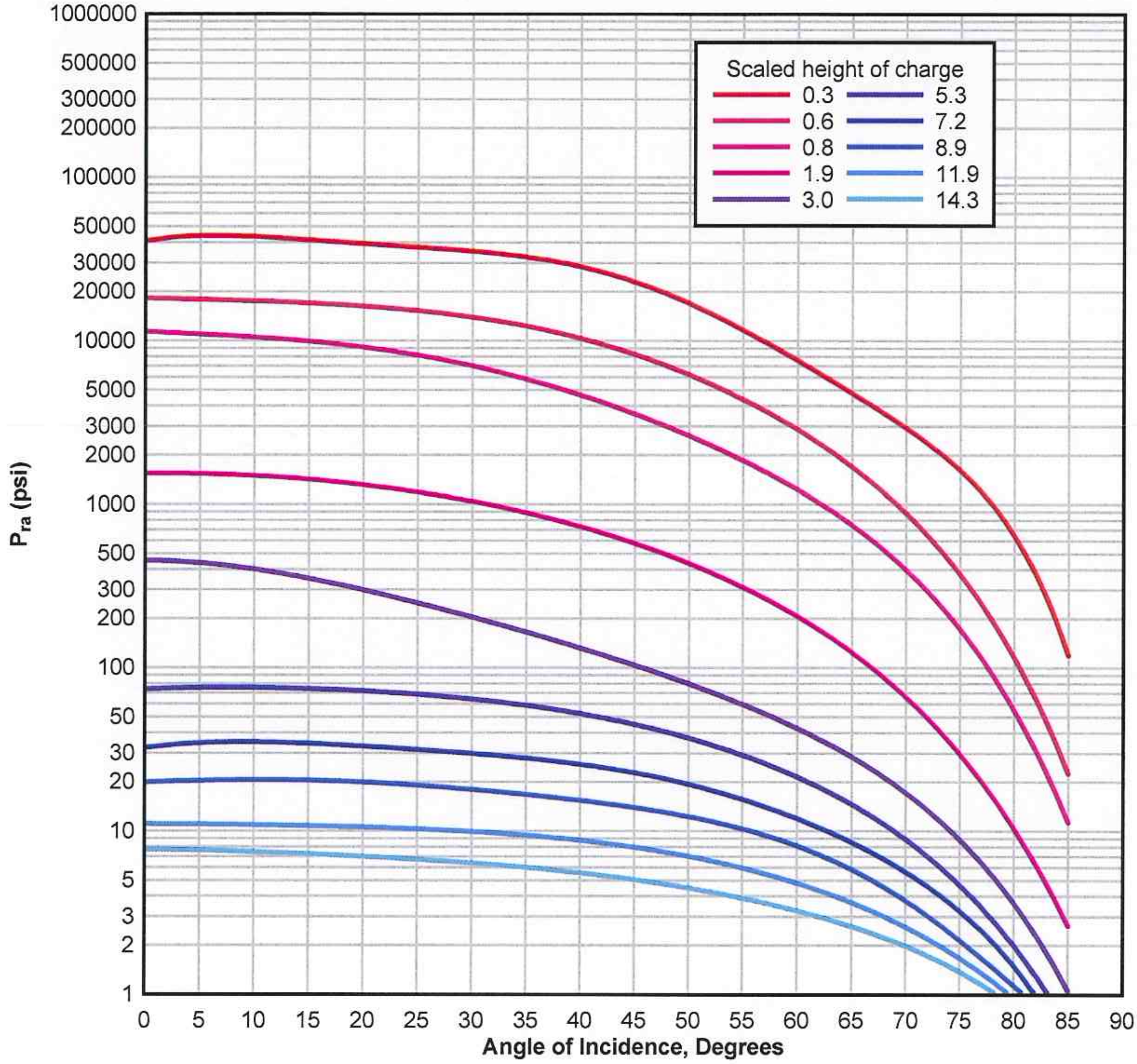


Figure 2-10 Variation of Scaled Reflected Impulse as a Function of Angle of Incidence

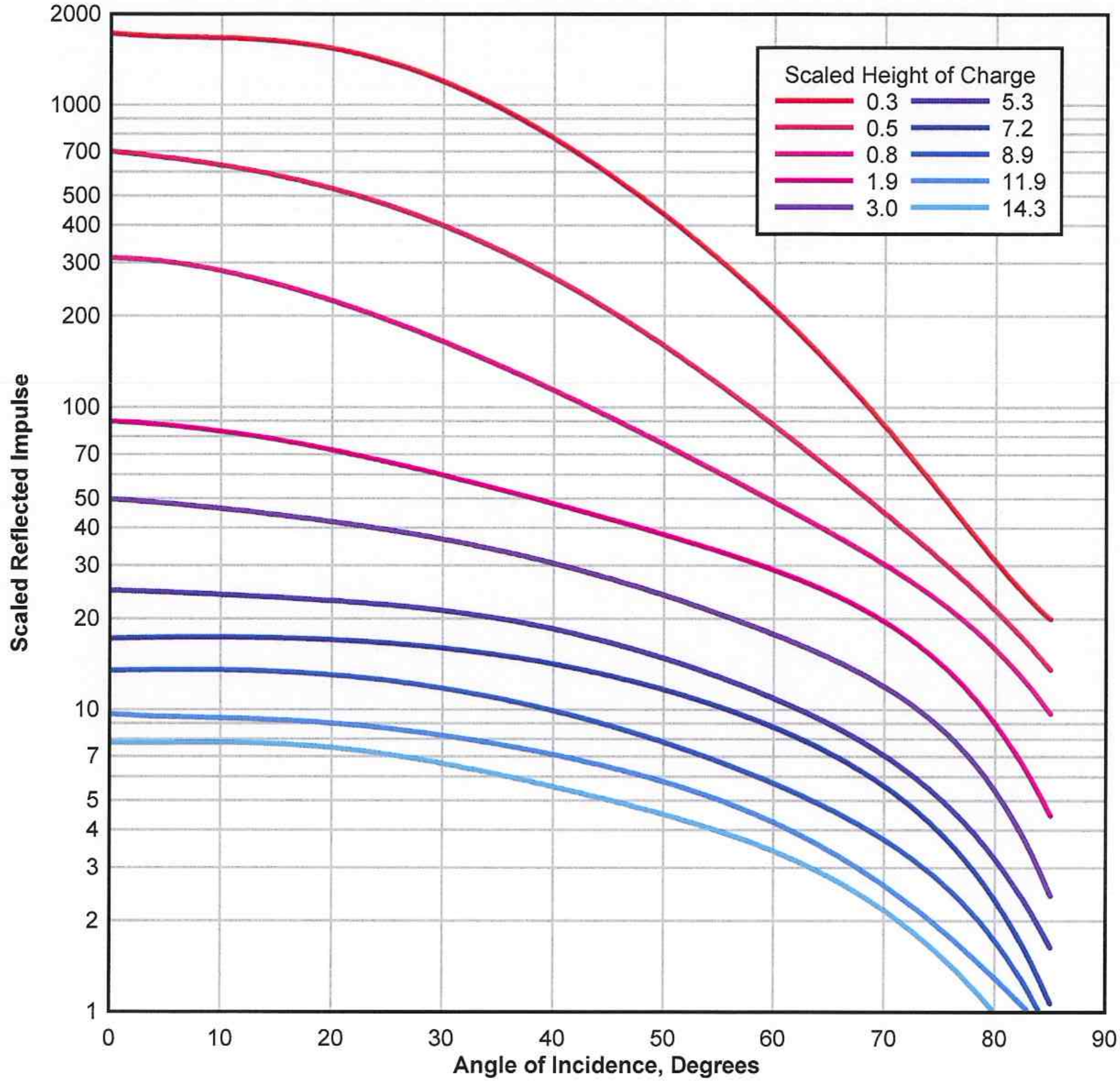


Figure 2-15 Positive Phase Shock Wave Parameters for a Hemispherical TNT Explosion on the Surface at Sea Level

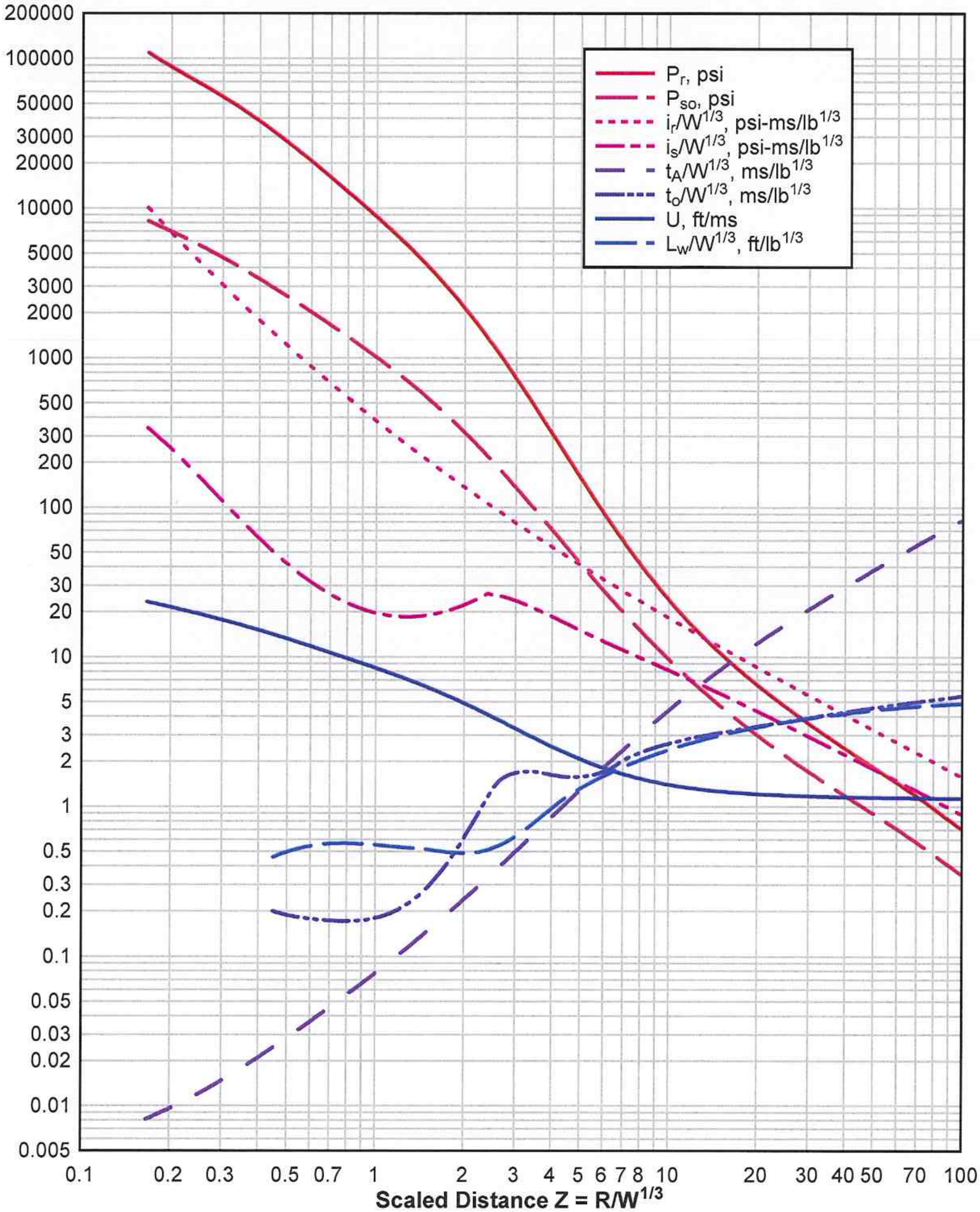


Figure 2-193 Reflected Pressure Coefficient versus Angle of Incidence

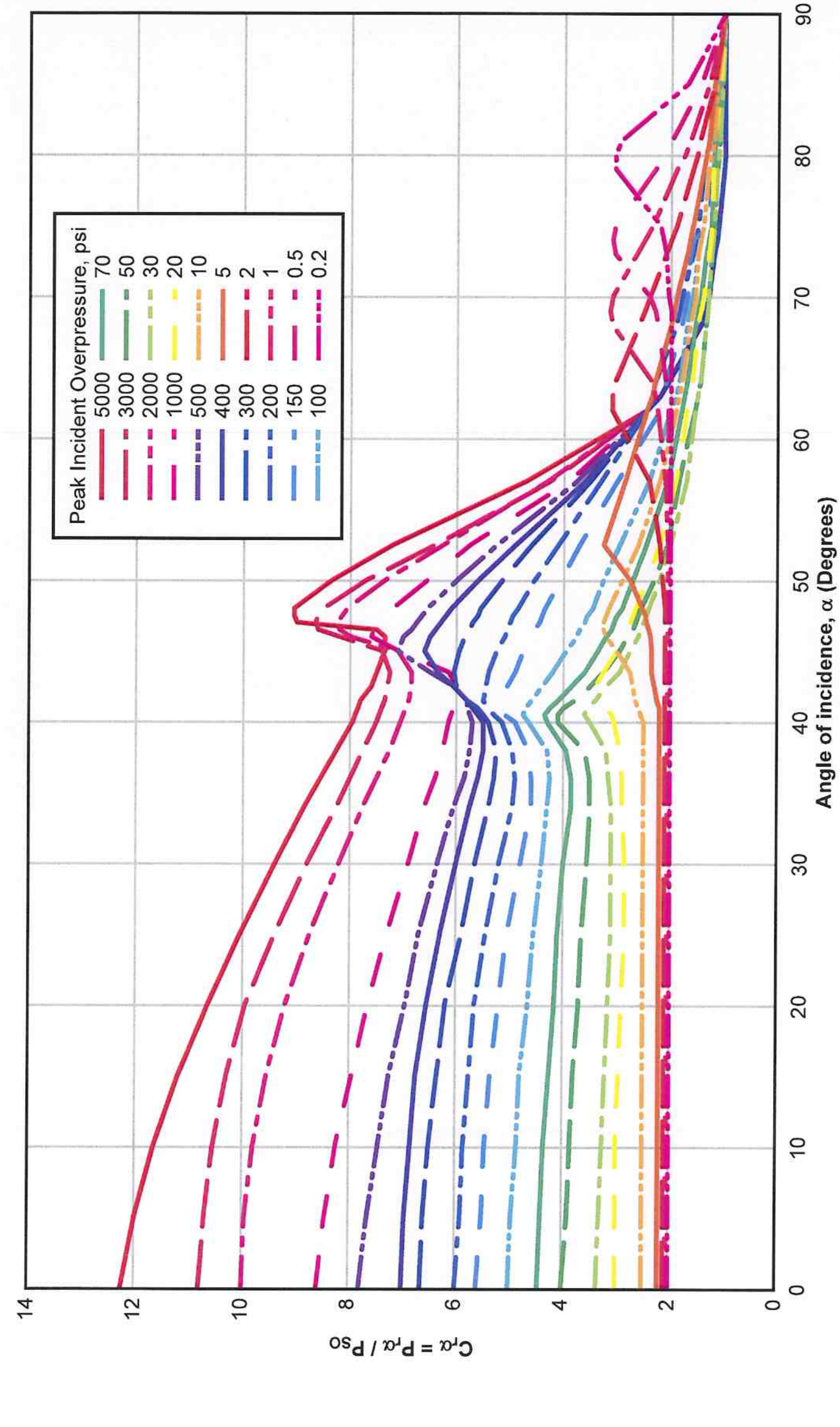


Figure 2-194(a) Reflected Scaled Impulse versus Angle of Incidence

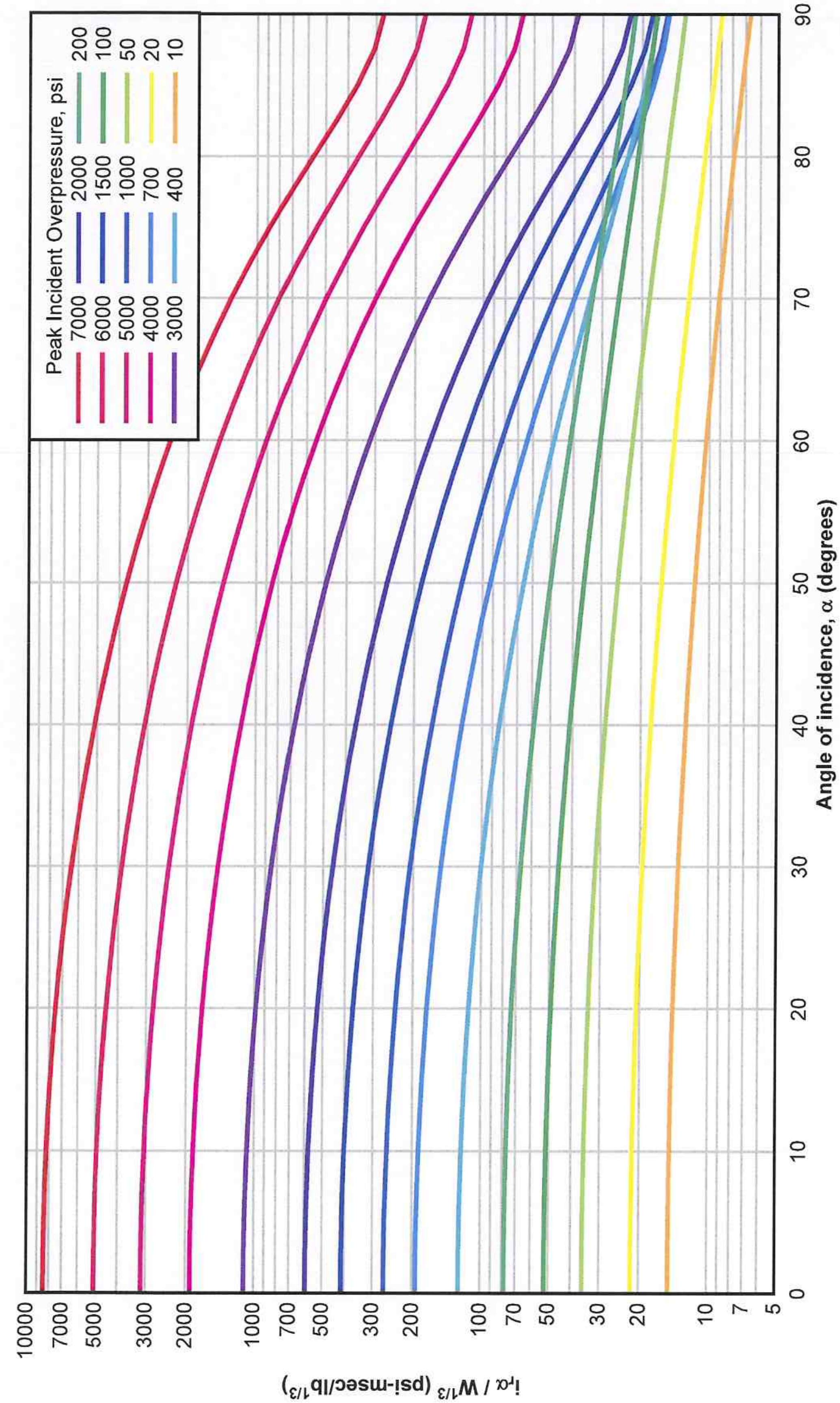


Figure 2-194(b) Reflected Scaled Impulse versus Angle of Incidence

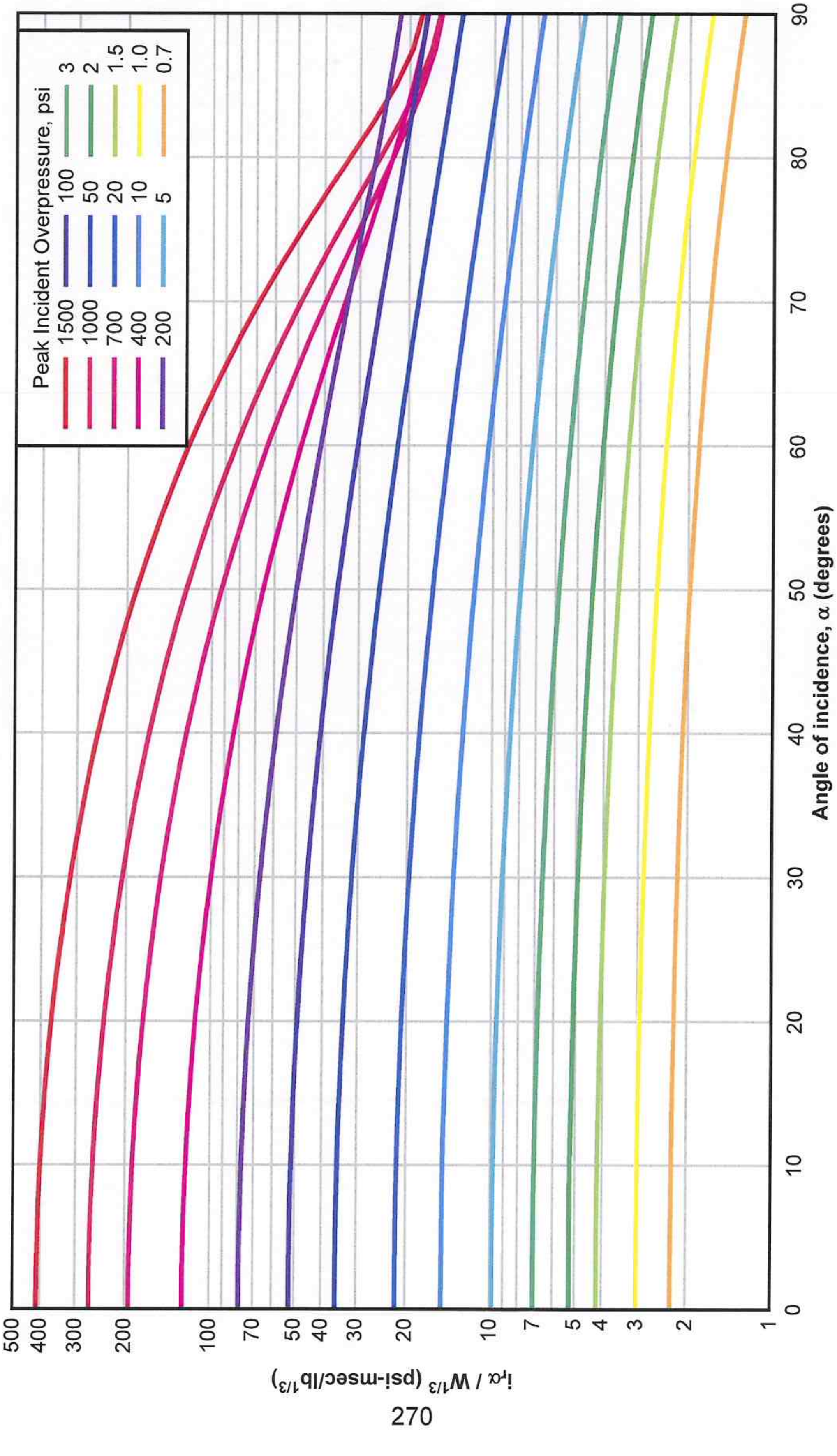


Figure 2-196 Peak Equivalent Uniform Roof Pressures

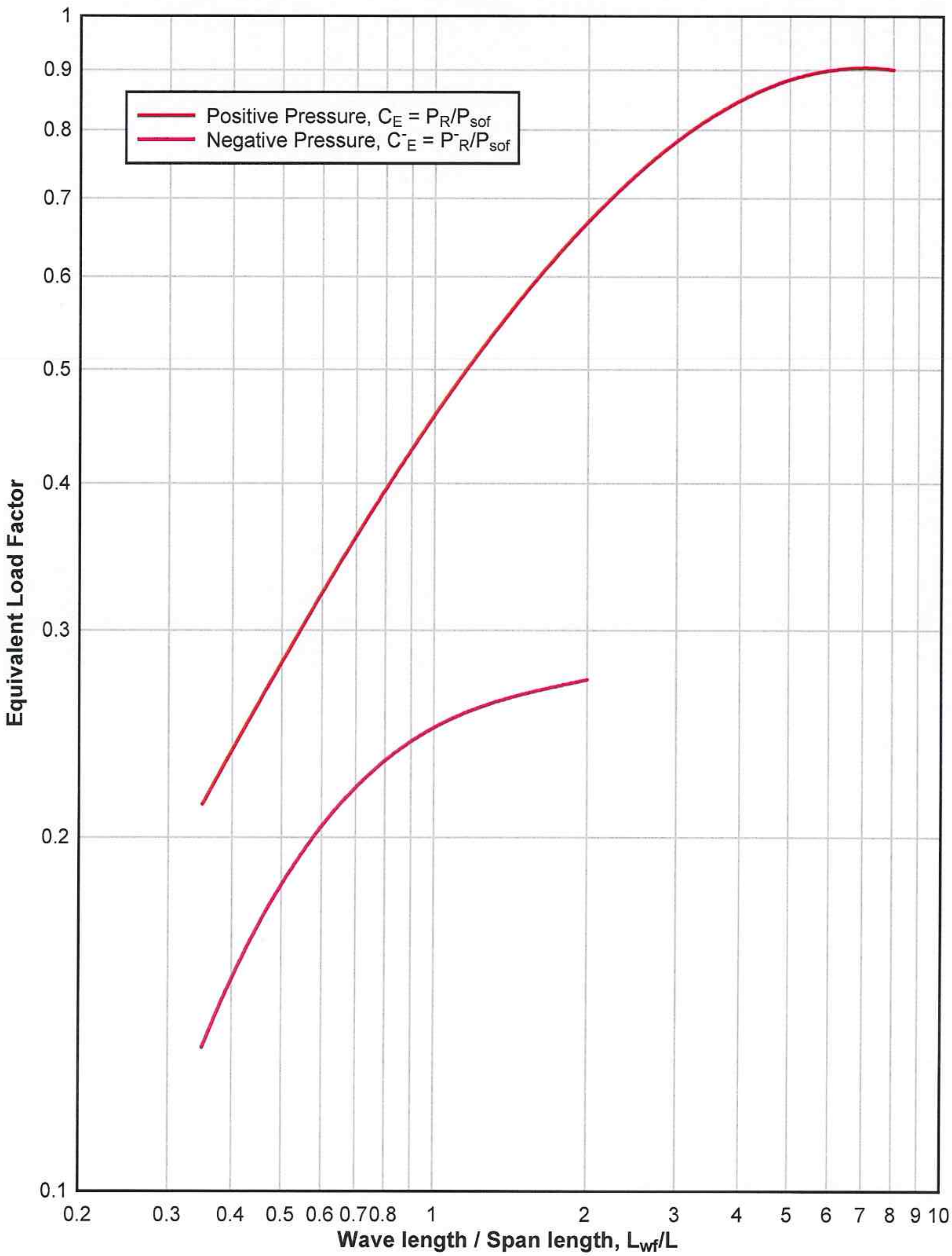


Figure 2-197 Scaled Rise Time of Equivalent Uniform Positive Roof Pressures

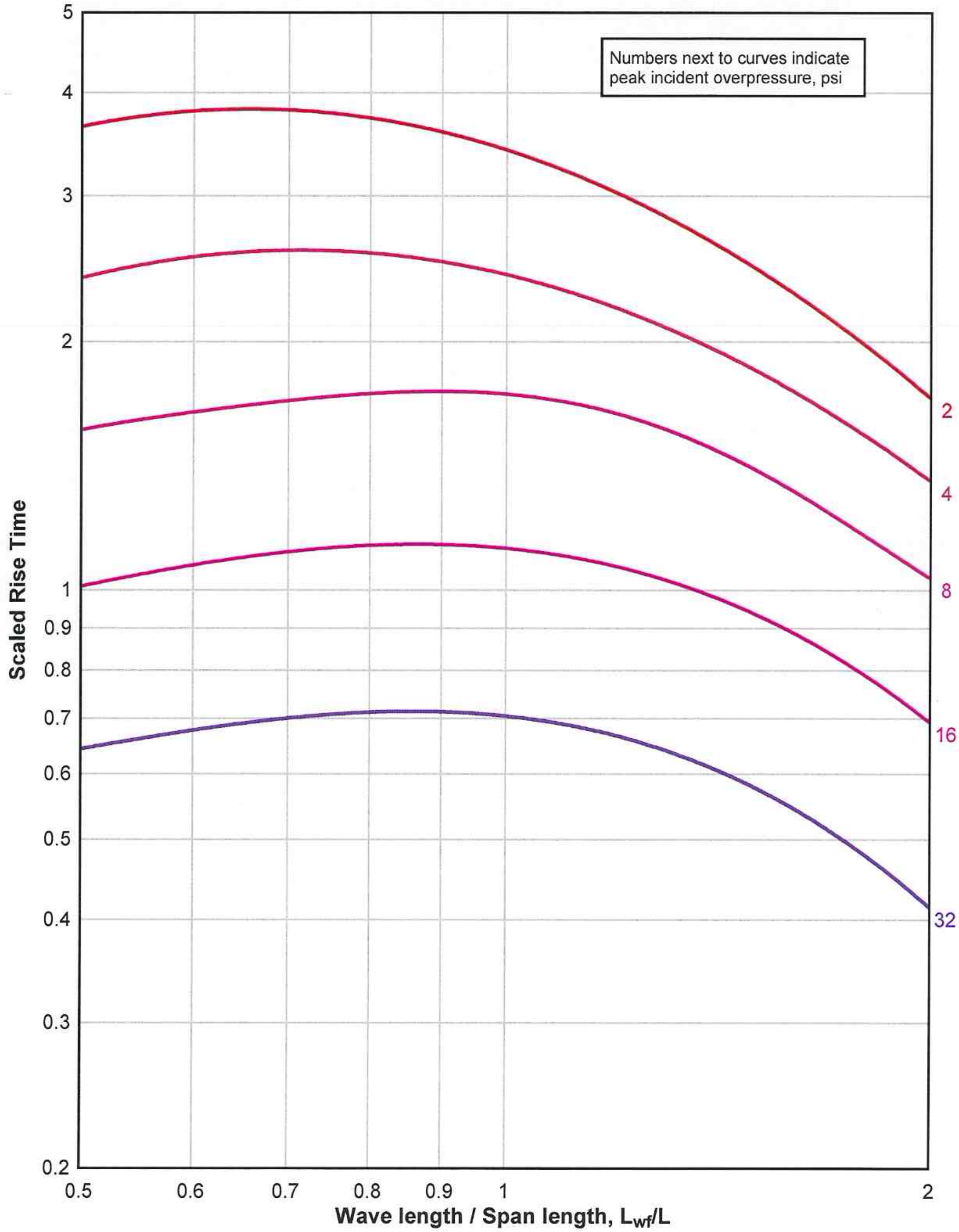


Figure 2-198 Scaled Duration of Equivalent Uniform Roof Pressures

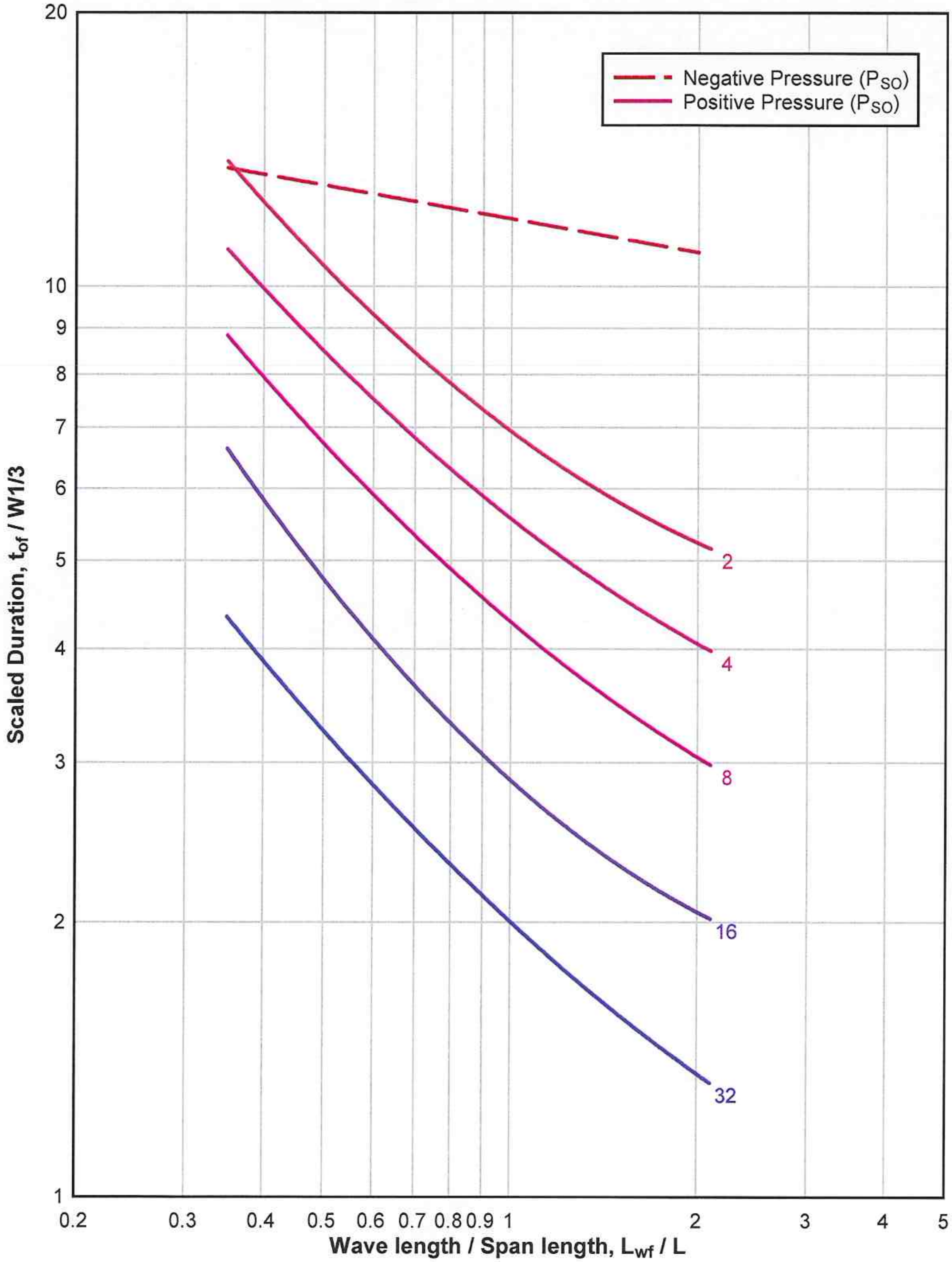


Table 3-1 Ultimate Unit Resistances for One-Way Elements

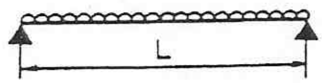
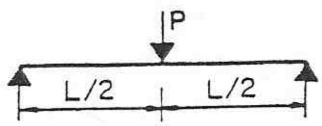
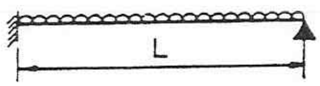
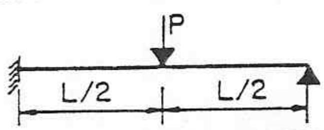
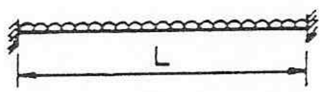
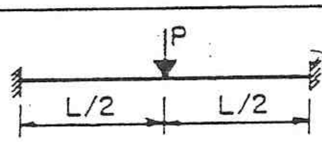
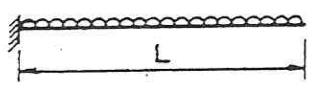
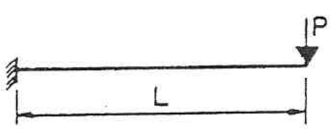
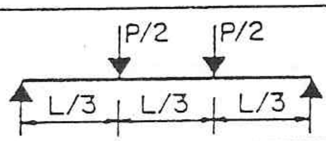
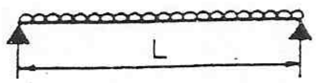
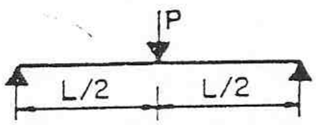
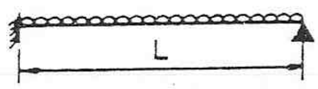
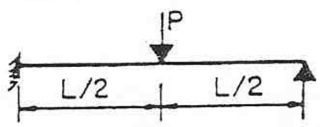
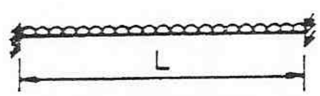
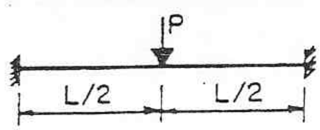
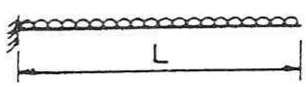
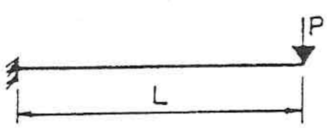
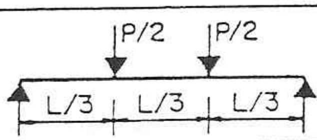
Edge Conditions and Loading Diagrams	Ultimate Resistance
	$r_u = \frac{8 M_p}{L^2}$
	$R_u = \frac{4 M_p}{L}$
	$r_u = \frac{4 (M_N + 2 M_p)}{L^2}$
	$R_u = \frac{2 (M_N + 2 M_p)}{L}$
	$r_u = \frac{8 (M_N + M_p)}{L^2}$
	$R_u = \frac{4 (M_N + M_p)}{L}$
	$r_u = \frac{2 M_N}{L^2}$
	$R_u = \frac{M_N}{L}$
	$R_u = \frac{6 M_p}{L}$

Table 3-8 Elastic, Elasto-Plastic and Equivalent Elastic Stiffnesses for One-Way Elements

Edge Conditions and Loading Diagrams	Elastic Stiffness, K_e	Elasto-Plastic Stiffness, K_{ep}	Equiv. Elastic Stiffness, K_E
	$\frac{384EI}{5L^4}$	—	$\frac{384EI}{5L^4}$
	$\frac{48EI}{L^3}$	—	$\frac{48EI}{L^3}$
	$\frac{185EI}{L^4}$	$\frac{384EI}{5L^4}$	$\frac{160EI^*}{L^4}$
	$\frac{107EI}{L^3}$	$\frac{48EI}{L^3}$	$\frac{106EI^*}{L^3}$
	$\frac{384EI}{L^4}$	$\frac{384EI}{5L^4}$	$\frac{307EI^*}{L^4}$
	$\frac{192EI}{L^3}$	$\frac{48EI^{**}}{L^3}$	$\frac{192EI^*}{L^3}$
	$\frac{8EI}{L^4}$	—	$\frac{8EI}{L^4}$
	$\frac{3EI}{L^3}$	—	$\frac{3EI}{L^3}$
	$\frac{56.4EI}{L^3}$	—	$\frac{56.4EI}{L^3}$

* Valid only if $M_N = M_p$

** Valid only if $M_N < M_p$

Table 3-12 Transformation Factors for One-Way Elements

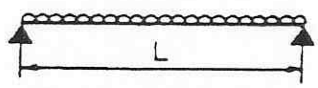
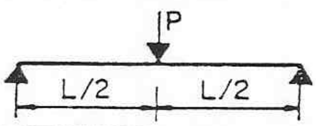
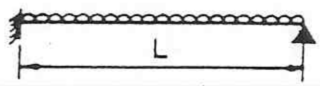
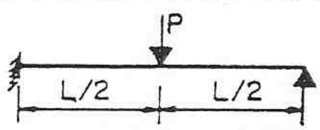
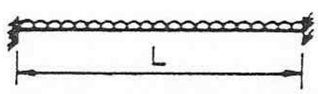
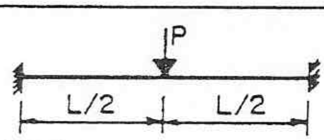
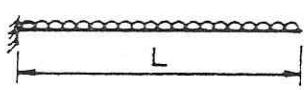
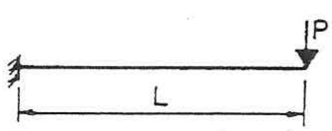
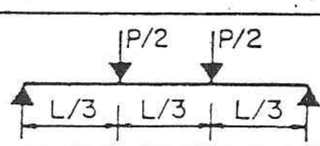
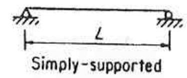
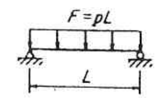
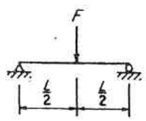
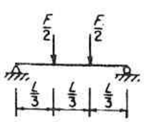
Edge Conditions and Loading Diagrams	Range of Behavior	Load Factor K_L	Mass Factor K_M	Load-Mass Factor K_{LM}
	Elastic Plastic	0.64 0.50	0.50 0.33	0.78 0.66
	Elastic Plastic	1.0 1.0	0.49 0.33	0.49 0.33
	Elastic Elasto-Plastic Plastic	0.58 0.64 0.50	0.45 0.50 0.33	0.78 0.78 0.66
	Elastic Elasto-Plastic Plastic	1.0 1.0 1.0	0.43 0.49 0.33	0.43 0.49 0.33
	Elastic Elasto Plastic Plastic	0.53 0.64 0.50	0.41 0.50 0.33	0.77 0.78 0.66
	Elastic Plastic	1.0 1.0	0.37 0.33	0.37 0.33
	Elastic Plastic	0.40 0.50	0.26 0.33	0.65 0.66
	Elastic Plastic	1.0 1.0	0.24 0.33	0.24 0.33
	Elastic Plastic	0.87 1.0	0.52 0.56	0.60 0.56

Table 5.1 Transformation Factors for Beams and One-way Slabs



Loading diagram	Strain range	Load factor K_L	Mass factor K_M		Load-mass factor K_{LM}		Maximum resistance R_m	Spring constant k	Dynamic reaction V
			Concentrated mass*	Uniform mass	Concentrated mass*	Uniform mass			
	Elastic	0.64	....	0.50	....	0.78	$\frac{8\mathfrak{M}_P}{L}$	$\frac{384EI}{5L^3}$	$0.39R + 0.11F$
	Plastic	0.50	....	0.33	....	0.66	$\frac{8\mathfrak{M}_P}{L}$	0	$0.38R_m + 0.12F$
	Elastic	1.0	1.0	0.49	1.0	0.49	$\frac{4\mathfrak{M}_P}{L}$	$\frac{48EI}{L^3}$	$0.78R - 0.28F$
	Plastic	1.0	1.0	0.33	1.0	0.33	$\frac{4\mathfrak{M}_P}{L}$	0	$0.75R_m - 0.25F$
	Elastic	0.87	0.76	0.52	0.87	0.60	$\frac{6\mathfrak{M}_P}{L}$	$\frac{56.4EI}{L^3}$	$0.525R - 0.025F$
	Plastic	1.0	1.0	0.56	1.0	0.56	$\frac{6\mathfrak{M}_P}{L}$	0	$0.52R_m - 0.02F$

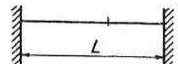
maximum resistance

* Equal parts of the concentrated mass are lumped at each concentrated load.

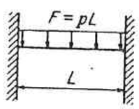
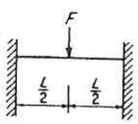
Source: "Design of Structures to Resist the Effects of Atomic Weapons," U.S. Army Corps of Engineers Manual EM 1110-345-415, 1957.

Table 5.2 Transformation Factors for Beams and One-way Slabs

$\mathfrak{M}_{Ps}$ = ultimate moment capacity at support
$\mathfrak{M}_{Pm}$ = ultimate moment capacity at midspan



Fixed ends

Loading diagram	Strain range	Load factor K_L	Mass factor K_M		Load-mass factor K_{LM}		Maximum resistance R_m	Spring constant k	Effective spring constant k_E†	Dynamic reaction V
			Concentrated mass*	Uniform mass	Concentrated mass*	Uniform mass				
	Elastic	0.53	...	0.41	...	0.77	$\frac{12\mathfrak{M}_{Ps}}{L}$	$\frac{384EI}{L^3}$	......	$0.36R + 0.14F$
	Elastic-plastic	0.64	...	0.50	...	0.78	$\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$	$\frac{384EI}{5L^3}$	$\frac{307EI}{L^3}$	$0.39R + 0.11F$
	Plastic	0.50	...	0.33	...	0.66	$\frac{8}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$	0	......	$0.38R_m + 0.12F$
	Elastic	1.0	1.0	0.37	1.0	0.37	$\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$	$\frac{192EI}{L^3}$	......	$0.71R - 0.21F$
	Plastic	1.0	1.0	0.33	1.0	0.33	$\frac{4}{L}(\mathfrak{M}_{Ps} + \mathfrak{M}_{Pm})$	0	......	$0.75R_m - 0.25F$

* Concentrated mass is lumped at the concentrated load.
† See Fig. 5.4.
Source: "Design of Structures to Resist the Effects of Atomic Weapons," U.S. Army Corps of Engineers Manual EM 1110-345-415, 1957.

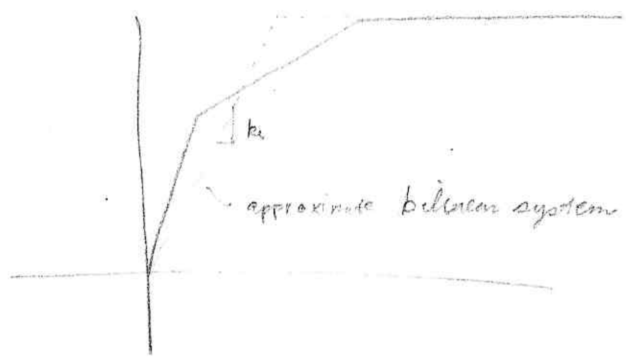
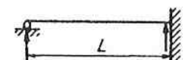
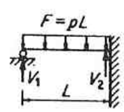
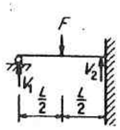
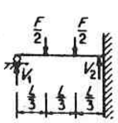


Table 5.3 Transformation Factors for Beams and One-way Slabs



Simply-supported and fixed

$\mathfrak{M}_{Ps}$ = ultimate bending capacity at support
$\mathfrak{M}_{Pm}$ = ultimate positive bending capacity

Loading diagram	Strain range	Load factor K_L	Mass factor K_M		Load-mass factor K_{LM}		Maximum resistance R_m	Spring constant k	Effective spring constant k_E†	Dynamic reaction V
			Concentrated mass*	Uniform mass	Concentrated mass*	Uniform mass				
	Elastic	0.58	....	0.45	....	0.78	$\frac{8\mathfrak{M}_{Ps}}{L}$	$\frac{185EI}{L^3}$	$\frac{160EI}{L^3}$	$V_1 = 0.26R + 0.12F$ $V_2 = 0.43R + 0.19F$
	Elastic-plastic	0.64	....	0.50	....	0.78	$\frac{4}{L}(\mathfrak{M}_{Ps} + 2\mathfrak{M}_{Pm})$	$\frac{384EI}{5L^3}$		$V = 0.39R + 0.11F \pm \mathfrak{M}_{Ps}/L$
	Plastic	0.50	....	0.33	....	0.66	$\frac{4}{L}(\mathfrak{M}_{Ps} + 2\mathfrak{M}_{Pm})$	0		$V = 0.38R_m + 0.12F \pm \mathfrak{M}_{Ps}/L$
	Elastic	1.0	1.0	0.43	1.0	0.43	$\frac{16M_{Ps}}{3L}$	$\frac{107EI}{L^3}$	$\frac{106EI}{L^3}$	$V_1 = 0.25R + 0.07F$ $V_2 = 0.54R + 0.14F$
	Elastic-plastic	1.0	1.0	0.49	1.0	0.49	$\frac{2}{L}(\mathfrak{M}_{Ps} + 2\mathfrak{M}_{Pm})$	$\frac{48EI}{L^3}$		$V = 0.78R - 0.28F \pm \mathfrak{M}_{Ps}/L$
	Plastic	1.0	1.0	0.33	1.0	0.33	$\frac{2}{L}(\mathfrak{M}_{Ps} + 2\mathfrak{M}_{Pm})$	0		$V = 0.75R_m - 0.25F \pm \mathfrak{M}_{Ps}/L$
	Elastic	0.81	0.67	0.45	0.83	0.55	$\frac{6M_{Ps}}{L}$	$\frac{132EI}{L^3}$	$\frac{122EI}{L^3}$	$V_1 = 0.17R + 0.17F$ $V_2 = 0.33R + 0.33F$
	Elastic-plastic	0.87	0.76	0.52	0.87	0.60	$\frac{2}{L}(\mathfrak{M}_{Ps} + 3\mathfrak{M}_{Pm})$	$\frac{56EI}{L^3}$		$V = 0.525R - 0.025F \pm \mathfrak{M}_{Ps}/L$
	Plastic	1.0	1.0	0.56	1.0	0.56	$\frac{2}{L}(\mathfrak{M}_{Ps} + 3\mathfrak{M}_{Pm})$	......		$V = 0.52R_m - 0.02F \pm \mathfrak{M}_{Ps}/L$

* Equal parts of the concentrated mass are lumped at each concentrated load.
† See Fig. 5.4.
Source: "Design of Structures to Resist the Effects of Atomic Weapons," U.S. Army Corps of Engineers Manual EM 1110-345-415, 1957.

Explanation of Notion on Tables for Response of Two-Way Elements

$\mathcal{M}_{Pfa}$ = total positive ultimate moment capacity along midspan section parallel to short edge

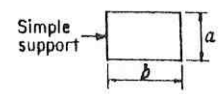
$\mathcal{M}_{Psa}$ = total negative ultimate moment capacity along short edge

$\mathcal{M}^{o}_{Psb}$ = negative ultimate moment capacity per unit width at center of long edge

I_a = moment of inertia per unit width

Note that the tables indicate both the maximum resistance and spring constant with respect to the *total* load on the slab

Table 5.4 Transformation Factors for Two-way Slabs: Simple Supports—Four Sides, Uniform Load

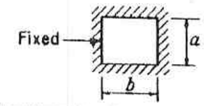


V_A = total dynamic reaction along short edge; V_B = total dynamic reaction along long edge.

Strain range	a/b	*Load factor* K_L	*Mass factor* K_M	*Load-mass factor* K_{LM}	*Maximum resistance*	*Spring constant* k	*Dynamic reactions* V_A	*Dynamic reactions* V_B
Elastic	1.0	0.46	0.31	0.67	$\frac{12}{a}(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Pfb})$	$\frac{252EI_a}{a^2}$	$0.07F + 0.18R$	$0.07F + 0.18R$
	0.9	0.47	0.33	0.70	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 11\mathfrak{M}_{Pfb})$	$\frac{230EI_a}{a^2}$	$0.06F + 0.16R$	$0.08F + 0.20R$
	0.8	0.49	0.35	0.71	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 10.3\mathfrak{M}_{Pfb})$	$\frac{212EI_a}{a^2}$	$0.06F + 0.14R$	$0.08F + 0.22R$
	0.7	0.51	0.37	0.73	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.8\mathfrak{M}_{Pfb})$	$\frac{201EI_a}{a^2}$	$0.05F + 0.13R$	$0.08F + 0.24R$
	0.6	0.53	0.39	0.74	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.3\mathfrak{M}_{Pfb})$	$\frac{197EI_a}{a^2}$	$0.04F + 0.11R$	$0.09F + 0.26R$
	0.5	0.55	0.41	0.75	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.0\mathfrak{M}_{Pfb})$	$\frac{201EI_a}{a^2}$	$0.04F + 0.09R$	$0.09F + 0.28R$
Plastic	1.0	0.33	0.17	0.51	$\frac{12}{a}(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Pfb})$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 11\mathfrak{M}_{Pfb})$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 10.3\mathfrak{M}_{Pfb})$	0	$0.07F + 0.13R_m$	$0.10F + 0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.8\mathfrak{M}_{Pfb})$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.3\mathfrak{M}_{Pfb})$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}(12\mathfrak{M}_{Pfa} + 9.0\mathfrak{M}_{Pfb})$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

Source: "Design of Structures to Resist the Effects of Atomic Weapons," U.S. Army Corps of Engineers Manual EM 1110-345-415, 1957.

Table 5.5 ***Transformation Factors for Two-way Slabs: Fixed Four Sides, Uniform Load***



V_A = total dynamic reaction along short edge; V_B = total dynamic reaction along long edge.

Strain range	*a/b*	*Load factor* K_L	*Mass factor* K_M	*Load-mass factor* K_{LM}	*Maximum resistance*	*Spring constant* k	*Dynamic reactions* V_A	*Dynamic reactions* V_B
Elastic	1.0	0.33	0.21	0.63	$29.2\mathfrak{M}^0_{Psb}$	$810EI_a/a^2$	$0.10F + 0.15R$	$0.10F + 0.15R$
	0.9	0.34	0.23	0.68	$27.4\mathfrak{M}^0_{Psb}$	$742EI_a/a^2$	$0.09F + 0.14R$	$0.10F + 0.17R$
	0.8	0.36	0.25	0.69	$26.4\mathfrak{M}^0_{Psb}$	$705EI_a/a^2$	$0.08F + 0.12R$	$0.11F + 0.19R$
	0.7	0.38	0.27	0.71	$26.2\mathfrak{M}^0_{Psb}$	$692EI_a/a^2$	$0.07F + 0.11R$	$0.11F + 0.21R$
	0.6	0.41	0.29	0.71	$27.3\mathfrak{M}^0_{Psb}$	$724EI_a/a^2$	$0.06F + 0.09R$	$0.12F + 0.23R$
	0.5	0.43	0.31	0.72	$30.2M^0_{Psb}$	$806EI_a/a^2$	$0.05F + 0.08R$	$0.12F + 0.25R$
Elastic-plastic	1.0	0.46	0.31	0.67	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 12(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	$252EI_a/a^2$	$0.07F + 0.18R$	$0.07F + 0.18R$
	0.9	0.47	0.33	0.70	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 11(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	$230EI_a/a^2$	$0.06F + 0.16R$	$0.08F + 0.20R$
	0.8	0.49	0.35	0.71	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 10.3(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	$212EI_a/a^2$	$0.06F + 0.14R$	$0.08F + 0.22R$
	0.7	0.51	0.37	0.73	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.8(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	$201EI_a/a^2$	$0.05F + 0.13R$	$0.08F + 0.24R$
	0.6	0.53	0.39	0.74	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.3(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	$197EI_a/a^2$	$0.04F + 0.11R$	$0.09F + 0.26R$
	0.5	0.55	0.41	0.75	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.0(\mathfrak{M}_{Pfb} + M_{Psb})]$	$201EI_a/a^2$	$0.04F + 0.09R$	$0.09F + 0.28R$
Plastic	1.0	0.33	0.17	0.51	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 12(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.09F + 0.16R_m$	$0.09F + 0.16R_m$
	0.9	0.35	0.18	0.51	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 11(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.08F + 0.15R_m$	$0.09F + 0.18R_m$
	0.8	0.37	0.20	0.54	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 10.3(M_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.07F + 0.13R_m$	$0.10F + 0.20R_m$
	0.7	0.38	0.22	0.58	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.8(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.06F + 0.12R_m$	$0.10F + 0.22R_m$
	0.6	0.40	0.23	0.58	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.3(M_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.05F + 0.10R_m$	$0.10F + 0.25R_m$
	0.5	0.42	0.25	0.59	$(1/a)\,[12(\mathfrak{M}_{Pfa} + \mathfrak{M}_{Psa}) + 9.0(\mathfrak{M}_{Pfb} + \mathfrak{M}_{Psb})]$	0	$0.04F + 0.08R_m$	$0.11F + 0.27R_m$

Source: "Design of Structures to Resist the Effects of Atomic Weapons," U.S. Army Corps of Engineers Manual EM 1110-345-415, 1957.

Table 3-2 Ultimate Unit Resistances for Two-Way Elements (Symmetrical Yield Lines)

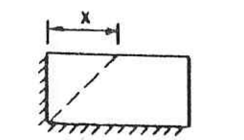
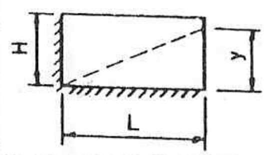
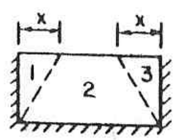
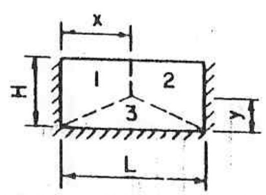
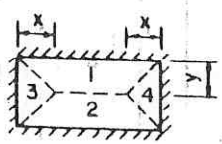
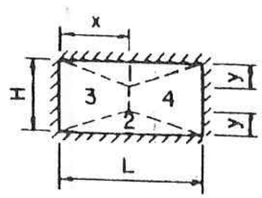
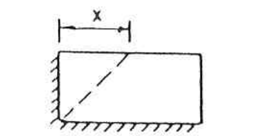
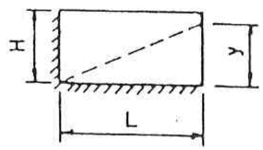
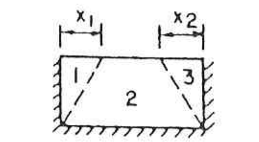
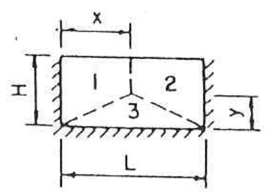
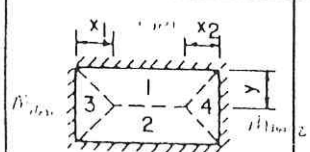
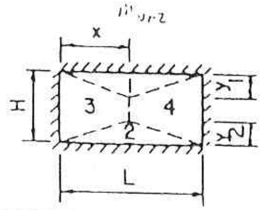
Edge Conditions	Yield Line Locations	Limits	Ultimate Unit Resistance
Two adjacent edges supported and two edges free		$x \leq L$	$\frac{5(M_{HN}+M_{HP})}{x^2}$ OR $\frac{6L\,M_{VN} + (5M_{VP} - M_{VN})x}{H^2(3L-2x)}$
		$y \leq H$	$\frac{5(M_{VN}+M_{VP})}{y^2}$ OR $\frac{6H\,M_{HN} + (5M_{HP} - M_{HN})y}{L^2(3H-2y)}$
Three edges supported and one edge free		$x \leq \frac{L}{2}$	$\frac{5(M_{HN}+M_{HP})}{x^2}$ OR $\frac{2M_{VN}(3L-x)+10x\,M_{VP}}{H^2(3L-4x)}$
		$y \leq H$	$\frac{5(M_{VN}+M_{VP})}{y^2}$ OR $\frac{4(M_{HN}+M_{HP})(6H-y)}{L^2(3H-2y)}$
Four edges supported		$x \leq \frac{L}{2}$	$\frac{5(M_{HN}+M_{HP})}{x^2}$ OR $\frac{8(M_{VN}+M_{VP})(3L-x)}{H^2(3L-4x)}$
		$y \leq \frac{H}{2}$	$\frac{5(M_{VN}+M_{VP})}{y^2}$ OR $\frac{8(M_{HN}+M_{HP})(3H-y)}{L^2(3H-4y)}$

Table 3-3 Ultimate Unit Resistances for Two-Way Elements (Unsymmetrical Yield Lines)

Edge Conditions	Yield Line Locations	Limits	Ultimate Unit Resistance
Two adjacent edges supported and two edges free		$x \le L$	Same as In Table 3-2
		$y \le H$	
Three edges supported and one edge free		$x \le \frac{L}{2}$	$\frac{5(M_{HN1} + M_{HP1})}{X_1^2}$ OR $\frac{5(M_{HN3} + M_{HP})}{X_2^2}$ OR $\frac{(5M_{VP} - M_{VN2})(X_1 + X_2) + 6M_{VN2}L}{H^2(3L - 2X_1 - 2X_2)}$
		$y \le H$	$\frac{(M_{HN1} + M_{HP})(6H - Y)}{X^2(3H - 2Y)}$ OR $\frac{(M_{HN2} + M_{HP})(6H - Y)}{(L - X)^2(3H - 2Y)}$ OR $\frac{5(M_{VN3} + M_{VP})}{Y^2}$
Four edges supported		$x \le \frac{L}{2}$	$\frac{(M_{VN1} + M_{VP})(6L - X_1 - X_2)}{Y^2(3L - 2X_1 - 2X_2)}$ OR $\frac{(M_{VN2} + M_{VP})(6L - X_1 - X_2)}{(H - Y)^2(3L - 2X_1 - 2X_2)}$ $\frac{5(M_{HN1} + M_{HP})}{X_1^2}$ OR $\frac{5(M_{HN2} + M_{HP})}{X_2^2}$
		$y \le \frac{H}{2}$	$\frac{5(M_{VN1} + M_{VP})}{Y_1^2}$ OR $\frac{5(M_{VN2} + M_{VP})}{Y_2^2}$ $\frac{(M_{HN1} + M_{HP})(6H - Y_1 - Y_2)}{X^2(3H - 2Y_1 - 2Y_2)}$ OR $\frac{(M_{HN2} + M_{HP})(6H - Y_1 - Y_2)}{(L - X)^2(3H - 2Y_1 - 2Y_2)}$

Dynamic Increase Factors (DIF) for Design of Reinforced Concrete Components

Type of Stress	Far Design Range			Close-In Design Range		
	Reinforcing Bars		Concrete	Reinforcing Bars		Concrete
	f_{dy}/f_y	f_{du}/f_u	f'_{dc}/f'_c	f_{dy}/f_y	f_{du}/f_u	f'_{dc}/f'_c
Bending	1.17	1.05	1.19	1.23	1.05	1.25
Diagonal Tension	1.00	----	1.00	1.10	1.00	1.00
Direct Shear	1.10	1.00	1.10	1.10	1.00	1.10
Bond	1.17	1.05	1.00	1.23	1.05	1.00
Compression	1.10	----	1.12	1.13	----	1.16

Notes:

- Far Design Range: $z \geq 1.0\, m/kg^{1/3}$ (2.5 ft/lb$^{1/3}$) — metric!
- Close-In Range Applies when $z < 0.4\, m/kg^{1/3}$ (1ft/lb$^{1/3}$)
- For combined bending and compression, use DIF=1.1 for steel and DIF=1.2 for concrete

APPENDIX 5.A
SUMMARY TABLES FOR DYNAMIC MATERIAL STRENGTH

TABLE 5.A.1: Strength Increase Factors (SIF)

Material	SIF
Structural Steel ($f_y \le 50$ ksi)	1.1
Reinforcing Steel ($f_y \le 60$ ksi)	1.1
Cold-Formed Steel	1.21
Concrete (1)	1.0

(1) The results of compression tests are usually well above the specified concrete strengths and may be used in lieu of the above factor. Some conservatism may be warranted because concrete strengths have more influence on shear design than bending capacity.

TABLE 5.A.2: Dynamic Increase Factors (DIF) for Reinforcing Bars, Concrete, and Masonry

Stress Type	DIF			
	Reinforcing Bars		Concrete	Masonry
	F_{dy}/F_y	F_{du}/F_u	f'_{dc}/f'_c	f'_{dm}/f'_m
Flexure	1.17	1.05	1.19	1.19
Compression	1.10	1.00	1.12	1.12
Diagonal Tension	1.00	1.00	1.00	1.00
Direct Shear	1.10	1.00	1.10	1.00
Bond	1.17	1.05	1.00	1.00

ASCE (1997)

TABLE 5.A.3: Dynamic Increase Factors (DIF) for Structural Steel, Cold-Formed Steel, and Aluminum

Material	DIF		
	Yield Stress		Ultimate Stress
	Bending/Shear	Tension/Compression	
	F_{dy}/F_y	F_{dy}/F_y	F_{du}/F_u
A36	1.29	1.19	1.10
A588	1.19	1.12	1.05
A514	1.09	1.05	1.00
A446	1.10	1.10	1.00
Stainless Steel Type 304	1.18	1.15	1.00
Aluminum, 6061-T6	1.02	1.00	1.00

TABLE 5.A.4: Dynamic Design Stress for Reinforced Concrete

Type of Stress	Type of Reinforcement	Maximum Support Rotation	Dynamic Design Stress (F_{ds})
Bending	Tension and Compression	$0 < \theta \le 2$	F_{dy}
		$2 < \theta \le 5$	$F_{dy} + (F_{du} - F_{dy})/4$
		$5 < \theta \le 12$	$(F_{dy} + F_{du})/2$
Diagonal Tension	Stirrups		F_{dy}
Direct Shear	Diagonal Bars	$0 < \theta \le 2$	F_{dy}
		$2 < \theta \le 5$	$F_{dy} + (F_{du} - F_{dy})/4$
		$5 < \theta \le 12$	$(F_{dy} + F_{du})/2$
Compression	Column	all	F_{dy}

TABLE 5.A.5: Dynamic Design Stress for Structural Steel

Type of Stress	Maximum Ductility Ratio	Dynamic Design Stress
all	$\mu \le 10$	F_{dy}
all	$\mu > 10$	$F_{dy} + (F_{du} - F_{dy})/4$